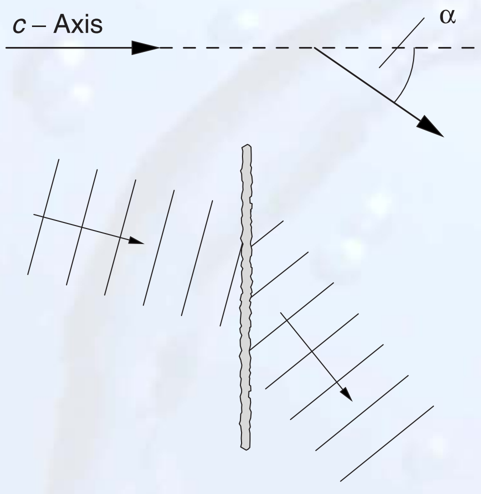
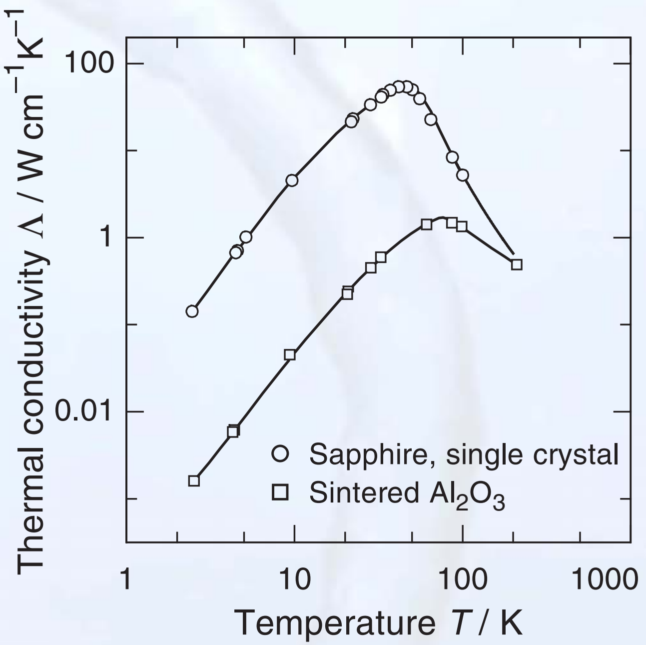
6.2 Heat Transport

c) grain boundaries



$$\ell \approx L/\langle \sin^2 \alpha \rangle$$

L — average size of crystallites



- sapphire single crystal: 1.5 mm
- sintered Al_2O_3 powder 5 … 30 µm

6.4 Influence of Normal Processes in Heat Transport

a) Poiseuille flow

N-processes are visible in heat transport experiments under special circumstances

→ ultralow temperatures (no umklapp processes) and low defect scattering

→ phonon flow in a thin crystal can be described like viscose flow of atoms in capillaries

condition:

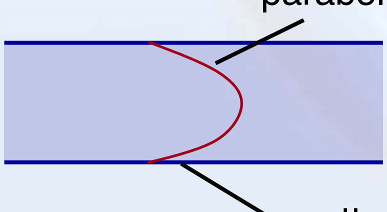
$$\ell_{\rm N} \ll d \ll \ell_{\rm R}$$

$$\ell_{\rm R}^{-1} = \ell_{\rm U}^{-1} + \ell_{\rm D}^{-1}$$

classical gas: Hagen-Poiseuille law

$$-\frac{\dot m}{\pi r^2}\frac{1}{|\nabla p\,|} = \frac{\varrho}{8}\frac{r^2}{\eta} = \frac{3}{8}\frac{1}{\overline{v}_{\rm th}}\frac{r^2}{\ell_{\rm g}}$$

$$\eta = \tfrac{1}{3}\varrho\,\overline{v}_{\rm th}\,\ell_{\rm g}$$



$\ell_g < r$

analog equation for phonon transport (phonon gas)

$$-\frac{\dot Q}{\pi r^2}\frac{1}{|\nabla T|} = \varLambda = \frac{1}{3}C_V\,v\,\ell_{\rm eff}$$

$$\ell_{\rm eff} \approx r^2/\ell_{\rm N}$$

$$\dot Q \mathrel{\hat=} \dot m$$

$$\Delta T \mathrel{\hat=} \Delta p$$

effective mean free path for N-processes for heat transport

6.4 Influence of Normal Processes in Heat Transport

Interpretation: heat resistance by scattering at the surface, but each phonon has to travel the statistical path $l_{\mathrm{eff}} \approx r^2/\ell_{\mathrm{N}}$ because of very frequent N-processes before it reaches the surface

