Casimir Force: Measurements

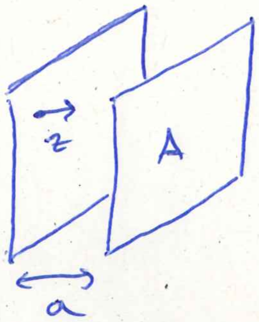
Theory reminder:

$$\boxed{\frac{F(a)}{A}} = \frac{\pi^2}{240}\frac{\hbar c}{a^4} \approx \underbrace{0.013 \overbrace{\frac{\text{dyn}}{\text{cm}^2}}^{10\,\mu\text{N}}}_{1.3\times10^{-7}\ \text{N/cm}^2} \cdot \frac{(\mu\text{m})^4}{a^4}$$

pressure

1.3×10^{-7} N/cm² vs. 1 atm ~ 10 N/cm²

$\Rightarrow 1.3\times10^{-5}$ mbar for 1 μm separation ($A = 1$ cm²)



- zero point energy
- vacuum fluctuations
- real photon not needed ($T=0$)

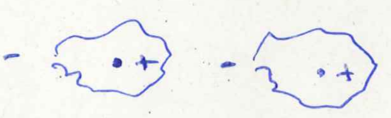
| 2005: >1000 theory
few ~10 expt |

* QED affects macroscopic classical objects

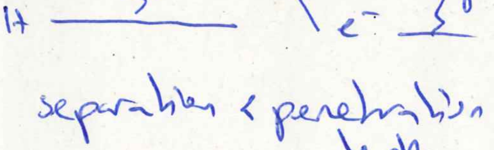
Casimir's "trick": introduce a cutoff

Note that we can view van der Waals, Casimir, Casimir-Polder as specific limits of a unified theory ("dispersion forces")

| van der Waals | (also London)



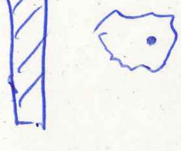
Å/nm-range, rel. retardation not important



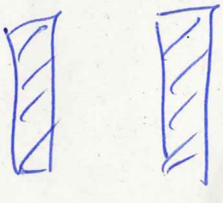
separation < penetration depth

polarizable atom/molecule induced dipole-dipole always attractive

| Casimir-Polder |



| Casimir |



larger separations, relativistic retardation is important ⇒ both QM and rel.

can also be repulsive → geom.-dependent

virtual photons do not reach 2nd body but QM fluctuations are correlated:

$$\langle E_k(t,\vec r_{1,2})\rangle = 0 \quad \text{but} \quad \langle E_k(t,\vec r_1)E_k(t,\vec r_2)\rangle \neq 0$$

Note Casimir imagined an electron w/ this force balancing electrostatic repulsion ⇒ falsified by experiment

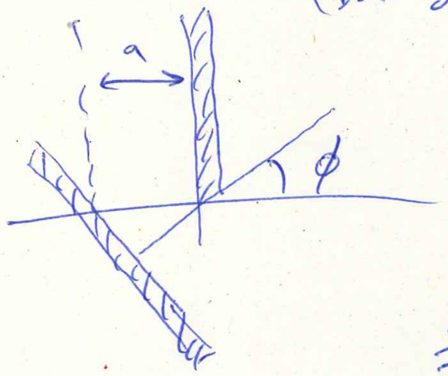
Lifschitz theory: (1956) considers dispersion forces from fluctuations in <u>dissipative</u> media. I.e., E-field fluctuates w/in and outside any medium.

Fluctuation-Dissipation theorem: relates fluctuation spectrum (at equilibrium) to the medium's generalized susceptibility (response to external influences).

