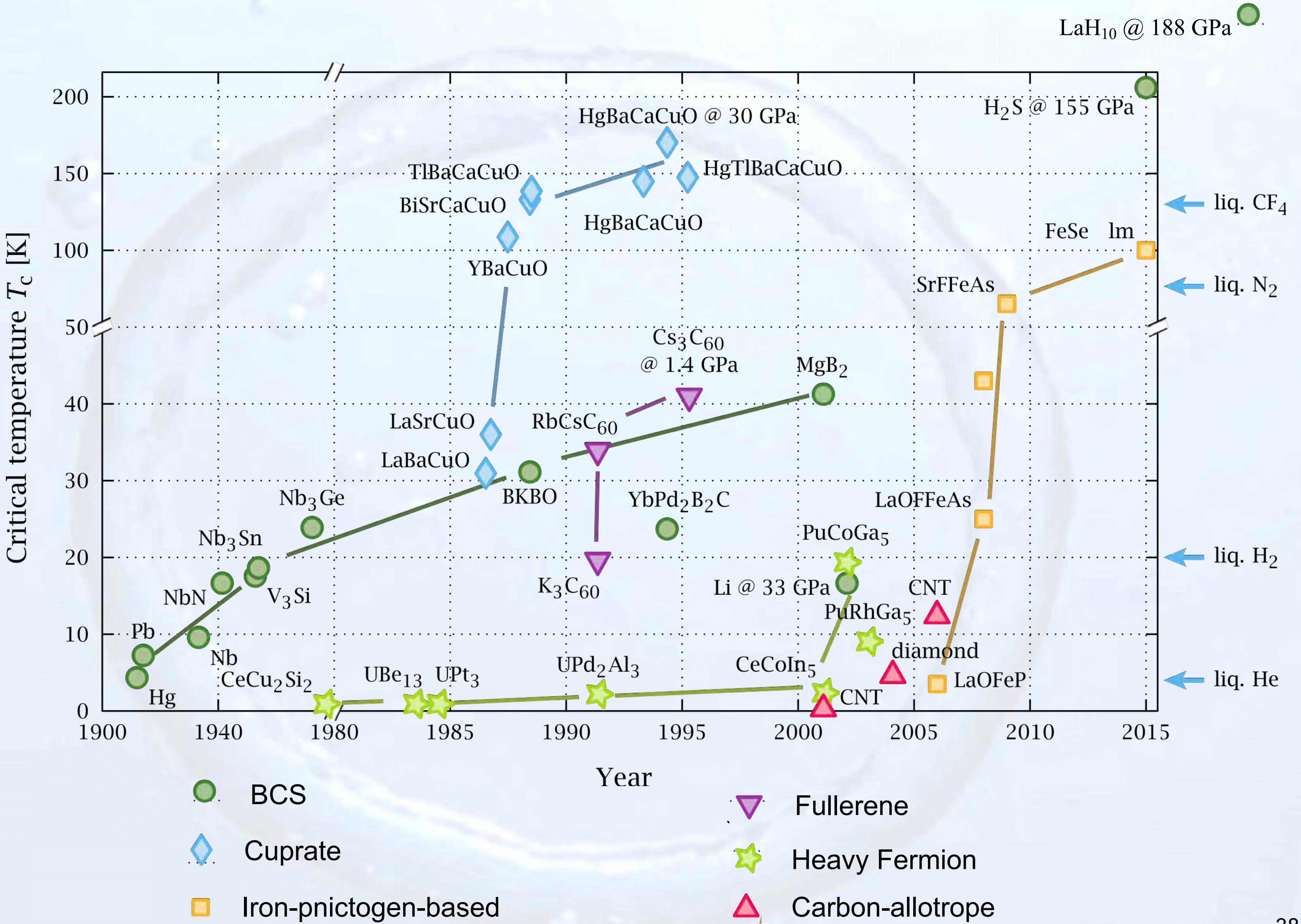
Bardeen – Josephson Debate

The Nobel Laureate Versus the Graduate Student

In a recent note, Josephson uses a somewhat similar formulation to discuss the possibility of superfluid flow across the tunneling region, in which no quasi-particles are created. However, as pointed out by the author [Bardeen, in a previous publication], pairing does not extend into the barrier, so that there can be no such superfluid flow.

Physics Today **54,** 46-51 (2001)

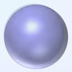
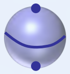
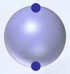
10.4 Unconventional Superconductors



10.4 Unconventional Superconductors

Quantum states

		S	L
s–wave	conventional superconductors	0	0
p–wave	(Sr_2RuO_4)	1	1
d–wave	cuprate high-T_c superconductors	0	2
f–wave	(UPt_3)	1	3

energy gap $\Delta_{\boldsymbol{k}} = \Delta_0(T) f(\widehat{\boldsymbol{k}})$		nodes	$f(\widehat{\boldsymbol{k}})$
conventional superconductors	isotropic		1
cuprate high-T_c superconductors	2D		$k_x^2 - k_y^2$
Sr_2RuO_4	axial 2D		$(\widehat{k}_x^2 - \widehat{k}_y^2)(\widehat{k}_x + \mathrm{i}\widehat{k}_y)$
UPt_3	3D		$\widehat{k}_z(\widehat{k}_x + \mathrm{i}\widehat{k}_y)^2$
UBe_{13}	axial 3D		$\widehat{k}_x + \mathrm{i}\widehat{k}_y$

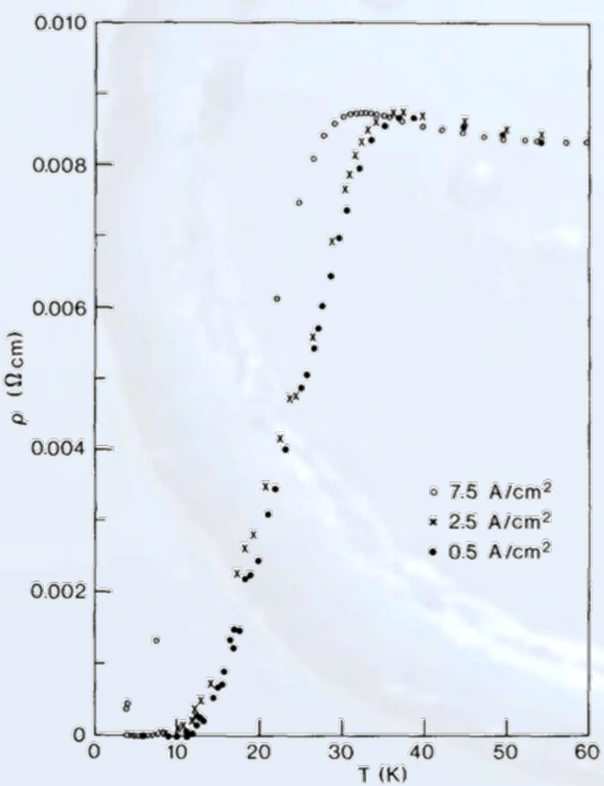
10.4 Unconventional Superconductors

cuprate high-T_c superconductors

discovered June 1986

Possible High T_c Superconductivity in the Ba – La – Cu – O System

J.G. Bednorz and K.A. Müller
IBM Zürich Research Laboratory, Rüschlikon, Switzerland



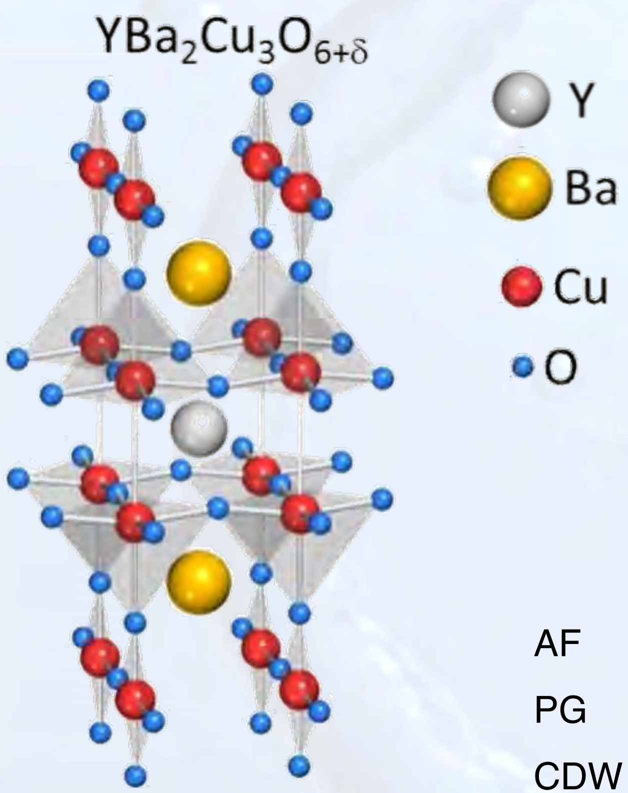
Georg Bednorz Karl Alexander Müller

APS March Meeting of 1987
The "Woodstock of Physics“ Hilton Hotel New York

10.4 Unconventional Superconductors

best investigated system: YBCO   Yttrium barium copper oxide

important are the Cu-O planes ⟶ different phases depending on hole concentration introduced by oxygen surplus



perovskite structure

$0 < \delta < 0.4$	insulator
$\delta \approx 0.4$	insulator-metal transition
$\delta > 0.4$	superconductor
$\delta = 0.92$	superconductor with T_c = 95 K

