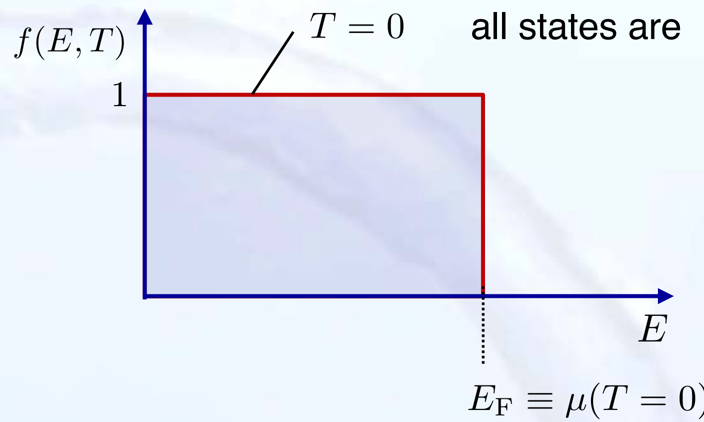
3.1 Ideal Fermi-Gas

Fermi-Dirac distribution

$$f(E,T) = \frac{1}{\mathrm{e}^{(E-\mu)/k_\mathrm{B}T} + 1}$$

chemical potential $\mu = f(E,T) = \frac{1}{2}$



all states are $E < \mu$ occupied

Fermi Energy

$$n = \frac{N}{V} = \int_0^\infty D(k) f(E,T)\,\mathrm{d}k = \int_0^\infty D(E) f(E,T=0)\,\mathrm{d}E$$

Fermi Temperature $E_\mathrm{F} = k_\mathrm{B} T_\mathrm{F}$

$$T_\mathrm{F} = \frac{\hbar^2}{2 m k_\mathrm{B}} \left(3\pi^2 n\right)^{2/3}$$

^{3}He: $T_\mathrm{F} \approx 4.9\,\mathrm{K}$

Internal energy

approximate solution for $T \ll E_\mathrm{F}/k_\mathrm{B}$

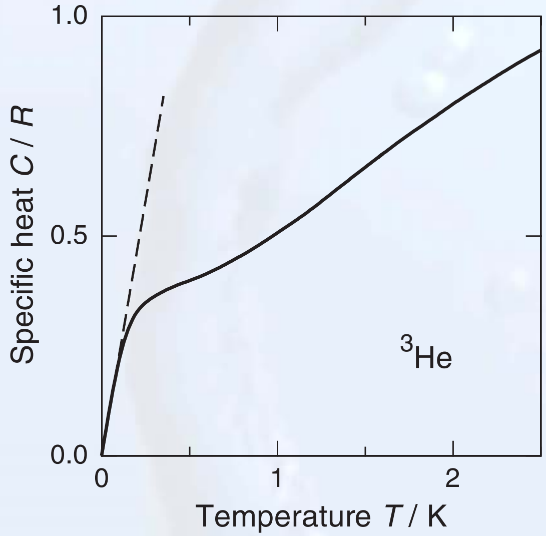
$$u = \frac{U}{V} = \int_0^\infty D(E)\, f(E,T)\, E\,\mathrm{d}E \quad \Rightarrow \quad u(T) = \underbrace{\frac{3}{5}\, n\, k_\mathrm{B} T_\mathrm{F}}_{\text{const.}} + \frac{\pi^2}{4} \frac{n}{E_\mathrm{F}} (k_\mathrm{B} T)^2$$

3.1 Ideal Fermi-Gas

a) Specific heat

$T \gg T_\mathrm{F}$ $\quad C_V = \frac{3}{2}R$

$T \ll T_\mathrm{F}$ $\quad C_V = \left(\frac{\partial u}{\partial T}\right)_V = \frac{\pi^2}{2}\frac{n}{E_\mathrm{F}}k_\mathrm{B}{}^2 T = \gamma T \quad \curvearrowright \quad C_V = \frac{\pi^2 R}{2}\left(\frac{T}{T_\mathrm{F}}\right)$



- $T > 1\,\mathrm{K}$ dense classical gas
- linear region for $T < 50\,\mathrm{mK}$
- expected at $T \approx T_\mathrm{F}/10 \approx 500\,\mathrm{mK}$

reason:

- effective mass $m \rightarrow m^*$
- nuclear spin fluctuations $\uparrow\downarrow \longleftrightarrow \uparrow\uparrow$

