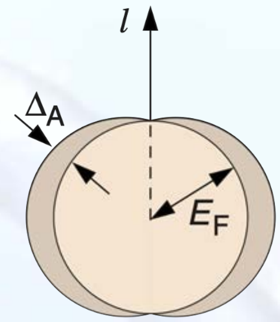
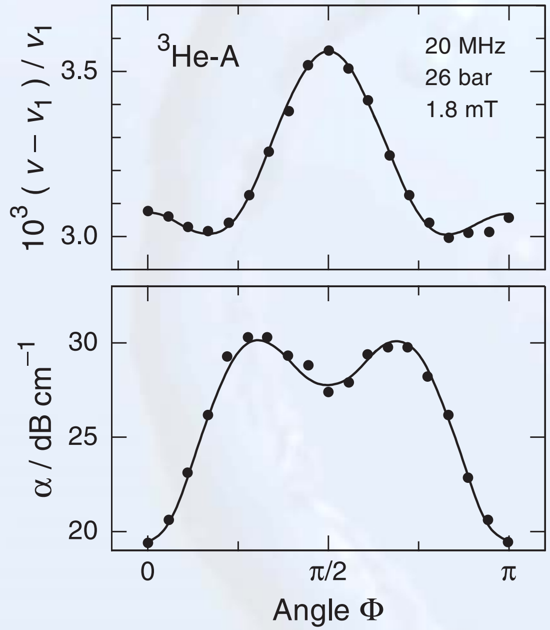
4.3 Quantum States of Superfluid ^{3}He

experimental determination of anisotropy of gap of ^{3}He-A



^{3}He–A

propagation of longitudinal zero sound

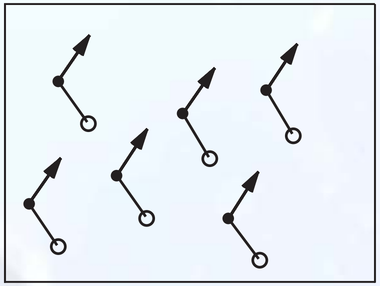


- in this experiment $\boldsymbol{l}$ is oriented by a small magnetic field 1.8 mT
- ϕ is the angle between $\boldsymbol{B}$ and $\boldsymbol{q}$

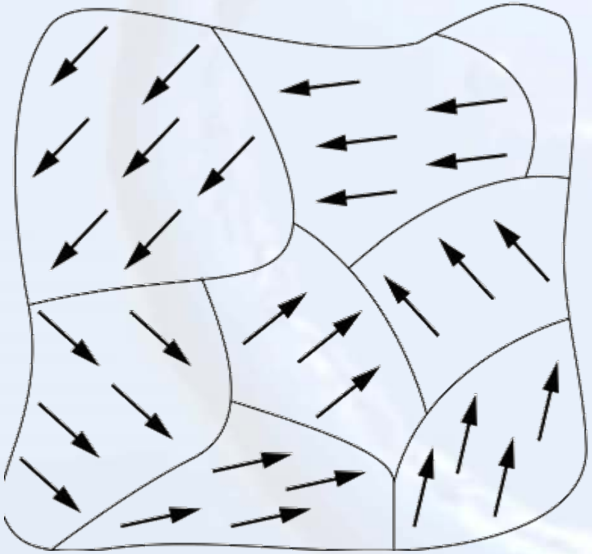
  wave vector of sound wave
- expected anisotropy is clearly observed

4.4 Orderparameter Orientation — Textures

Textures:



- this term was introduced by de Gennes (similar to liquid crystals)
- denotes orientational effects of $\boldsymbol{l}$ and $\boldsymbol{d}$
- texture depends on many things: dipole-dipole interaction, magnetic and electric fields, geometry, …
- often no uniform texture ⟶ texture domains



4.4 Orderparameter Orientation — Textures

a) orientation of $\boldsymbol{l}, \boldsymbol{d}$ without external field

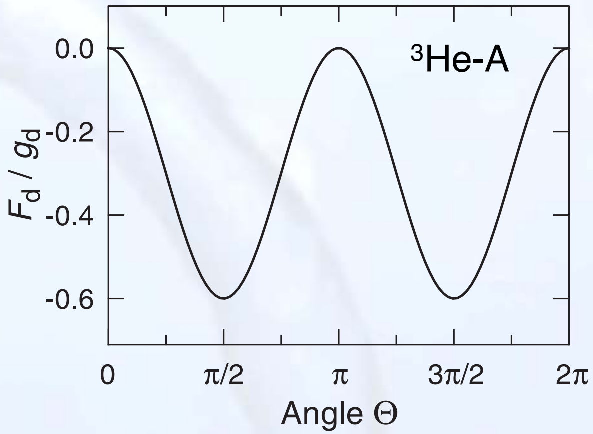
^{3}He-A: macroscopic orientation

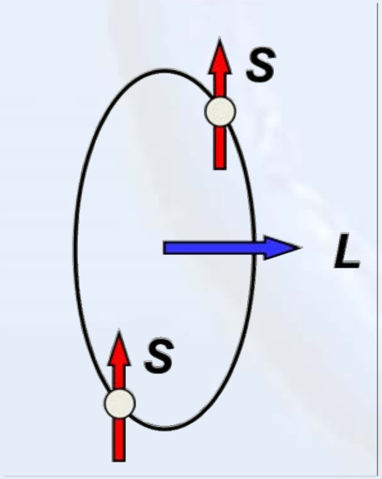
dipole-dipole energy is minimal, if $\boldsymbol{l} \parallel \boldsymbol{d} \;\widehat{=}\; \boldsymbol{l} \perp \boldsymbol{S}$