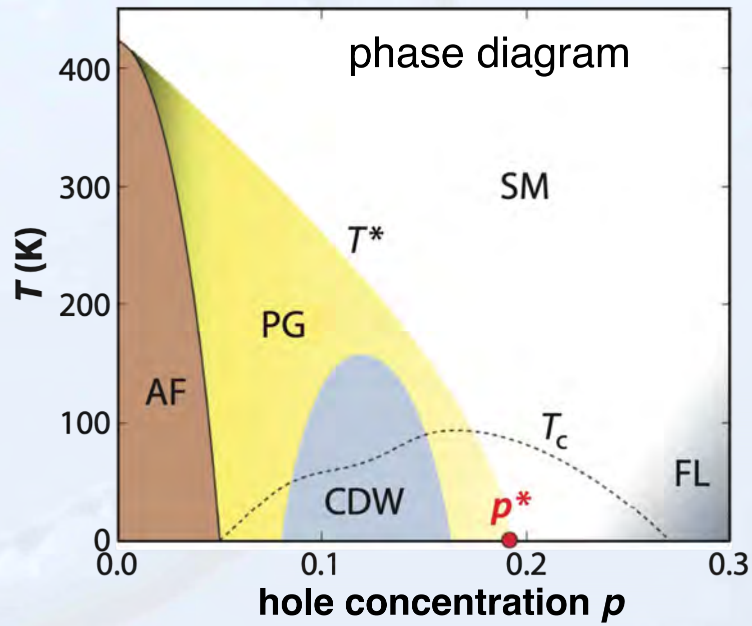
AF   anti-ferromagnetic phase

PG   pseudo gap

CDW charge density wave

SM   strange metal

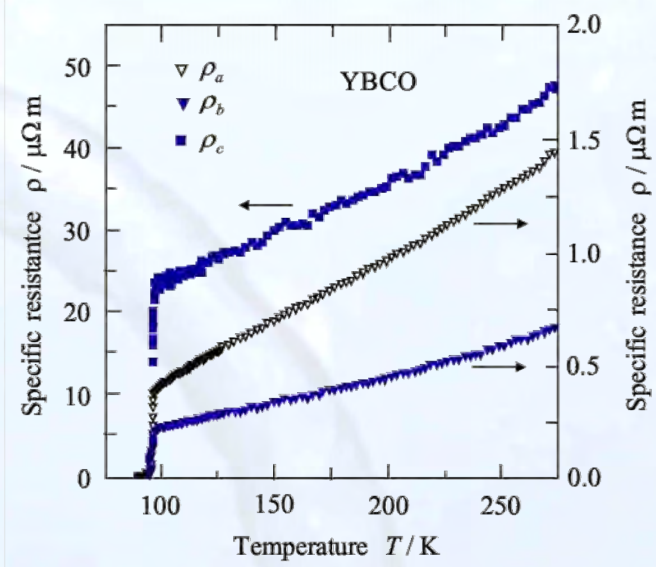
FL   Fermi liquid phase



10.4 Unconventional Superconductors

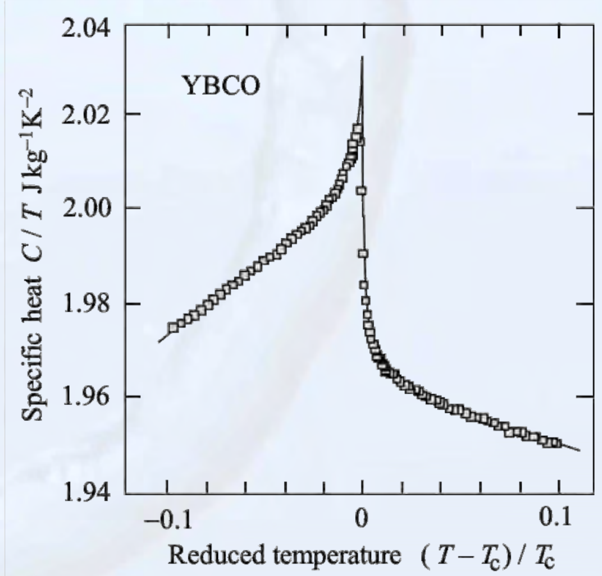
superconducting transition

a) resistance



- resistivity depends on crystal direction
- in *c*-direction (perpendicular to CuO_2 planes) resistivity is much higher

b) specific heat



- second order phase transition
- rounded onset of transition caused by large fluctuation

$\longrightarrow$ indicates short coherence length

10.4 Unconventional Superconductors

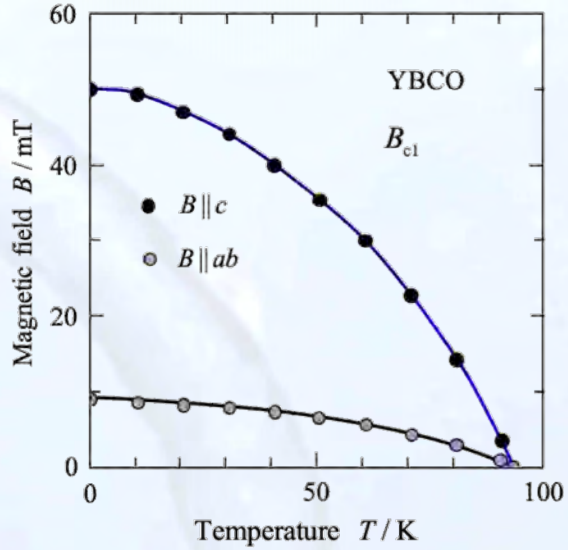
Critical fields

$$B_{c1} = \frac{\Phi_0}{4\pi\lambda_L^2}$$

- B_{c1} depends on crystal direction
- B_{c1} is very small at $T = 0$

→ $\lambda_L = 150$ nm → factor 10 larger as for Al

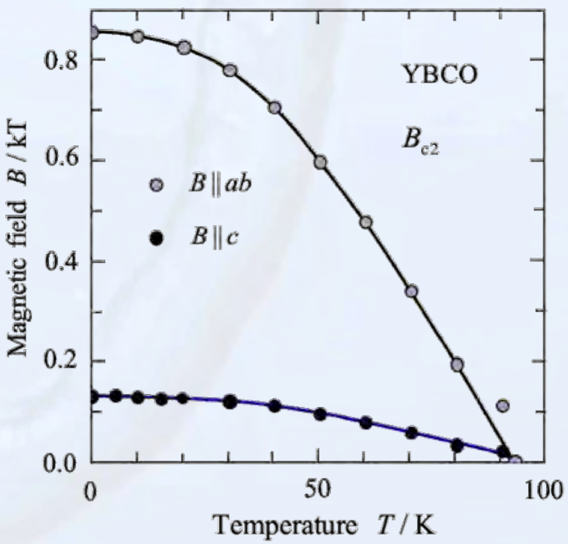
→ $\lambda_L^2 \propto 1/n_s$ → factor 100 less Cooper pairs



$$B_{c2} = \frac{\Phi_0}{2\pi\xi_{GL}^2}$$

- B_{c2} depends on crystal direction
- B_{c2} is very large at $T = 0$ → $B_{c2} >$ 800 T for $B \| ab$

→ very short coherence length $\xi_{GL} = 1.5$ nm



→ $\lambda_L \gg \xi_{GL}$ → extreme type II superconductor

10.4 Unconventional Superconductors

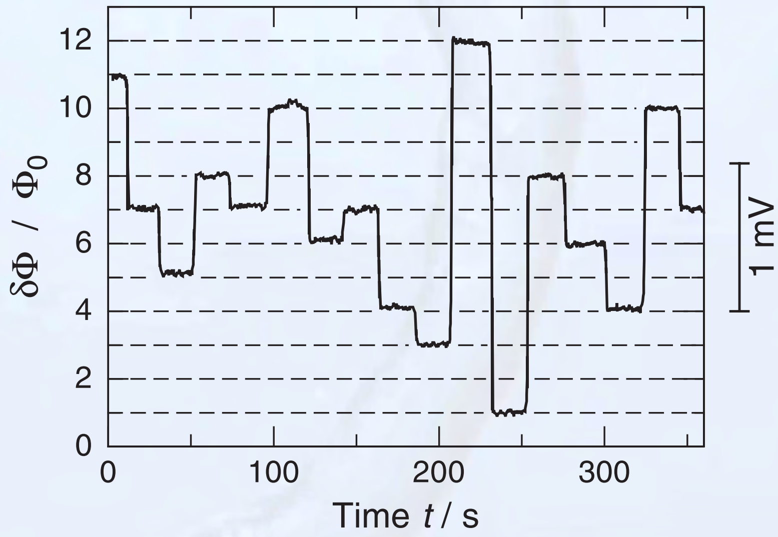
Experimental determination of *d*-wave nature of Cooper pairs inYBCO

