Experimental Tests of electroweak Standard Model

1. Precision test of Z sector
2. Precision tests of the W sector
3. Higher order corrections
4. Discovery of the Higgs boson and properties

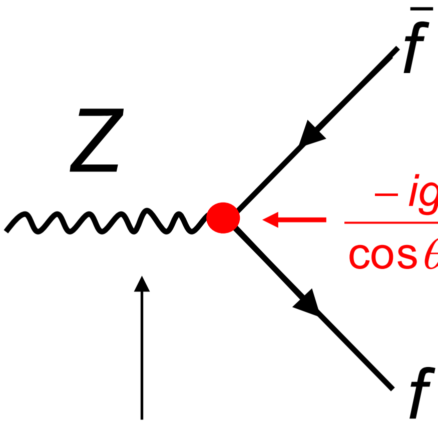
1. Precision Z-physics

All measurements and plots (if not mentioned differently) from:

Precision Electroweak Measurements on the Z Resonance

ALEPH, DELPHI, OPAL, L3, SLD Collaborations,
Phys.Rept.427:257-454,2006. arXiv:hep-ex/0509008

Recap: Z couplings



$$\frac{-ig}{\cos\theta_w}\gamma^\mu \frac{1}{2}\left(g_V^f - g_A^f\gamma^5\right)$$

$$\frac{-i\left(g_{\mu\nu} - q_\mu q_\nu / M^2\right)}{q^2 - M^2}$$

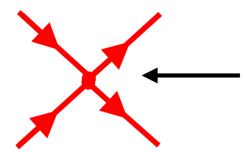
$$g_Z = \frac{g}{\cos\theta_w}$$

$$g = \frac{e}{\sin\theta_w} \qquad g' = \frac{e}{\cos\theta_w}$$

$$J_f = \frac{1}{2}\frac{e}{\sin\theta_w \cos\theta_w}\bar{\psi}_f \gamma^\mu \left(g_V^f - g_A^f\gamma^5\right)\psi_f$$

$$g_v^f = I_3^f - 2Q_f \sin^2\theta_W \quad \text{and} \quad g_A^f = I_3^f$$

Observation:
Relative strength of neutral current and charged current interactions in low energy neutrino interactions is equal:



$$\frac{g^2}{M_W^2}, \frac{g_Z^2}{M_Z^2}$$

$$\rho = \frac{g_Z^2}{M_Z^2} \Big/ \frac{g^2}{M_W^2} = \frac{g^2}{M_Z^2\cos^2\theta_w} \Big/ \frac{g^2}{M_W^2} = \frac{M_W^2}{M_Z^2\cos^2\theta_w}$$

$\rho = 1$ (automatically achieved by definition of θ_w)

$$g_V^f = I_3^f - 2Q_f \sin^2\theta_W \qquad g_A^f = I_3^f$$

$$g_L = \frac{1}{2}(g_V + g_A) \qquad g_R = \frac{1}{2}(g_V - g_A)$$

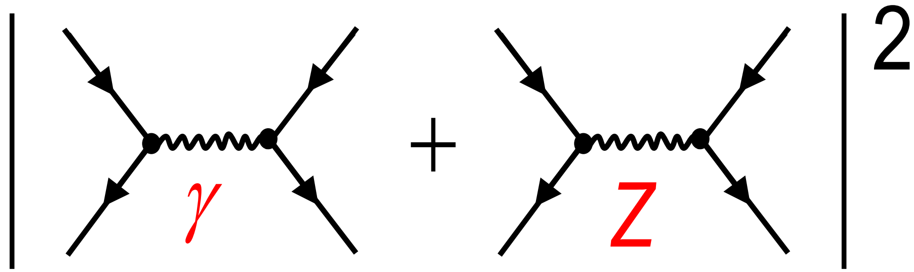
$\sin^2\theta_w \approx 0.231$ →

	g_V	g_A	g_V	g_A	g_L	g_R
ν	$1/2$	$1/2$	$1/2$	$1/2$	$1/2$	0
ℓ^-	$-1/2 + 2\sin^2\theta_W$	$-1/2$	-0.04	$-1/2$	-0.27	$+0.23$
$u-quark$	$+1/2 - 4/3\sin^2\theta_W$	$1/2$	$+0.19$	$1/2$	$+0.35$	-0.15
$d-quark$	$-1/2 + 2/3\sin^2\theta_W$	$-1/2$	-0.35	$-1/2$	-0.42	$+0.08$

1.1 Z-Boson parameters

Cross section for $e^+e^- \rightarrow \gamma / Z \rightarrow f\bar{f}$

$$|M|^2 =$$



for $e^+e^- \rightarrow \mu^+\mu^-$

$$M_\gamma = -ie^2(\bar{u}_\mu \gamma^\nu v_\mu)\frac{g_{\rho\nu}}{q^2}(\bar{v}_e \gamma^\rho u_e)$$

$$M_Z = -i\frac{g^2}{\cos^2\theta_W}\left[\bar{u}_\mu \gamma^\nu \frac{1}{2}(g_V^\mu - g_A^\mu \gamma^5)v_\mu\right]\underbrace{\frac{g_{\rho\nu} - q_\rho q_\nu / M_Z^2}{(q^2 - M_Z^2) + iM_Z\Gamma_Z}}\left[\bar{v}_e \gamma^\rho \frac{1}{2}(g_V^e - g_A^e \gamma^5)u_e\right]$$

Unphysical pole: Z propagator must be modified to account for finite Z width for $q^2 \approx M_Z^2$ (real particle w/ finite lifetime)

With a "little bit" of algebra similar as for M_γ in QED.

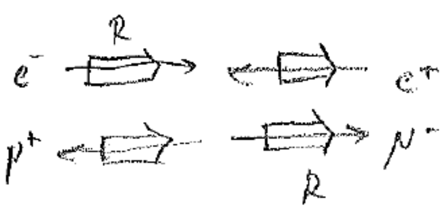
If you want to do the calculation yourself - here is the Z amplitude:

$$\mathcal{M}_{fi} = -\underbrace{\frac{g_Z^2}{(s-M_Z^2)+iM_Z\Gamma_Z}}_{P_Z(s)} \cdot \left[\bar{v}_e(p_2)\gamma^{\mu}\frac{1}{2}(c_V^e - c_A^e\gamma^5)\, u_e(p_1)\right] g_{\mu\nu} \left[\bar{u}_\nu(p_3)\gamma^{\nu}\frac{1}{2}(c_V^f - c_A^f\gamma^5)\, v_\nu(p_4)\right]$$

with $c_V = c_L + c_R$ and $c_A = c_L - c_R$ and projectors $P_{L,R}\, u = \frac{1}{2}(1 \mp \gamma^5)\, u = u_{L,R}$

one can calculate the different chiral / helicity combinations assuming massless fermions.

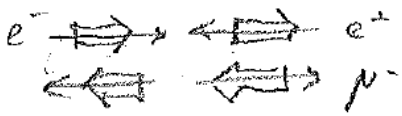
(3)



$$\mathcal{M}_{RR} = P_Z(s)\, g_Z^2 \cdot c_R^e\, c_R^\mu\, \underbrace{g_{\mu\nu} \cdot \left[v_L(p_2)\gamma^\mu u_R(p_1)\right]\left[\bar{u}_R(p_3)\gamma^\nu v_L(p_4)\right]}_{s\cdot(1+\cos\theta)}$$

$$|M_{RR}|^2 = |P_Z(s)|^2 \, g_Z^4 \, s^2 \cdot (c_R^e c_R^\mu)^2 \cdot (1+\cos\theta)^2$$

$$|M_{LL}|^2 = \cdots\cdots\cdots (c_L^e c_L^\mu)^2 \cdot (1+\cos\theta)^2$$



$$|M_{RL}|^2 = \cdots\cdots\cdots (c_R^e c_L^\mu)^2 \, (1-\cos\theta)^2$$

$$|M_{LR}|^2 = \cdots\cdots\cdots (c_L^e c_R^\mu)^2 \, (1-\cos\theta)^2$$

$$\Rightarrow \quad \langle |M|^2 \rangle = \frac{1}{4} \sum |M_{ij}|^2 \qquad (\text{see } e^+e^- \to \mu^+\mu^-)$$

with $\dfrac{d\sigma}{d\Omega} = \dfrac{1}{64\pi^2} \cdot \dfrac{1}{s} \cdot \underbrace{\left(\dfrac{p_f^*}{p_i^*}\right)}_{=1 \text{ for mass less fermions}} \cdot \langle |M|^2 \rangle$ one finds:

$$\frac{d\sigma}{d\Omega}\left(e^+e^- \to Z \to \mu^+\mu^-\right) = \frac{1}{256\pi^2 s} \cdot \frac{g_Z^4 \cdot s^2}{(s-m_Z^2)^2 + m_Z^2 \Gamma_Z^2} \cdot$$

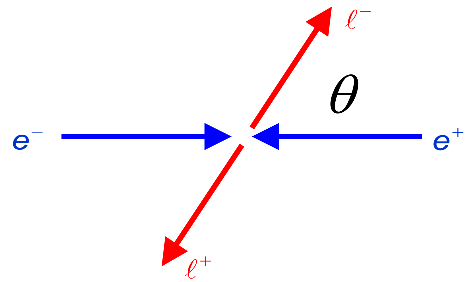
$$\left\{ \frac{1}{4}\left[(c_V^e)^2 + (c_A^e)^2\right]\left[(c_V^\mu)^2 + (c_A^\mu)^2\right](1+\cos^2\theta) + 2 c_V^e c_A^e c_V^\mu c_A^\mu \cdot \cos\theta \right\}$$

... one finds for the differential cross section:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[F_\gamma(\cos\theta) + F_{\gamma Z}(\cos\theta)\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2} + F_Z(\cos\theta)\frac{s^2}{(s-M_Z^2)^2+M_Z^2\Gamma_Z^2}\right]$$

γ — γ/Z interference — Z

Vanishes at $\sqrt{s}\approx M_Z$: finite lifetime decouples initial/final state.