Fermi statistic (↑↓) — ferromagnetic exchange interaction (↑↑)

3.1 Ideal Fermi-Gas

large distances (low density) ⟶ Fermi statistic dominates

short distances (high density) ⟶ strong ferromagnetic exchange

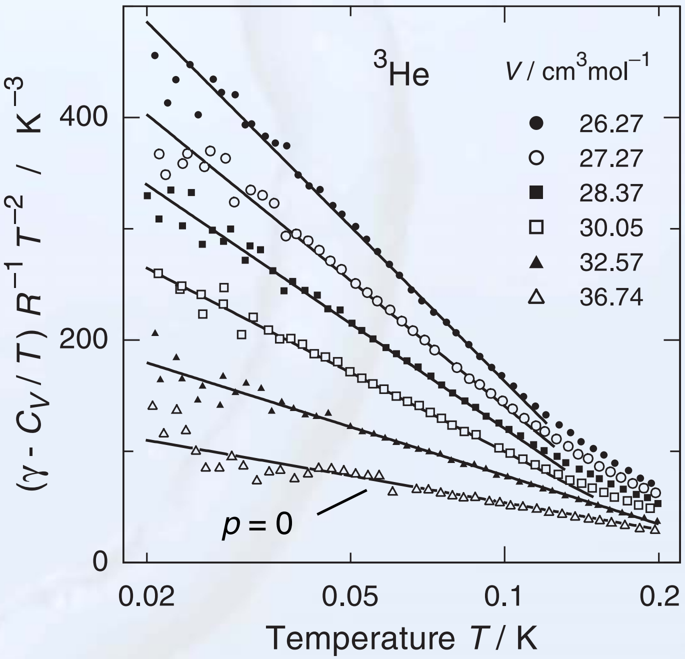
Paramagnon model (phenomenological description)

idea: fluctuating ferromagnetic regions ⟶ size and concentration depend on *T*

$$T < 0.2\,\mathrm{K}$$

$$C_V = \gamma T + \Gamma T^3 \ln\left(\frac{T}{\Theta_c}\right)$$

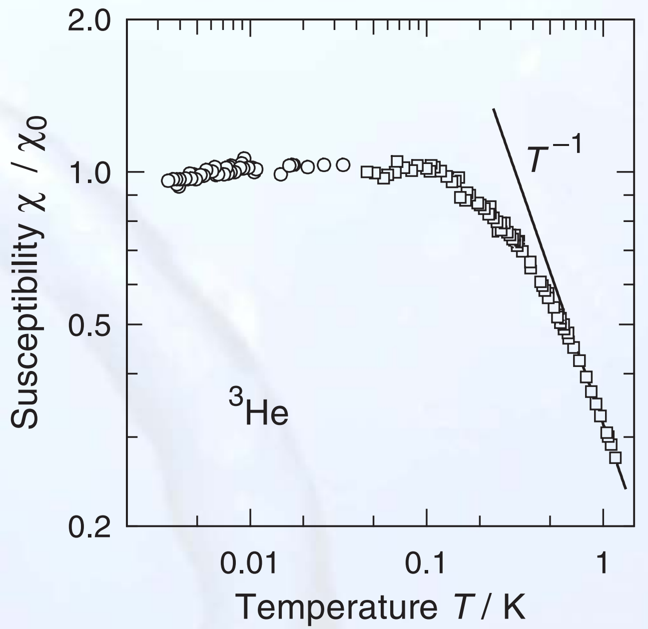
- plotted as $(\gamma - C_V/T)/(RT^2)$ vs $\log T$
- different pressure ⟶ different density
- $\log(T/\Theta_c)$ is visible
- slope proportional to spin correlation contribution



