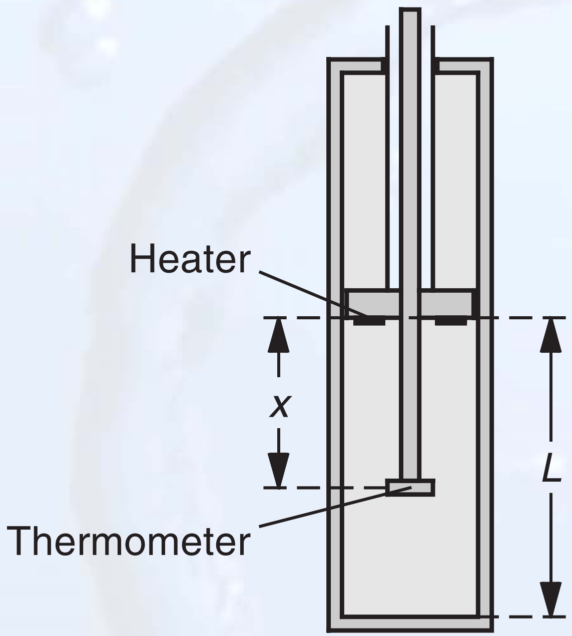
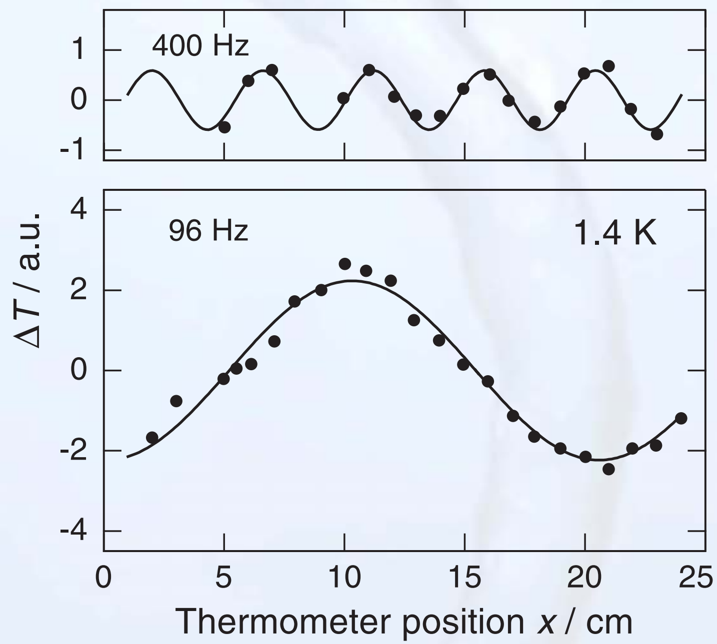
f) Second Sound

Propagation of temperature waves similar to sound waves

suggested by Kapitza
first seen by Peshkov 1944





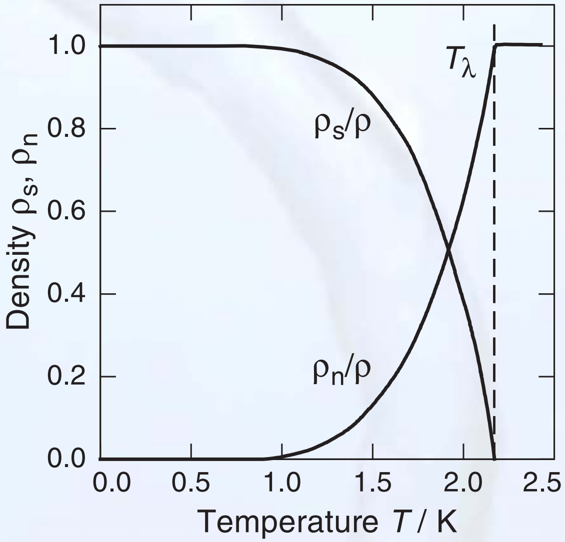
resonance condition $v_2 = 2L\nu/n$

- Seen up to 100 kHz (experimental limit)
- v_2 independent of frequency

2.2 Two-Fluid Model

Basic idea: He-II has to components → normalfluid, superfluid

Tisza 1938
London 1938
Landau 1941, 1947
Feynman 1953



Assumptions and Properties:

$$\varrho = \varrho_n + \varrho_s \qquad (1)$$

$T = T_\lambda$: $\varrho_s = 0$ and $\varrho_n = \varrho$

$T = 0$: $\varrho_s = \varrho$ and $\varrho_n = 0$

	density	viscosity	entropy
normal-fluid component	ϱ_n	$\eta_n = \eta$	$S_n = S$
superfluid component	ϱ_s	$\eta_s = 0$	$S_s = 0$