<u>First experiments:</u>

		error
1958,	Sparnaay (parallel plates)	~100%
1997,	Lamoreaux (plate-sphere)	5-10%
1998,	Mohideen + Roy (plate-sphere)	1%

<u>Sparnaay</u>: "...do not contradict Casimir's theoretical prediction" (but does rule out his model of the electron)



<u>Alignment</u> was major challenge

$$\frac{E}{A} = -C\,\frac{\hbar c}{a^3} \quad \text{at } \phi = 0$$

$$\frac{\pi^2}{720} = C \approx 0.014 \longrightarrow \frac{C(\phi)}{\sin\phi} \quad \text{for } \phi \neq 0$$

$C(\phi) \lesssim 0.007 - 0.012$

Effective distance is <u>also</u> a problem: $F \sim \frac{1}{a^4}$

$\Rightarrow$ dominated by zones of closest approach

also hard to know what the precise distance <u>is</u>

Measurement: capacitance, potential $V \Rightarrow \frac{E}{A} = \frac{1}{2}\left(\frac{C}{A}\right)V^2$

small attractive force $\Rightarrow$ increase C $\quad = \frac{1}{2}\left(\frac{\epsilon_0}{a}\right)V^2$

Geometrical effects depend on surface energy

$\Rightarrow$ similar for Casimir + electrostatic

Surface roughness, let
$$\left.\begin{aligned} z_1 &= A_1 f_1(x_1, y_1) \\ z_2 &= a + A_2 f_2(x_2, y_2) \end{aligned}\right\} \max|f_i(x_i, y_i)| = 1$$

expansion in $\frac{A_i}{a}$ can treat roughness as small angles:

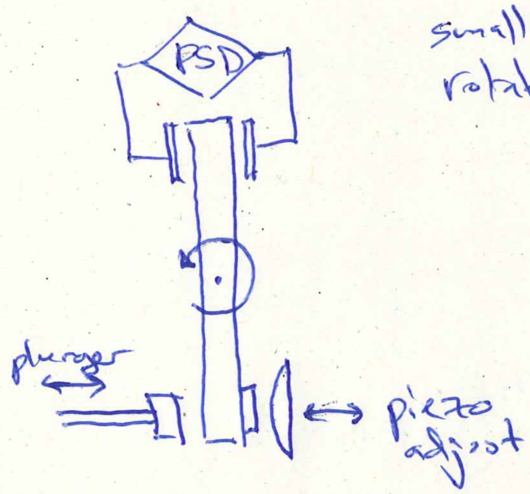
$$\frac{F(a)}{A} \times \Big(1 + \underbrace{\frac{10}{a^2}\left[A_1^2\langle f_1^2\rangle + A_2^2\langle f_2^2\rangle - 2\langle f_1 f_2\rangle A_1 A_2\right]} + \mathcal{O}\big(\big(\tfrac{A_i}{a}\big)^3\big)\Big)$$

small angle α: $1 + \frac{10}{3}\left(\frac{\alpha L}{a}\right)^2 + 7\left(\frac{\alpha L}{a}\right)^4 + \dots$ where $2L$ is the plate's characteristic length

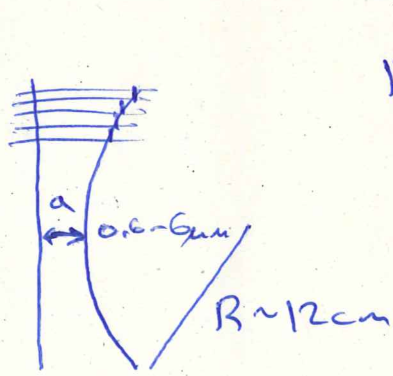
Practical challenges:
- surface potentials/static charge
- thin films (insulating oxides)
- roughness, dust
- anharmonic vibrations
- hysteresis

Note: conductive plates $\Rightarrow$ optical alignment methods problematic

Lamoreaux: torsion pendulum, with one curved plate



small Casimir force produces torque $\Rightarrow$ pendulum rotates to finite angle
- adjust the offset voltage to balance the dual capacitor $\Rightarrow$ compensate torque
- measurement records offset voltage (capacitors compared/balanced via phase-sensitive AC circuit)



Highly tolerant of misalignment but $\frac{F(a)}{A}$ no longer uniform $\Rightarrow$ calculate total

Proximity Force Approximation ("theorem")
calculate total force as sum over infinitesimal parallel plates

$$\frac{F(a)}{A} \to \int r\,dr\,d\phi\,\frac{F(a)}{dA} \Rightarrow F_{tot} = \underbrace{2\pi r}_{\text{factor from geom. curvature}} \cdot \underbrace{\frac{\pi^2}{720}\frac{\hbar c}{a^3}}_{E/A \text{ for } \parallel \text{ plates}}$$

This works surprisingly well, but with the disadvantage that errors are not reliably evaluated