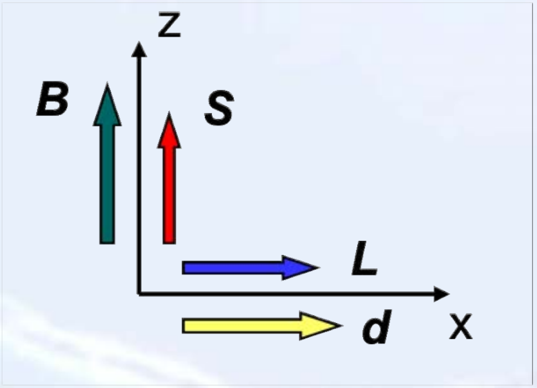
free energy: dipole-dipole interaction

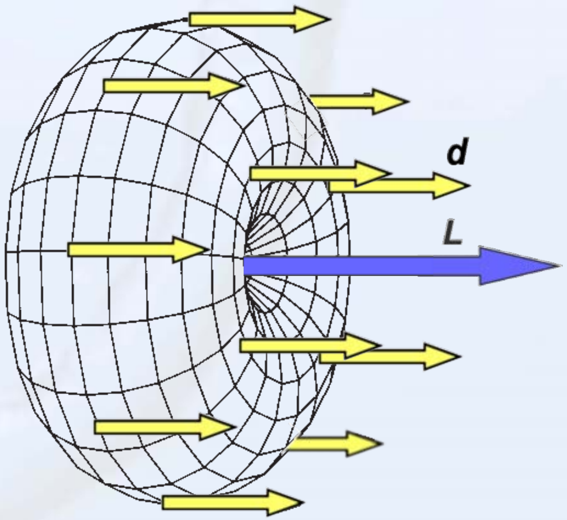
$$F_\mathrm{d} = -\frac{3}{5}\, g_\mathrm{d}(T) \left[1 - (\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{l}})^2\right] = -\frac{3}{5}\, g_\mathrm{d}(T)\, \sin^2 \Theta$$

$$g_\mathrm{d} \approx 10^{-10}(1 - T/T_\mathrm{c})\,\mathrm{J\,cm^{-3}} \propto \varrho_\mathrm{s}$$

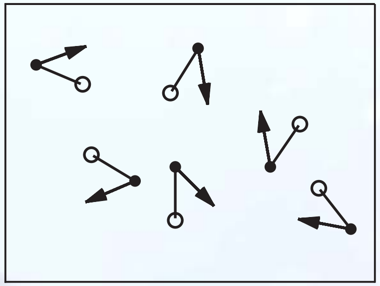








4.4 Orderparameter Orientation — Textures

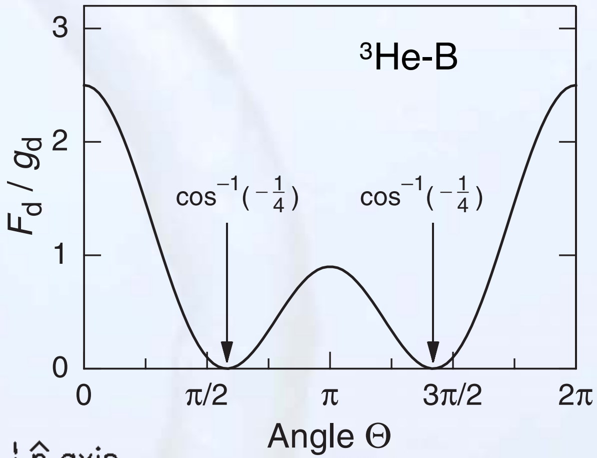


^{3}He-B: isotropic regarding spin and orbital momentum

⟶ no macroscopic orientation

but: dipole-dipole interaction leads to a relative orientation of $\boldsymbol{l}$, $\boldsymbol{d}$ locally for each point on the Fermi surface described by a rotation about $\hat{\boldsymbol{n}}$ described by $\hat{\boldsymbol{d}} = \overleftrightarrow{\mathbf{R}}\,(\hat{\boldsymbol{n}}, \Theta)\hat{\boldsymbol{k}}$

⟶ leads to weak texture effects

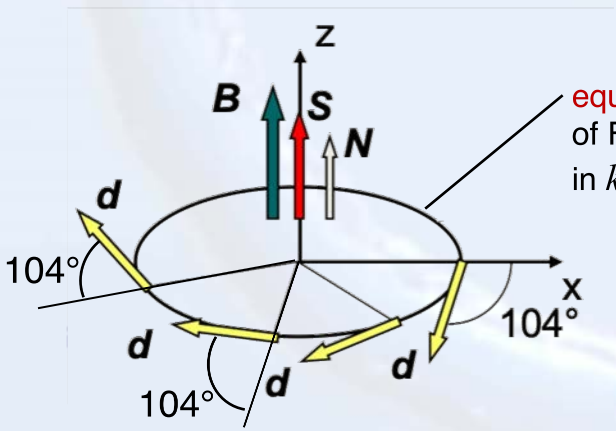


free energy: dipole-dipole interaction

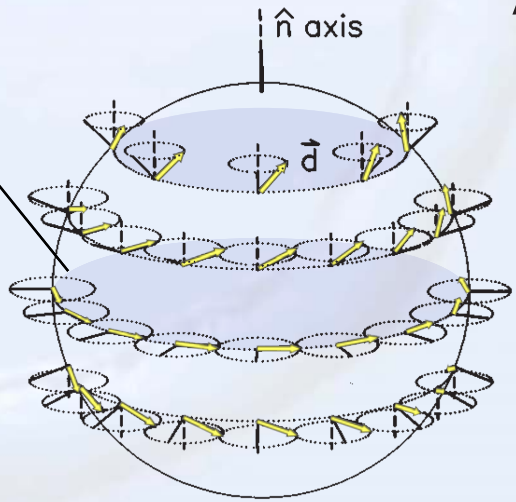
$$F_{\mathrm{d}} = \frac{8}{5}\, g_{\mathrm{d}}(T) \left(\cos\Theta + \frac{1}{4}\right)^2$$