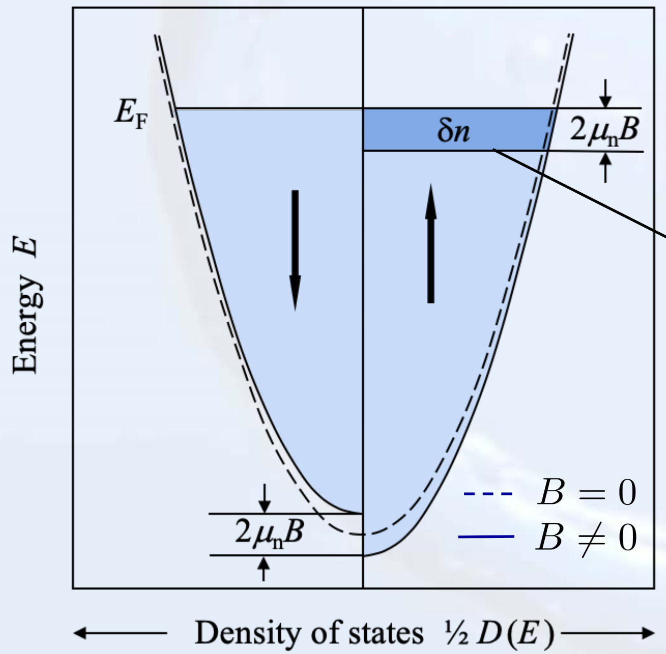
3.1 Ideal Fermi-Gas

b) Magnetic nuclear spin susceptibility $\chi = \frac{M}{H}$

- high temperatures: $\chi \propto \frac{1}{T}$
- low temperatures: $\chi = I(I+1)\,\mu_0\,\mu_n^2\,g_n^2\,\frac{2}{3}\frac{n}{E_F} = \beta^2\,D(E_F)$



Low temperatures: Pauli susceptibility



$$M = \delta n \mu_n = D(E_F)\mu_n B$$
$$= \frac{3 n \mu_n^2 B}{2 k_B}$$

3.1 Ideal Fermi-Gas

c) Transport properties

Boltzmann equation ⟶ kinetic gas theory

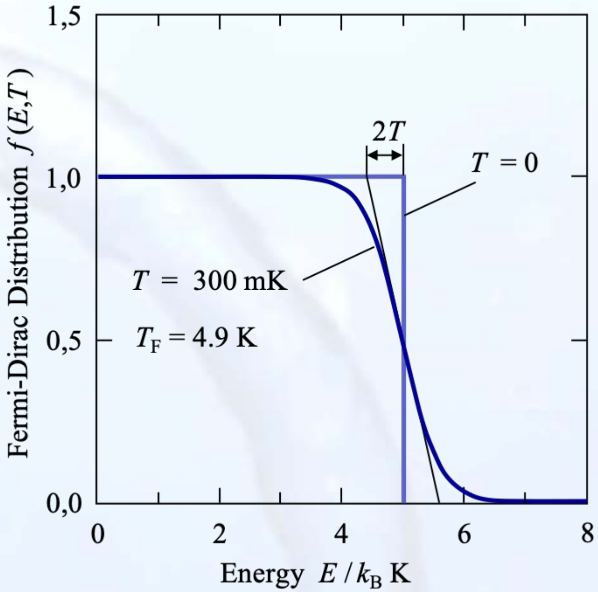
(i) viscosity $\eta = \frac{1}{3}\,\varrho\, v\, \ell = \frac{1}{3}\,\varrho \tau v_F^2$

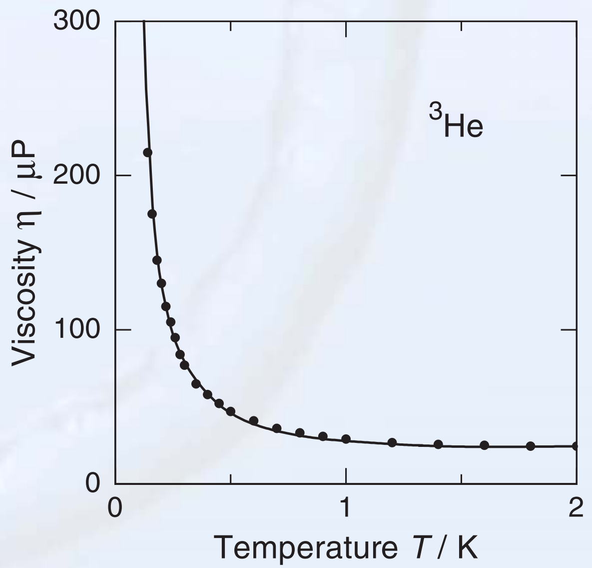
$\tau = v_F / \ell$

$v_F = (\hbar/m)(3\pi^2 n)^{1/3}$

$$\tau^{-1} \propto \left(\frac{k_B T}{E_F}\right)^2 \quad \curvearrowright \quad \tau \propto \left(\frac{T_F}{T}\right)^2 \propto \frac{1}{T^2}$$

- high temperatures: $\eta = 25\,\mu\mathrm{P} = \mathrm{const}$
- low temperatures: $\eta^{-1} \propto T^2$
- 2 mK: $\eta = 0.2\,\mathrm{P}$ like honey!