$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1+\cos^2\theta) = (1+\cos^2\theta)$$

symmetric in cosθ (QED part)

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W\cos^2\theta_W}\left[2g_V^e g_V^\mu(1+\cos^2\theta) + 4g_A^e g_A^\mu\cos\theta\right]$$

asymmetric in cosθ

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W\cos^4\theta_W}\left[(g_V^{e\,2}+g_A^{e\,2})(g_V^{\mu\,2}+g_A^{\mu\,2})(1+\cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu\cos\theta\right]$$

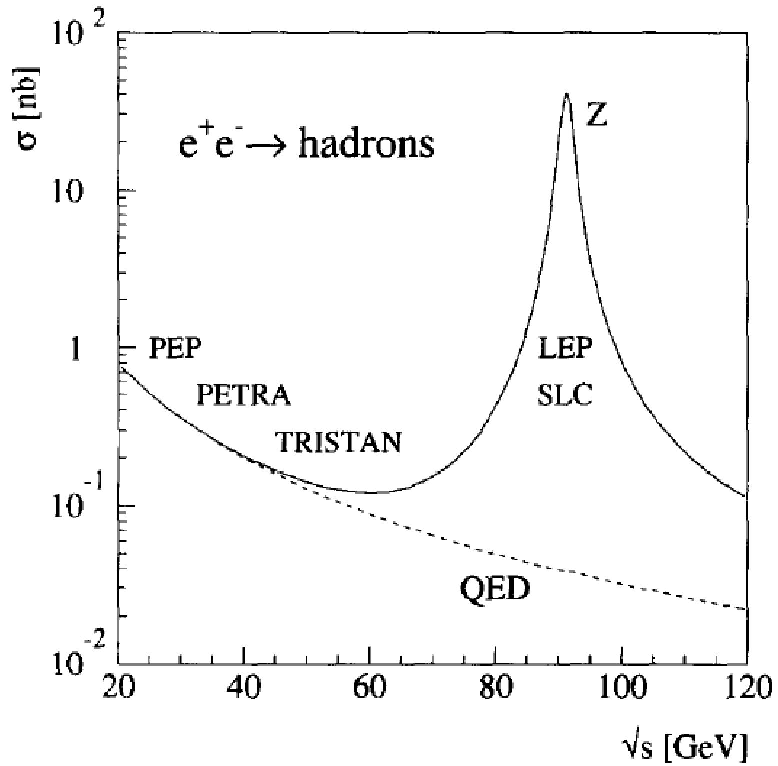
Asymmetric angular distribution → forward-backward asymmetry

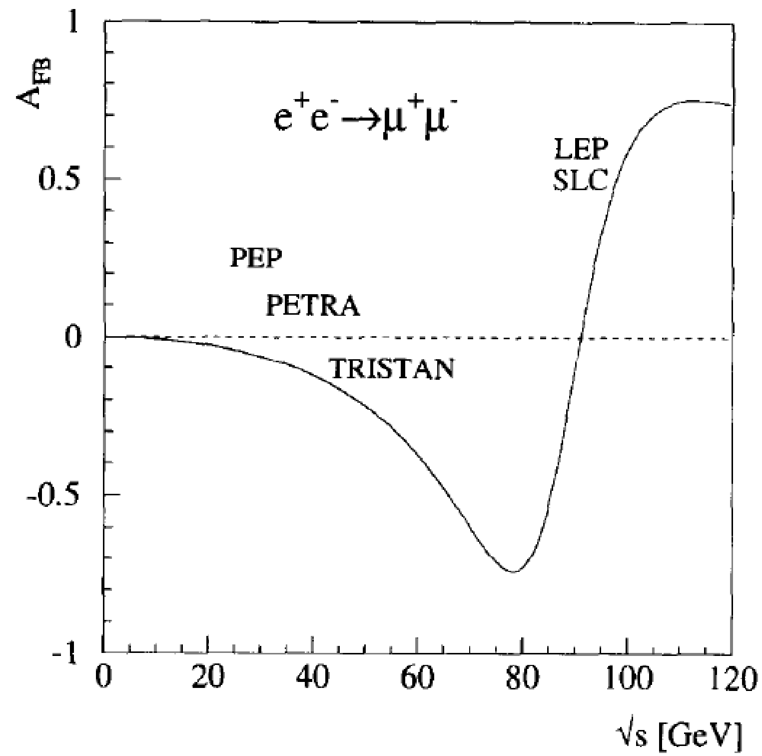
$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) + \frac{8}{3}A_{FB}\cos\theta \qquad \text{with} \quad \begin{cases} A_{FB} = \dfrac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \\ \sigma_{F(B)} = \displaystyle\int_{0\,(-1)}^{1\,(0)} \frac{d\sigma}{d\cos\theta}\, d\cos\theta \end{cases}$$

At this point A_{FB} is an observable → linear in couplings.

Expectations:

J.Mnich, Experimental Tests of Standard Model in ee→ff at the Z resonance





Large forward-backward asymmetries away from the Z pole caused by γ/Z interference.

Cross section at the Z-pole $\sqrt{s} \approx M_Z$: Breit-Wigner Resonance

(ignore QED contribution, interference vanishes)

$$\sigma_{tot} \approx \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16\sin^4\theta_w \cos^4\theta_w} \cdot \left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^\mu)^2 + (g_A^\mu)^2\right] \cdot \frac{s^2}{(s - M_Z^2)^2 + (M_Z \Gamma_Z)^2}$$

With partial and total widths: $\Gamma_f = \frac{\alpha M_Z}{12\sin^2\theta_w \cos^2\theta_w} \cdot \left[(g_V^f)^2 + (g_A^f)^2\right]$ $\qquad \Gamma_Z = \sum_i \Gamma_i$

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Breit-Wigner Resonance: $\sigma_Z\left(\sqrt{s} \approx M_Z\right) \approx \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2} = \frac{12\pi}{M_Z^2} BR(Z \to ee) BR(Z \to ff)$

Cross sections and widths can be calculated within the Standard Model if $\sin^2\theta_w$ and M_Z are known.

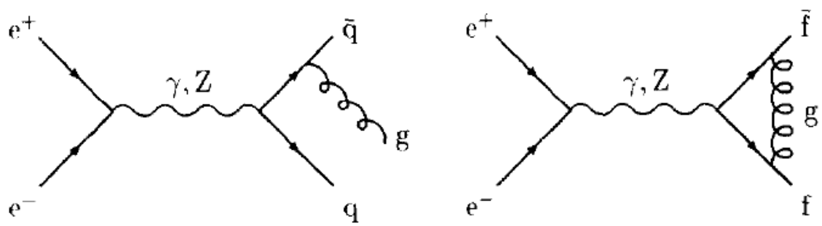
From the couplings one expects the following BR (independent of M_Z)

	$BR = \Gamma_f / \Gamma_Z$
e, μ, τ	3.5%
ν_e, ν_μ, ν_τ	7%
hadrons $(= \sum_q q\bar{q})$	69%

Remind color factor: N_C=3

No final state photon bremsstrahlung and no gluon bremsstrahlung considered.

Large corrections for hadronic final states from gluon final state bremsstrahlung:



$$R_{\mathrm{QCD}} = 1 + \frac{\alpha_S(m_Z^2)}{\pi} + \cdots .$$

Opens a way to measure α_s (M_Z).

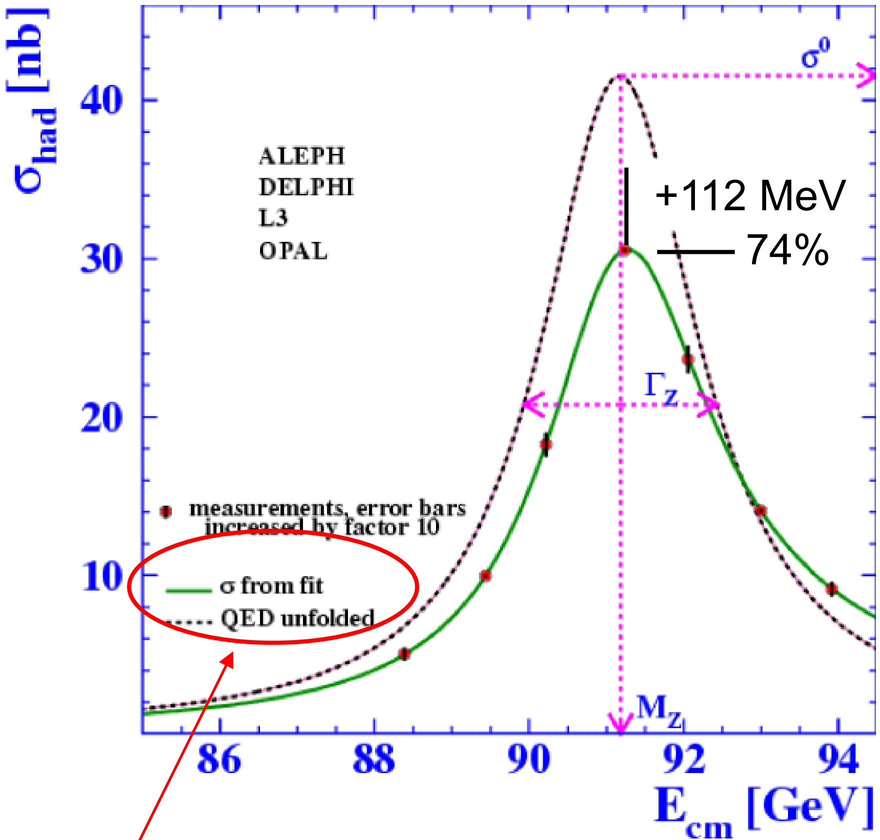
Similarly there are final state QED corrections to take into account (fomally similar but much smaller):

$$R_{QED} = 1 + \frac{\alpha(m_Z^2)}{\pi} + \ldots$$

Important: $\alpha(m_Z^2) = \frac{1}{129}$

Measurement of Z-lineshape

Strongly influenced by Bremsstrahlung



$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

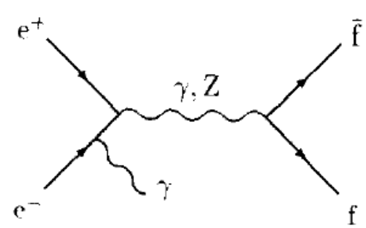
Peak: $$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