Leggett angle

dipole-dipole energy is minimal, if $\Theta = \arccos\left(-1/4\right) \approx 104°$



equatorial plane of Fermi sphere in k-space



4.4 Orderparameter Orientation — Textures

b) external influences on the orientation of $\boldsymbol{l}$, $\boldsymbol{d}$

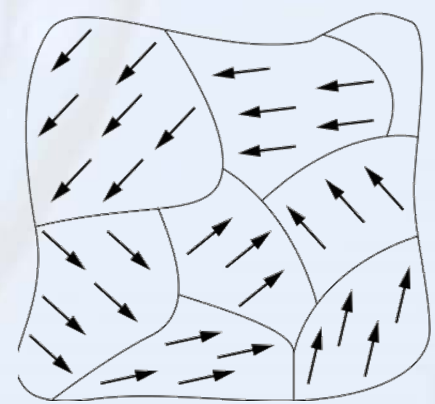
⟶ changes of the texture

textures in ^{3}He-A

preferred alignment and relative strength of different influences

	Preferred Alignment	$\Delta E/(1 - T/T_c)$ $(\mathrm{J\,m^{-3}})$
magnetic dipole interaction	$\boldsymbol{d} \parallel \boldsymbol{l}$	$-6 \times 10^{-5}\,(\widehat{\boldsymbol{d}} \cdot \widehat{\boldsymbol{l}})^2$
electric field	$\boldsymbol{l} \perp \boldsymbol{\mathcal{E}}$	$2 \times 10^{-7}\,(\widehat{\boldsymbol{l}} \cdot \boldsymbol{\mathcal{E}})^2$
magnetic field	$\boldsymbol{d} \perp \boldsymbol{B}$	$5\,(\widehat{\boldsymbol{d}} \cdot \boldsymbol{B})^2$
mass flow	$\boldsymbol{l} \parallel \boldsymbol{v}_s$	$-10\,(\widehat{\boldsymbol{l}} \cdot \boldsymbol{v}_s)^2$
wall alignment	$\boldsymbol{l} \parallel \boldsymbol{N}$	$-30\,(\widehat{\boldsymbol{l}} \cdot \widehat{\boldsymbol{N}})^2$

- most important are walls $\boldsymbol{l} \parallel \boldsymbol{N}$ and mass flow $\boldsymbol{l} \parallel \boldsymbol{v}_s$
- strength compared to intrinsic alignment:
  $\mathcal{E} = 17\,\mathrm{V\,m^{-1}}$, $B = 3.3\,\mathrm{mT}$ and $v_s = 2.4\,\mathrm{mm\,s^{-1}}$
- for in homogenies textures ⟶ gradient energy must be considered



4.4 Orderparameter Orientation — Textures

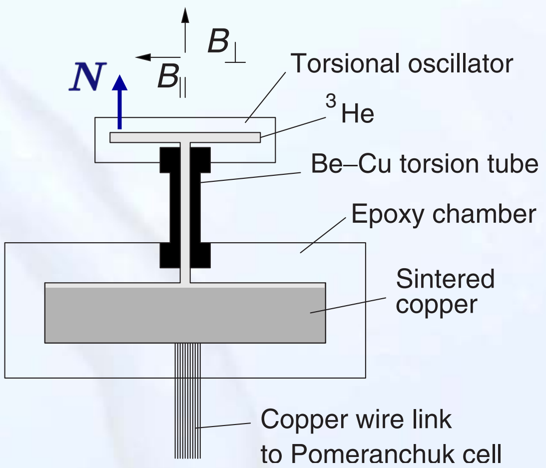
Example for influence of wall and magnetic field

Determination of ϱ_s/ϱ with with a disc like resonator

- ϱ_n is dragged with resonator because of η_n
- mass of ϱ_n adds to moment of inertia
- resonance frequency depends on ϱ_n/ϱ

$\longrightarrow \varrho_s/\varrho$

Andronikasvili-like experiment



(i) $\boldsymbol{B}$ parallel to wall $\boldsymbol{B}_{||} \perp \boldsymbol{N}$

$$\left.\begin{array}{l} \boldsymbol{l} \parallel \boldsymbol{N} \\ \boldsymbol{S} \parallel \boldsymbol{B}_{||} \curvearrowright \boldsymbol{d} \perp \boldsymbol{B}_{||} \end{array}\right\} \boldsymbol{d} \parallel \boldsymbol{l}$$

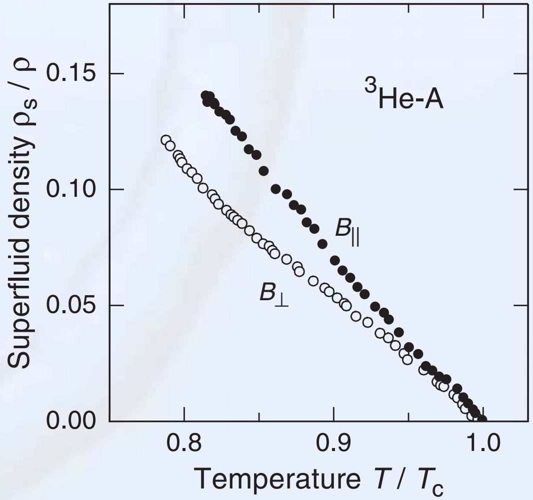
optimal even without external field

(ii) $\boldsymbol{B}$ perpendicular to wall $\boldsymbol{B}_{\perp} \parallel \boldsymbol{N}$

$$\left.\begin{array}{l} \boldsymbol{l} \parallel \boldsymbol{N} \\ \boldsymbol{S} \parallel \boldsymbol{B}_{\perp} \curvearrowright \boldsymbol{d} \perp \boldsymbol{B}_{\perp} \end{array}\right\} \boldsymbol{d} \perp \boldsymbol{l}$$

