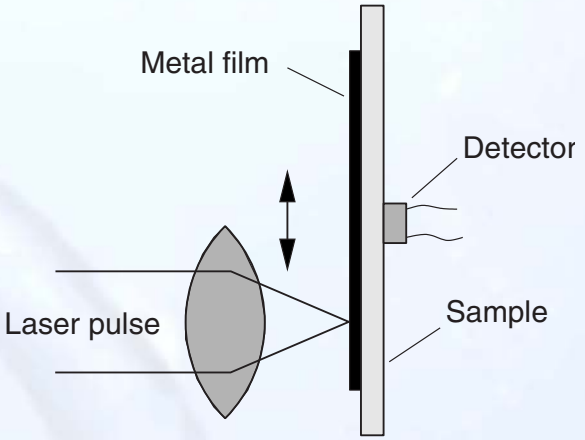
6.5 Ballistic Propagation of Phonons

b) position depended measurements

single crystals are elastically anisotropic

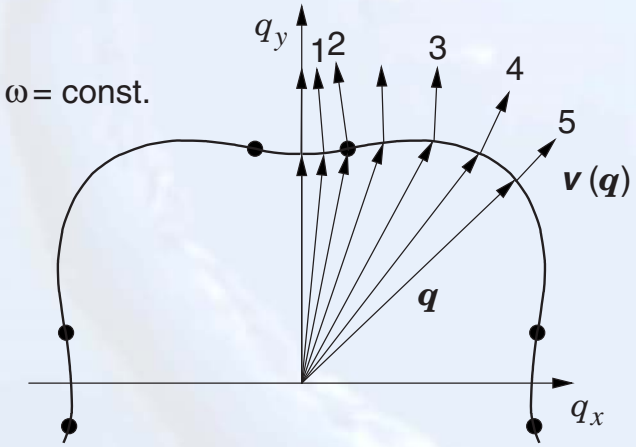
→ sound waves are propagating not always in direction of wave vector, but in direction of the group velocity

$$\boldsymbol{v} = \mathrm{d}\omega(\boldsymbol{q})/\mathrm{d}\boldsymbol{q}$$

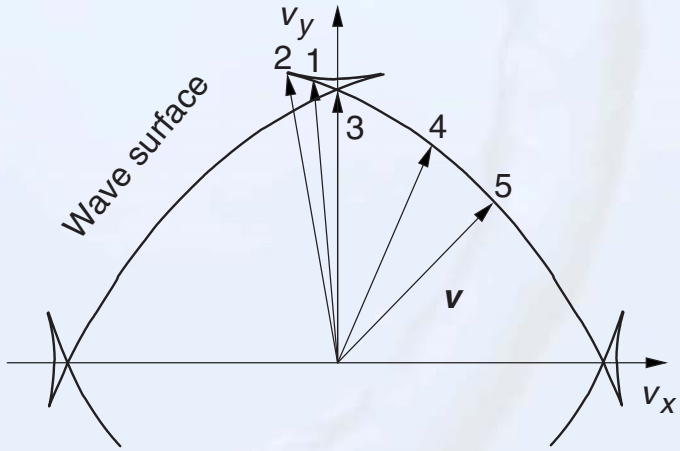


Construction of wave front

constant ω curve in *k* space



wave front reconstruction



→ propagation in preferred directions

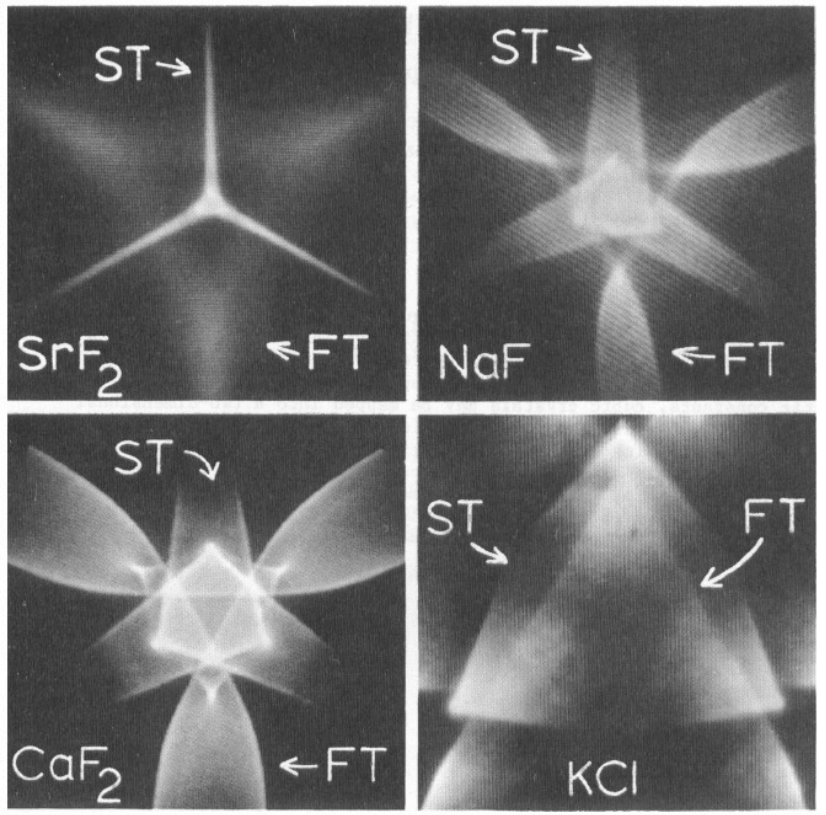
→ phonon focusing

Phonon Focussing on Cubic Crystals in ⟨111⟩ direction

example:

four cubic crystals, [111] direction

→ 3-fold symmetry



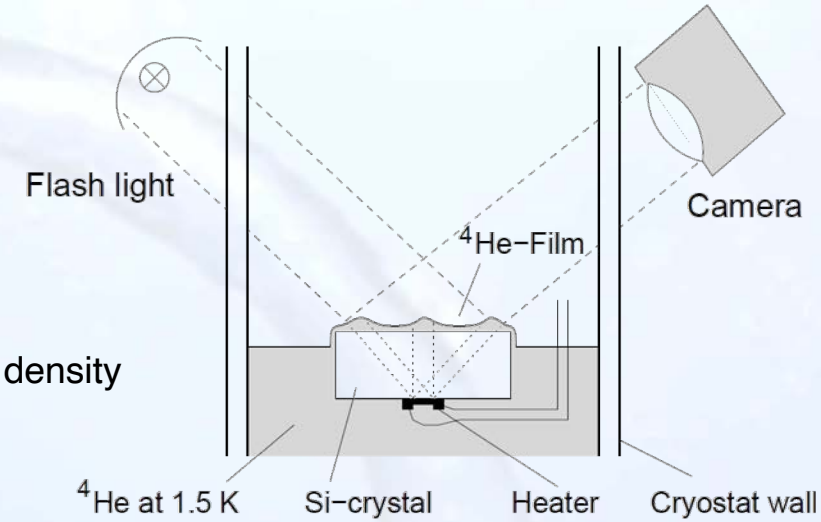
Phonon Focusing made visible by ^{4}He Films

