Anomalous Magnetic Moments of Leptons – tests of QED

Fundamental quantities in elementary particle physics
- can be measured precisely
- can calculate from first principles

} $g-2$ of $\mu^{\pm}, e^{\pm}$ are exceptional and complementary

Recall $\mu_e = \frac{e}{m_e}\vec{S} = g_e \frac{e}{2m_e}\vec{S}$ where $g_e \approx 2.0023193\ldots$

Current status: $a_{e,\mu,\tau} = \frac{1}{2}(g_{e,\mu,\tau} - 2)$ "magnetic anomaly"

<u>electron</u>, $a_e = 0.00115965218059(13) \Rightarrow 0.13$ ppt

single electron "quantum cyclotron": PRL 130, 071801 (2023)

<u>muon</u>, $a_{\mu^+} = 0.00116592057(25) \Rightarrow 0.21$ ppm

μ^+ storage ring at FNAL: PRL 131, 161802 (2023)

<u>tau</u>, $-0.052 < a_\tau < 0.013$ (95% c.l.)

$\gamma\gamma \to \tau\tau$ using SM+EFT

DELPHI at LEP: EPJC 35, 159–170 (2004)

This year at LHC: ATL-PHYS-PROC-2024-024

It is common to write $\frac{g_e}{2} = 1 + C_2\frac{\alpha}{\pi} + C_4\left(\frac{\alpha}{\pi}\right)^2 + \cdots$

But note that only one parameter is needed for QED (we could take α for example)

recall $\alpha = \frac{e^2}{4\pi} = \frac{g_W^2 \sin^2\theta_W}{4\pi}$ or $\underbrace{\frac{e^2}{4\pi\epsilon_0\hbar c}}$ in SI units

<u>Testing the Standard Model</u>

$\Rightarrow$ need α as input from other experiments (atom interferometers), compare experiment vs. theory for value of g_e

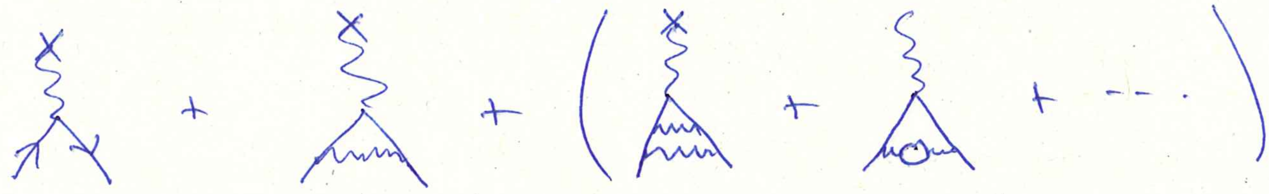
Note: only ϵ_0 is not <u>exact</u> in the 2019 SI redefinition

... unfortunately the atom interferometer experiments presently disagree on the value of α

Precision measurement of α

$\Rightarrow$ assume the SM is correct: invert the equation to solve for α using the experimental g_e

The leading interaction of a magnetic field + bare lepton is identical in QED:



2 (Dirac) $\qquad \frac{\alpha}{\pi}$ (Schwinger) $\qquad \sim\alpha^2 \qquad \sim\alpha^2$

Radiative corrections: charge is carried on the lepton, but virtual particles account for part of the mass-energy $\Rightarrow$ $\frac{e}{m}$ coupling to external $\vec{B}$ slightly increases

But electron is much less sensitive to short distance scales, including SM strong and weak interactions.

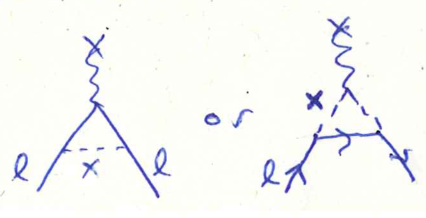
Even within QED, vacuum polarization looks very different for $e^\pm$ and $\mu^\pm$: muons easily create virtual $e^\pm$ pairs, but due to lower electron mass this situation is not symmetric.

$\Rightarrow$ $e^\pm$ is better for testing "pure" QED

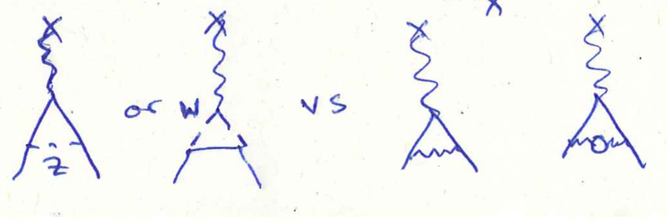
$\mu^\pm$ has greater sensitivity to high-mass particles

$\tau^\pm$ experimentally not yet possible to resolve $\frac{\alpha}{\pi}$ ...