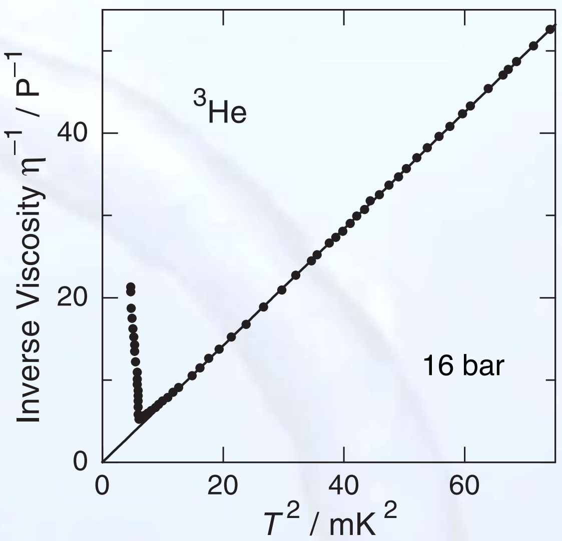


3.1 Ideal Fermi-Gas

viscosity at ultra-low temperatures

→ $\eta^{-1} \propto T^2$ as expected

→ phase transition occurring at ~ 2 mK

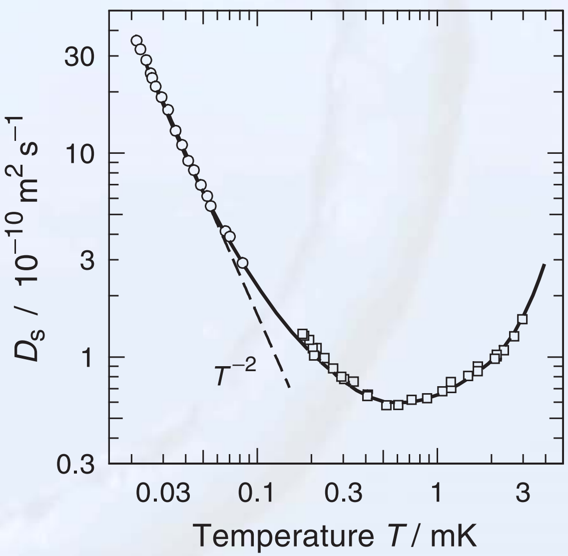


(ii) Self-diffusion coefficient

diffusion of nuclear spins

$$D_s = \frac{1}{3} v\,\ell \quad \longrightarrow \quad D_s = \frac{1}{3}\tau v_F^2$$

- low temperatures: $D_s \propto \tau \propto \frac{1}{T^2}$
- high temperatures: dense classical gas $D_s \propto T$

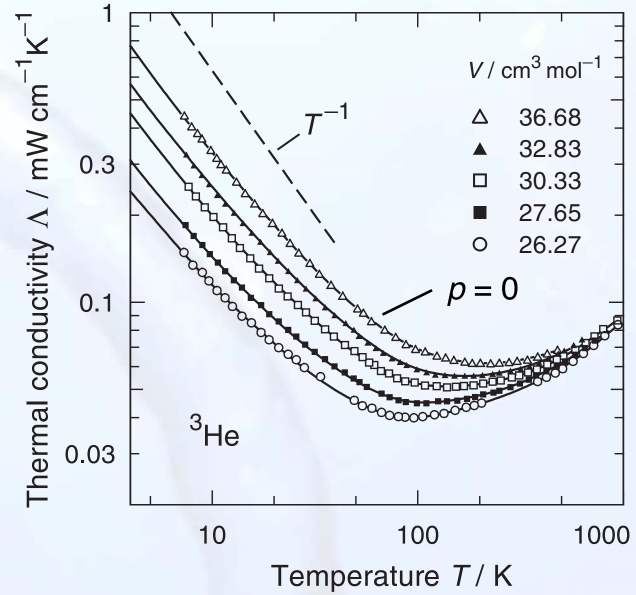


3.1 Ideal Fermi-Gas

(iii) Thermal conductivity

$$\Lambda = \frac{1}{3} C_V \, v \, \ell \quad \longrightarrow \quad \Lambda = \frac{1}{3} C_V \, \tau \, v_F^2$$

- low temperatures: $C \propto T$, $\tau \propto T^{-2}$ $\Big\}$ $\Lambda \propto T^{-1}$ and paramagnon contributions
- high temperatures: dense classical gas
- very small absolute value: $\Lambda \approx 10^{-4}\,\mathrm{W\,cm^{-1}K^{-1}}$ at 200 mK



Is ^{3}He a Fermi gas?