→ flux quantization

→ Josephson effect

a) flux quantization

normal geometry -- YBCO ring

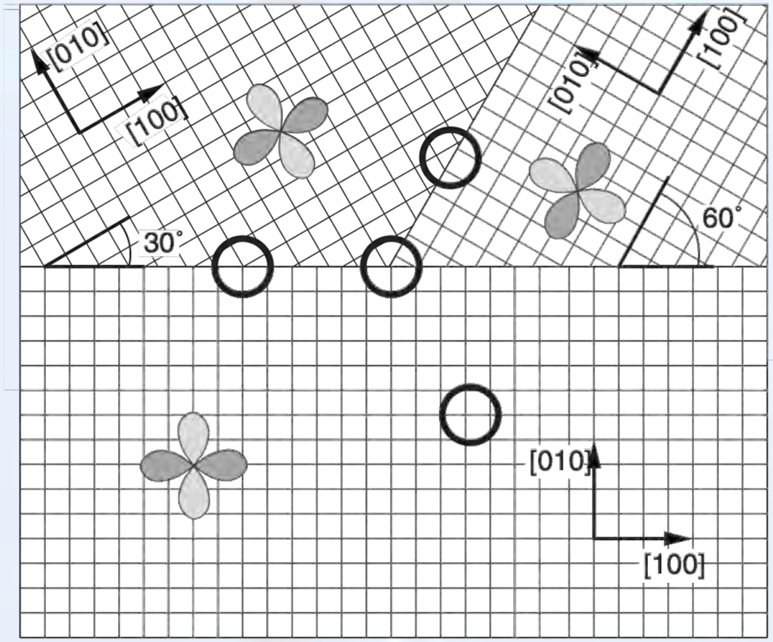


- flux quantization measured with SQUID
- result: Φ_0 = h/2e

Experimental determination of *d*-wave nature of Cooper pairs in YBCO

unconventional flux quantization

tri-crystalline rings



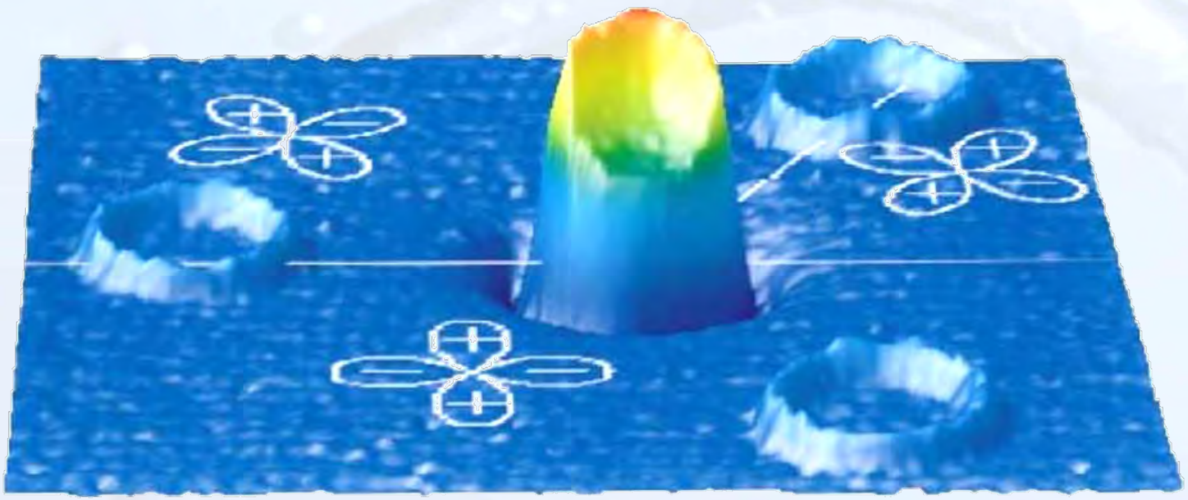
- YBCO film is grown epitaxially on $SrTiO_3$ substrate
- $SrTiO_3$ is suitably tailored having 3 crystal orientations
- YBCO is patterned by ion milling to produces rings
- rings are positioned that they contain grain boundaries
- macroscopic quantum states in different parts of the rings have different orientations
- the grain boundaries produce π junctions

0 or even number of π junctions $\Phi = n\Phi_0$

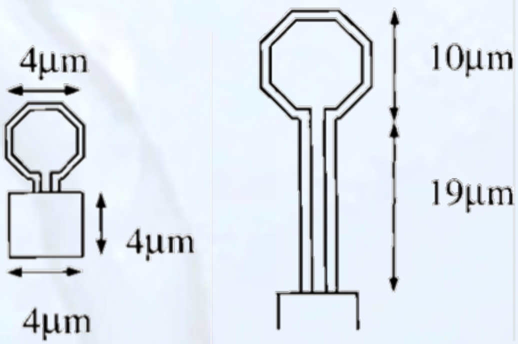
odd number of π junctions $\Phi = \left(n + \frac{1}{2}\right)\Phi_0$ $\longrightarrow$ half-flux quantum

10.4 Unconventional Superconductors

Experimental result

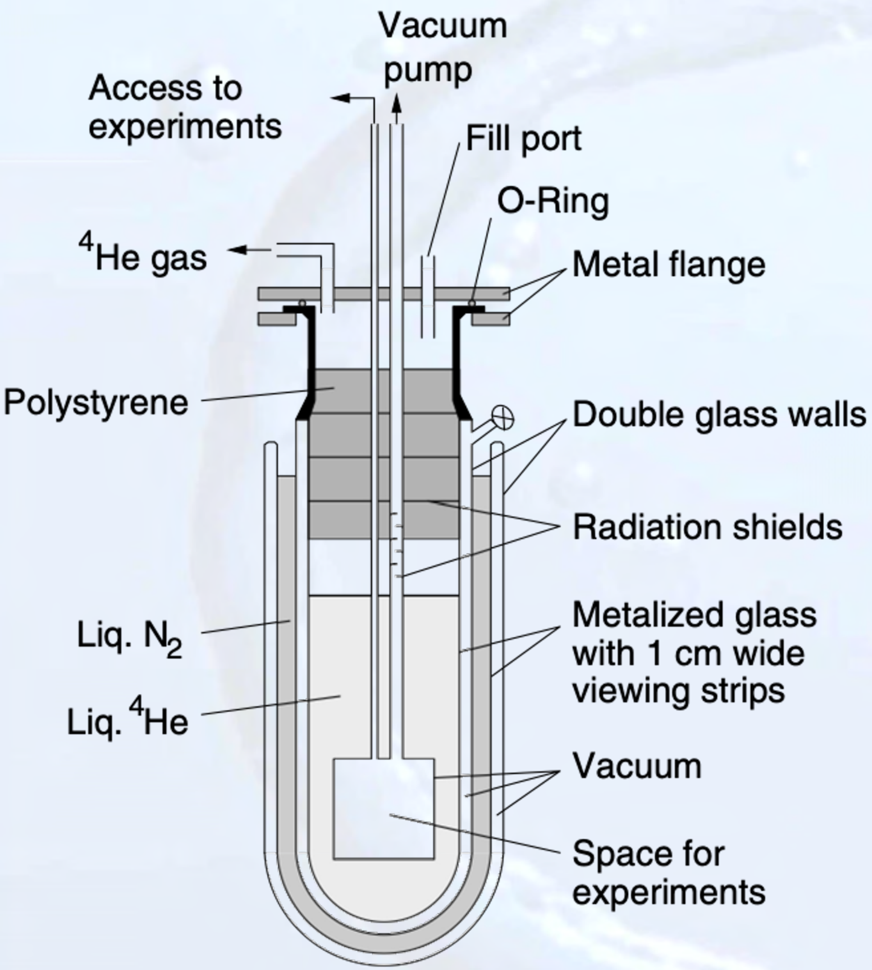


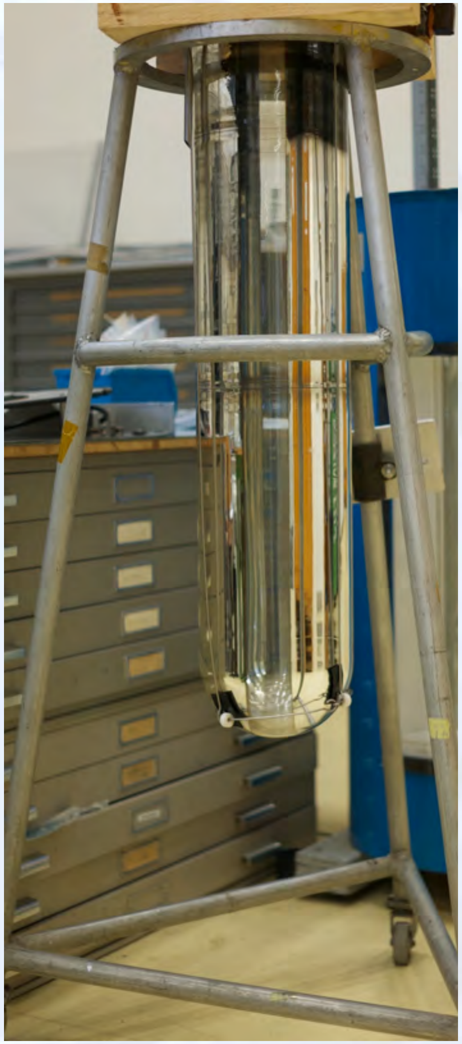
scanning SQUID microscopy



- rings with even number of π junctions show no flux
- ring in the middle with 3 π junctions shows spontaneous formation of half-flux quantum