Corrections for a heavy particle X enter as $\frac{m_\ell^2}{m_X^2}$ $\quad \ell = e, \mu, \tau$



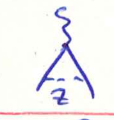
and for the same reasons:



So keeping in mind that $m_\mu^2/m_e^2 \approx 40\,000$

more complicated...

$$\frac{g_e}{2} = 1 + \underbrace{\sum_{n=1}^{\infty}\left(\frac{\alpha}{\pi}\right)^n C_{2n} + a_{\mu\tau}} + \boxed{a_{weak}} + \boxed{a_{hadronic}}$$



$$a_{QED} = \sum_{n=1}^{\infty}\left(\frac{\alpha}{\pi}\right)^n \Big[A_{2n}^{(1)} + A_{2n}^{(2)}\left(\frac{m_e}{m_\mu}\right) \sim 4.8\times10^{-3}$$
$$+ A_{2n}^{(2)}\left(\frac{m_e}{m_\tau}\right) \sim 2.9\times10^{-4}$$
$$+ A_{2n}^{(3)}\left(\frac{m_e}{m_\mu}, \frac{m_e}{m_\tau}\right) \sim 10^{-6}$$

universal for e, μ, τ and known analytically for $2n = 2, 4, 6, 8$ (numerically for 10)

Experimentally Larmor precession (spin) and cyclotron motion are described by two closely related frequencies:

$$\hbar\omega_s = |2\mu_e B| = \frac{g_e}{2}\underbrace{\frac{\hbar e B}{m_e}}_{\hbar\omega_c \text{ cyclotron energy}} \quad \text{where } \mu_e = -\frac{g_e}{2}\frac{e\hbar}{2m_e} = -\frac{g_e}{2}\mu_B$$

spin

$$\Rightarrow \frac{g_e}{2} = \frac{\omega_s}{\omega_c} = 1 + \frac{\omega_a}{\omega_c}$$

where $\omega_a = \omega_s - \omega_c$ is the "anomaly frequency"

For the electron, any of these frequencies can be measured with $\sim 10^{-10}$ precision. Why measure a difference of two large frequencies? Use standard error propagation for $g_e/2$:

$$\frac{\Delta(g_e/2)}{g_e/2} = \frac{2}{g_e}\sqrt{\left(\frac{\Delta\omega_a}{\omega_c}\right)^2 + \left(\frac{\omega_a \Delta\omega_c}{\omega_c^2}\right)^2}$$

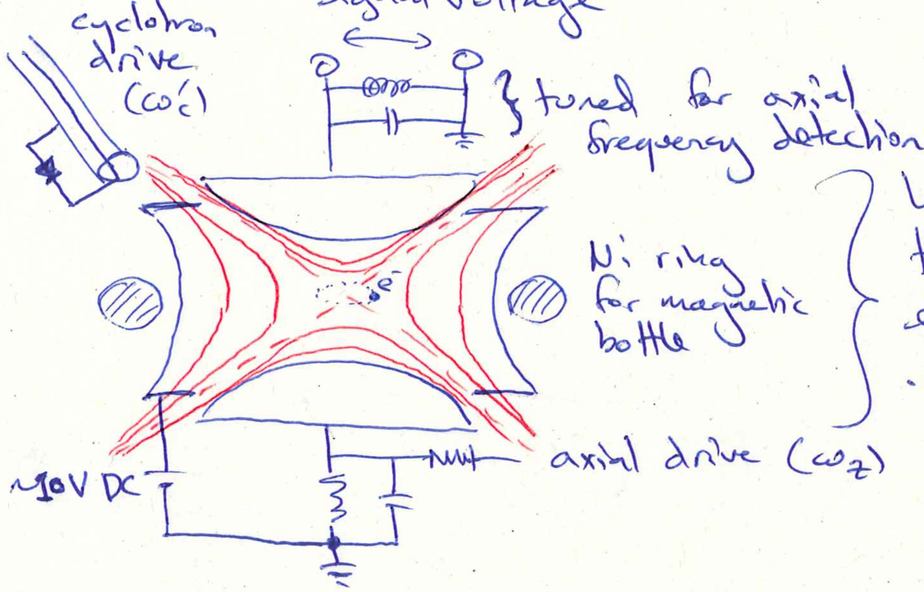
$$= \underbrace{\frac{2}{g_e}\frac{\omega_a}{\omega_c}}_{\approx 0.00115\ldots}\sqrt{\left(\underbrace{\frac{\Delta\omega_a}{\omega_a}}_{10^{-10}}\right)^2 + \left(\underbrace{\frac{\Delta\omega_c}{\omega_c}}_{10^{-10}}\right)^2} \approx 10^{-13}$$