⟶ the observed mean free path in transport measurements are longer than the sample diameter

exact calculation

$$\Lambda = \frac{1}{3} C_V v \frac{5\,d^2}{32\,\ell_{\mathrm{N}}} \longrightarrow \boxed{\Lambda \propto T^8}$$

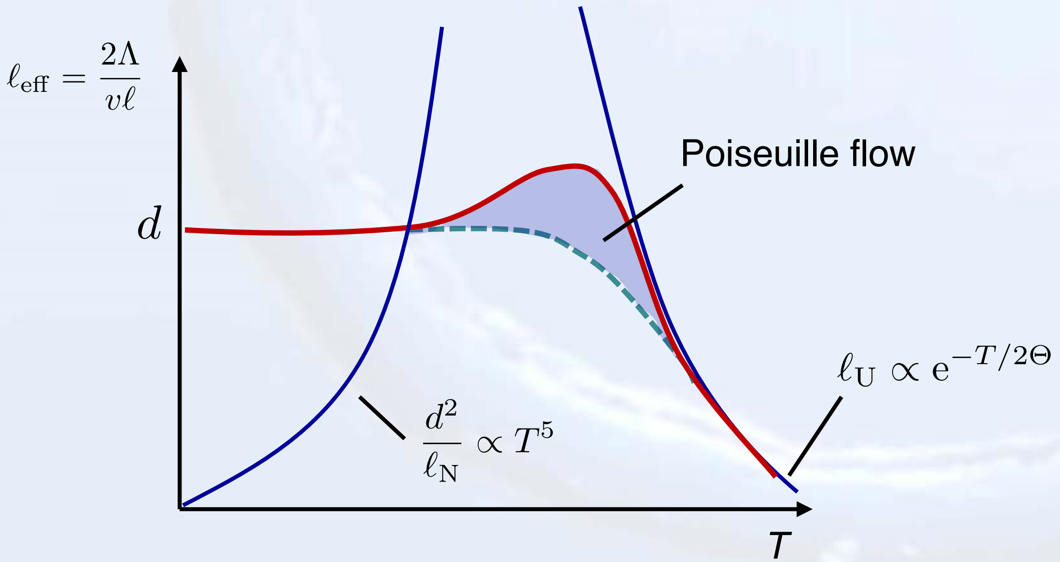
$C_V \propto T^3$, $\ell_{\mathrm{N}} \propto T^{-5}$

3 phonon process

$\sigma \propto \prod_i \omega_i \propto \omega_1\omega_2\omega_3 \propto T^3$

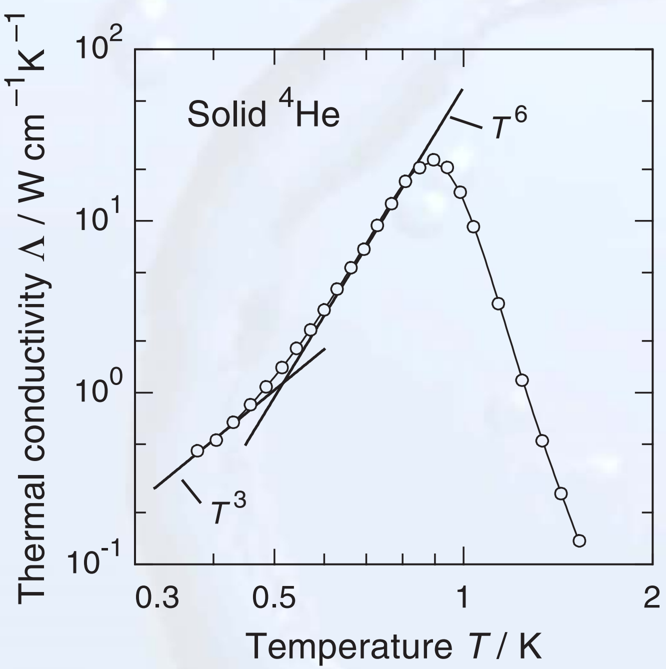
$D(\omega) \propto \omega^2 \propto T^2$