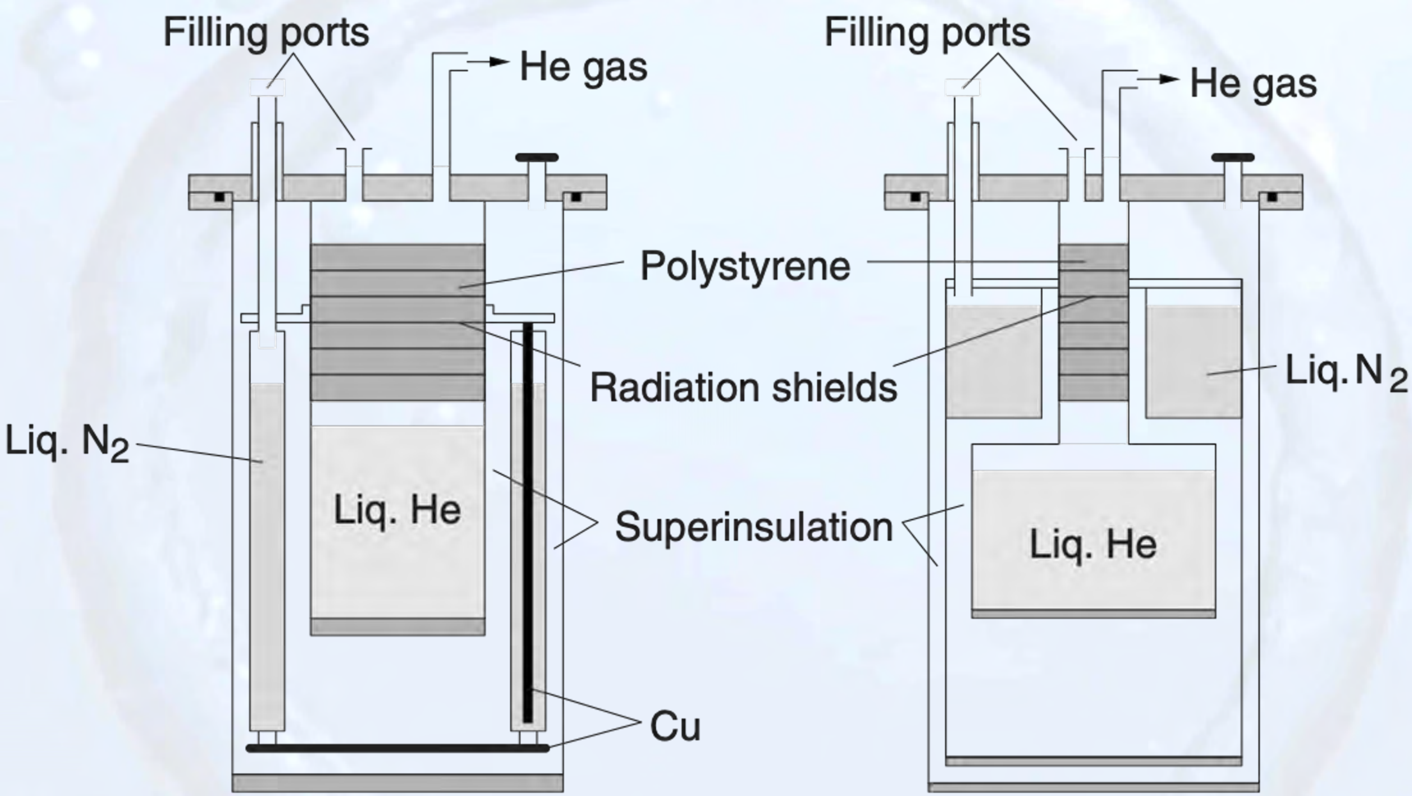
^{4}He bath cryostat: glass dewar





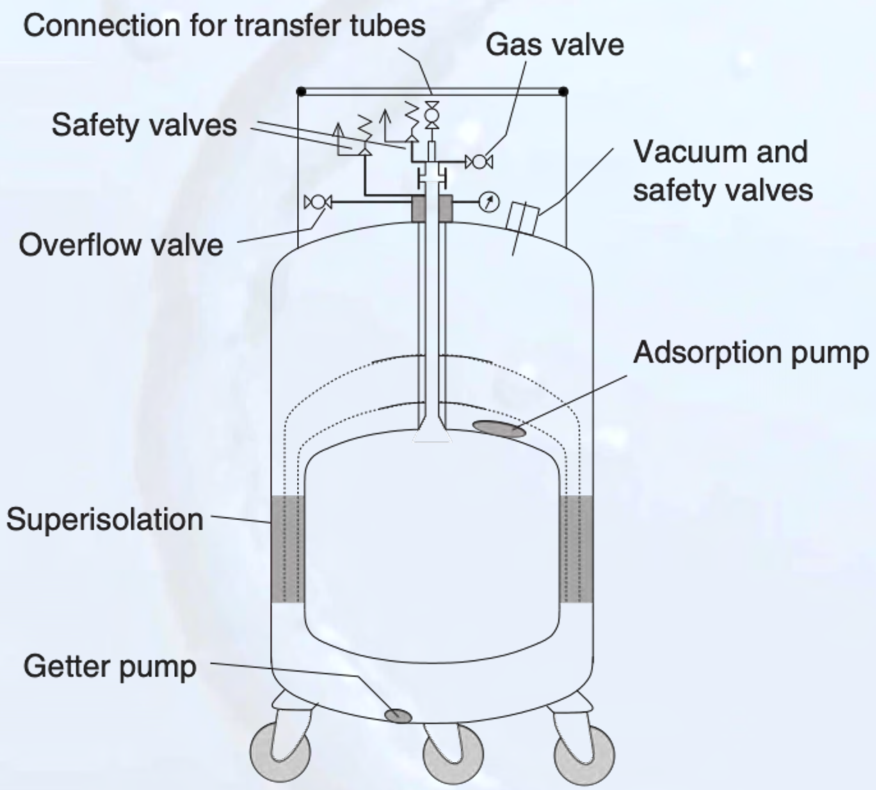
11.1 Bath Cryostats

^{4}He Bath cryostat: metal dewar

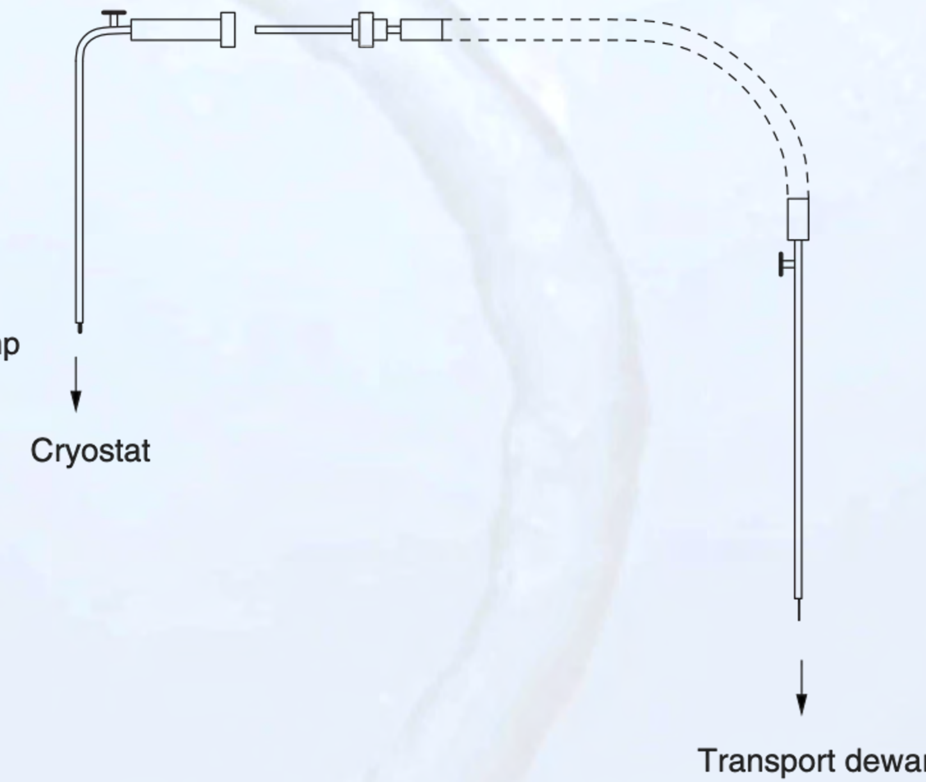


11.1 Bath Cryostats

helium transport vessel



helium transfer tube



11.1 Bath Cryostats

Radiation shields – super insulation

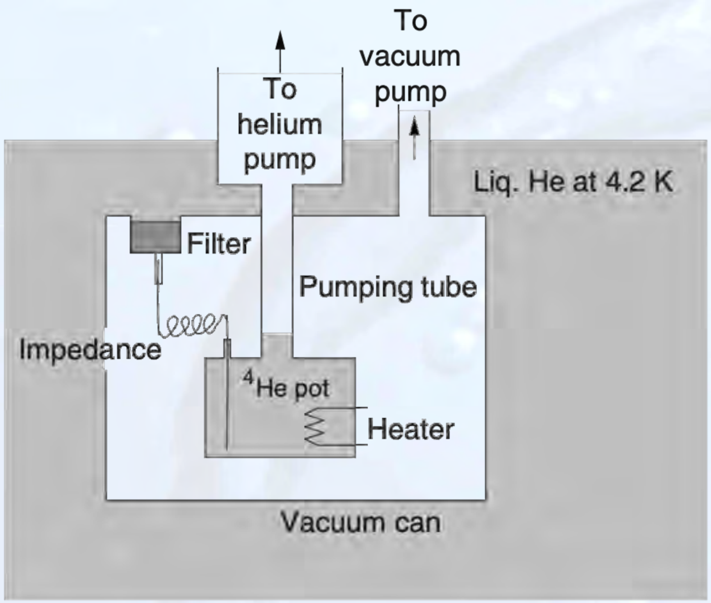
multiple radiation shields → smaller steps → reduction of heat flow

30 to 80 layers of low conductivity high reflection material → aluminized Mylar

apparent thermal conductivity
$\sim 10^{-4}$ to 10^{-5} W/(m K)

11.1 Bath Cryostats

Cryostats with 1-K-Pot



Vapor pressure curve of various cryogenic liquids

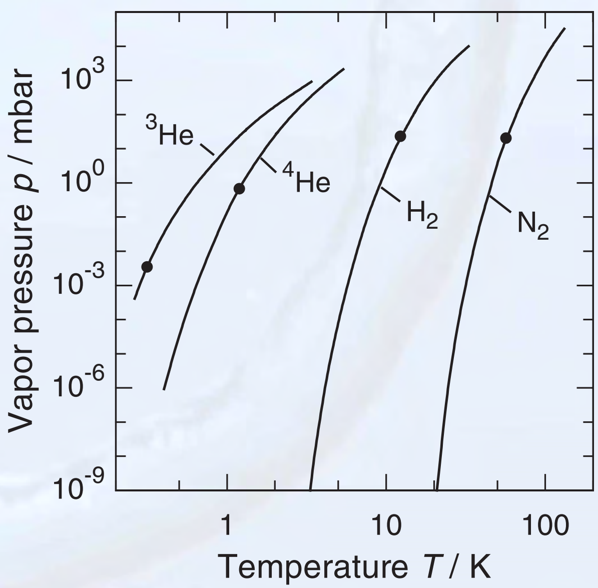
Clausius-Clapeyron equation

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{\Delta V\, T}$$

$$\Delta V = V_g - V_\ell \approx V_g \qquad pV_g = RT$$

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{RT^2}\, p \quad \longrightarrow \quad \boxed{p(T) = p_0\, \mathrm{e}^{-L/RT}}$$