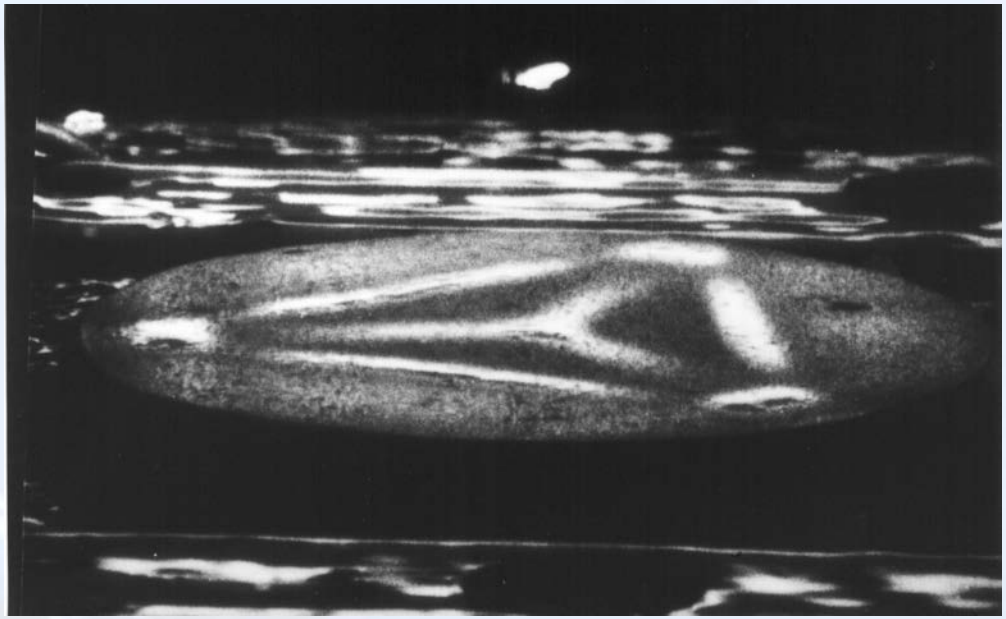
demonstration using superfluid ^{4}He

ballistic phonons ⟶ substrate locally heated

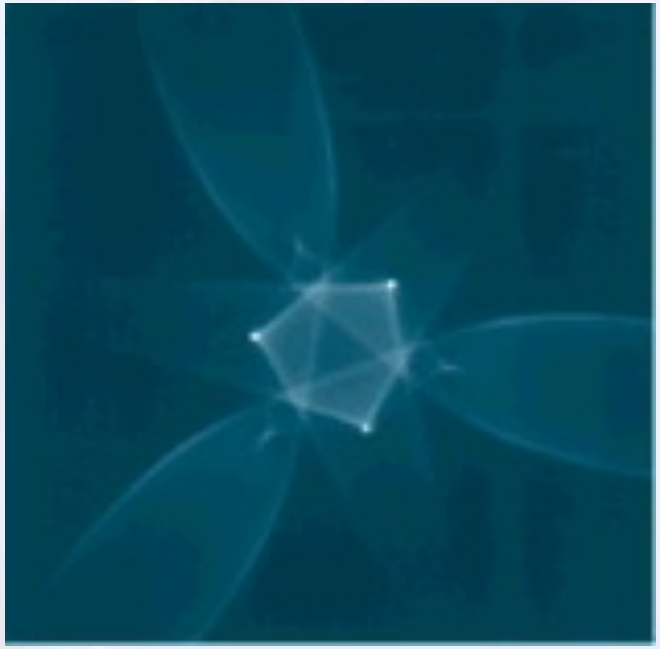
⟶ fountain effect

⟶ film thickness changes ≙ phonon density

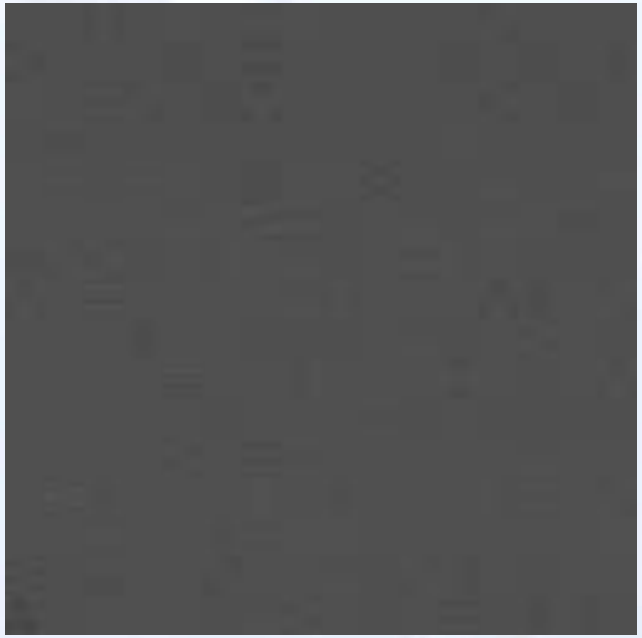




Phonon Focusing

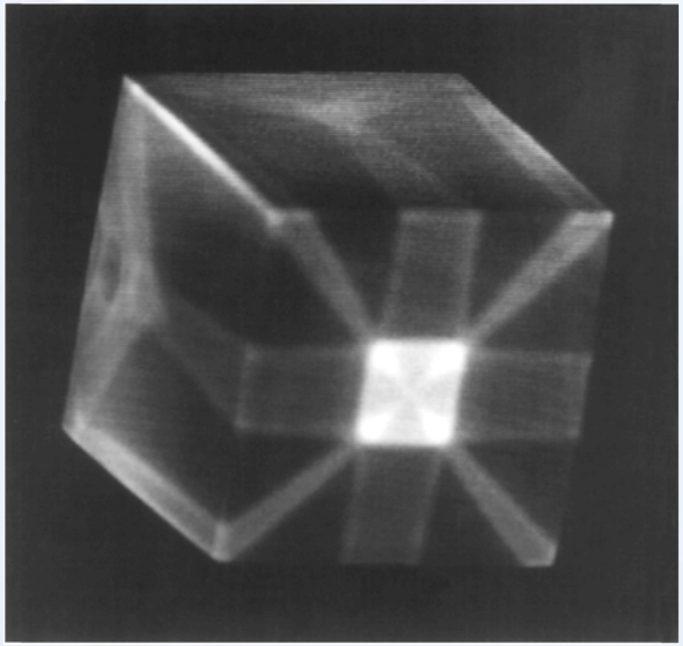


CaF_2

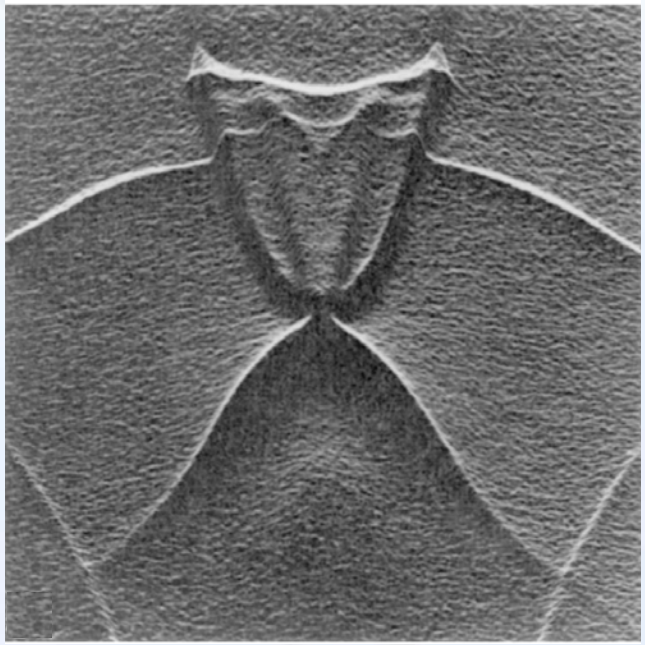


Nb at 1.8 K

Phonon Focusing



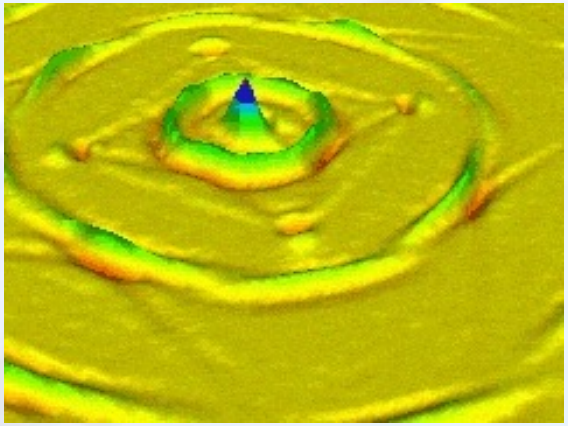
Ge
wide-angle measurement

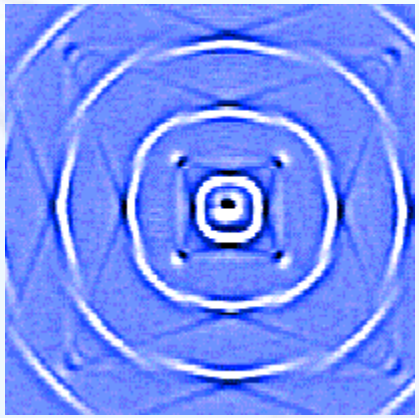


y-cut quartz
e-beam scanning

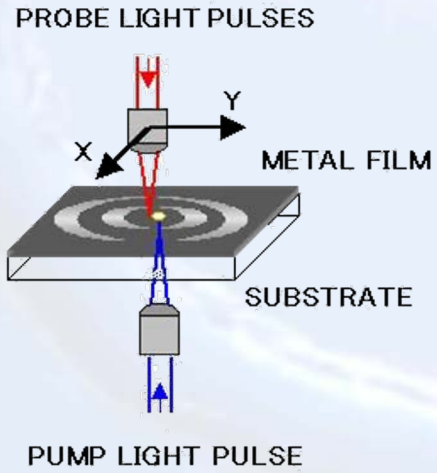
Phonon Focusing

TeO_2

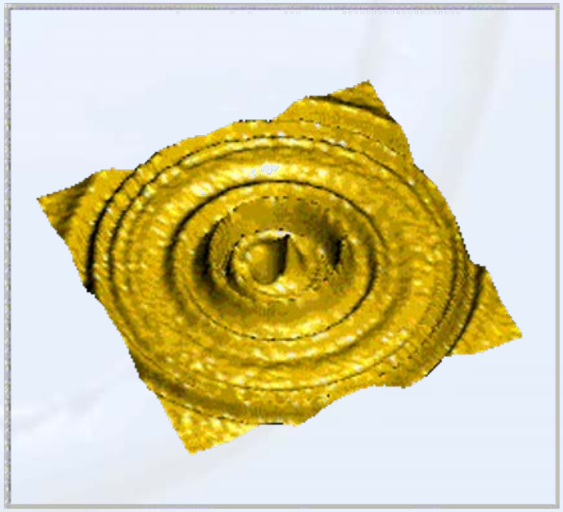




experimental technique