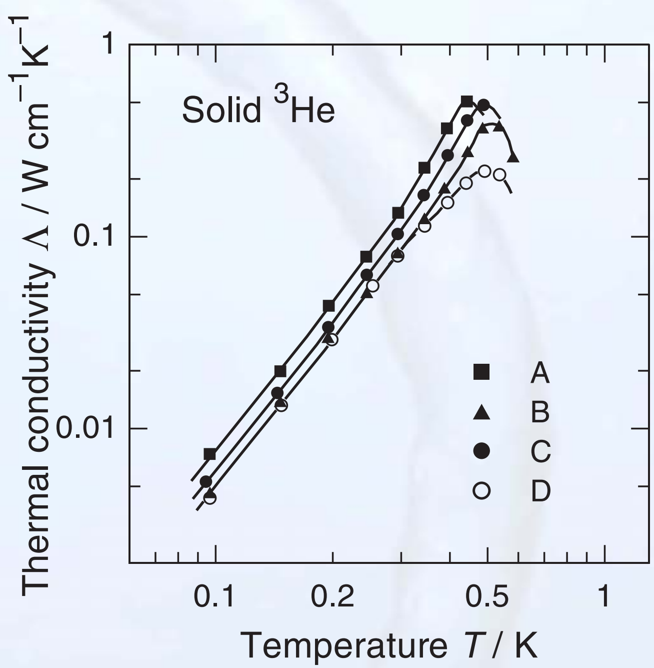
⟶ $\ell_{\mathrm{N}} \propto T^{-5}$



6.4 Influence of Normal Processes in Heat Transport

experimental evidence



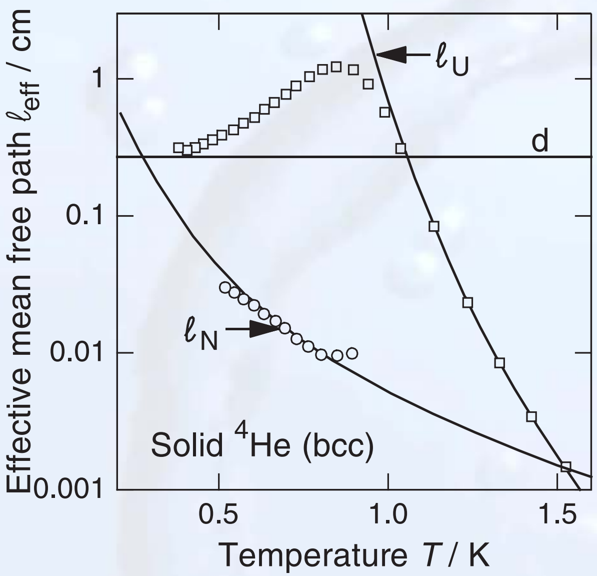


- steeper than T^3 temperature dependence observed
- temperature dependence is T^6 instead of T^8
- only small temperature range

- A,B,C crystals with different densities
- D contains 100 ppm ^{4}He
- slightly steeper temperature dependence as T^3

6.4 Influence of Normal Processes in Heat Transport

analysis of ^{4}He data



- separation of ℓ_{N} and ℓ_{U}
- $\ell_{\mathrm{N}} \propto T^{-3}$ instead of $\ell_{\mathrm{N}} \propto T^{-5}$
- possible explanation: influence of dislocations reduces the temperature dependence

Why is the Poiseuille flow not always observed?

$\ell_{\mathrm{N}} < d$ ⟶ means high temperature, can be fulfilled easily

in addition: momentum must be carried to wall ⟶ random walk $r \approx \sqrt{p}\,\ell_{\mathrm{N}}$

crystal radius (r)

number of scattering processes necessary to reach surface (p)

6.4 Influence of Normal Processes in Heat Transport

at the same time no scattering processes leading to heat resistance should occur

→ $\ell_\mathrm{R} > p\,\ell_\mathrm{N}$ (i)

$\ell_\mathrm{R}^{-1} = \ell_\mathrm{U}^{-1} + \ell_\mathrm{D}^{-1}$

and $\ell_\mathrm{R} > \ell_\mathrm{eff} \approx r^2/\ell_\mathrm{N}$ → $\ell_\mathrm{R} > r^2/\ell_\mathrm{N}$ (ii)

mean free path for Poiseuille flow

conditions (i) and (ii) are difficult to fulfill at the same time

b) Second sound

second sound in "phonon gas" $\widehat{=}$ first sound in classical gas

density wave in classical gases $\ell < \lambda_\mathrm{sound}$: $v_\mathrm{s} \approx \frac{1}{\sqrt{3}}\,\overline{v}_\mathrm{th}$ sound velocity

analog for phonon gas: $v_2 \approx \frac{1}{\sqrt{3}}\,v_\mathrm{s}$ velocity of second sound

6.4 Influence of Normal Processes in Heat Transport

conditions:

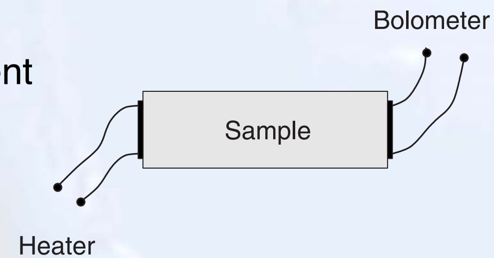
- no umklapp processes, no defect scattering
- $\ell_\mathrm{N} \ll v_\mathrm{s} t_\mathrm{p} \ll \ell_\mathrm{R}$ ↷ $\tau_\mathrm{N} \ll t_\mathrm{p} \ll \tau_\mathrm{R}$

pulse length

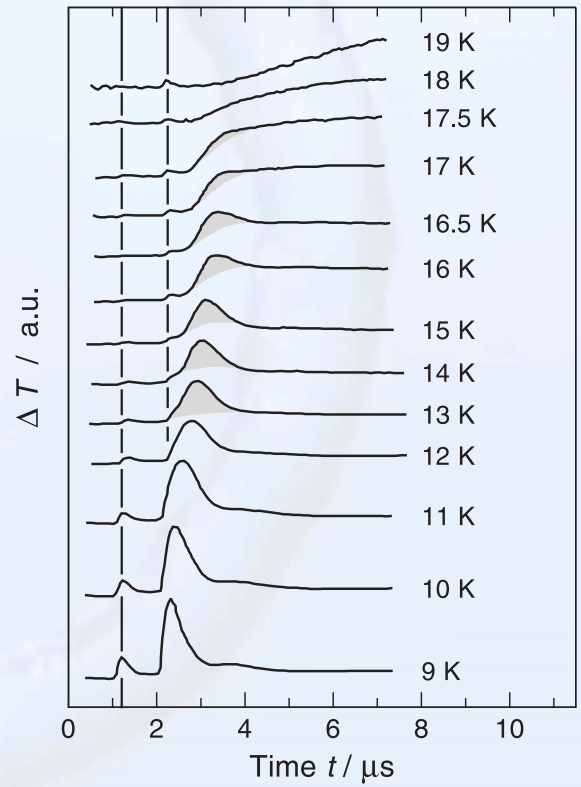
➡ can be fulfilled more easily than condition of Poiseuille flow if $\ell_\mathrm{R} \gg \ell_\mathrm{N}$

pulse length can be adjusted

experiment



second sound in NaF

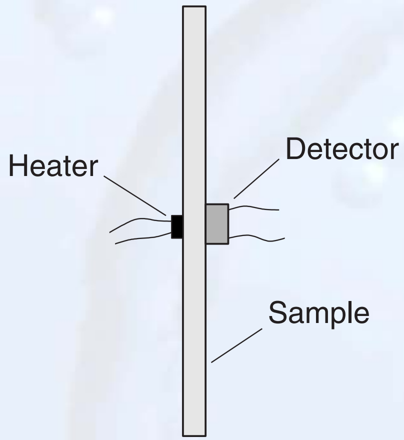


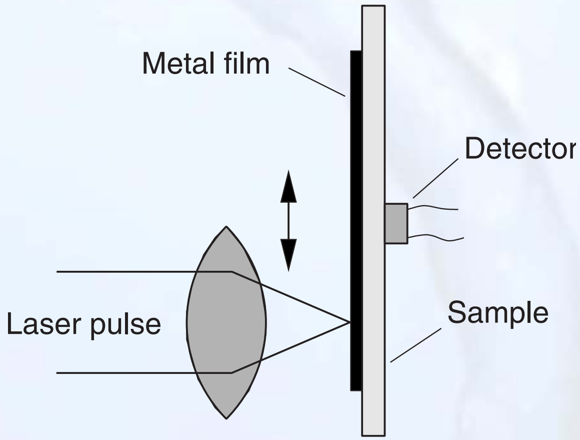
- T = 9 K → ballistic propagation: τ_N too long
- $T >$ 17 K → diffusive phonon scattering
- 9 K < T < 17 K → indications for second sound