i.e. $1000\times$ better than for ω_c and ω_s

... so it is advantageous to measure $\frac{\omega_a}{\omega_c} = \frac{g_e - 2}{2}$ $\left(\to \frac{\omega_a}{\gamma\omega_c}\right)$ (4)
rather than $\frac{\omega_s}{\omega_c}$.   relativistic

<u>But</u> two separate measurements are still needed for the two frequencies (so very stable $\vec{B}$) and the most important correction (cavity shift) is not affected by this choice.

Experimental environment for g_e measurement: <u>Penning Trap</u>
(see detail also on slides)



$\vec{B} = B\hat{z}$ (uniform)

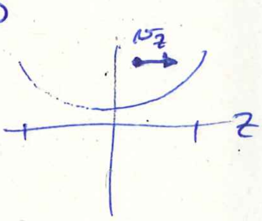
$\Phi = -\frac{\Phi_0}{2}\left[z^2 - \frac{x^2+y^2}{2}\right]$
(Quadrupole)

... all inside a superconducting solenoid to supply $B = 5.3T$ w/ drift of 0.2ppb/hr

Equation of motion: (electron, $-e$)
$$m_e \frac{d^2}{dt^2}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = e\frac{\Phi_0}{2}\begin{pmatrix} x \\ y \\ -2z \end{pmatrix} + eB\begin{pmatrix} -dy/dt \\ dx/dt \\ 0 \end{pmatrix}$$

3 independent/orthogonal simple harmonic oscillators

Axial frequency: $\omega_z = \sqrt{\frac{e\Phi_0}{m_e}}$ $E_z = \frac{1}{2} m_e \omega_z^2 z^2$

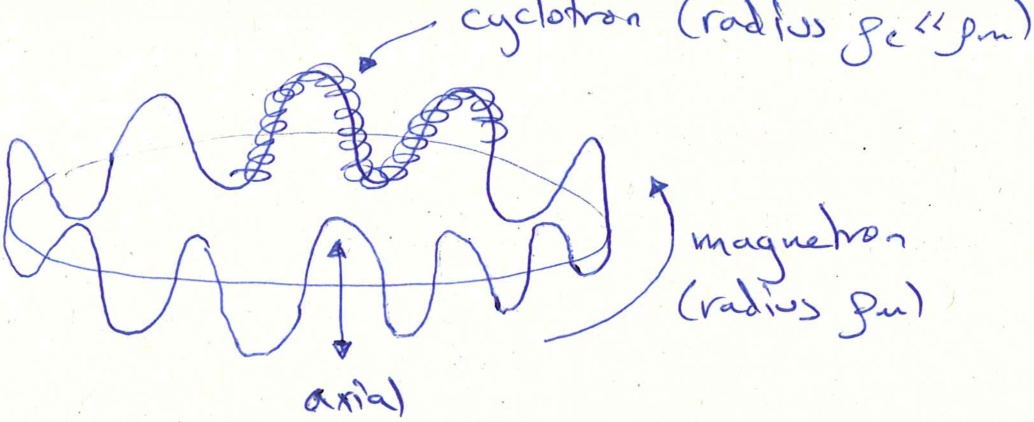


Transverse motion: two circular motions with eigenfrequencies

$$\omega_\pm = \frac{1}{2}\left(\omega_c \pm \sqrt{\omega_c^2 - 2\omega_z^2}\right)$$

ω_+ ~ modified cyclotron
ω_- ~ magnetron (slower)

approximate expansions:
$$\omega_+ \approx \omega_c - \frac{\omega_z^2}{2\omega_c} = \omega_c - \omega_m \equiv \omega_c'$$
$$\omega_- \simeq \omega_z^2 / 2\omega_c \equiv \omega_m$$
} ok up to the 10^{-12} level