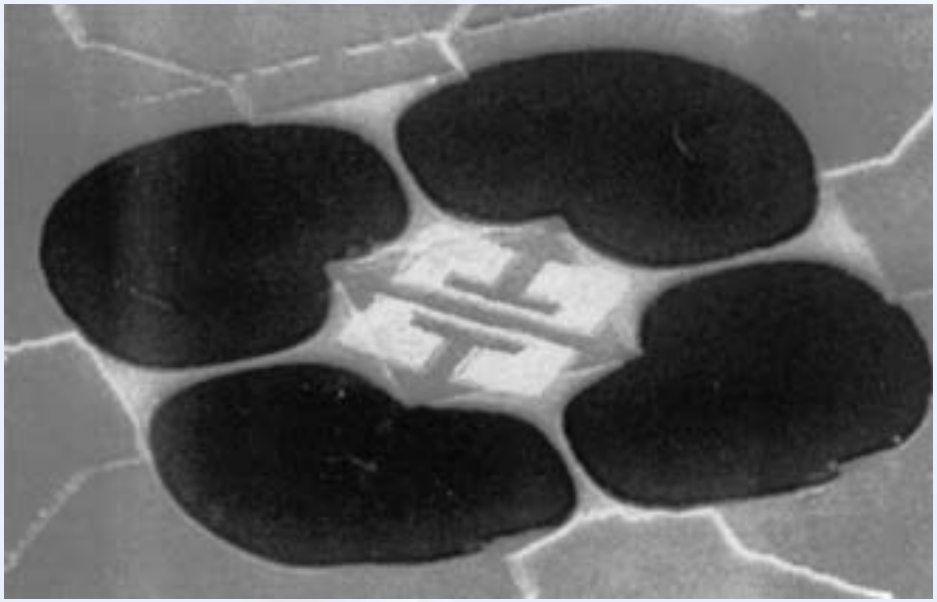
isotrop phonon propagation in glass

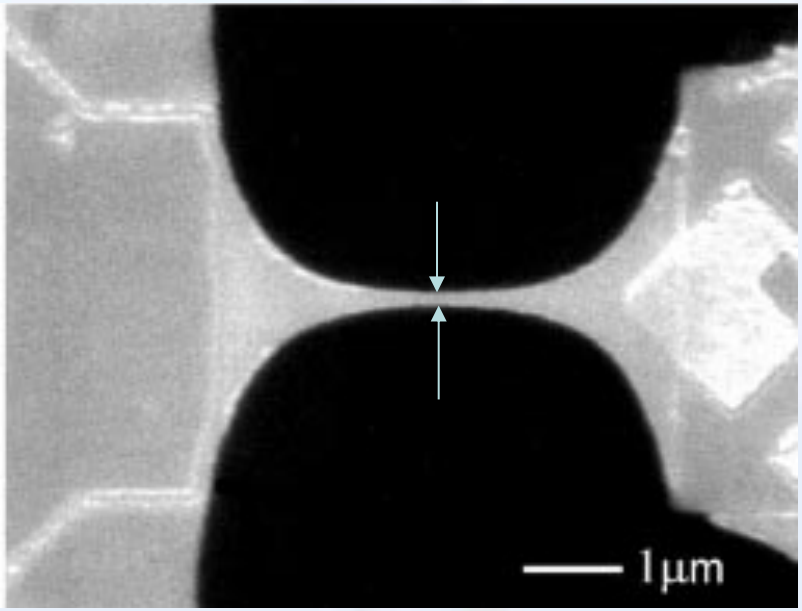


6.3 Thermal Conductivity in One-dimensional Samples

Geometry of setup:

Si_3N_4 silicon nitride





4 × 4 µm^3 island with gold resistors as heaters and thermometers

minimal width of bridge $w < 200$ nm

6.3 Thermal Conductivity in One-dimensional Samples

heat flow: $J = \frac{1}{L} \sum_{q} \hbar \omega_q v_q$

L: length of sample; q: all thermally excited phonons

summation ⟶ integration

$$J = \sum_i \frac{1}{L} \int_0^\infty \mathcal{D}_i^1(q)\, \hbar\omega_i\, v_i\, [f_\mathrm{h}(\omega, T) - f_\mathrm{c}(\omega, T)]\, \mathrm{d}q$$

