# 5.1 Specific heat and phase diagram

dilute solutions of $^3$He in He-II ($c_3 < 0.15$, $T < 0.5$ K)

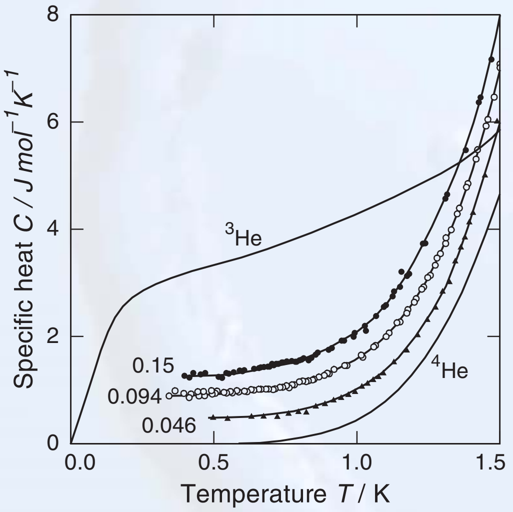
→ $^4$He: passive background fluid

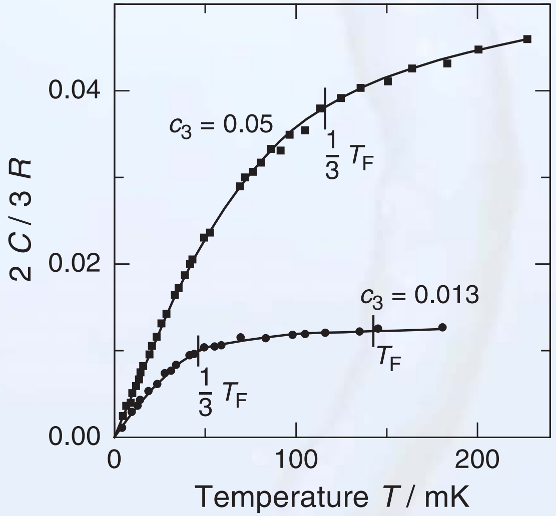
→ $^3$He: “free” atoms in a quasi vacuum and effective mass $m_3^* = 2.4\, m_3$

Fermi gas $$T_\mathrm{F} = \frac{\hbar^2}{2 m_3^* k_\mathrm{B}} \left(3\pi^2 n_3\right)^{2/3} \propto c_3^{2/3}$$

$C$ → $T > T_\mathrm{F}, \quad C \propto c_3 T^0 \quad (\hat{=}\, \frac{3}{2} R)$ high $T$

$C$ → $T < \frac{1}{3} T_\mathrm{F}, \quad C \propto T$ low $T$





- $T_\lambda$ depends on $c_3$
- pure $^3$He: transition Fermi gas → Fermi liquid
- high $T$, dilute solution: classical gas with $m^*$

- low $T$: transition classical gas → Fermi gas
- lines correspond to theory

# 5.1 Specific heat and phase diagram

Finite solubility of $^3$He in liquid $^4$He at $T = 0$

reason: difference in zero-point motion of $^3$He and $^4$He

v. Waals interaction identical for $^3$He-$^3$He and $^3$He-$^4$He

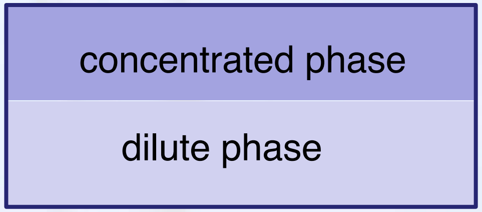
but: larger zero-point motion of $^3$He weakens the bonding

} stronger effective binding for $^3$He–$^4$He

compared to $^3$He–$^3$He

in equilibrium on finds
$$\mu_{3,\mathrm{d}}(T, c_{3,\mathrm{d}}) = \mu_{3,\mathrm{c}}(T, c_{3,\mathrm{c}})$$

dilute          concentrated



- ► $T = 0 \longrightarrow c_3 = 1$ for concentrated phase (pure $^3$He)
- ► necessary energy to bring one $^3$He atom into “vacuum” $L_3(T = 0)$

  $$\longrightarrow \mu_{3,\mathrm{c}}(0,1) = \mu_3(0) = -L_3(0) = -2.473\,\mathrm{K}$$

  latent heat
- ► dilute phase: $E_3 = -\mu_{3,\mathrm{d}}(0,0) \longrightarrow$ binding energy