In addition: no turbulence associated with ϱ_s → rot $\boldsymbol{v}_s = 0$

Two-Fluid Hydrodynamics

density $$\varrho = \varrho_\mathrm{n} + \varrho_\mathrm{s} \quad (1)$$

mass flow $$\boldsymbol{j} = \varrho_\mathrm{n}\boldsymbol{v}_\mathrm{n} + \varrho_\mathrm{s}\boldsymbol{v}_\mathrm{s} \quad (2)$$

continuity eqn.
(mass conservation) $$\frac{\partial \varrho}{\partial t} = -\mathrm{div}\,\boldsymbol{j} \quad (3)$$

He-II is ideal fluid $\eta_\mathrm{n} < 10^{-5}\,\mathrm{P} \sim 0$

➡ Euler eqn. (Newton's 2nd law of motion for continua)

$$\frac{\partial \boldsymbol{j}}{\partial t} + \underbrace{\varrho \boldsymbol{v}\ \mathrm{div}\,\boldsymbol{v}}_{\approx 0} = -\mathrm{grad}\,p$$

for small velocities since quadratic in v
(approximation for linear regime)

$$\frac{\partial \boldsymbol{j}}{\partial t} = -\mathrm{grad}\,p \quad (4)$$

Two-Fluid Hydrodynamics

entropy conservation

motion is reversible since no dissipative processes ➡ He-II is an ideal fluid (in first approximation)

entropy / mass

only ϱ_n contributes

$$\frac{\partial(\varrho S)}{\partial t} = -\mathrm{div}(\underbrace{\varrho S}\boldsymbol{v}_\mathrm{n}) \qquad (5)$$

entropy density

One more equation is needed ➡ an equation of motion for ϱ_s (or ϱ_n)

this is difficult to derive ➡ see R.B. Dingle, Proc. Phys. Soc. A62, 648 (1949) (40 pages)

here: Gedankenexperiment according to Landau

idea: Superfluid component is added at "constant" volume in the system

Two-Fluid Hydrodynamics

Consider change of internal energy

$$\mathrm{d}U = T\,\mathrm{d}S - p\,\mathrm{d}V + G\,\mathrm{d}m$$

$\mathrm{d}S = 0$ reversible

$\mathrm{d}V = 0$ ← V = constant

G: Gibbs free energy per unit mass

$$\mathrm{d}U = G\,\mathrm{d}m$$

Gibbs free energy is potential energy of superfluid component / mass

➡ $-\mathrm{grad}\,G$ is corresponding force

$$\frac{\mathrm{d}\boldsymbol{v}_\mathrm{s}}{\mathrm{d}t} = -\mathrm{grad}\,\mu$$

$\mu = G/m$: Chemical potential

and

$$\mathrm{d}\mu = -S\,\mathrm{d}T + \frac{1}{\varrho}\,\mathrm{d}p$$

➡ $$\frac{\partial \boldsymbol{v}_\mathrm{s}}{\partial t} = S\,\mathrm{grad}\,T - \frac{1}{\varrho}\,\mathrm{grad}\,p \qquad (6)$$

Two-Fluid Hydrodynamics

density	$$\varrho = \varrho_\mathrm{n} + \varrho_\mathrm{s}$$	(1)
mass flow	$$\boldsymbol{j} = \varrho_\mathrm{n}\boldsymbol{v}_\mathrm{n} + \varrho_\mathrm{s}\boldsymbol{v}_\mathrm{s}$$	(2)
mass conservation continuity eqn.	$$\frac{\partial\varrho}{\partial t} = -\mathrm{div}\,\boldsymbol{j}$$	(3)
ideal fluid	$$\frac{\partial\boldsymbol{j}}{\partial t} = -\mathrm{grad}\,p$$	(4)
entropy conservation	$$\frac{\partial(\varrho S)}{\partial t} = -\mathrm{div}(\varrho S\boldsymbol{v}_\mathrm{n})$$	(5)
an equation of motion for superfluid component	$$\frac{\partial\boldsymbol{v}_\mathrm{s}}{\partial t} = S\,\mathrm{grad}\,T - \frac{1}{\varrho}\,\mathrm{grad}\,p$$	(6)

