7.2 Electrical Conductivity

metals, no superconductors, no semiconductors

Boltzmann equation $\longleftrightarrow$ kinetic gas theory

- starting point: equilibrium distribution without external fields $f_0(\boldsymbol{k})$

Fermi-Dirac distribution

- with field: stationary non-equilibrium value of $f(\boldsymbol{k}, \boldsymbol{r}, t)$

- expand $f_0(\boldsymbol{k}) - f(\boldsymbol{k}, \boldsymbol{r}, t)$ in linear order + relaxation ansatz for collisions

$\Rightarrow$ linearized Boltzmann equation $f(\boldsymbol{k}) \approx f_0(\boldsymbol{k}) + \frac{e\tau(\boldsymbol{k})}{\hbar} \boldsymbol{\mathcal{E}} \cdot \frac{\partial f_0(\boldsymbol{k})}{\partial \boldsymbol{k}}$

scattering time

electric field

$\Rightarrow$ $$j_x = -e \int D(k)\, v_x(k)\, f(k)\, \mathrm{d}k = -\frac{e}{\pi^2} \int k^2\, v_x(k)\, f(k)\, \mathrm{d}k$$

$\Rightarrow$ $\sigma = \frac{1}{3} e^2 D(E_\mathrm{F})\, v_\mathrm{F}^2\, \tau(E_\mathrm{F}) \longrightarrow \boxed{\sigma = \frac{n\, e^2}{m} \tau(E_\mathrm{F})}$

7.2 Electrical Conductivity

scattering time determined by:

- defect scattering
- phonon scattering
- magnon scattering (in ferromagnets)
- electron-electron scattering (can be neglected in most cases)

a) defect scattering

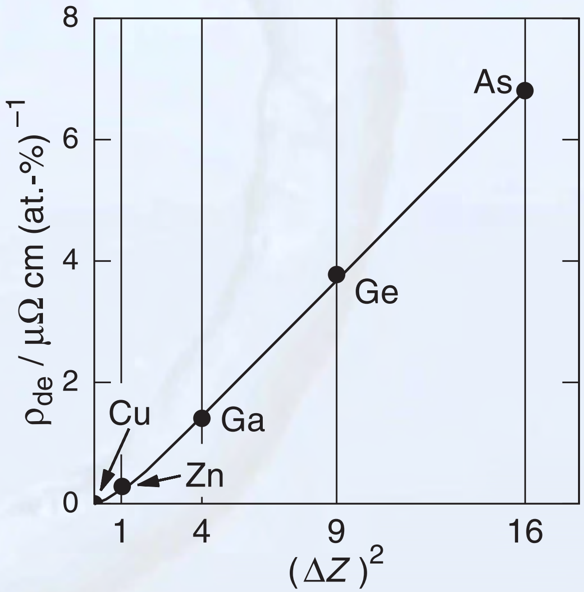
↗ local charge density variations

↘ local strain fields (less important)

Local charge variations

- Rutherford scattering on ionic cores of impurity atoms
- scattering cross section : $\sigma_{\mathrm{cross}} \propto (\Delta Z)^2$
- resistivity $\varrho_{\mathrm{D}} \propto (\Delta Z)^2$

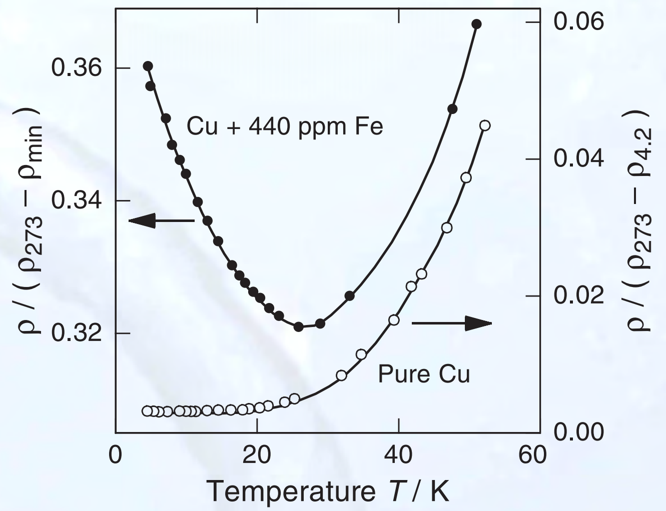
- residual resistance of copper with 1 at% impurities with different valence electrons configurations
- agrees well with: $\varrho_{\mathrm{D}} \propto (\Delta Z)^2$



Interaction of Conduction Electrons and Localized Magnetic Moments

1930 Meissner and Voigt observe a resistance minimum for Au and Cu with magnetic impurities

- example: Cu + 440 ppm Fe
- resistance minimum at 27 K



1964 explanation by Kondo by spin dependent scattering of electrons on magnetic impurities

a) Influence of conduction electrons on localized magnetic moments

example: d-levels of transition metals in simple metals

- d-d interaction ⟶ splitting and polarization of d-levels, because of crystal field
- interaction of d-electrons with conduction electrons (s) ⟶ hybrid states