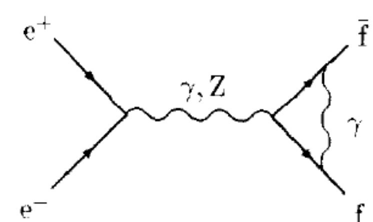
- Resonance position → M_Z
- Height → $\Gamma_e \Gamma_\mu$
- Width → Γ_Z

Bremsstrahlung corrections

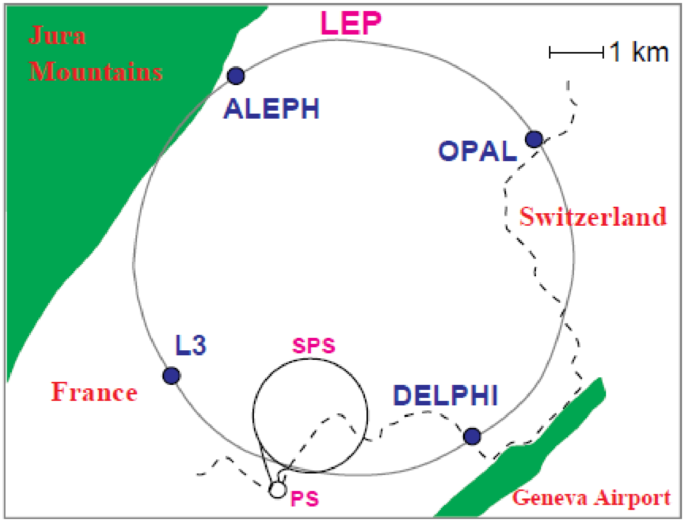
$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^{1} G(z,s)\sigma_{ff}^0(zs)dz \qquad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$

Radiator function G(z,s)





Large Electron Positron Collider (LEP)

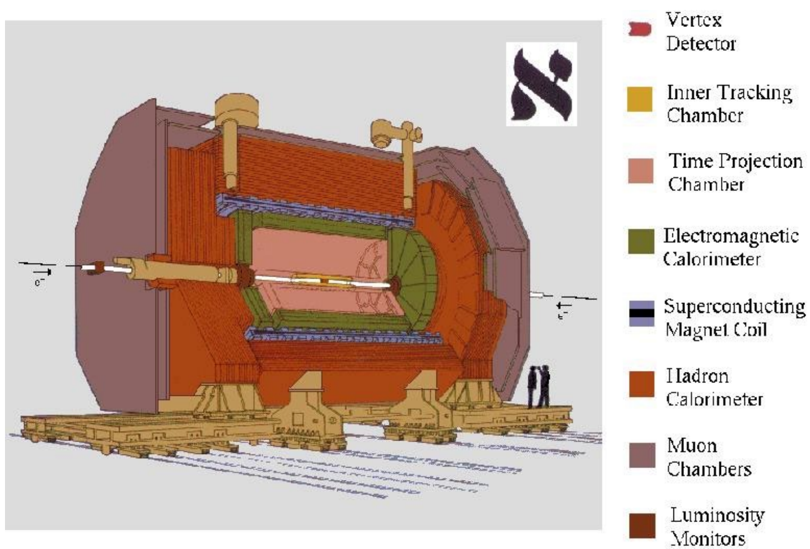


Circumference	~27 km
Centre-of-mass energy	92.1 GeV(LEP1) to 209 GeV(LEP 2)
Accelerating gradient	Up to 7 MV/m (SC cavities)
Number of bunches	4 x 4
Current per bunch	~ 750 μA
Luminosity (at Z0)	$\sim 24 \times 10^{30} cm^{-2} s^{-1}$ (~1 Z0/sec)
Luminosity (at LEP2)	$\sim 50 \times 10^{30} cm^{-2} s^{-1}$ (3 WW/hour)
Interaction regions	4 (ALEPH,DELPHI,L3,OPAL)
Energy calibration	< 1 MeV (at Z0)

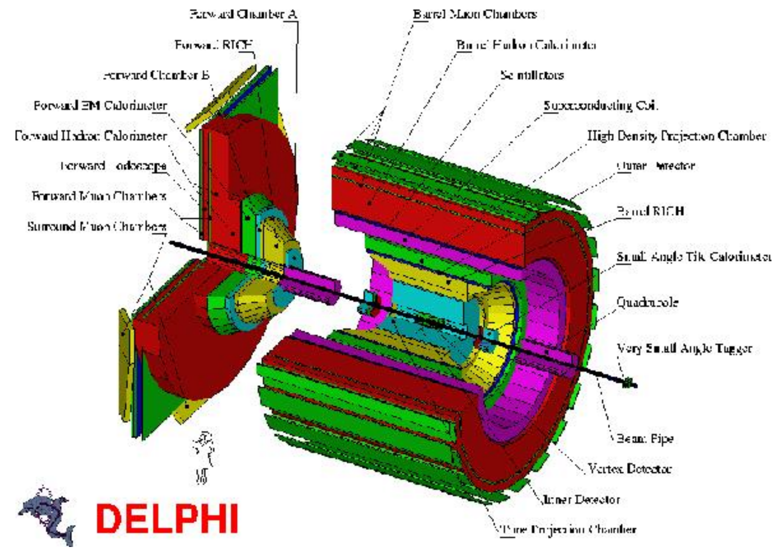
Number of Events										
	$Z \to q\bar{q}$					$Z \to \ell^+\ell^-$				
Year	A	D	L	O	LEP	A	D	L	O	LEP
1990/91	433	357	416	454	1660	53	36	39	58	186
1992	633	697	678	733	2741	77	70	59	88	294
1993	630	682	646	649	2607	78	75	64	79	296
1994	1640	1310	1359	1601	5910	202	137	127	191	657
1995	735	659	526	659	2579	90	66	54	81	291
Total	4071	3705	3625	4096	15497	500	384	343	497	1724