not optimal for dipole dipole interaction

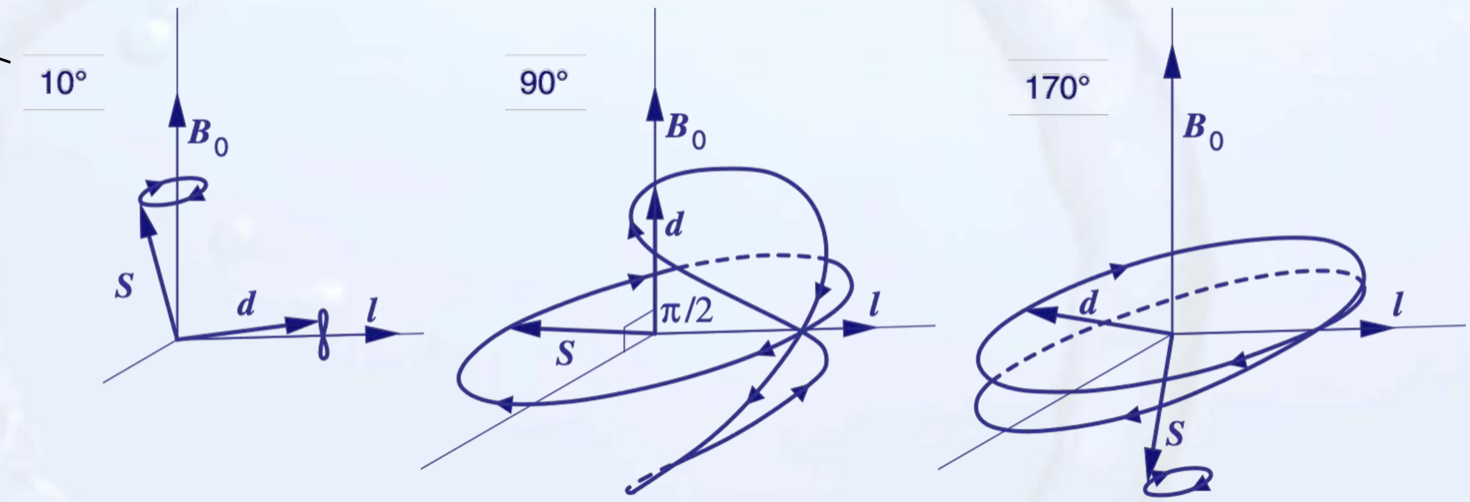
$\longrightarrow \varrho_{s\perp} < \varrho_{s||}$



4.5 Spin Dynamics — NMR Experiments

- static field $\boldsymbol{B}_0$ ⟶ $\omega_\mathrm{L} = \gamma|\boldsymbol{B}_0|$ Lamor frequency
- rf pulse ⟶ tipping of the magnetization $\langle \boldsymbol{S} \rangle$
- ^{3}He: coupling of $\boldsymbol{S}, \boldsymbol{d}$ $(\boldsymbol{d} \cdot \boldsymbol{S} = 0)$ ⟶ additional restoring force

angle between $\boldsymbol{B}_0$ and $\boldsymbol{S}$ (tipping angle)



- without external influences: state of minimal dipole-dipole energy $\boldsymbol{d} \parallel \boldsymbol{l}$
- any deviation from $\boldsymbol{d} \parallel \boldsymbol{l}$ costs energy proportional to $\sin^2(\boldsymbol{d}, \boldsymbol{l})$

⟶ resonance frequency increases

4.5 Spin Dynamics — NMR Experiments

Leggett equations:

$$\boldsymbol{R}_{\rm d}(T) = \frac{6}{5}\, g_{\rm d}(T)\, (\boldsymbol{d}\times\boldsymbol{l})\,(\boldsymbol{d}\cdot\boldsymbol{l})$$ additional restoring force

$$\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \gamma \boldsymbol{S}\times\boldsymbol{B}_0 + \boldsymbol{R}_{\rm d}$$

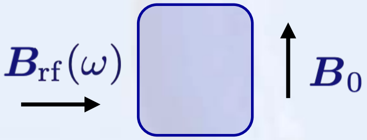
$$\frac{\mathrm{d}\boldsymbol{d}}{\mathrm{d}t} = \boldsymbol{d}\times\gamma\boldsymbol{B}_{\rm eff} = \boldsymbol{d}\times\gamma\left(\boldsymbol{B} - \frac{\mu_0\gamma\boldsymbol{S}}{\chi_{\rm N}}\right)$$

comment:

”Bloch equations” for superfluid ^{3}He-A

all predictions from these equations are precisely observed

transversal resonance

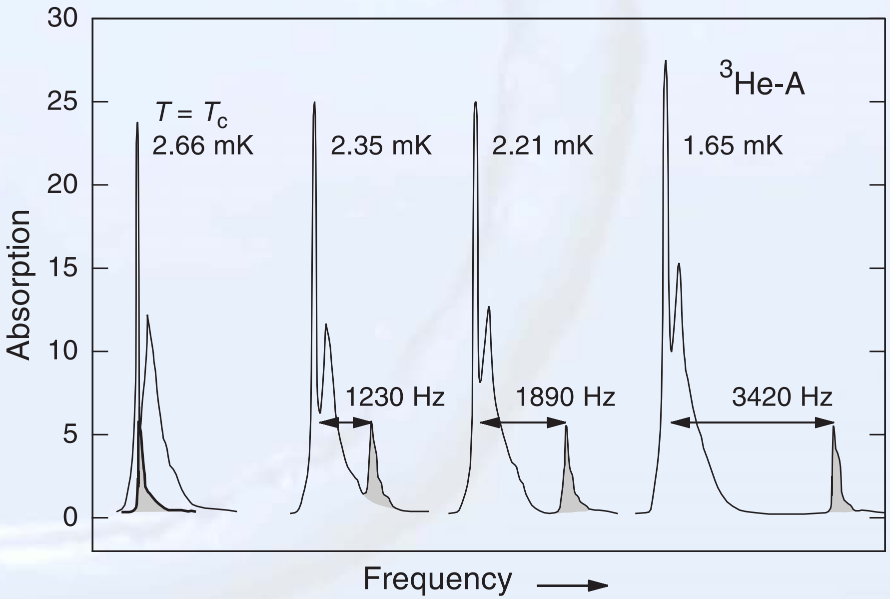


small (tipping) angle solution:

$$\omega_{\rm t}^2 = (\gamma B_0)^2 + \frac{\gamma^2\mu_0\langle H_{\rm d}\rangle}{\chi_{\rm N}} = \omega_{\rm L}^2 + \Omega_{\rm A}^2(T)$$