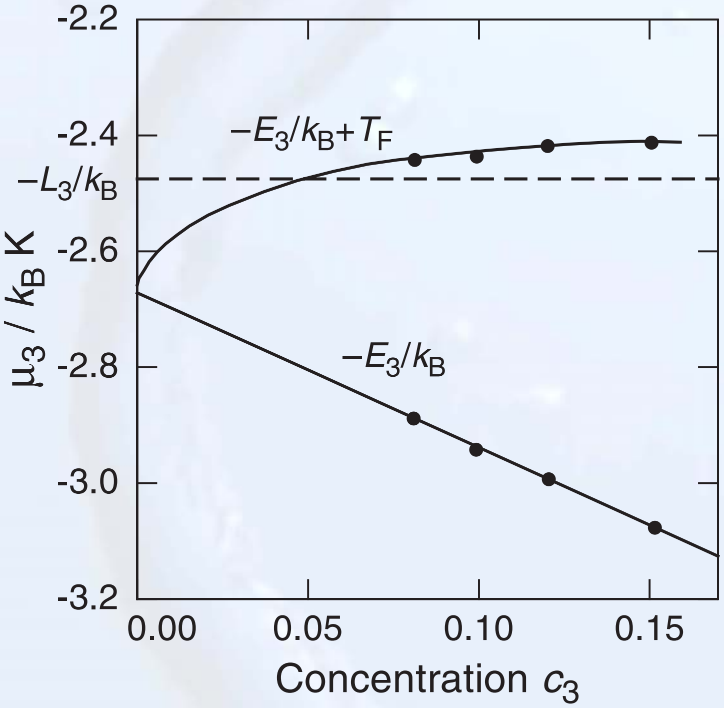
  $c_{3,\mathrm{d}} \rightarrow 0$
- ► with increasing concentration, the effective binding energy for $^3$He is reduced because of the Pauli principle $\longrightarrow$ Fermi gas: $E_\mathrm{F} = k_\mathrm{B} T_\mathrm{F}(c_3)$