vapor pressure curve

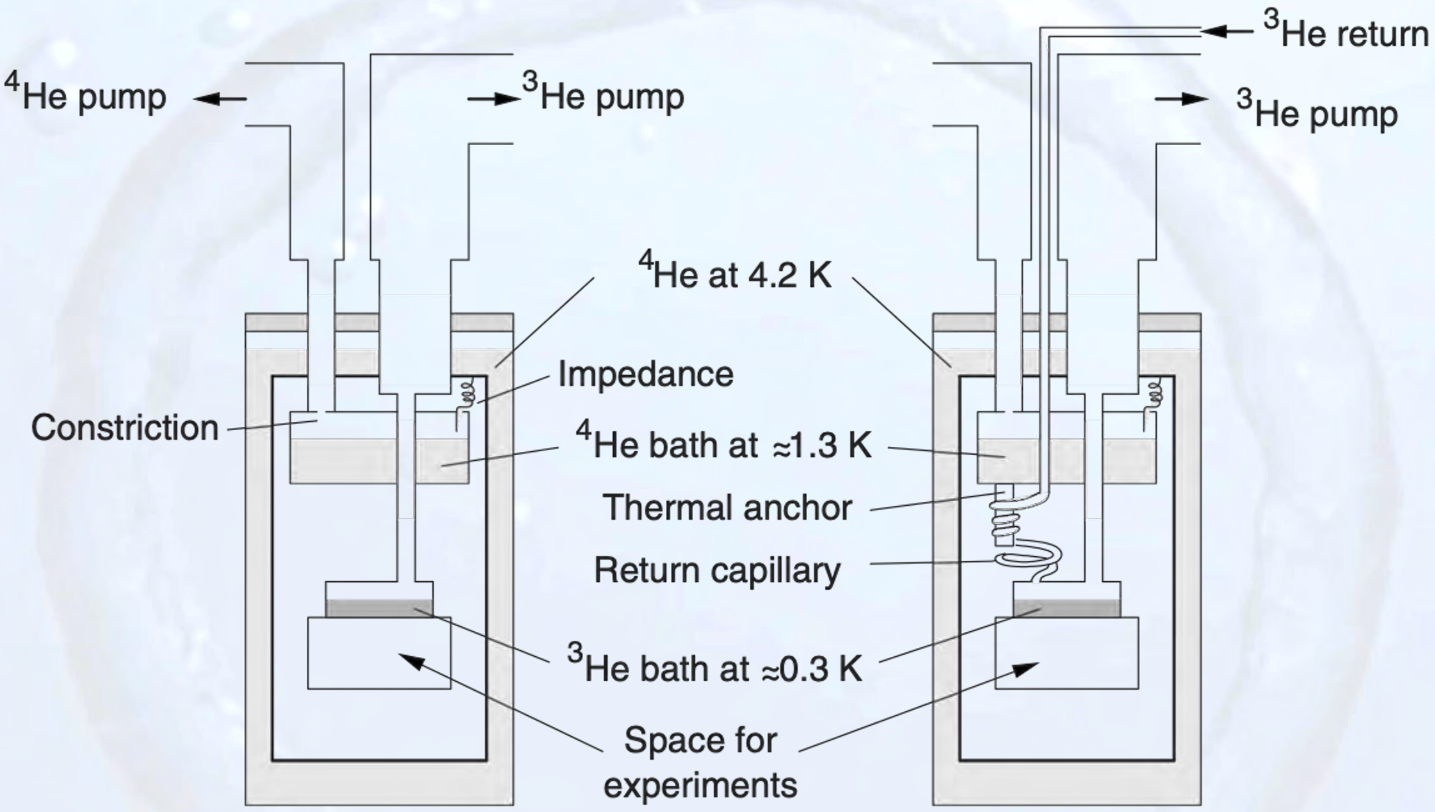


$$^{4}\mathrm{He} \quad L = 90\,\mathrm{J\,mol^{-1}}$$

$$^{3}\mathrm{He} \quad L = 40\,\mathrm{J\,mol^{-1}}$$

11.1 Bath Cryostats

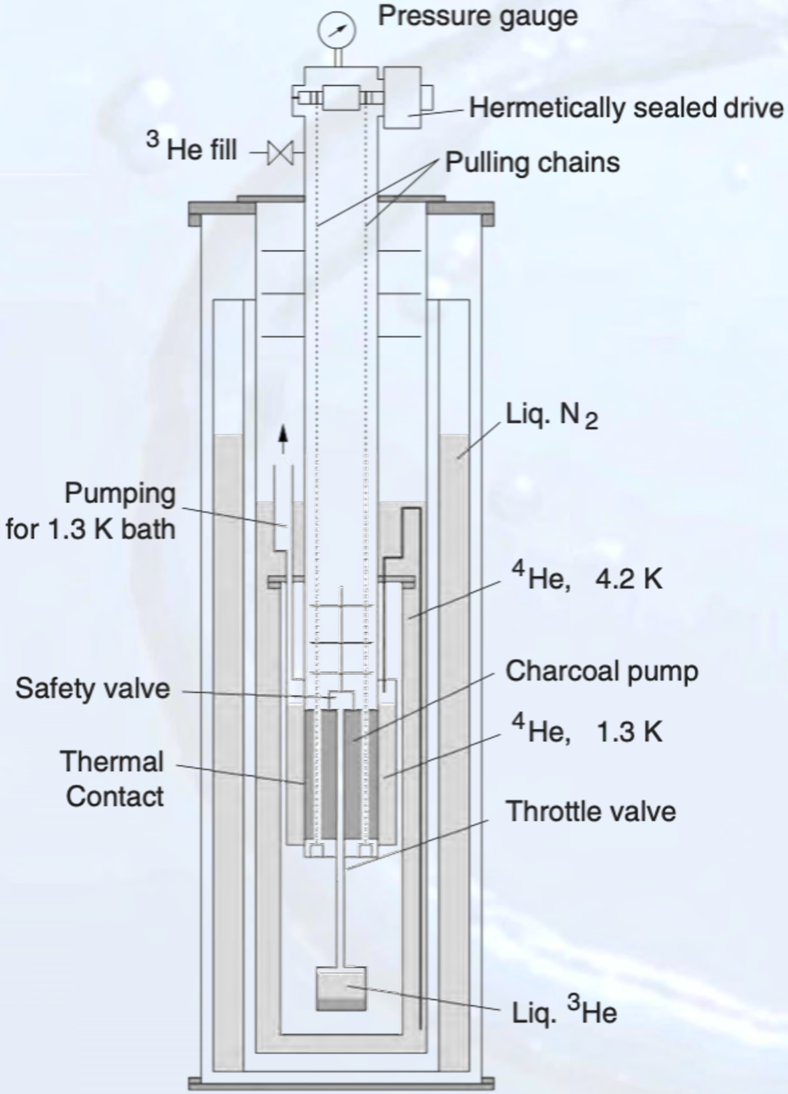
^{3}He cryostats

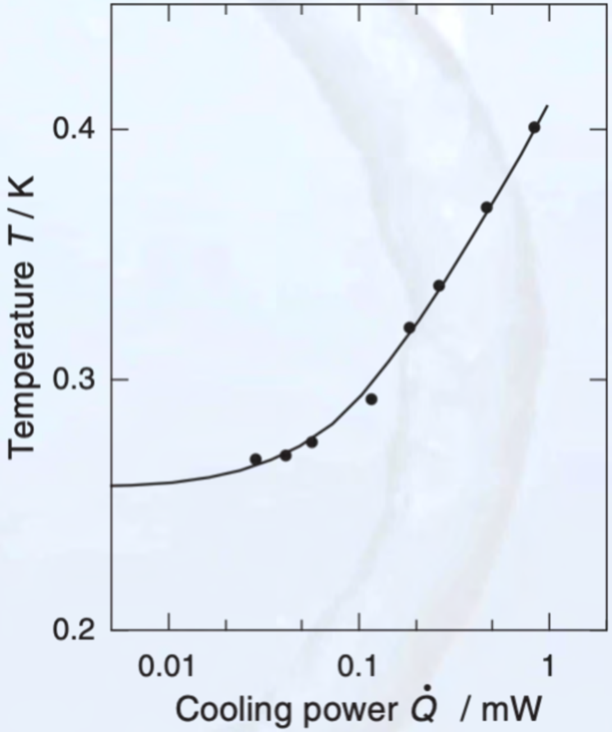


cooling power $\dot{Q} = \dot{n}_{\mathrm{g}}\, L \propto p \propto \mathrm{e}^{-L/RT}$

11.1 Bath Cryostats