spatial mean of dipole-dipole coupling

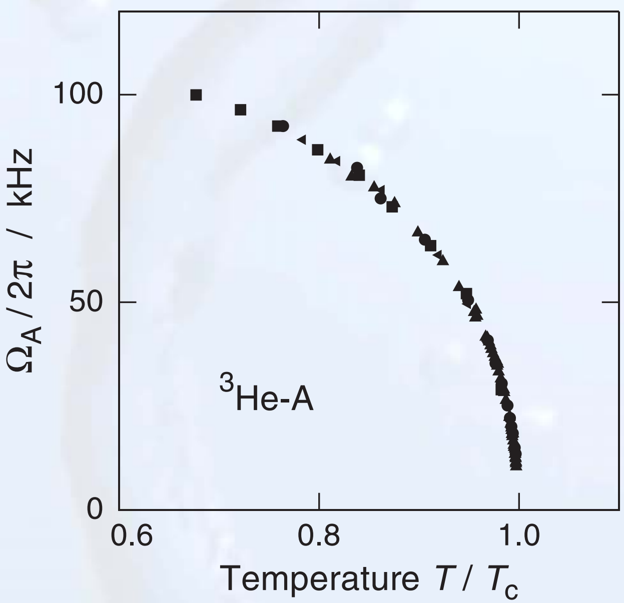
original observations by Doug Osheroff



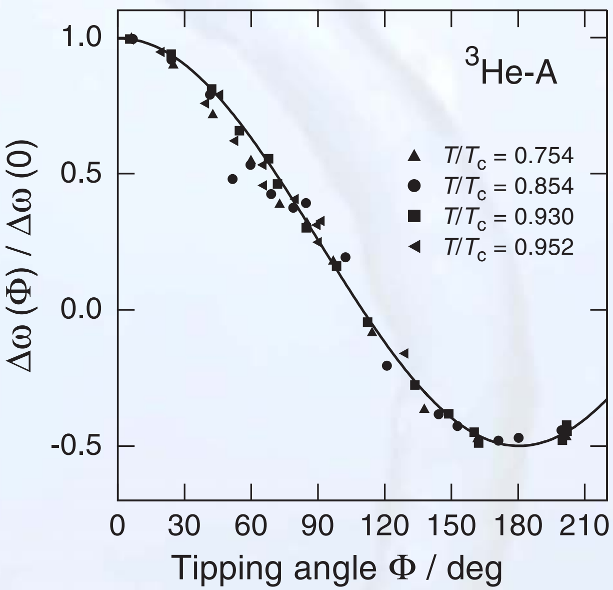
4.5 Spin Dynamics — NMR Experiments

extended NMR experiments with transvers geometry

small angle measurement



tipping angle dependence



- resonance frequency increases proportional to ϱ_s
- temperature dependence of order parameter
- line shows prediction from Leggett equations
- excellent agreement with theory

4.5 Spin Dynamics — NMR Experiments

Longitudinal resonance

$\boldsymbol{B}_{\rm rf}(\omega) \uparrow$ $\uparrow \boldsymbol{B}_0$

modulation of static field $B_0 \longrightarrow B_z = B_0 + B_{\rm rf}(\omega)$

$\downarrow$

oscillation of $\boldsymbol{d} \longrightarrow$ resorting force

in ordinary liquids $\boldsymbol{R}_{\rm d} = 0$

because of $(\boldsymbol{S} \times \boldsymbol{B})_z = 0 \longrightarrow {\rm d}S_z/{\rm d}t = R_{\rm d,z}$ first Leggett equation

^{3}He-A: $R_{\rm d} \neq 0$ $\quad |\uparrow\uparrow\rangle \longleftrightarrow |\downarrow\downarrow\rangle$

$$\Delta S_z = \frac{\chi}{\mu_0 \gamma} \Delta B_0 \, (1 - \cos \Omega_{\rm A} t)$$

^{3}He-B:

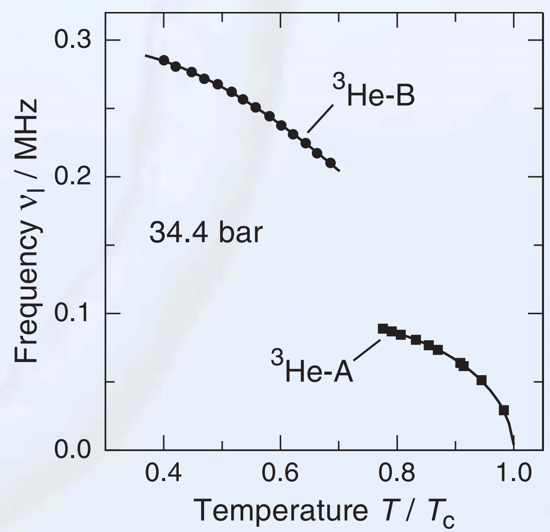
$$\Omega_{\rm B}^2(T) = \Omega_{\rm A}^2(T) \, \frac{5}{2} \frac{\chi_{\rm B}}{\chi_{\rm A}}$$

reduced, since 1/3 in $|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$

$\chi_{\rm A} \;\widehat{=}\;$ ^{3}He-N

^{3}He-A$_1$:

no effect, since only one spin configuration $|\uparrow\uparrow\rangle$



4.6 Macroscopic Quantum Effects

macroscopic wave function

$$\Psi_{\alpha\beta}(\boldsymbol{r}) = \mathcal{A}_{\alpha\beta}(\boldsymbol{r})\, \mathrm{e}^{\mathrm{i}\varphi(\boldsymbol{r})}$$

3 x 3 matrix

} 18 degrees of freedom

i) quantization of circulation

→ ^{4}He circulation is quantized

→ ^{3}He behavior is more complicated

^{3}He-A: → circulation is only irrotational under ideal conditions, which means without external influences

→ if $\mathrm{curl}\,\boldsymbol{v}_\mathrm{s} \neq 0$ → $\boldsymbol{v}_\mathrm{s} \neq \frac{\hbar}{2m_3}\nabla\varphi$

► in general $\mathrm{curl}\,\boldsymbol{v}_\mathrm{s} = \frac{\hbar}{2m_3 r}\,\widehat{\boldsymbol{l}} \cdot \left(\frac{\partial\widehat{\boldsymbol{l}}}{\partial\phi} \times \frac{\partial\widehat{\boldsymbol{l}}}{\partial r}\right)$