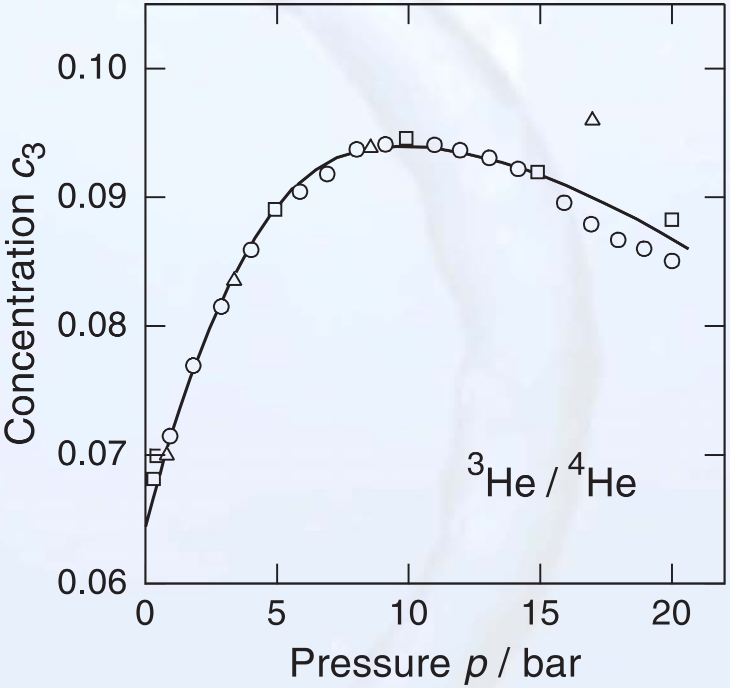
# 5.1 Specific heat and phase diagram

equilibrium concentration at $T = 0$

$$-L_3(0) = -E_3(0, c_3) + k_\mathrm{B} T_\mathrm{F}(c_3)$$

pressure dependence





- calculation of $E_3(0,c_3)$ is not trivial

→ Bardeen, Baym, Pines model

- maximum at 8.7 bar
- concentration $c_3 = 0.096$

# 5.2 Normalfluid Component

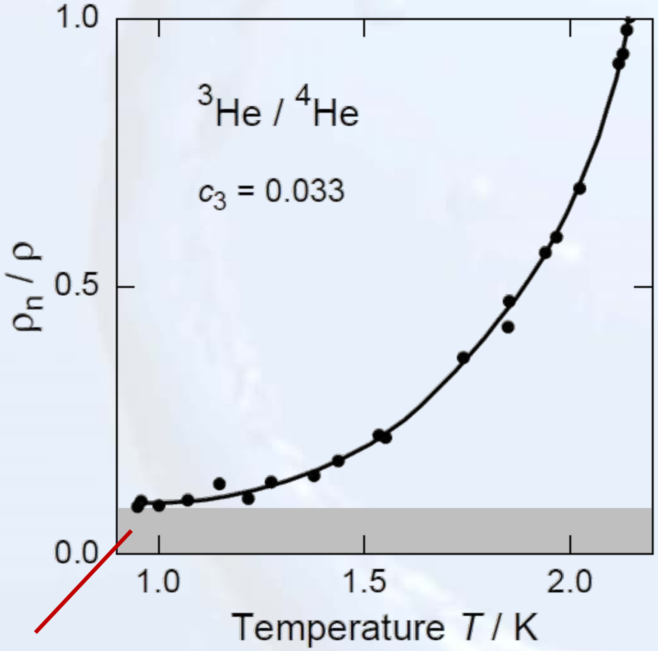
determination of $\varrho_\mathrm{n}$ → Andronikasvili-type experiment

15 mica sheets
4 cm diameter
190 $\mu$m spacing

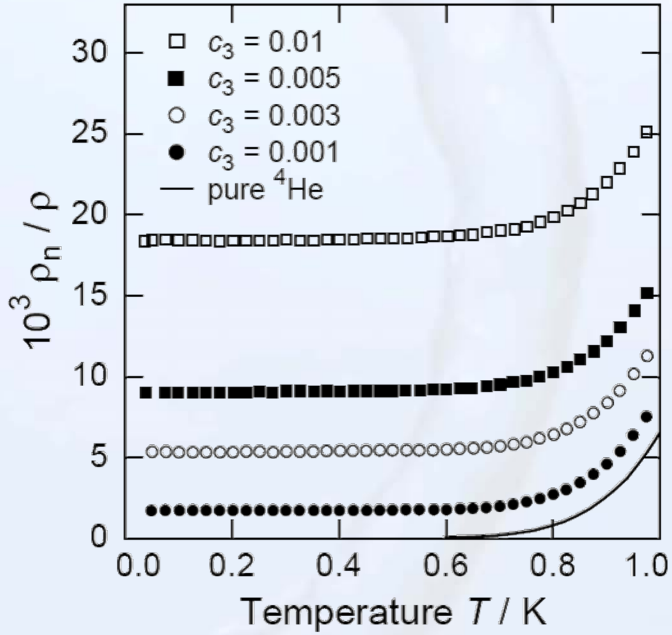
$$\varrho_\mathrm{n} = \varrho_{\mathrm{n},4} + \varrho \frac{m_3^*}{m_4} c_3$$