Cooling power of a ^{3}He cryostat with charcoal absorption pump





11.2 Dilution Refrigerators

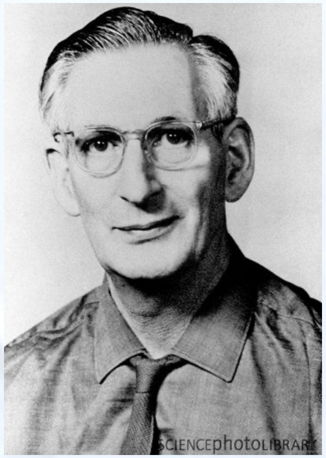
History

1951 basic idea suggested by Heinz London

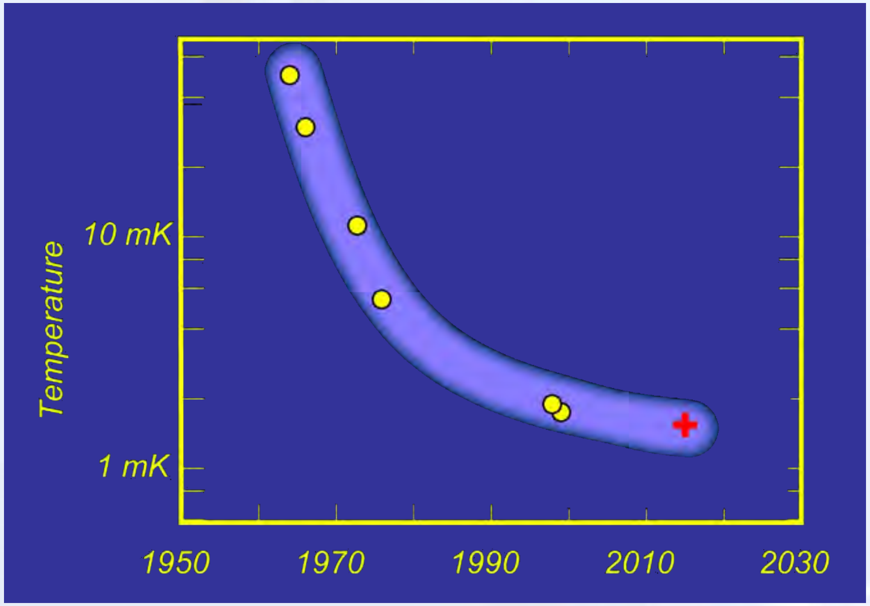
1962 detailed concept worked out by London, Clark, Mendoza

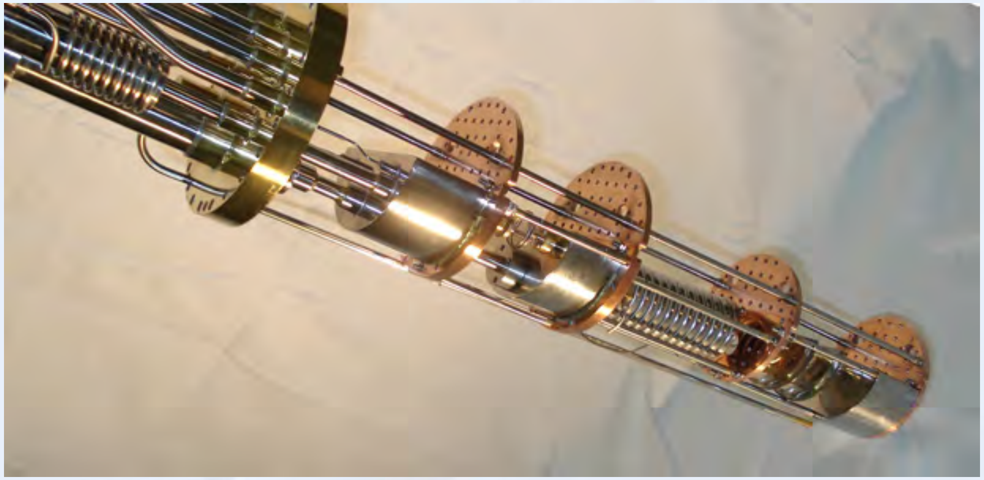
1965 first realization Das, De Bruyn Ouboter, Taconis T_{min} = 220 mK