i: phonons modes; v_i: group velocity

assumptions

- transmission coefficient for coupling between bath and thin bar = 1
- $\mathcal{D}_i^1(q) = L/2\pi\,, \quad q \longleftrightarrow \omega\,, \quad \frac{\partial q}{\partial \omega}$ cancels with $v = \frac{\partial \omega}{\partial q}$
- small temperature difference ΔT

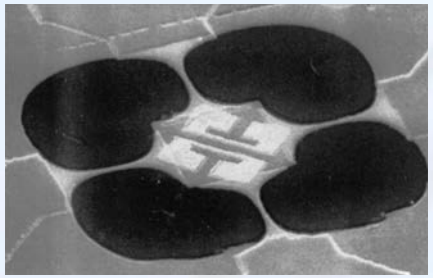
  ⟶ $[f_\mathrm{h}(\omega, T) - f_\mathrm{c}(\omega, T)]$ can be expanded, keep only terms linear in ΔT

$$G = \frac{J}{\Delta T} = \frac{k_\mathrm{B}^2 T}{h} \sum_i \int_0^\infty \frac{x^2 \mathrm{e}^x}{(\mathrm{e}^x - 1)^2} \mathrm{d}x = N_i \underbrace{\frac{\pi^2}{3} \frac{k_\mathrm{B}^2 T}{h}} = N_i G_0 \qquad \boxed{G = N_i G_0}$$