pure He-II

const



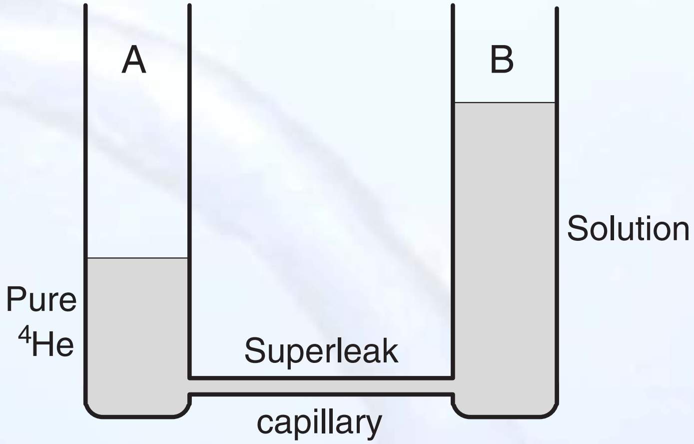
$^3$He



→ $\varrho_\mathrm{n}(T \to 0) = \mathrm{const} \propto c_3$

# 5.2 Normalfluid Component

Osmotic pressure



- $^4$He flows to solution to thin the $^3$He concentration
- $^3$He is blocked

→ osmotic pressure

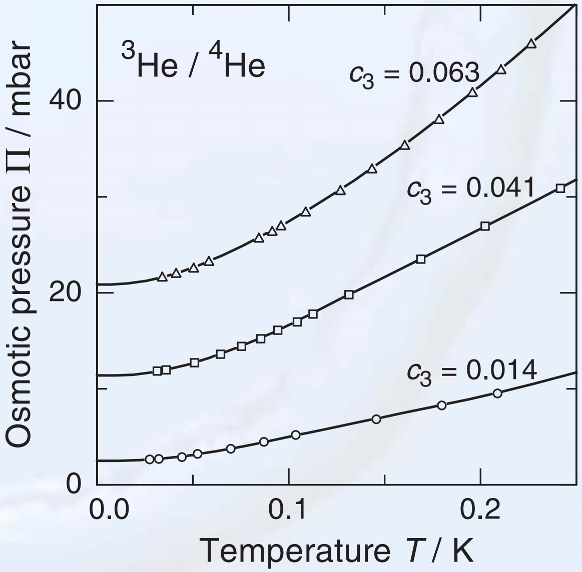
van't Hofft law ($T \gg T_\mathrm{F}$, classical regime)

$$\Pi = n_3 k_\mathrm{B} T \propto c_3 T$$

$T \ll T_\mathrm{F}$ , degenerate Fermi gas

$$\Pi = \frac{2}{5} \underbrace{n_3 k_\mathrm{B} T_\mathrm{F}}_{\text{depends on } c_3} \propto c_3^{5/3} = \text{const}$$