1999 lowest temperature obtained , J.C. Cousins *et al.* T_{min} = 1.75 mK

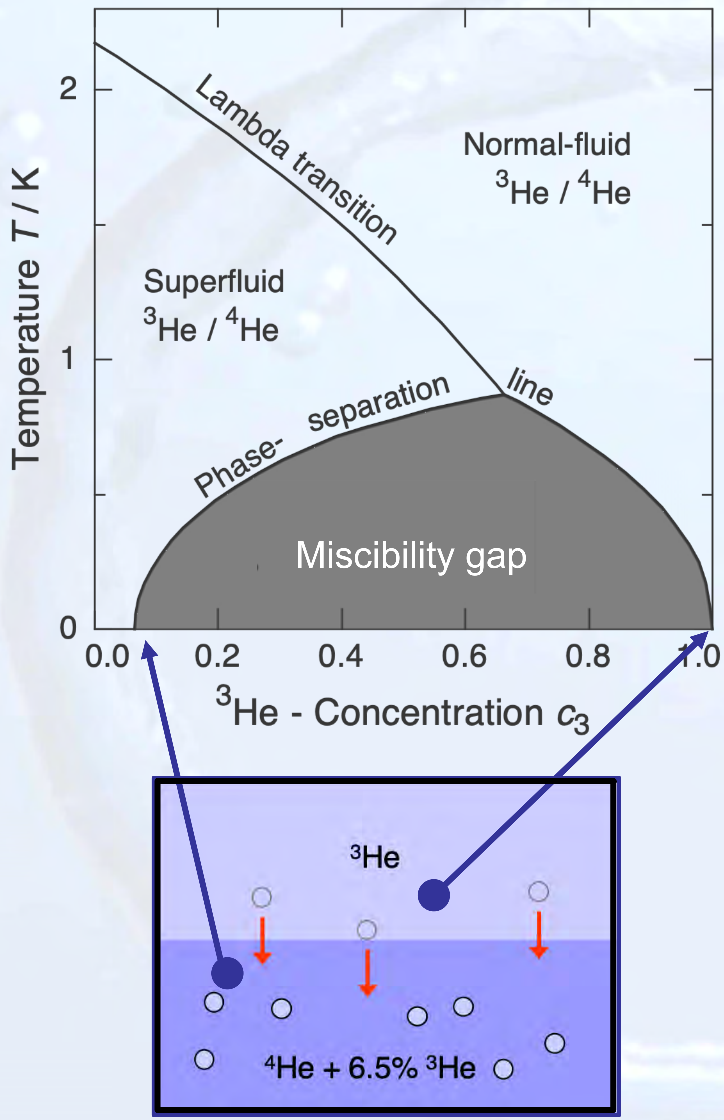


Heinz London





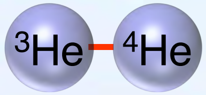
11.2 Dilution Refrigerators



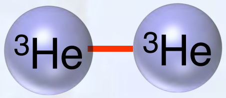
occurrence of miscibility gap

but 6.5 % ^{3}He in ^{4}He at $T = 0$ K

reason:
zero-point motion weakens binding



stronger binding

as 

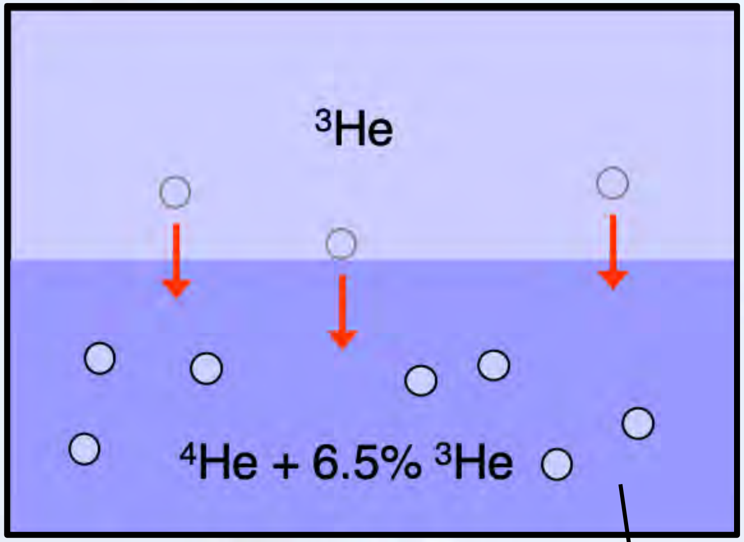
but: Fermi energy

max. 6.5% ^{3}He in ^{4}He at $T = 0$ K

11.2 Dilution Refrigerators

principal of cooling by mixing ^{3}He/^{4}He

- transition of ^{3}He into the ^{4}He rich phase
- cooling by „evaporation“ of ^{3}He into ^{4}He quasi vacuum



superfluid ^{4}He

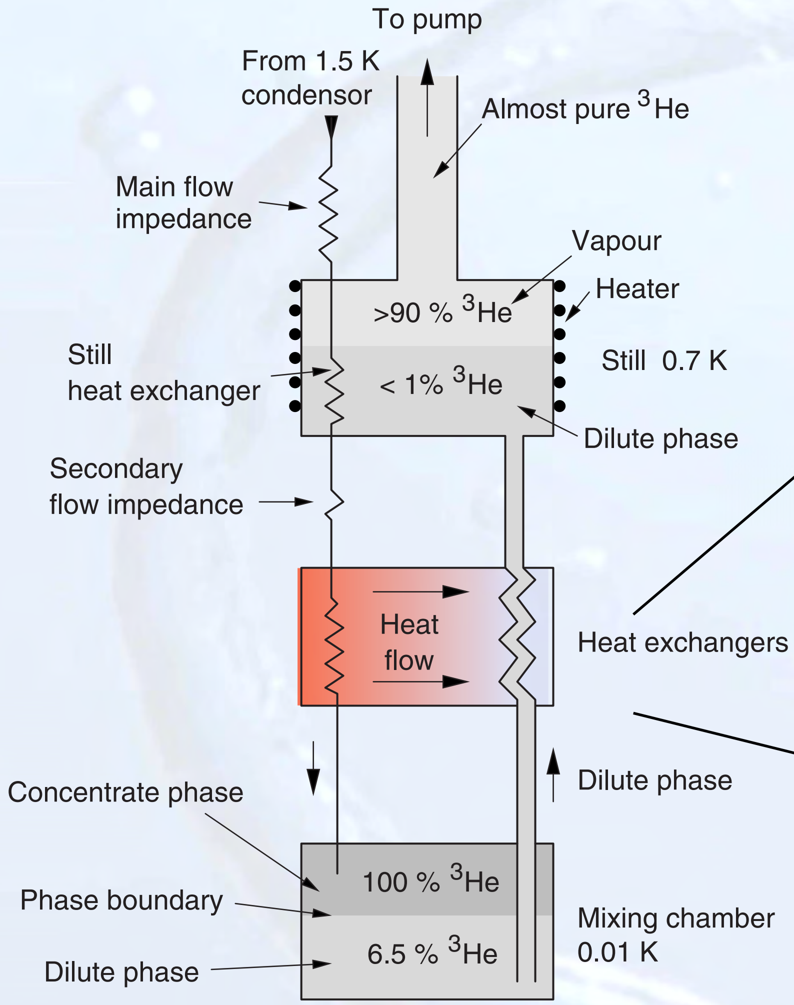
heat of solubility per Mol:

$$\Delta Q = T \Delta S = a T^2$$

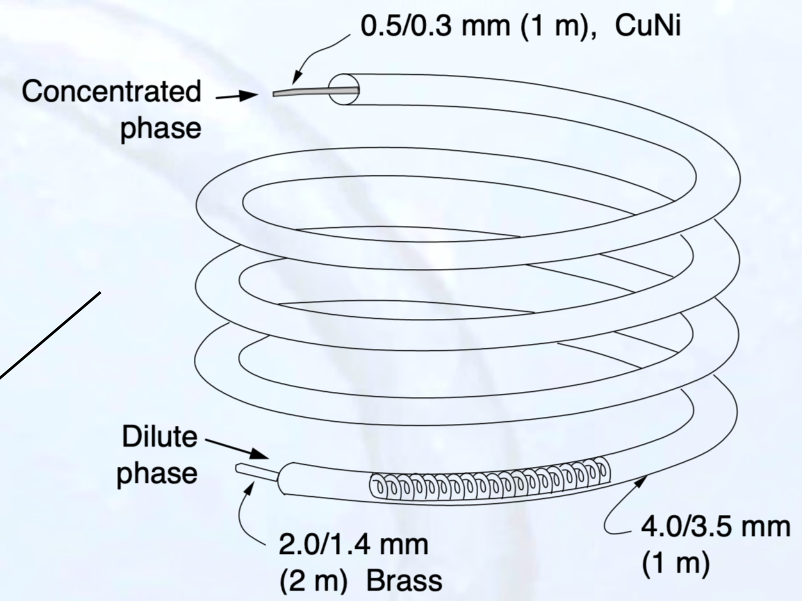
$$a = -84\,\mathrm{J/K^2}$$

11.2 Dilution Refrigerators

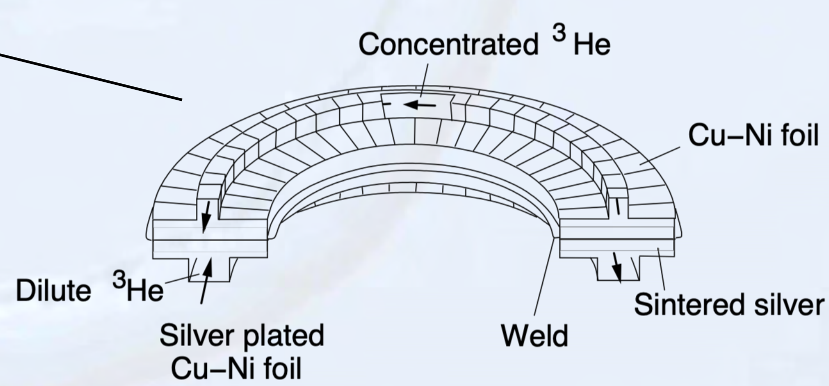
Realisation of $^3He/^4He$ cooling cycle



continuous heat exchanger



step heat exchanger



11.2 Dilution Refrigerators

Kapitza Resistance — thermal boundary resistance

Snell's law of refraction $$\frac{\sin\alpha_\ell}{\sin\alpha_\mathrm{s}} = \frac{v_\ell}{v_\mathrm{s}}$$