► phase can be adjusted by modification of $\boldsymbol{l}$

→ structure of vortices depend on $\boldsymbol{l}(\boldsymbol{r})$

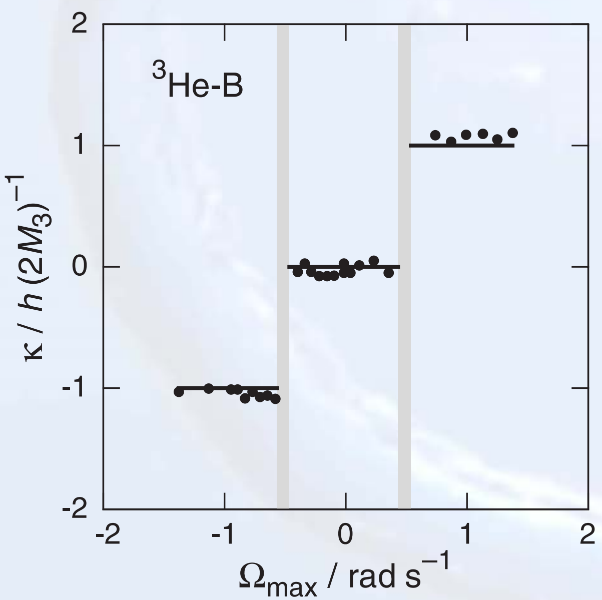
4.6 Macroscopic Quantum Effects

^{3}He-B: → circulation is quantized

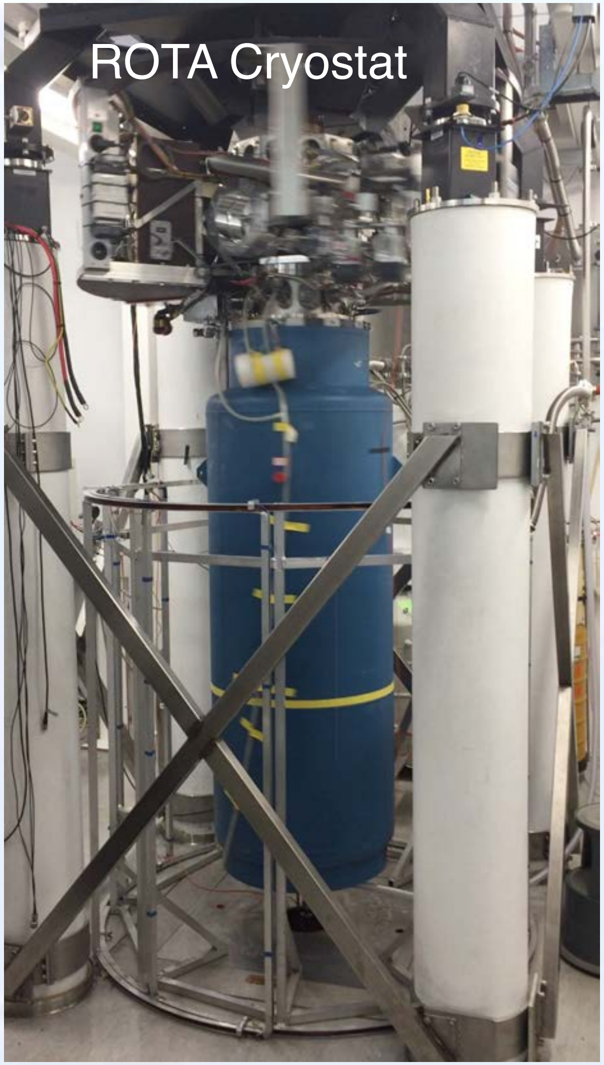
→ $$\boldsymbol{v}_\mathrm{s} = \frac{\hbar}{2m_3}\nabla\varphi$$

→ $$\boldsymbol{\kappa}_3 = \frac{h}{2m_3}$$

- Vinen-type experiment
- 1 rad/s = 0.16 revolutions /s

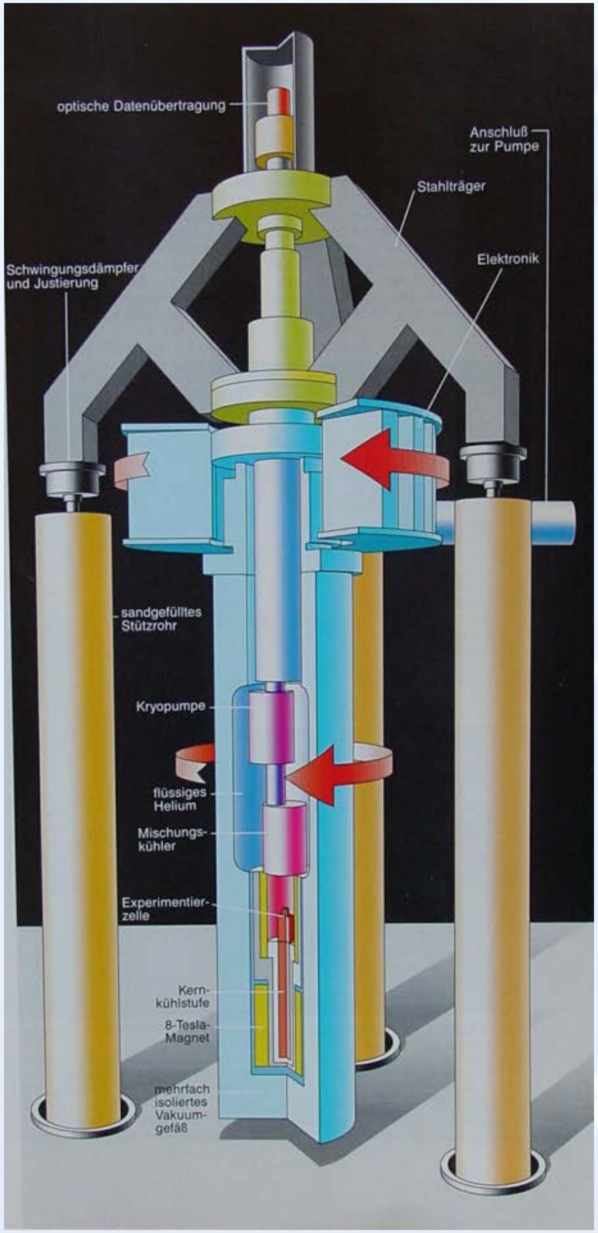


experimental problem:
rotation at very low temperatures



up to 3 revolutions / s

4.6 Macroscopic Quantum Effects



4.6 Macroscopic Quantum Effects

Quantized Vortices (structure much more complicated as in He-II)