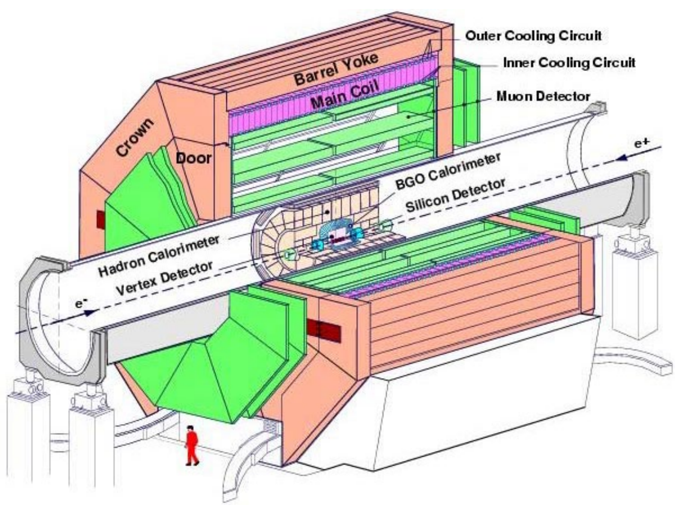
Events per experiment

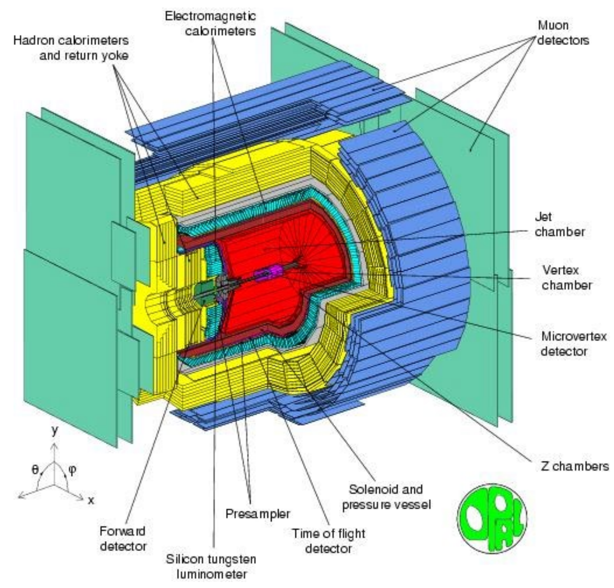
LEP Detectors



The ALEPH Detector

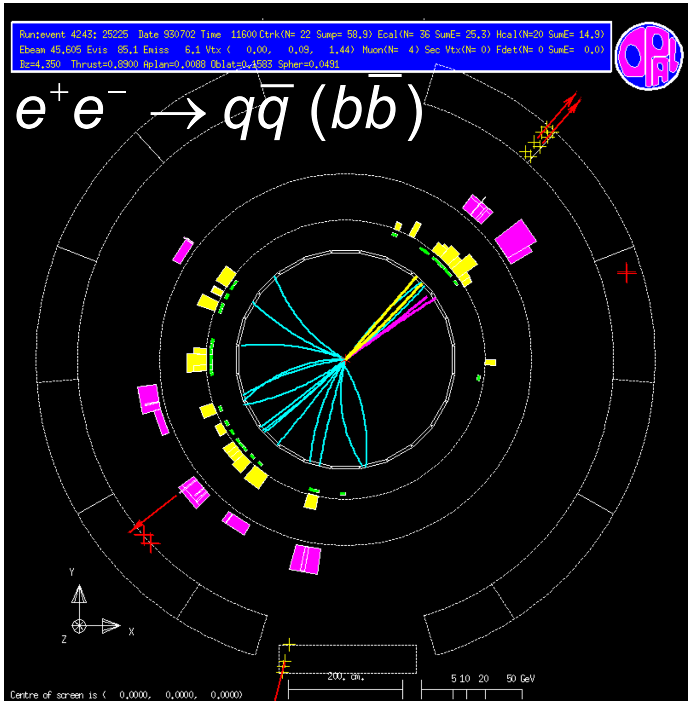


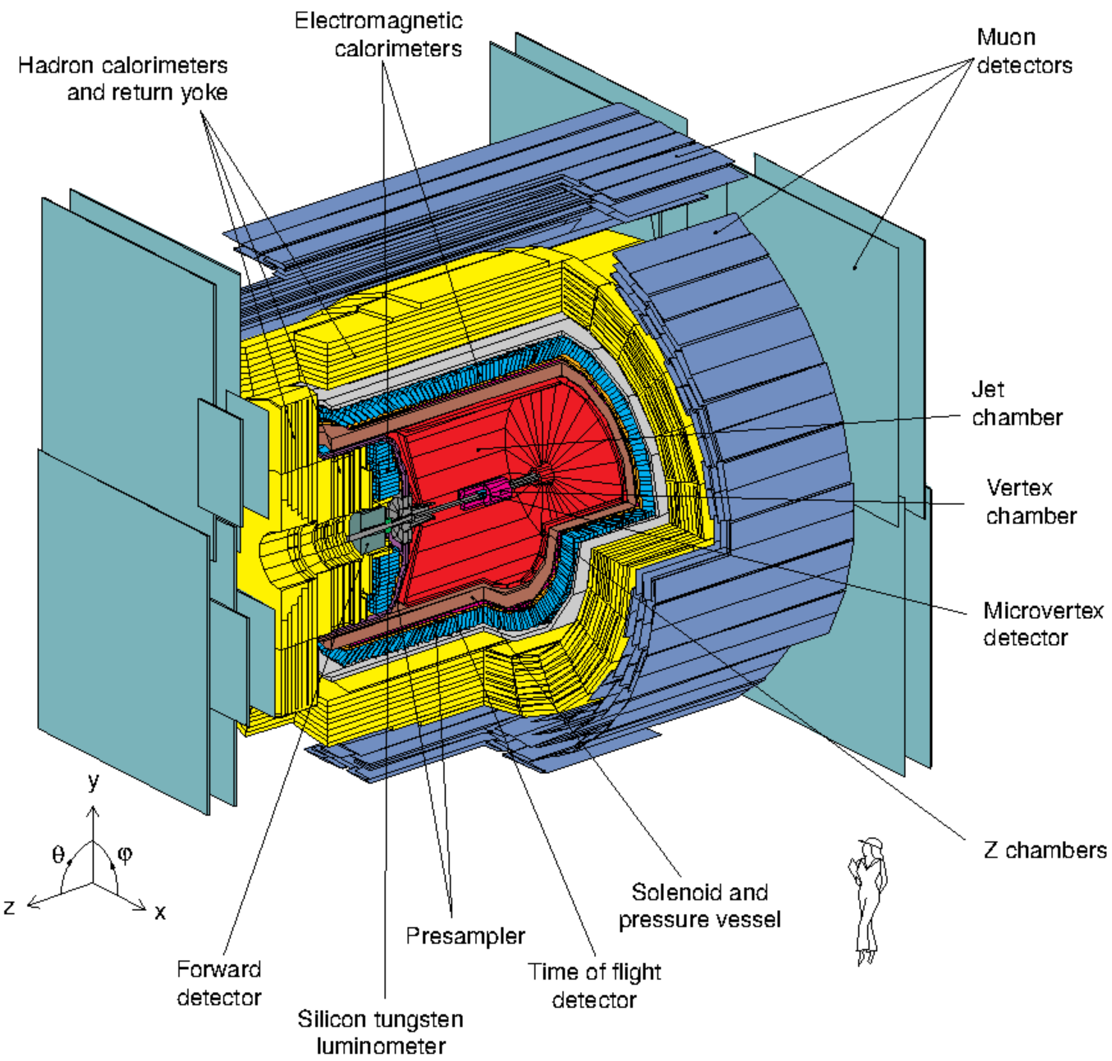




Example: OPAL Detector

Heidelberg:
Jet-Chamber





Measurement of Cross Section $\sigma(\sqrt{s})$:

$$\sigma(\sqrt{s}) = \frac{N_{signal} - N_{back}}{\varepsilon \cdot L_{\text{int}}}$$

"Cut and count". Uncertainties do to background description.

$= 2E_B$
Requires calibration of beam energy and experiment dependent correction (synchrotron loss). Uncertainties in the energy scale translates into an absolute error of M_Z.

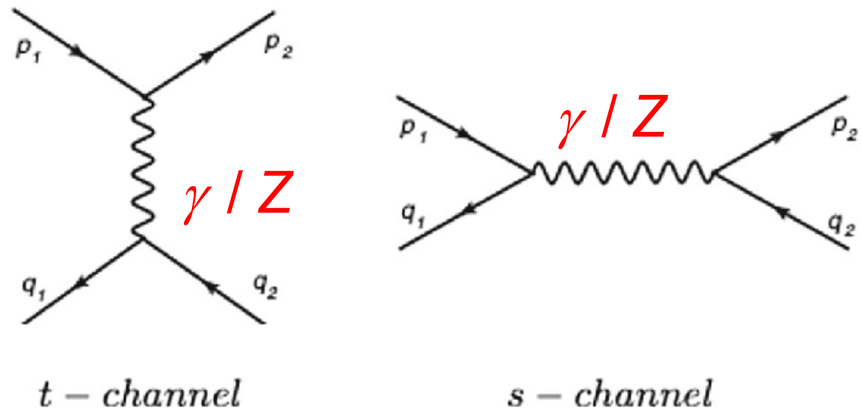
Recorded "integrated luminosity" determined through reference process: Determines the min. error of cross section measurements.

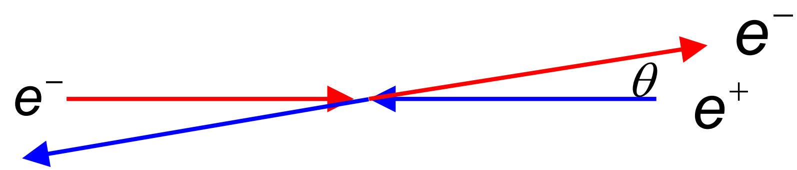
Detector acceptance $\times$ efficiency: Usually determined from simulation. Uncertainties related to detector description and uncertainties from radiative corrections.

Luminosity determination at e^+e^- - collider

Reference channel:

$$e^+e^- \rightarrow e^+e^-(\gamma)$$





QED part:

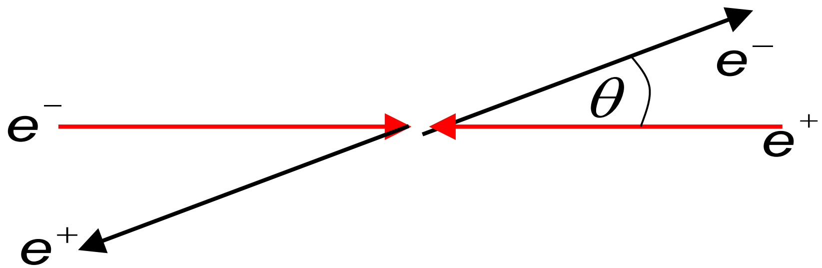
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2+u^2}{s^2}\right) = \frac{\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2 \sim \frac{1}{\theta^4}$$

For small scattering angle

Remark: Calculation more complicated if one includes the Z exchange. However, for the scattering at very small scattering angles, contribution from Z exchange is very small, and process can be used as “Z-independent” reference process.

Interference between Z-s-channel and γ t-channel vanishes at the Z pole!

CM system:

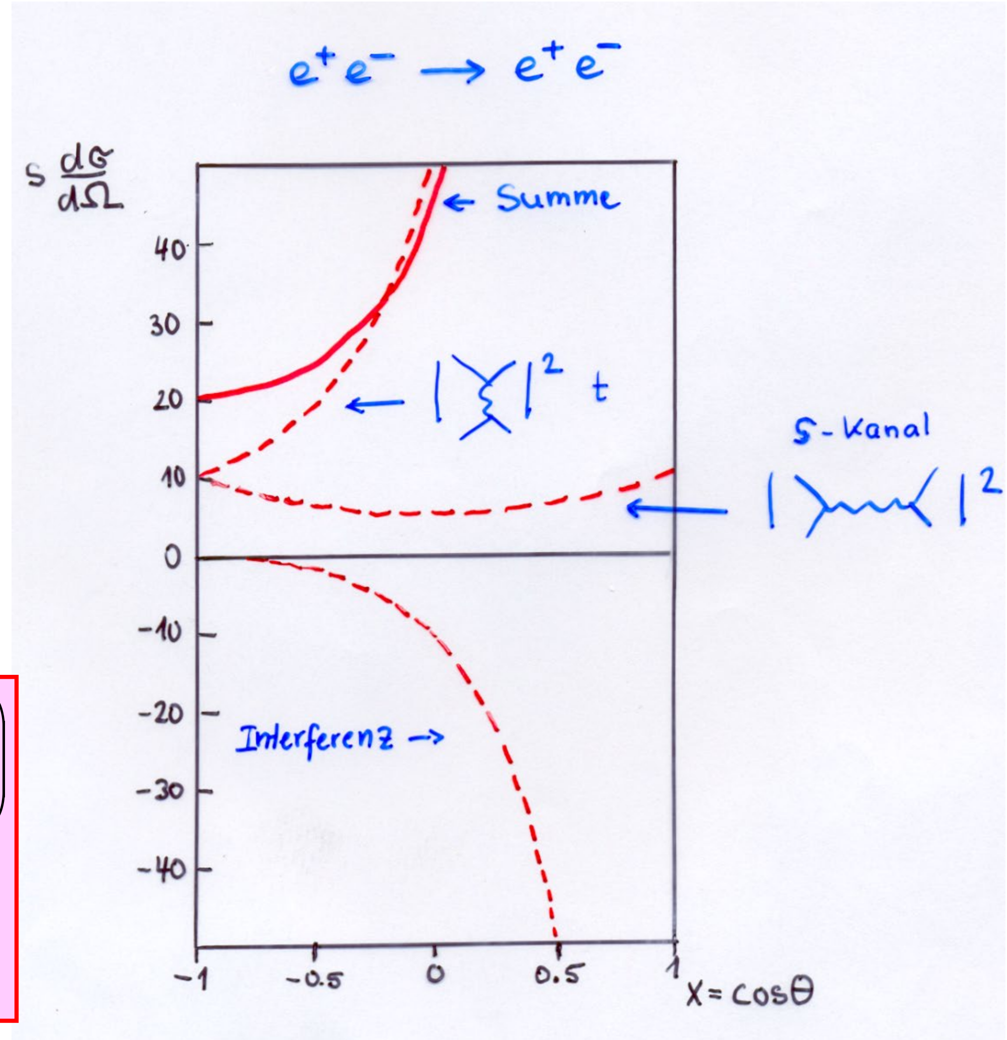


with x=cosθ

t s

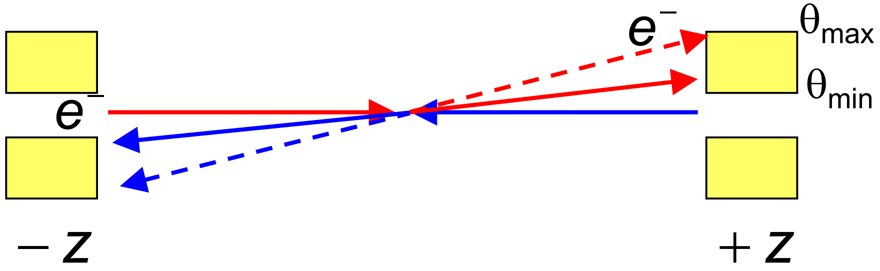
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{4+(1+x)^2}{(1-x)^2} - \frac{(1+x)^2}{1-x} + \frac{1+x^2}{2} \right)$$

$$= \frac{\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta} \right)^2$$



Luminosity measurement:

(exploits small angle scattering)



$$\frac{d\sigma}{d\Omega} \approx \frac{4\alpha^2(\hbar c)^2}{E^2\theta^4} \longrightarrow \frac{d\sigma}{d\theta} \sim \frac{1}{\theta^3} \to \sigma \sim \left(\frac{1}{\theta_{min}{}^2} - \frac{1}{\theta_{max}{}^2}\right)$$

	distance (m)	R_{min} (cm)	R_{max} (cm)	Θ_{min} (mrad)	Θ_{max} (mrad)	technology
ALEPH LCAL	2.7	10	52	45	190	lead+prop. wire ch.
DELPHI SAT	2.5	10	40	43	135	lead+sc. fibers
L3 BGO	2.8	6.8	19	25	70	BGO
OPAL FD	2.4	11.5	29	48	120	lead+scintillator

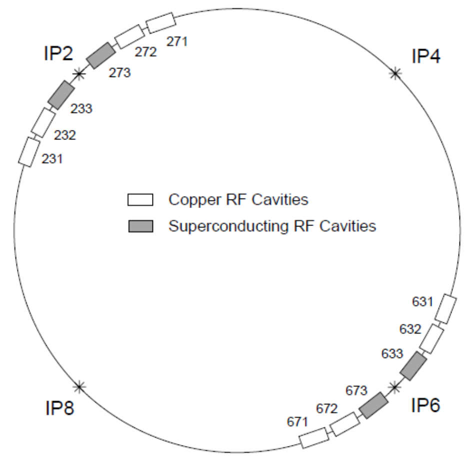
Table 1: Basic parameters of the first generation detectors at LEP.

Typical luminosity error achieved: 0.3 - 0.5 % (1st generation lumi detector)
0.07 – 0.15 % (2nd generation: Si strips)

LEP Beam Energy Calibration

$$E_B = \frac{e\,c}{2\pi} \oint_s B(s)ds$$

Beam energy calibration requires precise measurements of the average B-field along the LEP ring. In addition interaction point dependent corrections are necessary to account for the energy loss by synchrotron radiation (260 MeV / turn) and the asymmetric position of the RF cavities during LEP-1.



Different measurement to determine B-field of dipole magnets have been used:

- Field display: NMR probe / rotating coil inside a reference magnet powered in series with the LEP dipole magnets.
  Problem: different position and different environment. Used to extrapolate from periods w/o other measurements
- Flux loop measurements: induction loops in all 8 octants → measure induction voltage when the B field is ramped.
- NMR probes inside the ring dipole magnets (installed only in 1995)

Good reproducibility but no absolute calibration.

Measurements to calibrate flux-loops / NMR probes:

- Proton calibration: LEP ring was filled with 20 GeV protons.
  → precise determination of proton velocity → proton momentum → B field
  Method reached absolute accuracy of 10^{-4} at 20 GeV.

- Resonant depolarization (ultimate method, precision better than 1 MeV (10^{-5}).
  Method is a "g-2 experiment" where the electron g-2 is known and the average B-field / average electron energy E_B is determined instead.

Spin-tune:

$$\Delta \nu = \frac{g-2}{2} \frac{E_B}{m_e c^2}$$

Method:
Due to the Sokolov-Ternov effect the electrons inside LEP build-up a transvers vertical polarization (~50% was reached).
Using a weak RF-magnet w/ horizontal B-field turns the spins by small amount → depolarization when RF frequency is equal to the spin-tune.

The beam polarization is measured using Compton polarimetry (Compton scattering of laser light).

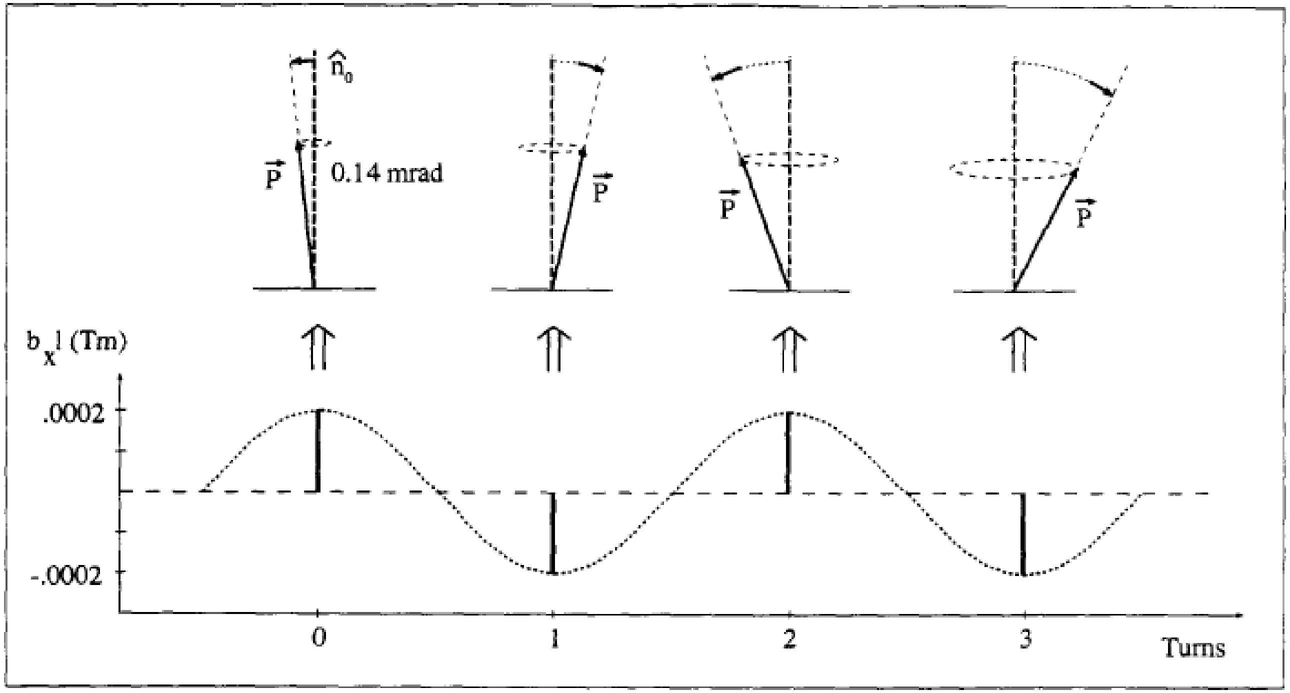


Figure 1: Resonance condition between the nominal spin precession with $[\nu] = 0.5$ and the radial perturbation $\int b_x l$ from the RF-magnet. In an ideal storage ring the polarization vector is initially along the vertical direction. After being tilted $\vec{P}$ precesses with ν about its initial direction. If the perturbation is in phase with the nominal spin precession (in this example $f_{dep} = 0.5 \cdot f_{rev}$) the polarization vector is resonantly rotated away from the vertical direction.