	^{3}He	Fermi Gas	Ratio
$C_V/\gamma T$	2.78	1.00	2.78
$v = v_F/\sqrt{3}$ $(\mathrm{m\,s^{-1}})$	188	95	1.92
χ/β^2 $(\mathrm{J\,m^3})^{-1}$	3.3×10^{51}	3.6×10^{50}	9.1

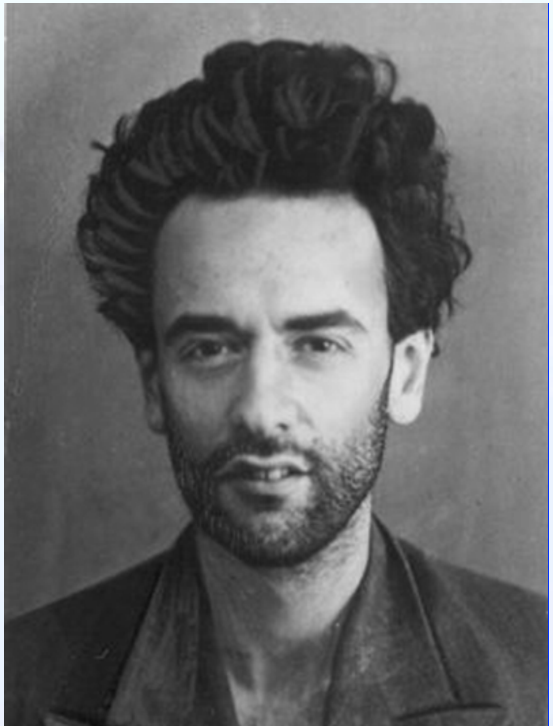
➡ deviations are not too big, but still significant and in addition differently large for different properties

3.2 The Landau Fermi-Liquid Theory

free Fermi gas ⟶ strongly interacting ^{3}He atoms

↓

collective excitations $\widehat{=}$ quasi particles


Lev Davidovic Landau 1938

Landau theory of Fermi liquids 1956-1958

⟶ prediction of zero sound and collision-less spin waves

Basic idea

- interaction does change the energy of particles, but not momentum!
- plausible since momentum states are given by boundary conditions

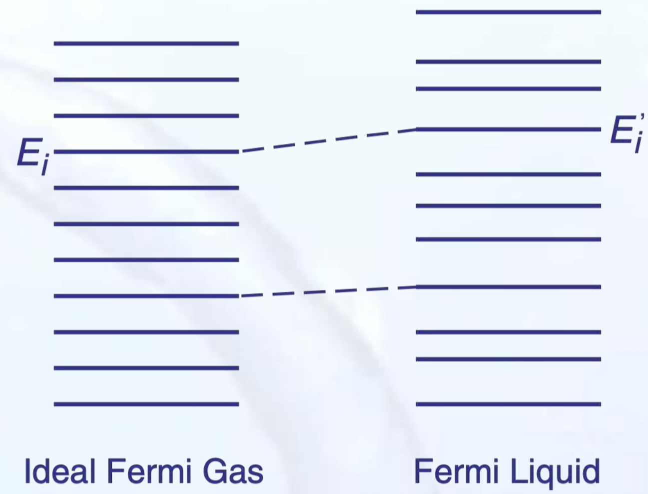
} for each state in the Fermi gas there is a corresponding state in the liquid, but with modified energy

3.2 The Landau Fermi-Liquid Theory

Quasi-particle concept

important: total energy is not given by the sum of all individual states (isolated atoms)

$$U \neq \sum_i f_i E_i$$



Landau's Gedankenexperiment

consider that the interaction is switched on slowly

➡ number of states does not change

$$n = 2\varrho_k \int f \, \mathrm{d}^3k = \int D(k)\, f \, \mathrm{d}k \longrightarrow$$

number of quasi particles per volume analog to Fermi gas

$2\varrho_k$ ↑ 2 spin states

d^3k → $4\pi k^2 \mathrm{d}k$

➡ energy of one quasi particle is defined by the change of energy of the complete system when a quasi particle is added:

$$\delta u = \int E \, \delta f \, \mathrm{d}^3k$$

δu: $\dfrac{\delta U}{V}$

δf: small change in occupation when one quasi particle is added