^{3}He-A:

coherence length

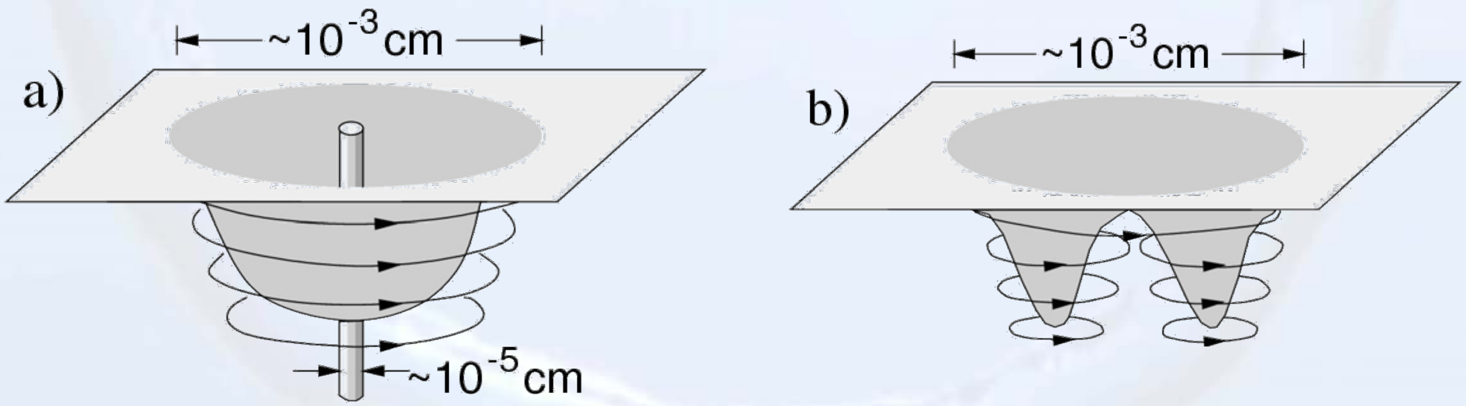
a) with uniform texture and orbital field $\boldsymbol{l}$ ⟶ vortices with normal-fluid hard core $\xi_0 \approx 100\,\mathrm{nm}$

extended soft region $\xi_\mathrm{d} \approx 6\,\mu\mathrm{m}$. $n = 1$

dipole healing length it describes over which distance $\boldsymbol{d} \parallel \boldsymbol{l}$ recovers

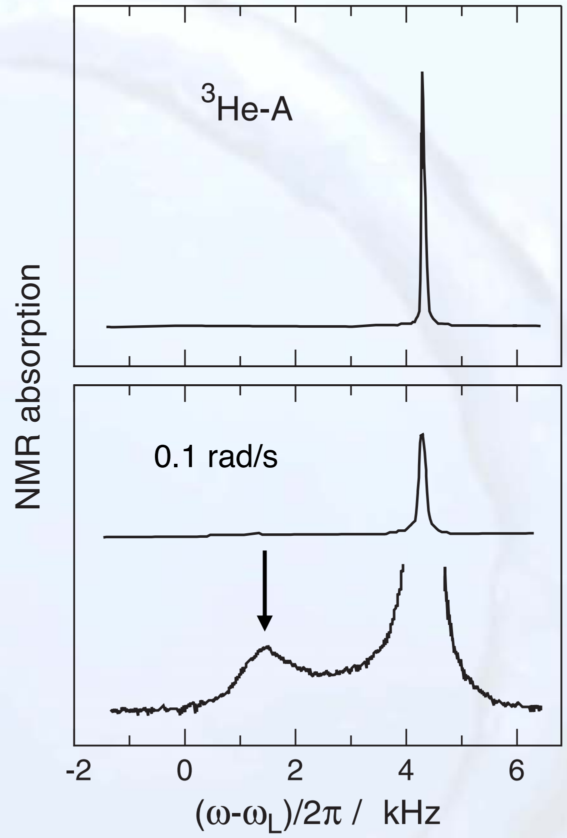
b) if $\boldsymbol{l}$ can adjust freely one finds continuous vortices with $n = 2$ without singularity (no hard core)

continuous velocity field



4.6 Macroscopic Quantum Effects

Investigation of vortices in ^{3}He-A with NMR



cryostat at rest

cryostat rotating

frequency shift because of localized spin waves in core!

container diameter 2.5 mm

4.6 Macroscopic Quantum Effects

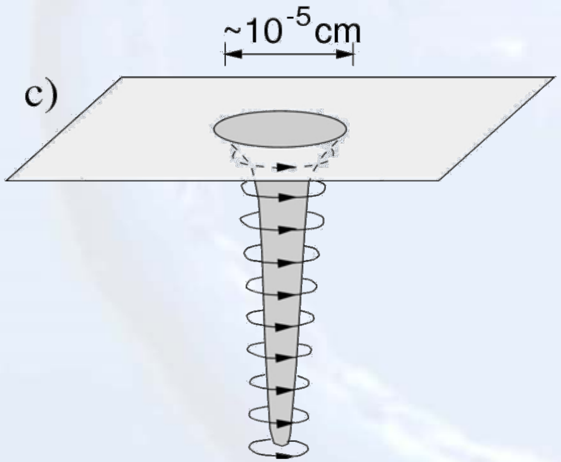
^{3}He-B: only vortices with hard core $\xi_0 \approx 10 \ldots\ 100\,\mathrm{nm}$

depends on pressure

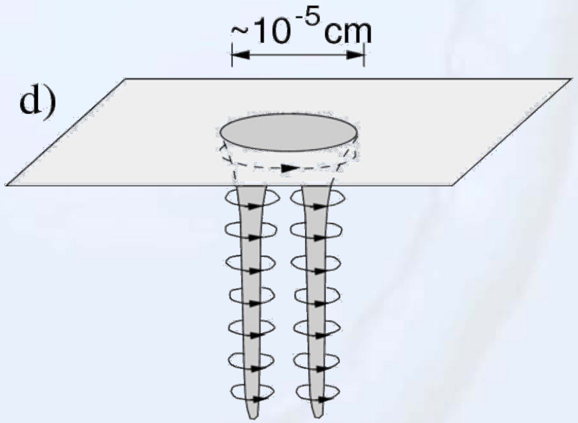
c) single vortices with A phase in core

d) double vortices with two half-quantum of circulation and normal-fluid core

c)



d)