N_i: number of contributing modes; G_0: quantum of thermal conductance

Quantized of Heat Conduction: Sample Geometry

for given geometry



$N_i = 4 \times 4$ legs $= 16$

modes:
1 longitudinal (dilatation)
1 torsional
2 bending

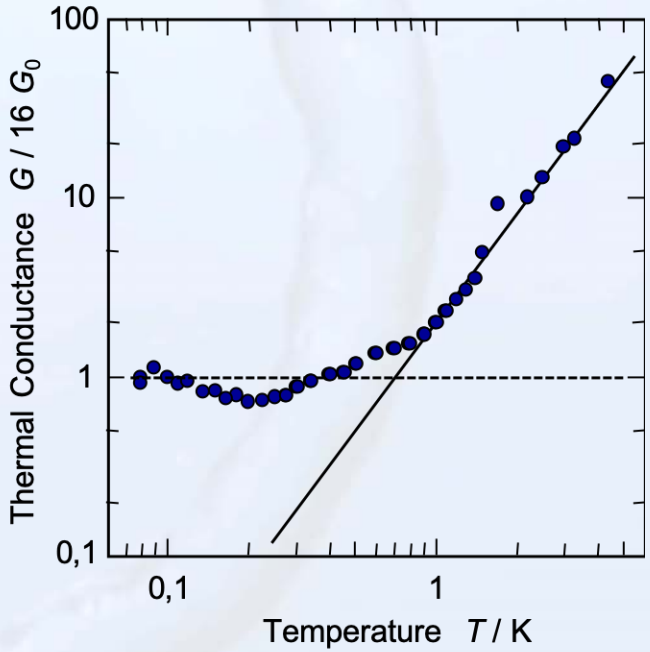
$$G_0 = (9.456 \times 10^{-13}\,\mathrm{W\,K^{-2}})\,T$$

- transition roughly at 0.8 K

$$T_{\mathrm{crossover}} \approx \frac{h\nu}{2wk_{\mathrm{B}}} \approx 0.8\,\mathrm{K}$$

expected for: $q_{\mathrm{th}} \approx k_{\mathrm{B}}T/(\hbar v) < \Delta q = \frac{\pi}{w}$

spacing between lowest lying modes



7. Conduction Electrons

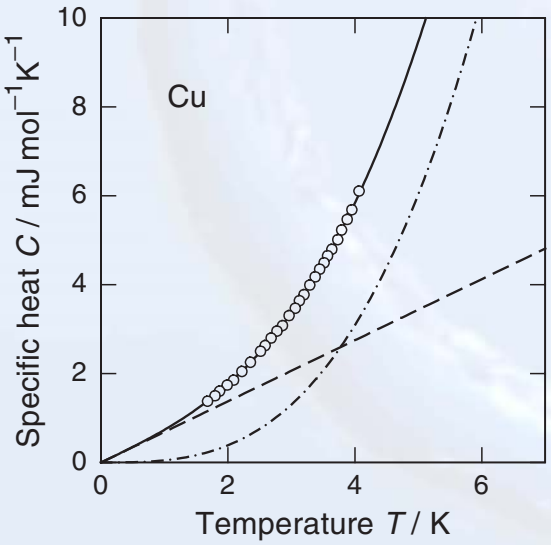
Drude (1900) ⟶ Sommerfeld (1927) ⟶ Bloch (1940)

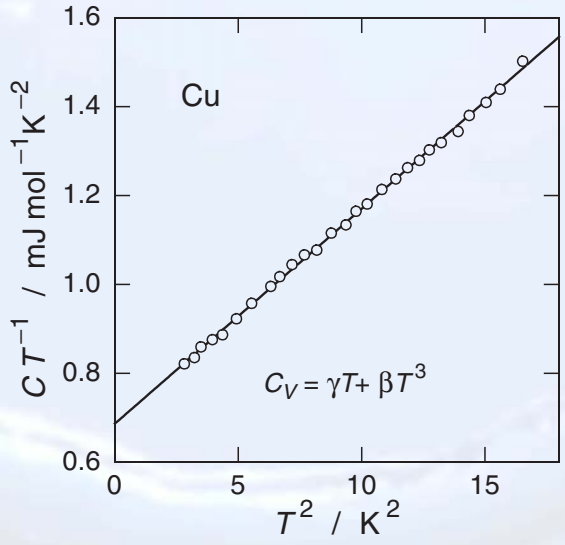
7.1 Specific heat

a) simple metals ⟶ free electrons gas

free electrons gas: $c_V = \gamma T + \beta T^3$ (phonons)

electrons: $\gamma = \dfrac{\pi^2 n k_B^2}{2E_F} \propto n^{1/3}\, m^*_{th}$





- electrons dominate below ~ 4 K
- very good qualitative agreement

7.1 Specific Heat

Element	γ_{exp}	γ_{theo}	m^*_{th}/m	Element	γ_{exp}	γ_{theo}	m^*_{th}/m
Ag	0.64	0.64	1.00	Cu	0.69	0.50	1.37
Al	1.35	0.91	1.48	Ga	0.60	1.02	0.59
Au	0.69	0.64	1.08	In	1.66	1.26	1.31
Ba	2.70	1.95	1.38	K	2.08	1.75	1.19
Be	0.17	0.49	0.35	Li	1.65	0.75	2.19
Ca	2.73	1.52	1.80	Mg	1.26	1.00	1.26
Cd	0.69	0.95	0.73	Na	1.38	1.3	1.22
Cs	3.97	2.73	1.46	Pb	2.99	1.50	1.99