critical angle of total reflection $$\alpha_\ell^\mathrm{c} = \arcsin\left(\frac{v_\ell}{v_\mathrm{s}}\right)$$

for liquid helium and copper $\alpha_\ell^\mathrm{c} \approx 4°$

fraction of phonons incident within critical angle

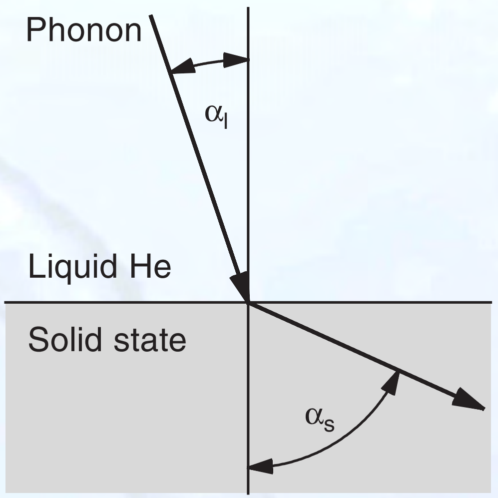
$$f = \frac{1}{2}\sin^2\alpha_\ell^\mathrm{c} = \frac{1}{2}\left(\frac{v_\ell}{v_\mathrm{s}}\right)^2 < 10^{-2}$$

transmission coefficient

$$t = \frac{4Z_\ell Z_\mathrm{s}}{(Z_\ell + Z_\mathrm{s})^2} \approx \frac{4Z_\ell}{Z_\mathrm{s}} = \frac{4\varrho_\ell v_\ell}{\varrho_\mathrm{s} v_\mathrm{s}}$$

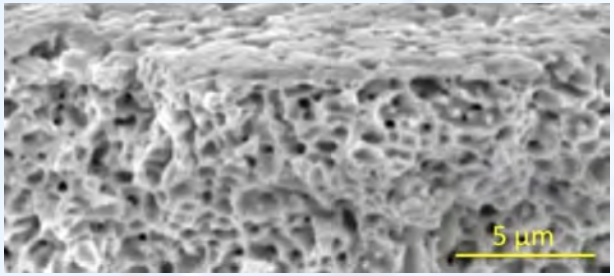
$Z_\ell = \varrho_\ell v_\ell$ $Z_\mathrm{s} = \varrho_\mathrm{s} v_\mathrm{s}$ acoustic impedances

fraction of phonons crossing the interface $$\boxed{ft = \frac{2\varrho_\ell v_\ell^3}{\varrho_\mathrm{s} v_\mathrm{s}^3}}$$



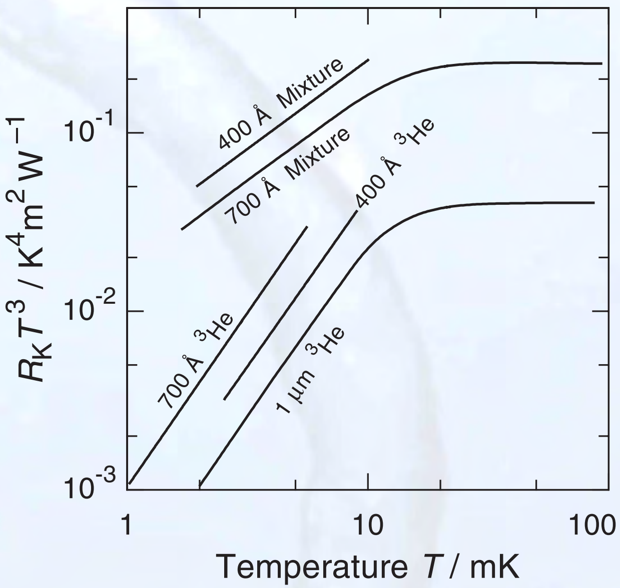
- Kaptiza resistance occurs at any solid-solid, liquid-solid interface
- particular problematic for liquid helium because of the low sound velocity
- helium-copper $ft < 10^{-5}$

11.2 Dilution Refrigerators



silver sinter
SEM image

Kapitza resistance between pure ^{3}He and ^{3}He/^{4}He mixtures and silver sinters of different grain sizes



heat flow from liquid to solid (using Debye model)

$$\dot{Q} = \frac{1}{2} f t u v_\ell A = \frac{\pi^2 k_B^4 \varrho_\ell v_\ell}{30\hbar^3 \varrho_s v_s^3} A T^4$$

$$u = U/V = \pi^2 k_B^4 T^4 / (30\hbar^3 v_\ell^3)$$

in equilibrium identical heat flow from solid to liquid

net flow in non-equilibrium (ΔT)

$$\dot{Q} = \frac{d\dot{Q}}{dT}\Delta T = \frac{2\pi^2 k_B^4 \varrho_\ell v_\ell}{15\hbar^3 \varrho_s v_s^3} A T^3 \Delta T$$

Kapitza resistance

$$\boxed{R_K = \frac{A\Delta T}{\dot{Q}} = \frac{15\hbar^3 \varrho_s v_s^3}{2\pi^2 {k_B}^4 \varrho_\ell v_\ell} \frac{1}{T^3}}$$

- $20\,\mathrm{mK} < T < 100\,\mathrm{mK}$ good agreement with Debye model $R_K \propto T^{-3}$
- below 20 mK $R_K \propto T^{-2}$ or $R_K \propto T^{-1}$

⟶ anomalous Kapitza resistance

origin: TLS, coupling to zero and second sound modes, phonon wavelength larger than sinter grains

11.2 Dilution Refrigerators

Cooling power

assuming 100% ^{3}He circulation one finds in equilibrium:

$$\dot{Q}_{\mathrm{mc}} + \dot{N}_3\left[H_3(T_{\mathrm{ex}}) - H_3(T_{\mathrm{mc}})\right] = \dot{N}_3\left[H_{3,\mathrm{d}}(T_{\mathrm{mc}}) - H_3(T_{\mathrm{mc}})\right]$$

enthalpy

$H = U + pV$

circulation rate

enthalpy of ^{3}He-dilute phase

mixing chamber temperature

heat leak and/or available cooling power

enthalpy of ^{3}He-rich phase

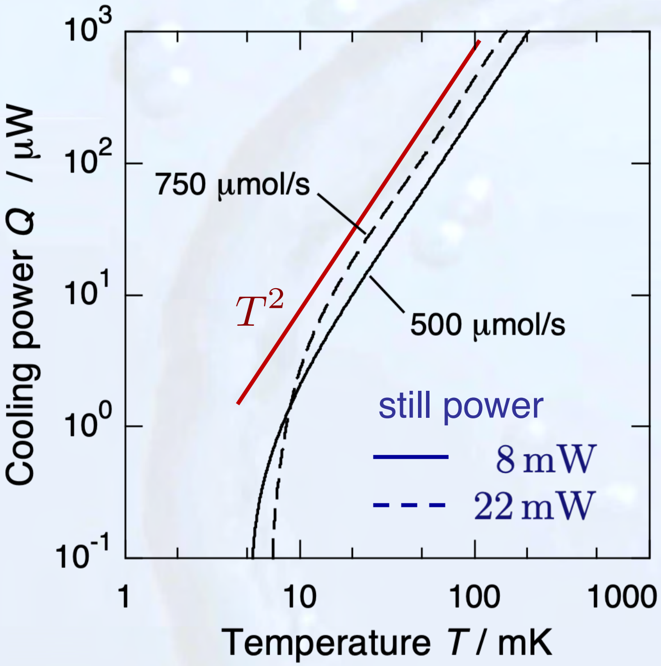
temperature after last heat exchanger

inserting the enthalpies

$$\boxed{\begin{aligned}\dot{Q}_{\mathrm{mc}} &= \dot{N}_3\left[H_{3,\mathrm{d}}(T_{\mathrm{mc}}) - H_3(T_{\mathrm{ex}})\right] \\ &= \dot{N}_3\left(95\,T_{\mathrm{mc}}^2 - 11\,T_{\mathrm{ex}}^2\right)\left(\frac{\mathrm{J}}{\mathrm{mol\ K^2}}\right)\end{aligned}}$$

11.2 Dilution Refrigerators

Temperature and circulation rate dependence of the cooling power



limiting case of vanishing cooling power: $\dot{Q}_{\mathrm{mc}} = 0$

$$95\, T_{\mathrm{mc}}^2 - 11\, T_{\mathrm{ex}}^2 = 0$$

$$\boxed{\frac{T_{\mathrm{ex}}}{T_{\mathrm{mc}}} = 2.8}$$

➡ this underlines the importance of the heat exchanger quality

- for $\dot{Q} \gg \dot{Q}_{\text{heat leak}} \longrightarrow \dot{Q} \propto T^2 \; , \quad \dot{Q} \propto \dot{N}_3$
- heat leak determines lowest temperature

circulation rate