second sound so far observed in: ^{3}He, ^{4}He, NaF, Bi

6.5 Ballistic Propagation of Phonons

Experimental techniques





time resolved measurements

position dependent measurement

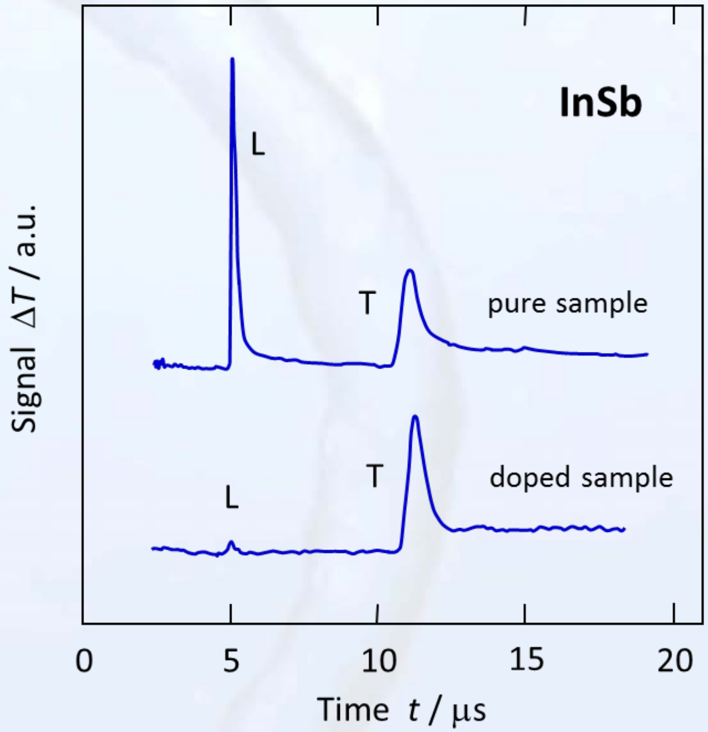
$$t_i = d/v_i$$

time of phonon with polarization i
from heater to detector

6.5 Ballistic Propagation of Phonons

a) time resolved measurements

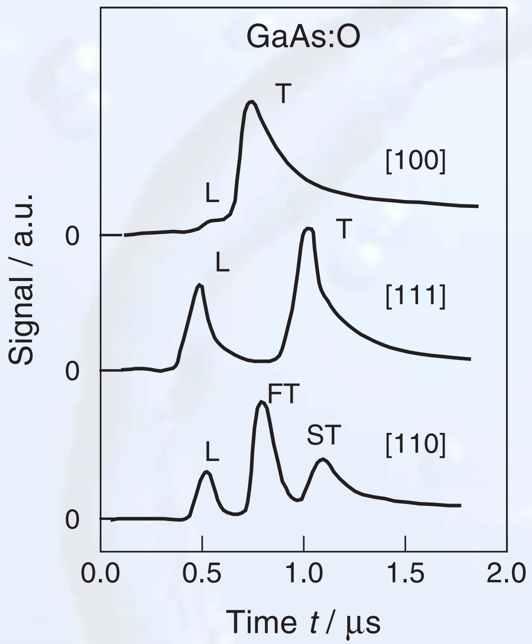
Example: InSb, investigation of electron-phonon coupling

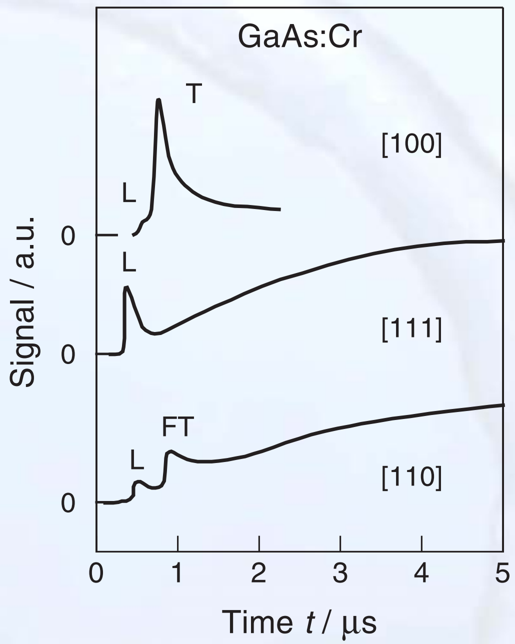


- samples with different doping level
- electrons interact with phonons via density variation
- longitudinal phonons: density variation → strong coupling
- transverse phonons: no density variation → no coupling

6.5 Ballistic Propagation of Phonons

2nd example: GaAs:O, GaAs:Cr, phonon-defect coupling is investigated
free electrons are unimportant





- [110] 3 branches L, FT, ST
- [100], [111], FT, ST degenerate

- [100] no change
- [111] FT, ST disappear
- [110] ST disappears, FT reduced
- diffused phonons appear in [110], [111]
- reason: resonant scattering with Cr defects