→ good qualitative agreement for simple metals

→ $\gamma_{exp}/\gamma_{theo} = m^*_{th}/m$ for quantitative agreement

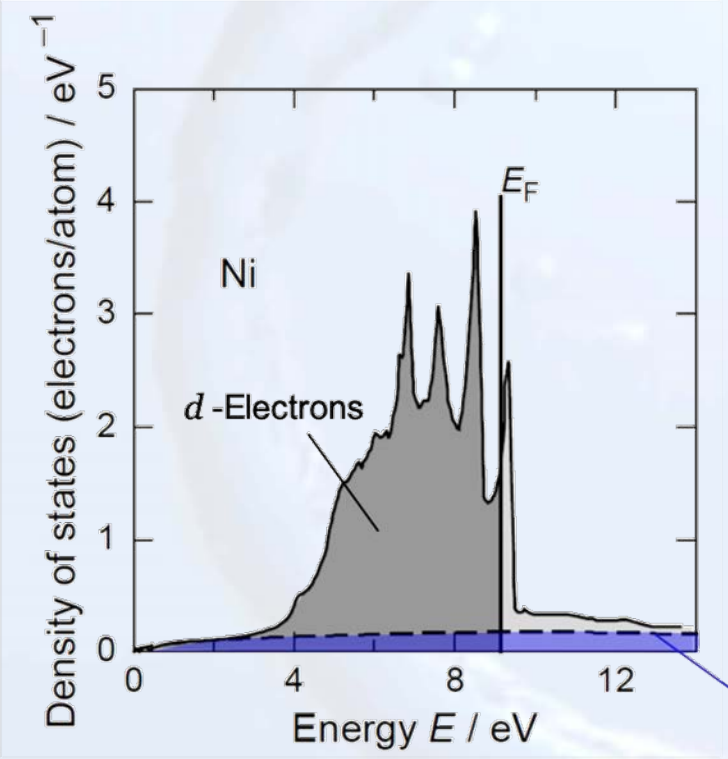
7.1 Specific Heat

but: transition series metals

example nickel: $m^*_{\mathrm{th}} \approx 15\,m$ ⟶ reason is *d*-electrons contribute, which are not (completely) free

involved in covalent bond, highly oriented

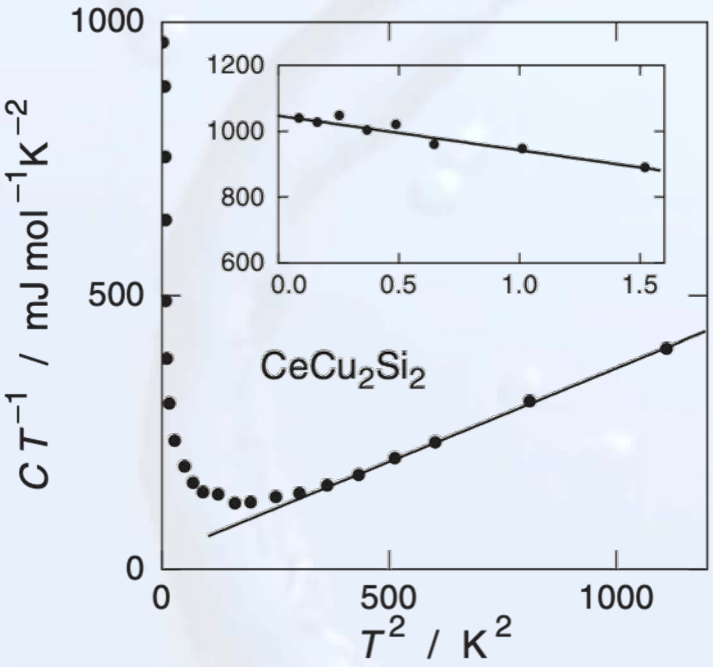
⟶ no spherical Fermi surface



- d-electrons with large density of state dominate at E_{F}
- d-electrons are localized

b) metal with heavy electrons

examples: $CeCu_2Si_2$ cer electronic configuration [Xe] $5d^1\ 4f^1\ 6s^2$



→ γ not constant below 15 K

$\gamma = C_{el}/T$

$\gamma \approx 30\,\mathrm{mJ\,(mol\,K^2)^{-1}}$ extrapolated from high T

$\gamma \approx 1050\,\mathrm{mJ\,(mol\,K^2)^{-1}}$ at low temperatures $T \rightarrow 0$

→ $4f$ electrons are localized at high T and form a conduction band at low T

→ effective mass: $m^* \approx 100\,m_e$

- $T > 15$ K, $D(E)$ and m^* are constant
- $T < 15$ K, C/T increase strongly with decreasing temperature