For transverse motion, energy is the sum of kinetic and electrostatic:

$$E_c = \frac{1}{2} m_e \left(\omega_c'^2 - \frac{1}{2}\omega_z^2 \right) \rho_c^2 \approx \frac{1}{2} m_e \omega_c'^2 \rho_c^2$$

$$E_m = \frac{1}{2} m_e \left(\omega_m^2 - \frac{1}{2}\omega_z^2 \right) \rho_m^2 \approx - \frac{1}{4} m_e \omega_z^2 \rho_m^2$$

orbit size grows in time

Numerically, for the most recent experiments there is a clear hierarchy:

$$\omega_c' \approx 2\pi \times 149\text{ GHz} \gg \omega_z \approx 2\pi \times 114\text{ MHz} \gg \omega_m \approx 2\pi \times 43\text{ kHz}$$

where the experimental anomaly frequency is $\omega_a' \approx 2\pi \times 173\text{ MHz}$

Each motion has a different timescale for reaching thermal equilibrium: synchrotron radiation is important

PRL 55, 1 (1985) p. 67-70

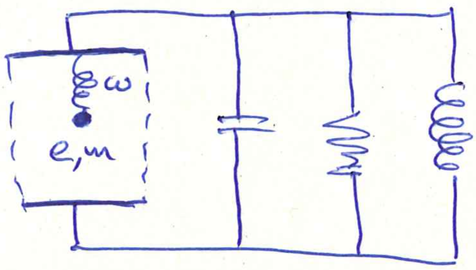
Cyclotron $\tau_c = \frac{1}{\gamma_c} = 4\pi\epsilon_0 \frac{3 m_e c^3}{4 e^2 \omega_c^2} \approx 90\text{ ms}$ at 5.3 T

this is too fast $\Rightarrow$ trick is used to extend $\tau_c \approx 5\text{-}10$ s
the cavity suppresses spontaneous radiation over certain frequency bands by factor of 50-70

$\Rightarrow$ τ_c now becomes long enough to detect single-quantum transitions! (uses "self-excitation feedback), one jump $\Rightarrow$ 1.3 Hz shift

Magnetron $\tau_m = 4\pi\epsilon_0 \frac{3 m_e c^3}{4 e^2 \omega_m^2} \frac{\omega_c' - \omega_m}{\omega_m} \approx 10^{10}$ years! ... so just cool initially to a reasonable ρ_m

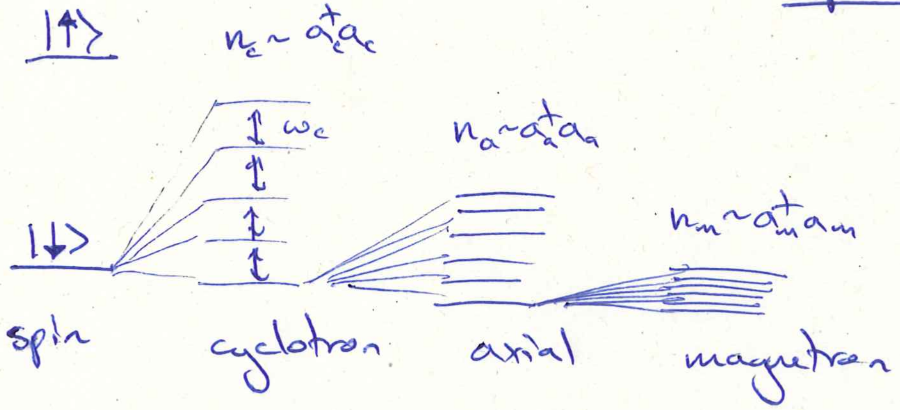
Axial: $\tau_z = \frac{1}{\gamma_z} \approx 30\,\text{ms}$ strongly and intentionally damped by the detection resonator $\Rightarrow$ QND measurement (quantum non-demolition) i.e., no change of state as a result of the measurement



$\rightarrow$ see slides for 1-minute Fourier transform of Johnson noise

There could also be radiative decay of the excited spin state, but this is also slow!

$$\tau_s = 4\pi\epsilon_0 \frac{6 m_e^2 c^5}{g_e^2 \hbar e^2 \omega_s^3} \simeq 5 \text{ years at } 5.3\,\text{T}$$