- naively expect deviations of order $\frac{a}{R} \sim 10^{-5}$ (here) but need exact/numerical calculation to confirm (may not be available) have this, e.g. for plate-cylinder

Finite-temperature: large ($\sim 100\%$) correction!

$$k_B T \cdot \frac{a}{\hbar c} \approx 0.126 \frac{a}{\mu m} \text{ at } 300K \begin{cases} \leq \frac{1}{2} \Rightarrow a \lesssim 4\mu m \\ > \frac{1}{2} \Rightarrow a > 4\mu m \end{cases}$$

$$F_{tot} \to F_{tot} \times \begin{cases} 1 + \frac{720}{\pi^2}\left(\frac{k_B T a}{\hbar c}\right)^3 \frac{\zeta(3)}{2\pi} - 16\left(\frac{k_B T a}{\hbar c}\right)^4 + \mathcal{O}(a^5) \\ \frac{720}{\pi^2}\left(\frac{k_B T a}{\hbar c}\right)\frac{\zeta(3)}{8\pi} + \mathcal{O}(a^{-1}) \end{cases} \qquad \zeta(3) \approx 1.2$$

$$= \begin{cases} \frac{\pi^3 R}{360} \frac{\hbar c}{a^3} + \underbrace{R\zeta(3)\frac{(k_B T)^3}{\hbar^2 c^2}}_{\text{indp of } a} - \underbrace{\frac{2\pi^3 R}{45} a \cdot \frac{(k_B T)^4}{\hbar^3 c^3}}_{\text{linear in } a} + \mathcal{O}(a^2) \\ \underbrace{\frac{R}{4a^2} k_B T \zeta(3)}_{\text{indp of } \hbar c} \end{cases}$$

cf. long-wavelength limit (R-J) of blackbody

Finite conductivity: recall $\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \Rightarrow$ 10-20% correction

plasma frequency in metals, $\omega_p = \frac{2\pi c}{\lambda_p}$ and for $\left(\frac{\lambda_p}{a}\right)^2 \ll 1$

$$F_{tot} \to F_{tot} \times \underbrace{\left(1 - \frac{2\lambda_p}{\pi a}\right)}_{\equiv \eta} = F_{tot} \times \left(1 - \frac{4c}{a\omega_p}\right)$$

In reality the correction to $F(a)$ varies more slowly
$\Rightarrow$ use experimental data for complex index of refraction

Indistinguishable from a calibration error!

For <u>ideal</u> metal plates

at smallest separation, $\eta(0.6\,\mu m) \approx 0.78$ (700nm Au)
0.79 Cu

under if Au diffused into Cu, or film was thinner

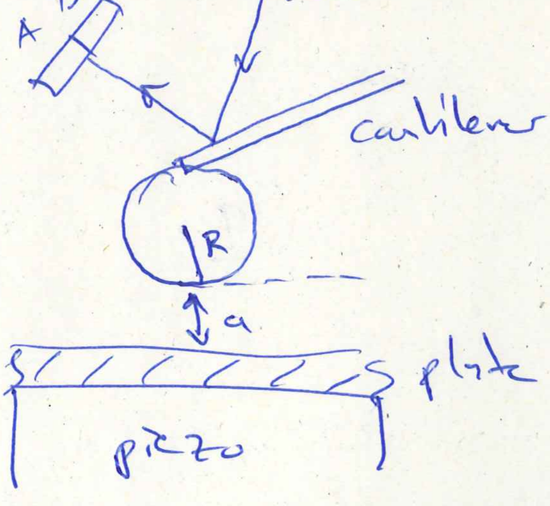
<u>Problems</u>: large residual potential offset, electrostatic force

data not accurate enough to demonstrate finite-T correction

best agreement around 1μm
small thermal correction
skin effect and roughness partially compensate.

measured force (Casimir) up to ~100 μdyn ≈ 1000 pN

<u>Mohideen + Roy</u> AFM: atomic force microscope



bending of cantilever
⇒ laser beam deflects, split photodiode signal

measured a ~100nm – 1μm
w/ R = 200μm
⇒ forces up to 120pN at closest approach

Required corrections:

- finite conductivity
- roughness
- temperature

1.6 pN deviation (rms) overall, but individual corrections w/ both signs and size up to 48 pN