2.3 Properties of He-II described using the two-fluid model

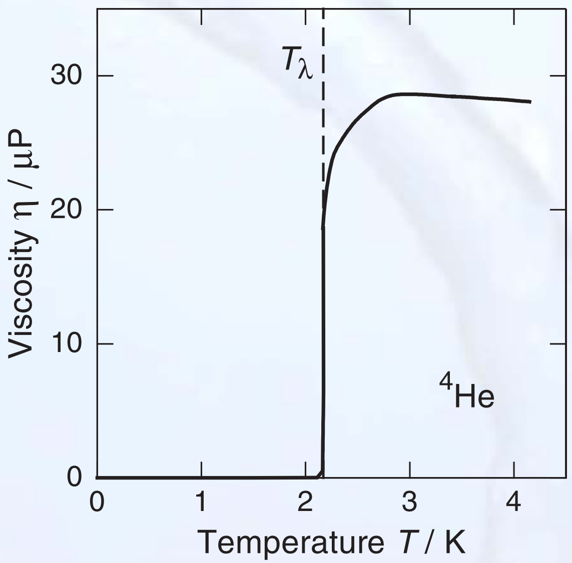
a) Viscosity

(i) capillaries (extremely thin)

Interpretation: $\boldsymbol{v}_{\mathrm{n}} \approx 0$

➡ only superfluid phase is observed

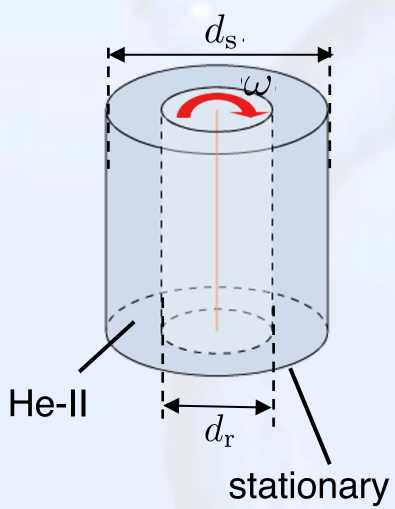
$\eta = \eta_{\mathrm{s}} = 0$



2.3 Properties of He-II described using the two-fluid model

a) Viscosity

(ii) rotary viscosimeter



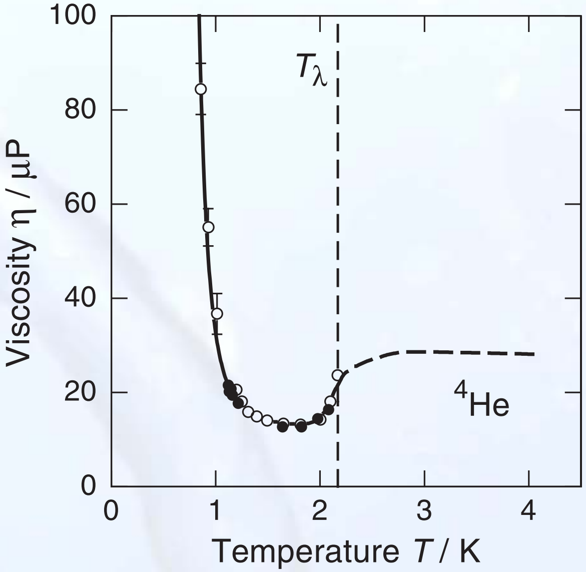
Torque acting on stationary cylinder is measured

$$M_r = \pi\eta\omega d_r^2 d_s^2/(d_s^2 - d_r^2)$$

since $\eta_s = 0$ no torque resulting from ϱ_s

$$\Rightarrow \quad M_r \propto \eta = \eta_n$$

two-fluid model



Temperature dependence

$\eta_n(T)$ at very low temperatures $T <$ 1.8 K ?

$\eta_n \propto \ell_n$ ➡ mean free path increases with decreasing temperature because thermal excitations disappear

Viscosity

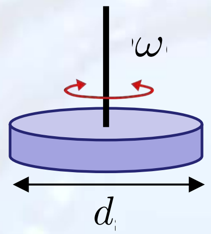
$$\eta = \frac{1}{3}\varrho v \ell$$

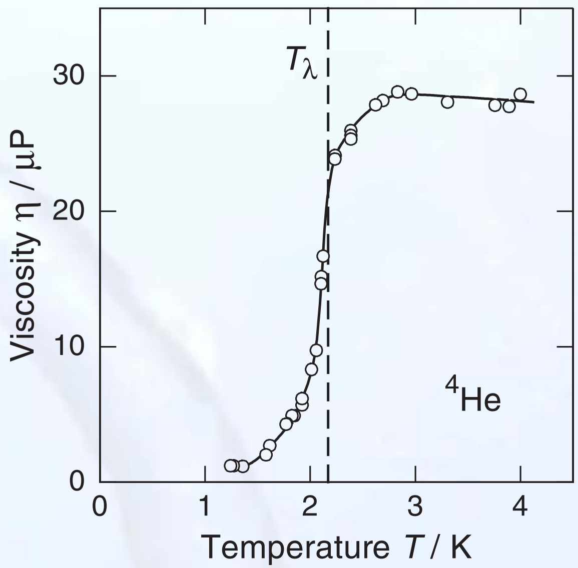
Landau-Chalatnikow Theory

2.3 Properties of He-II described using the two-fluid model

a) Viscosity

(iii) oscillating disc





Torque acting on the disc:

$$M_\mathrm{d} = \pi\sqrt{\varrho\eta}\,\omega^{3/2}r^4\;\Theta(\omega)$$

$$\Theta(\omega) = \Theta_0\cos(\omega t - \pi/4)$$

$$M_\mathrm{d} \propto \sqrt{\varrho\eta}$$

product is important for M_d

$T < T_\lambda$ ➡ $\eta_\mathrm{s} = 0$ ➡ $\eta_\mathrm{n}\varrho_\mathrm{n}$ is measured

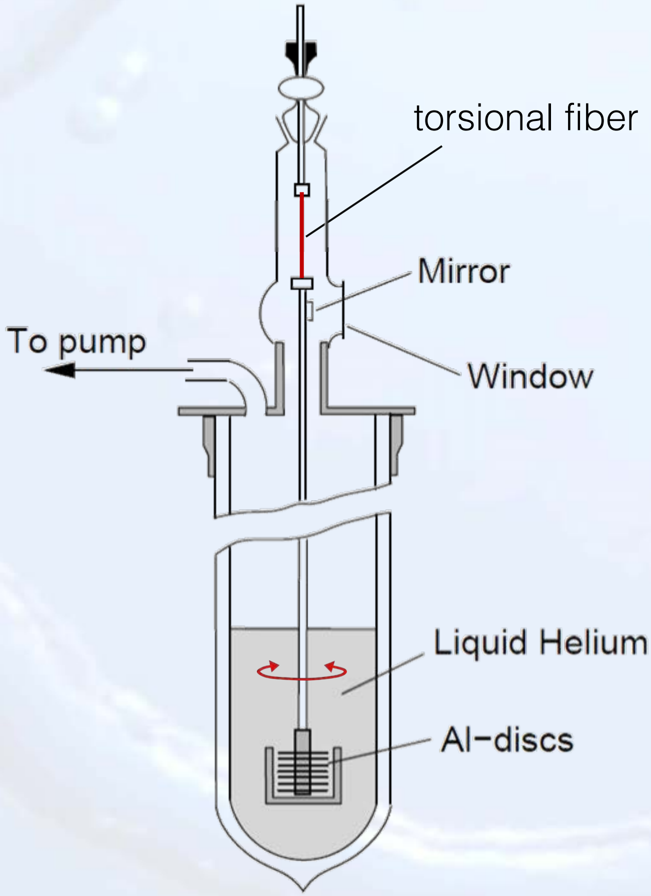
for $T \rightarrow 0$ ➡ $\varrho_\mathrm{n} \rightarrow 0$ ➡ $\varrho_\mathrm{n}\,\eta_\mathrm{n} \rightarrow 0$

2.3 Properties of He-II described using the two-fluid model

Determination of ϱ_n

Experiment of Andronikasvili (1948)

First direct observation of ϱ_n

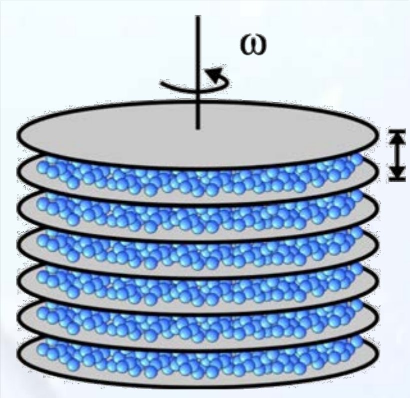


Elepter Luarsabovich Andronikashvili (1910-1989)

50 aluminum discs

thickness 13 µm
diameter 3.5 cm
spacing 210 µm

2.3 Properties of He-II described using the two-fluid model



observation → slow resonant oscillations (mass and torsion fiber)

Important parameter is the viscos penetration depth
for wave with frequency ω

$$\delta = \sqrt{2\eta_n/\varrho_n\omega}$$

$d < \delta$:
- ϱ_n is dragged along with torsion oscillator above and below T_λ
- ϱ_s remains stationary
- period of oscillation determined by mass of torsion oscillator (and spring constant)

→ ϱ_n can be determined

temperature dependence (empirical relation)

$$\varrho_n = \varrho_\lambda \left(\frac{T}{T_\lambda}\right)^{5.6}$$

comparison with 2nd Sound → fits well