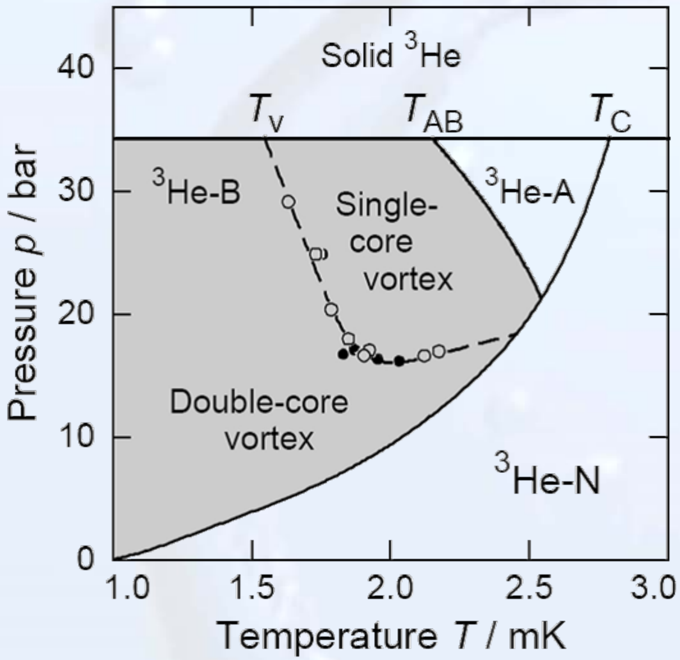


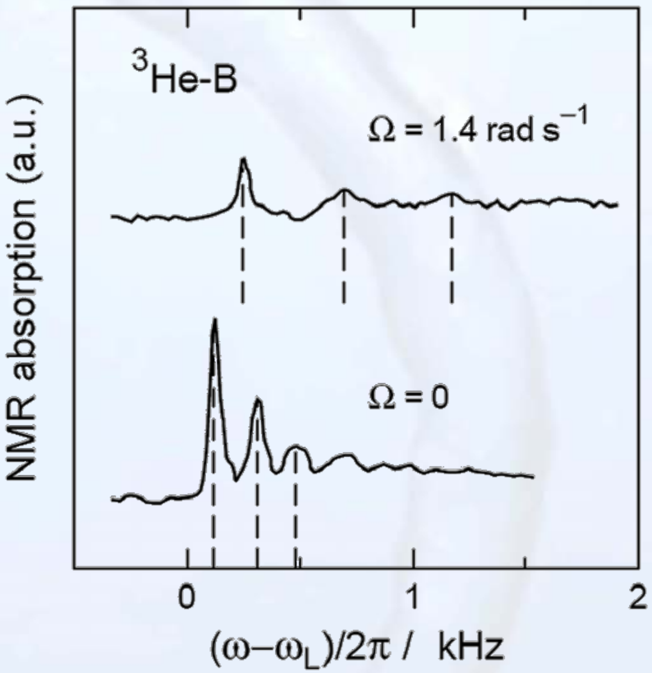
these vortices exist in distinct parts of the phase diagram

4.6 Macroscopic Quantum Effects

^{3}He-B: phase diagram under rotation

spin waves resonances (collision-less)





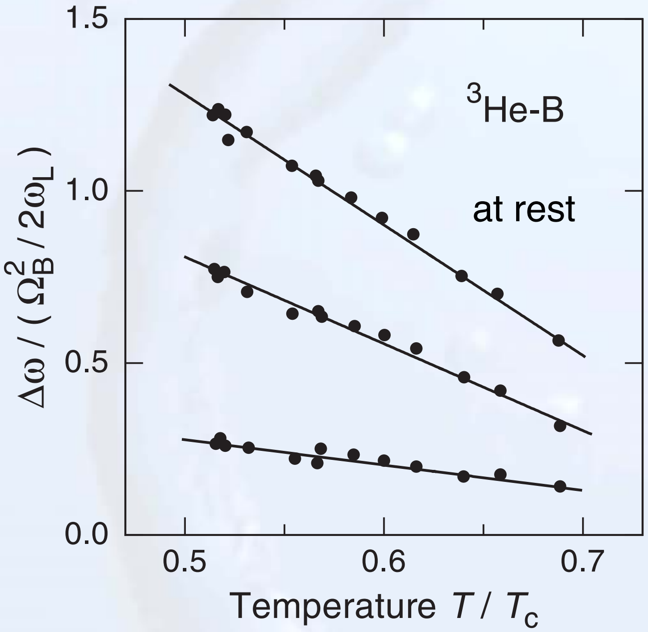
first order phase transition

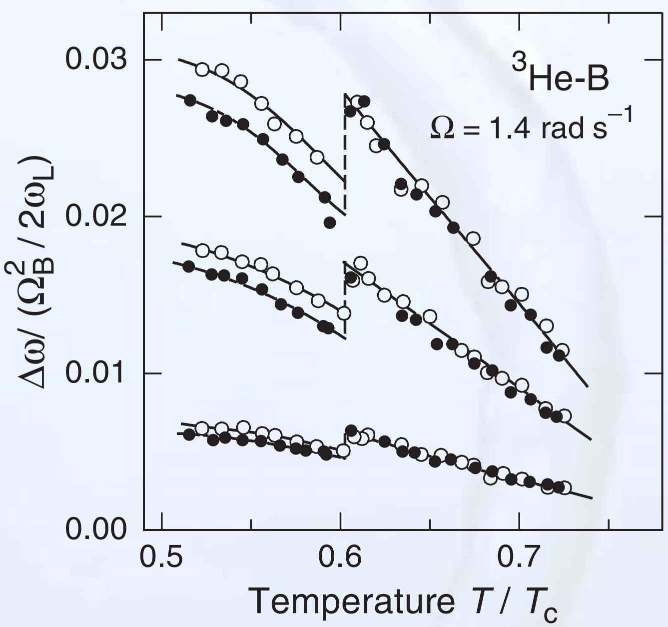
under rotation ⟶ larger spacing
because additional term in free energy

4.6 Macroscopic Quantum Effects

^{3}He-B: phase diagram under rotation

spin waves resonances (collision-less)





hysteresis is observed

⟶ 1st order transition