7.1 Specific Heat

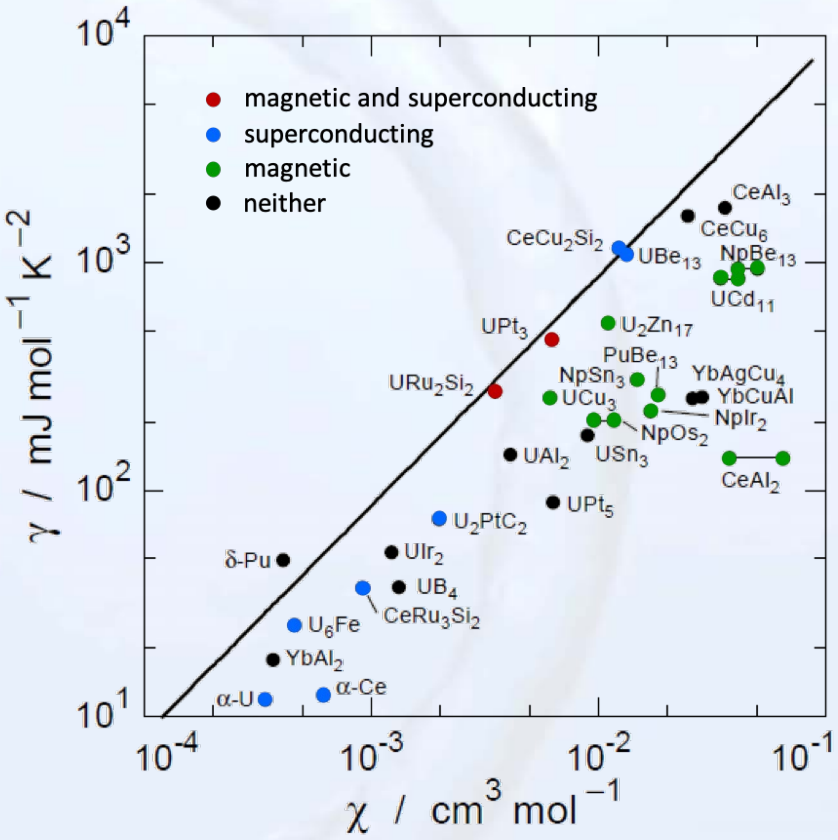
Heavy fermion systems

- interesting class of solids with strongly correlated electrons
- effective masses m^* up to 2000 m_e observed
- origin: interaction with localized spins

$$\chi = \mu_0 \mu_B^2 D(E_F) \propto n^{1/3} m^*$$

Wilson ratio: $$R = \frac{\chi}{\gamma} \frac{\pi^2 k_B^2}{\mu_0 \mu_{eff}^2}$$

$$\gamma \propto n^{1/3} m_{th}^*$$



important: Fermi liquid theory

$$C = \frac{m^*}{m} C_{FG} = \left(1 + \frac{1}{3} F_1\right) C_{FG}$$

analogy to ^{3}He reaches even further

⟶ some heavy fermion systems show unconventional superconductivity (S ≠ 0) : UPt_3, URu_2Si_2 …

7.2 Electrical Conductivity

metals, no superconductors, no semiconductors

Boltzmann equation ⟷ kinetic gas theory

- starting point: equilibrium distribution without external fields $f_0(\boldsymbol{k})$

  Fermi-Dirac distribution

- with field: stationary non-equilibrium value of $f(\boldsymbol{k}, \boldsymbol{r}, t)$

- expand $f_0(\boldsymbol{k}) - f(\boldsymbol{k}, \boldsymbol{r}, t)$ in linear order + relaxation ansatz for collisions

→ linearized Boltzmann equation $f(\boldsymbol{k}) \approx f_0(\boldsymbol{k}) + \frac{e\tau(\boldsymbol{k})}{\hbar}\, \boldsymbol{\mathcal{E}} \cdot \frac{\partial f_0(\boldsymbol{k})}{\partial \boldsymbol{k}}$

scattering time; electric field

→ $j_x = -e \int D(k)\, v_x(k)\, f(k)\, \mathrm{d}k = -\frac{e}{\pi^2} \int k^2\, v_x(k)\, f(k)\, \mathrm{d}k$

→ $\sigma = \frac{1}{3}\, e^2\, D(E_\mathrm{F})\, v_\mathrm{F}^2\, \tau(E_\mathrm{F})$ → $\boxed{\sigma = \frac{n\, e^2}{m}\, \tau(E_\mathrm{F})}$

7.2 Electrical Conductivity

scattering time determined by:

- defect scattering
- phonon scattering
- magnon scattering (in ferromagnets)
- electron-electron scattering (can be neglected in most cases)

a) defect scattering

↗ local charge density variations

↘ local strain fields (less important)

Local charge variations

- Rutherford scattering on ionic cores of impurity atoms
- scattering cross section : $\sigma_{\text{cross}} \propto (\Delta Z)^2$
- resistivity $\varrho_{\text{D}} \propto (\Delta Z)^2$

- residual resistance of copper with 1 at% impurities with different valence electrons configurations
- agrees well with: $\varrho_{\text{D}} \propto (\Delta Z)^2$