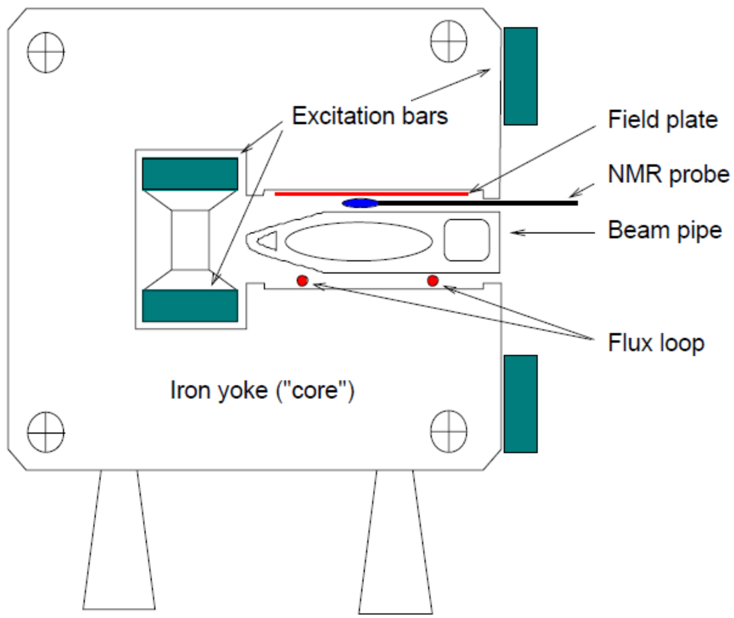


Fig. 1. Diagram of a concrete-reinforced dipole cross-section. The approximate positioning of the NMR probe is also shown

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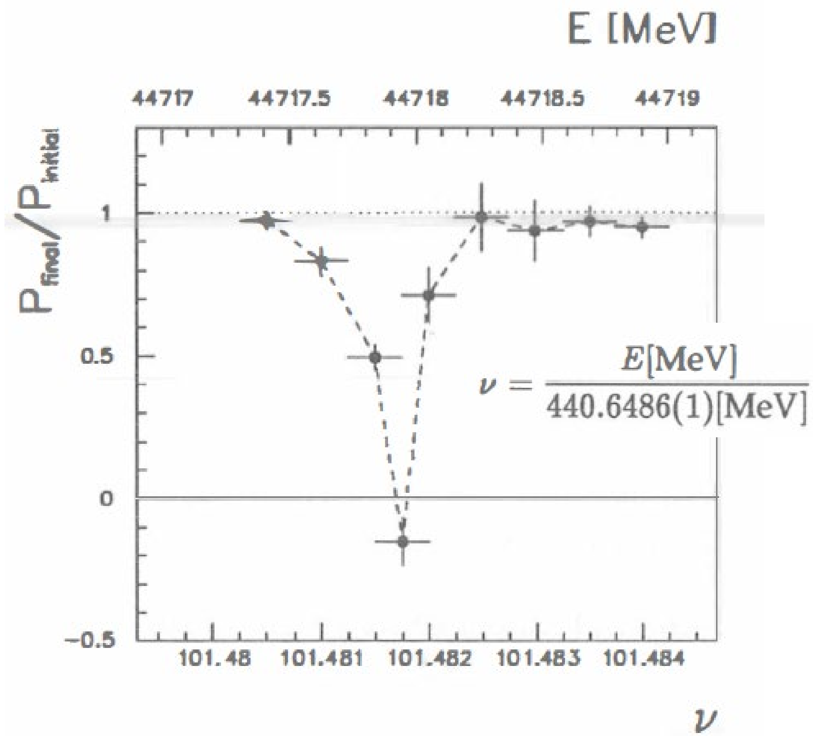


Figure 1: Measurement of the width of the artificially excited spin resonance which is used for energy calibration by resonant depolarization. The vertical axis represents the ratio of the polarization after and before a depolarization. The width of the resonance is about 200 keV.

J. Wenninger, Energy calibration at LEP

Unexpected effects seen at the ppm-level

Eur. Phys. J. C 6, 187{223 (1999)

Changes of the circumference of the LEP ring changes the energy of the electrons and thus the CM energy (shifts M_Z) :

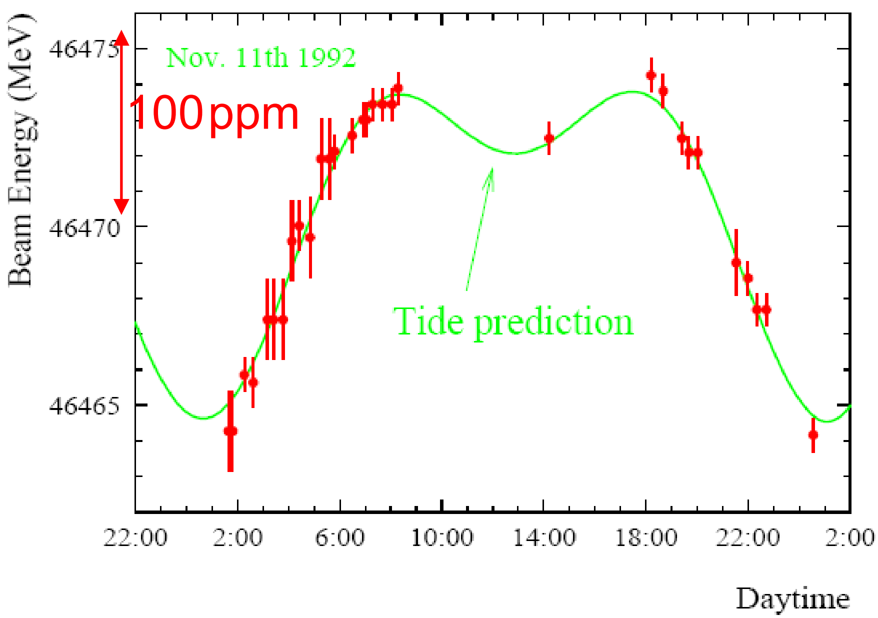
- tide effects
- water level in lake Geneva

Changes of LEP circumference
ΔC=1…2 mm/27km (4…8x10^{-8})

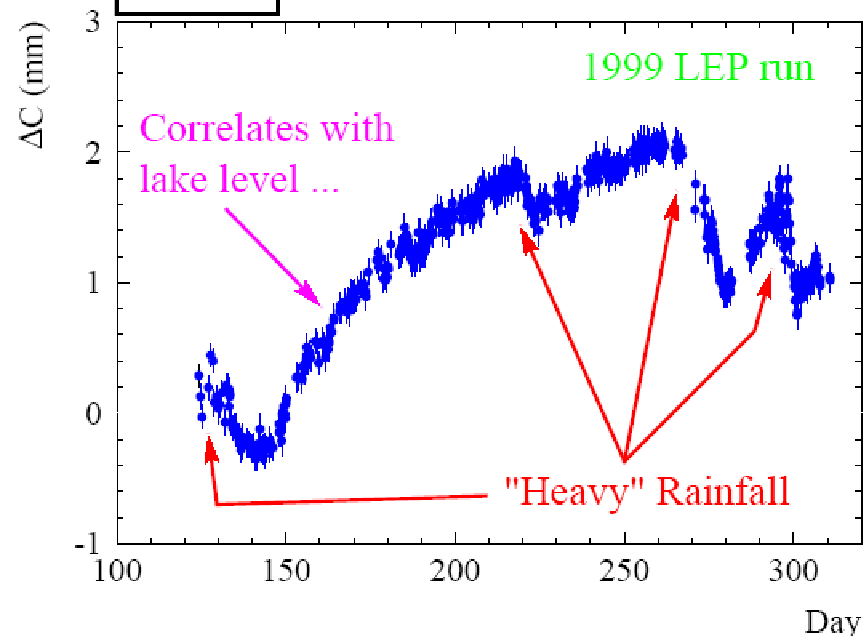
$$\frac{\Delta E(t)}{E_0} = -\frac{1}{\alpha}\frac{\Delta C(t)}{C_0}$$

(effect: off-center in quadrupoles)

1992 Effect of moon



1993 Effect of lake



$$4 \cdot 10^{-8} \times 27\text{km} \sim 1.2\text{mm} \rightarrow \pm 5\text{MeV}$$

Effect of the French "Train a Grande Vitesse" (TGV)

Eur. Phys. J. C 6, 187{223 (1999)