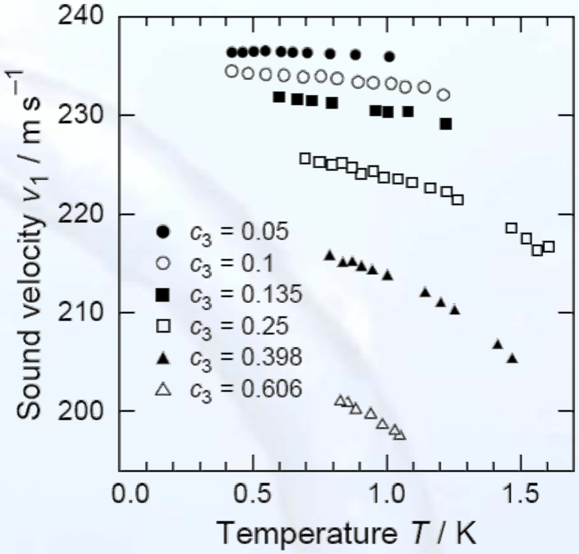
→ transition from FG to classical gas



# 5.3 Sound Propagation

## First sound

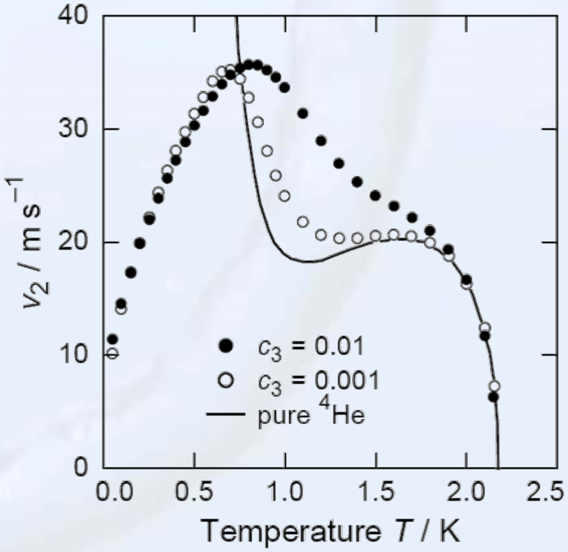
$$v_1^2 = \left(\frac{\partial p}{\partial \varrho}\right)_{S,c_3} \left[1 + \frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}}\left(\frac{\partial \varrho}{\partial c_3}\,\frac{c_3}{\varrho}\right)^2\right]$$



## Second sound

$$v_2^2 = \frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}}\left[\overline{S}\left(\frac{\partial T}{\partial S}\right)_{\varrho,c_3} + c_3^2\,\frac{\partial(\mu_3-\mu_4)}{\partial c_3}\right]\left[1+\frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}}\left(\frac{\partial\varrho}{\partial c_3}\,\frac{c_3}{\varrho}\right)^2\right]^{-1}$$

$$\overline{S} = S_{4,0} - \frac{k_\mathrm{B}}{m_4}\left[c_3+\ln(1-c_3)\right] + \frac{k_\mathrm{B}}{m_3}\,c_3$$

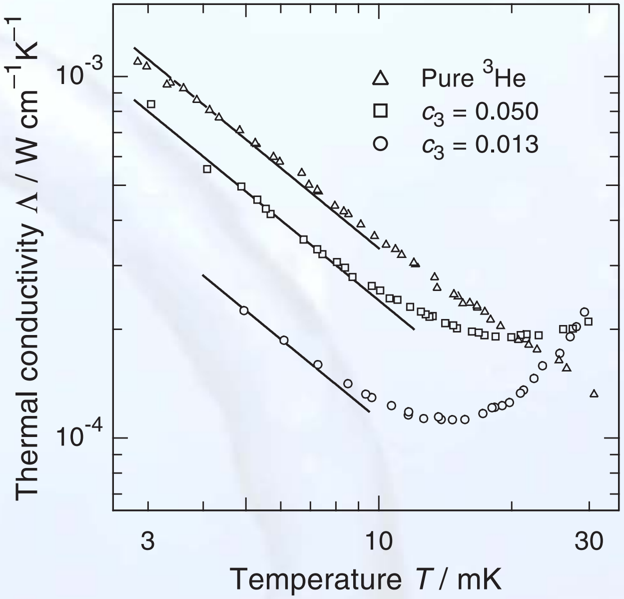


# 5.4 Transport Properties

Thermal transport (rather complex)

high Temp.: $\varrho_\mathrm{n}$ flow leads to $^3$He concentration gradient

- ⟶ $^3$He atoms diffuse back
- ⟶ $^3$He form scattering centers for $\varrho_\mathrm{n}$
- ⟶ reduction of heat transport



low Temp.: $^3$He atoms form FG

$$\Lambda = \frac{1}{3} C\, v_\mathrm{F}\, \ell \propto \frac{c_3}{T}$$

$$\ell = v_\mathrm{F}\tau \propto (T_\mathrm{F}/T)^2$$

$$v_\mathrm{F} = (\hbar/m_3^*)(3\pi^2 n_3)^{1/3}$$

$$C \propto T/T_\mathrm{F}$$

$$m_3^* = (1 + F_1/3)m_3$$

# 5.5 Search For a Superfluid Phase of $^3$He in Mixture

very interesting: 3 superfluid phases in the same container ⟶ $^4$He, $^3$He, and dilute $^3$He

Problem: $^3$He/$^4$He mixtures are hard to cool to below 200 μK because of Kapitza resistance

↓

acoustic mismatch hinders cooling

new initiative:

⟶ cooling by melting of $^4$He crystal

⟶ lowest temperature so far 90 μK