Temperatures: Cyclotron ~ 100 mK (physical trap T)

Axial ~ 0.5 K (from amplifier)

Magnetron (metastable)

$$T_m = -\frac{\omega_m}{\omega_z} T_z \simeq -10^{-3} T_z$$

Quantum numbers:

$m_s = \pm\frac{1}{2}$

$\bar{n}_c \approx 10^{-32} \approx 0$ (cooled to ground state)

$\bar{n}_z \approx 100$

$\bar{n}_m \approx 100$

Practical challenges: microwave cavity resonances near cyclotron frequency

- inhibits spontaneous emission
- alters measured cyclotron frequency from free-space value

Measured (cavity-shifted) cyclotron frequency:

$$\bar{\omega}_c^{cav} = \bar{\omega}_c \left(1 + \frac{\Delta\bar{\omega}_c^{cav}}{\bar{\omega}_c}\right)$$

shift

also changes ω_a but <u>not</u> ω_s: $\bar{\omega}_a \to \bar{\omega}_a - \Delta\bar{\omega}_c^{cav}$

So the following correction must be applied to the measured g_e (few 10^{-12} depending on frequency)

$$\left.\frac{\Delta g_e}{2}\right|_{cav} \approx \left(1 + \frac{\bar{\omega}_a}{\bar{\omega}_c}\right) \frac{\Delta\bar{\omega}_c^{cav}}{\bar{\omega}_c}$$

this approximation is ok to $\sim 10^{-15}$ and can be derived from the invariance theorem (below)

Relations among measured frequencies are simplified by the Brown-Gabrielse invariance theorem:

$$\omega_c = \sqrt{\bar{\omega}_c^2 + \bar{\omega}_z^2 + \bar{\omega}_m^2} \approx \bar{\omega}_c + \frac{\bar{\omega}_z^2}{2\bar{\omega}_c}$$

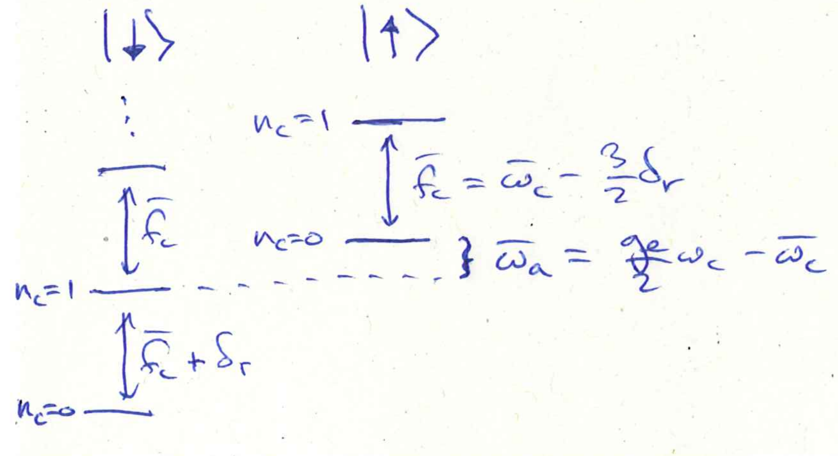
e.g. for small field distortions or electrode misalignments

there is also a relativistic mass shift that alters the cyclotron frequency:

$$\Delta\omega_c = -\delta_r (n_c + 1 + m_s) \quad \text{where} \quad \frac{\delta_r}{2\pi} = \frac{h\left(\frac{\omega_c}{2\pi}\right)^2}{m_e c^2} \approx 180\,\text{Hz} \times \left(\frac{B}{5.3\,\text{T}}\right)^2$$

$\Rightarrow \sim 10^{-9}$ correction to ω_c

define $\bar{f}_c = \bar{\omega}_c - \frac{3}{2}\delta_r$



Measurement is then:

$$-\frac{\mu_e}{\mu_B} = \frac{g_e}{2} = 1 + \frac{\bar{\omega}_a - \frac{\bar{\omega}_z^2}{2\bar{f}_c}}{\bar{f}_c + \frac{3}{2}\delta_r + \frac{\bar{\omega}_z^2}{2\bar{f}_c}} + \left.\frac{\Delta g_e}{2}\right|_{cav}$$

by expanding the invariance theorem using $\bar{\omega}_c \gg \bar{\omega}_z \gg \bar{\omega}_m \gg \delta_r$