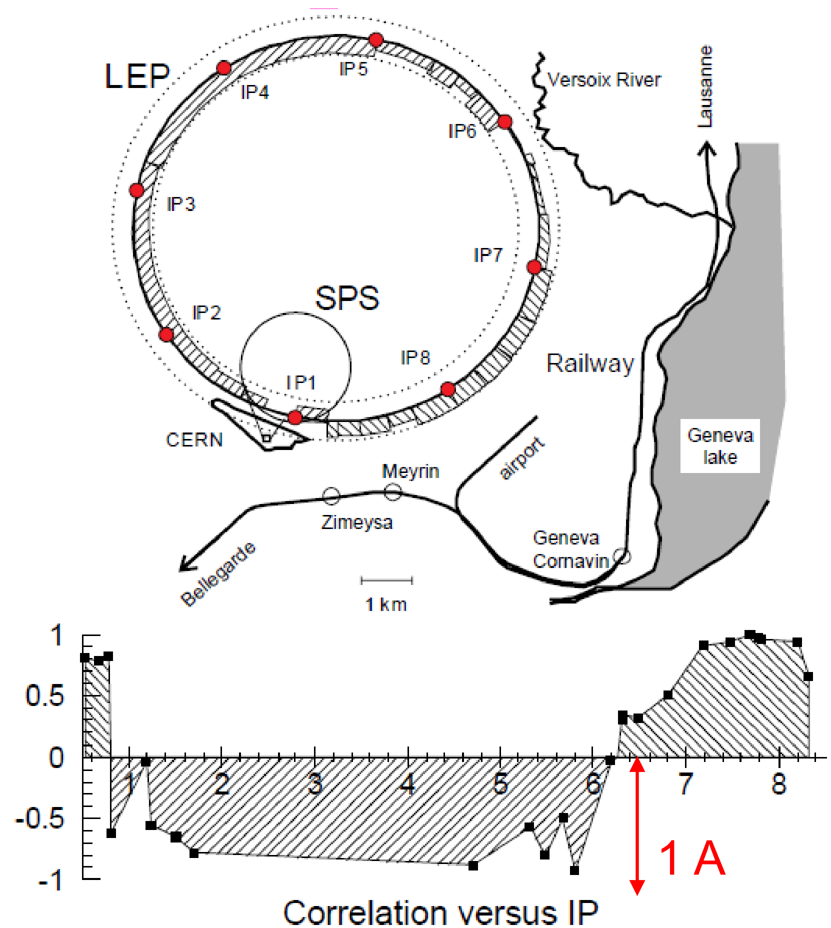
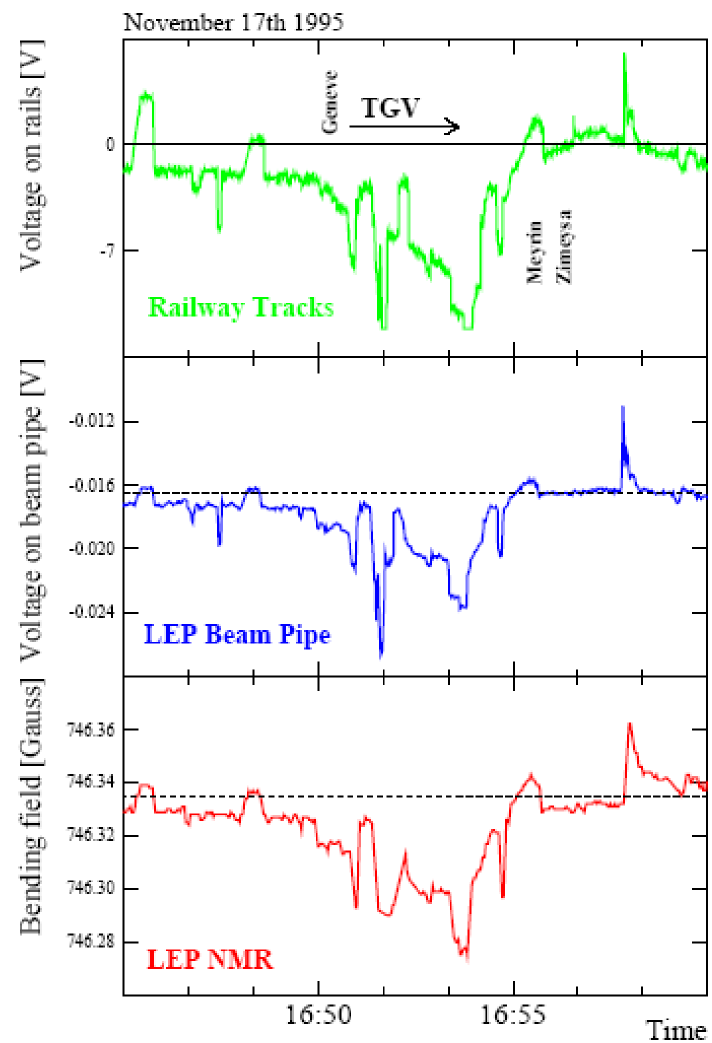


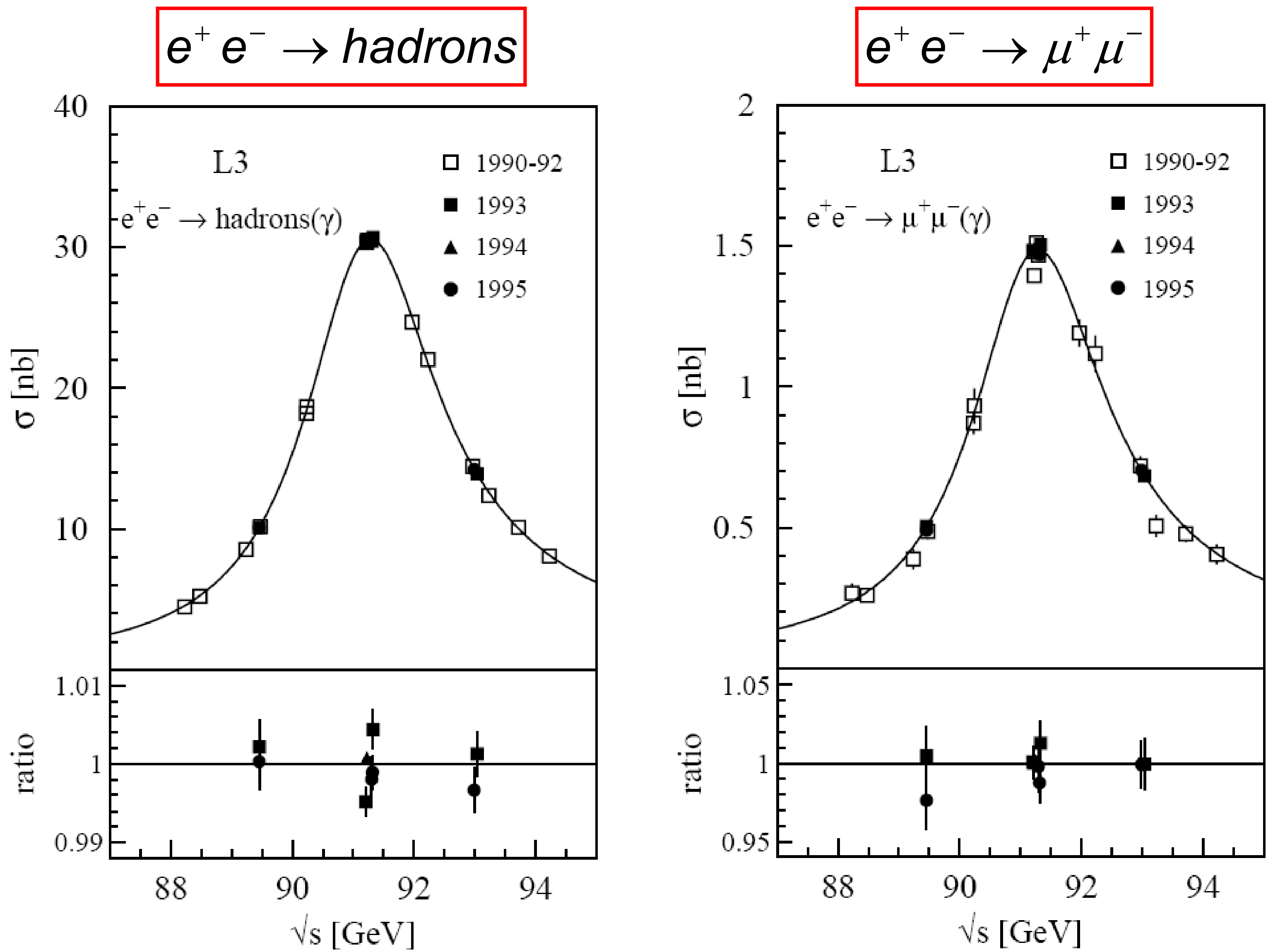
Fig. 4. Diagram of CERN accelerators and surroundings. The measured correlation pattern of the parasitic currents along the ring is shown, proving that they enter and leave LEP near IP6 and IP1

V



In conclusion: Measurements at the ppm level are difficult to perform. Many effects must be considered!

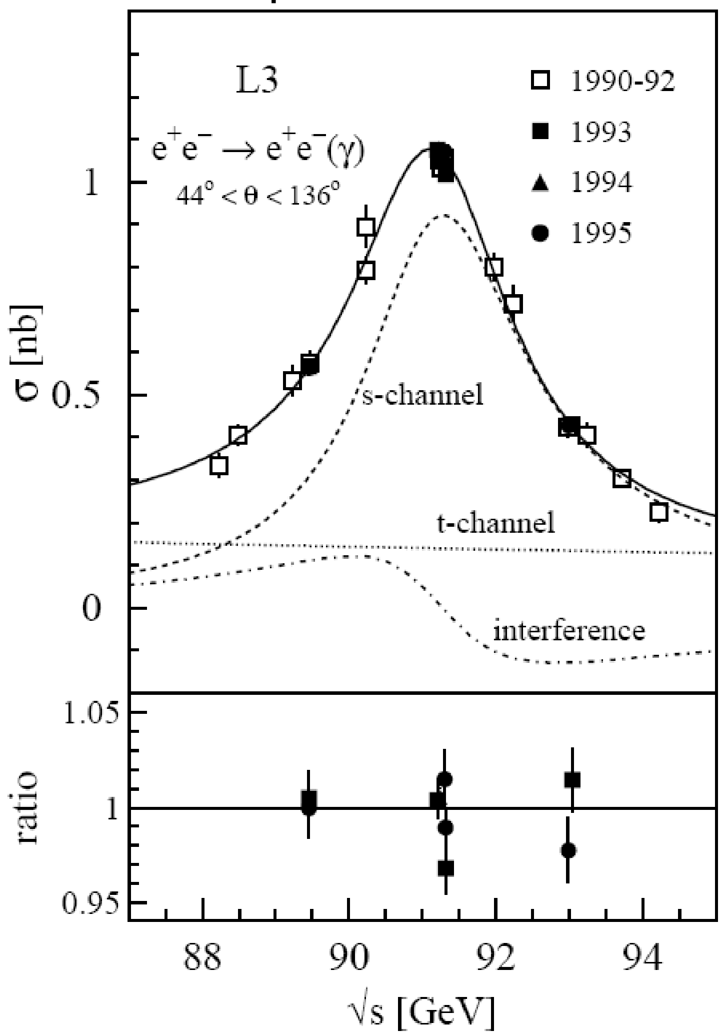
Cross section measurements $\sigma(\sqrt{s})$



Resonance shape is the same, independent of final state: Propagator the same!

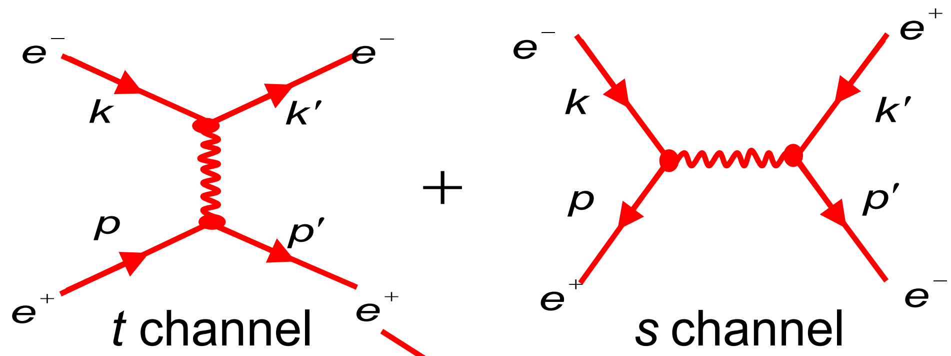
$e^+ e^- \to e^+ e^-$

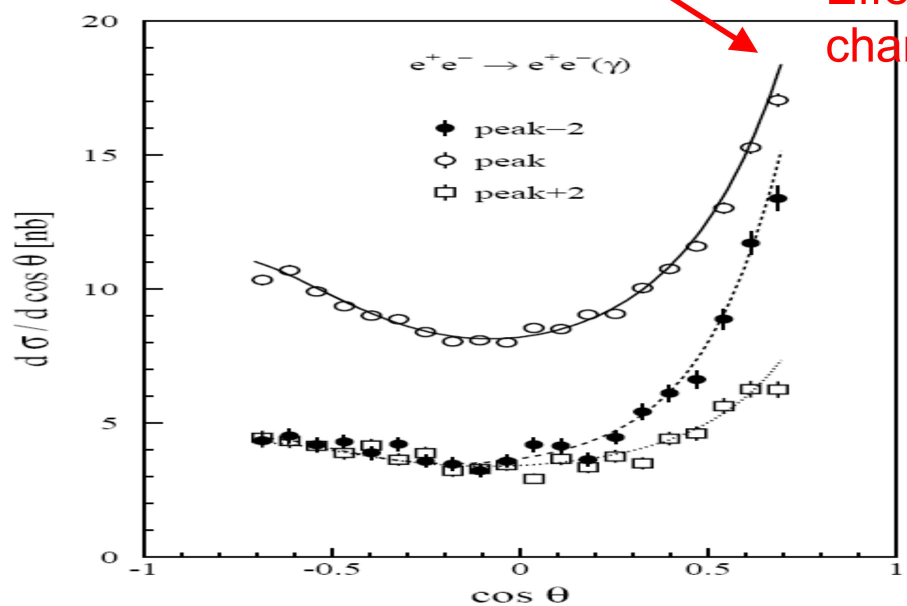
Z pole contribution.



s-channel contribution ~ $(\Gamma_e)^2$

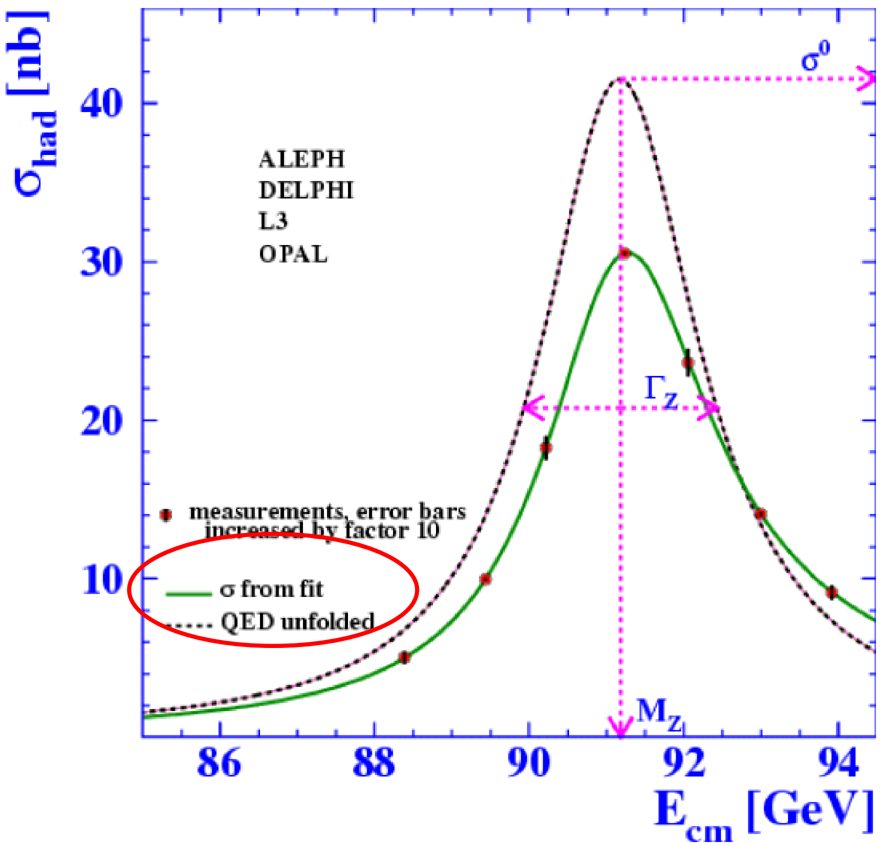
t channel contribution → forward peak





Effect of t channel

Reminder:



$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak: $$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

- Resonance position → M_Z
- Height → $\Gamma_e \Gamma_\mu$
- Width → Γ_Z

Z line shape parameters (LEP average)

M_Z	$= 91.1876 \pm 0.0021$ GeV	± 23 ppm (*)

Γ_Z	$= 2.4952 \pm 0.0023$ GeV	± 0.09 %
Γ_{had}	$= 1.7458 \pm 0.0027$ GeV	
Γ_e	$= 0.08392 \pm 0.00012$ GeV	
Γ_μ	$= 0.08399 \pm 0.00018$ GeV	
Γ_τ	$= 0.08408 \pm 0.00022$ GeV	

3 leptons are treated independently

test of lepton universality

Γ_Z	$= 2.4952 \pm 0.0023$ GeV
Γ_{had}	$= 1.7444 \pm 0.0022$ GeV
Γ_e	$= 0.083985 \pm 0.000086$ GeV

Assuming lepton universality: $\Gamma_e = \Gamma_\mu = \Gamma_\tau$

(predicted by SM: g_A and g_V are the same)

*) error of the LEP energy determination: ± 1.7 MeV (19 ppm)

http://lepewwg.web.cern.ch/

Number of light neutrinos

In the Standard Model:

$$\Gamma_Z = \Gamma_{had} + 3 \cdot \Gamma_\ell + \underbrace{N_\nu \cdot \Gamma_\nu}_{\text{invisible}:\ \Gamma_{inv}} \rightarrow \begin{cases} e^+ e^- \to Z \to \nu_e \bar{\nu}_e \\ e^+ e^- \to Z \to \nu_\mu \bar{\nu}_\mu \\ e^+ e^- \to Z \to \nu_\tau \bar{\nu}_\tau \end{cases}$$

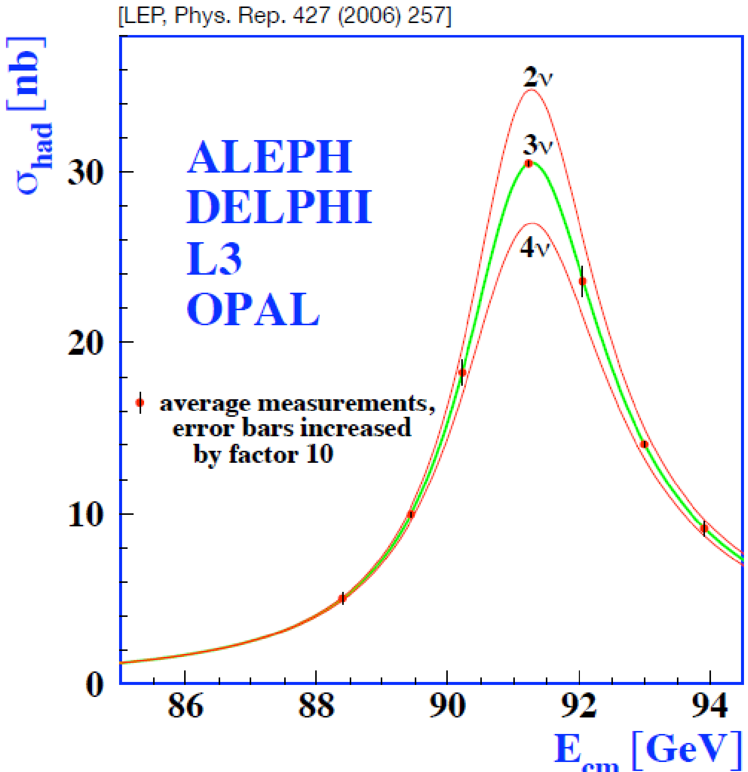
$$\Gamma_{inv} = 0.4990 \pm 0.0015 \text{ GeV}$$

To determine the number of light neutrinos:

$$N_\nu = \frac{\Gamma_{inv}}{\Gamma_{\nu,SM}} = \underbrace{\left(\frac{\Gamma_{inv}}{\Gamma_\ell}\right)_{exp}}_{5.9431 \pm 0.0163} \cdot \underbrace{\left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)_{SM}}_{=1/1.991 \pm 0.001}$$

5.9431±0.0163

=1/1.991±0.001 (small theo. uncertainties from m_{top} M_{H})



$$N_\nu = 2.9840 \pm 0.0082$$

No room for new physics: $Z \not\rightarrow$ new invisible particles