↓

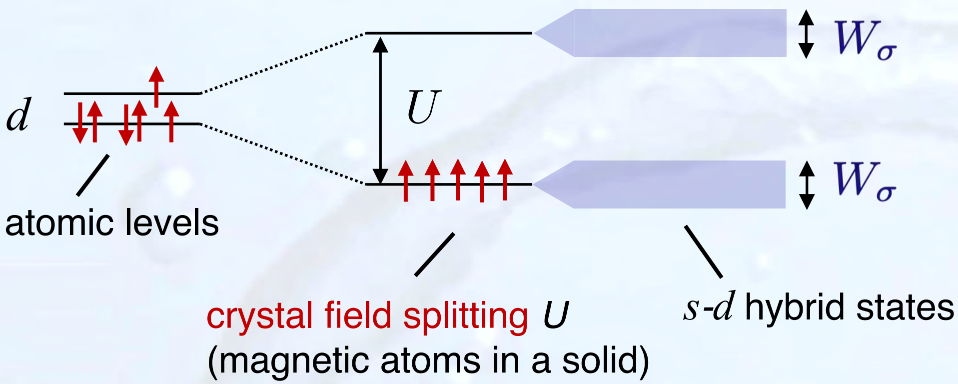
width determined by $s \leftrightarrow d$ transition rate

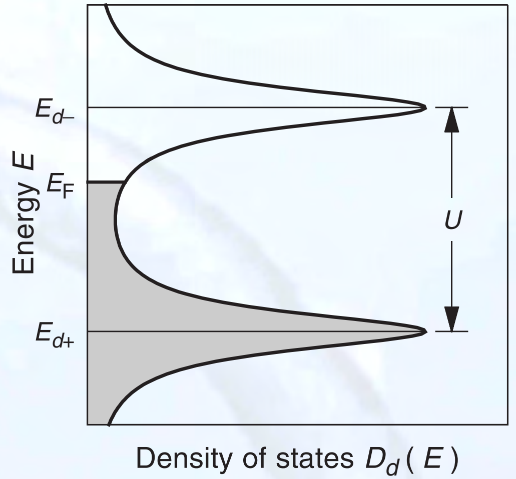
matrix element

⟶ golden rule: $$\frac{W_\sigma}{\hbar} = \frac{\pi}{\hbar}\,\mathcal{V}^2\, D_s(E_{d\sigma})$$

Density of states of s-electrons at $E_{d\sigma}$

Interaction of Conduction Electrons and Localized Magnetic Moments





AgMn:

$W_\sigma = 0.5\,\mathrm{eV}$

$U = 4.8\,\mathrm{eV}$

localized moments remain if interaction (W_σ) is not too strong $W_\sigma < U$

spin-dependent interaction of s electrons with d-electrons

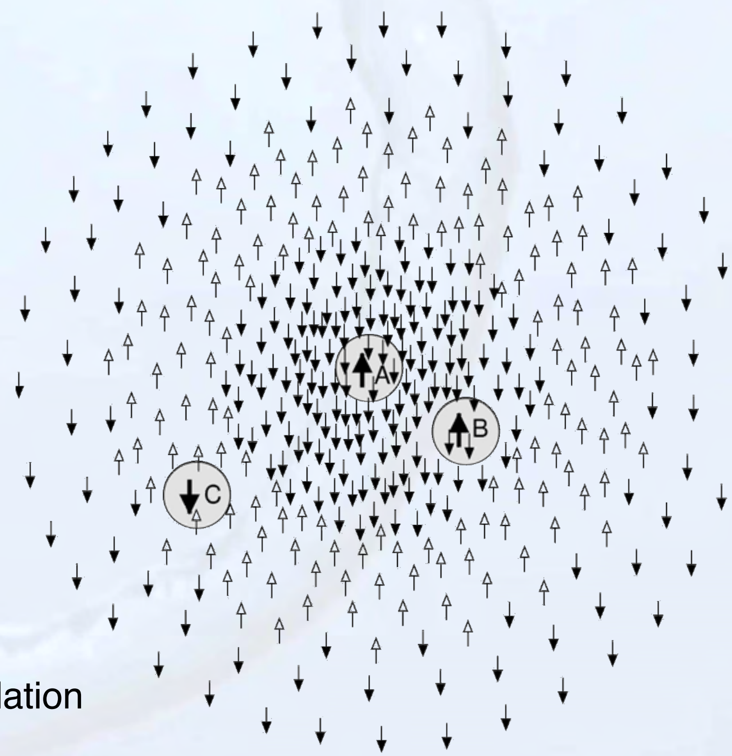
$J \approx -\mathcal{V}^2/U$

position of localized spin

$$\mathcal{H}_{sd} = -J\,\boldsymbol{S}\cdot\boldsymbol{s}\;\delta(\boldsymbol{r}-\boldsymbol{R}) \qquad (\star)$$

localized spins

conduction electron



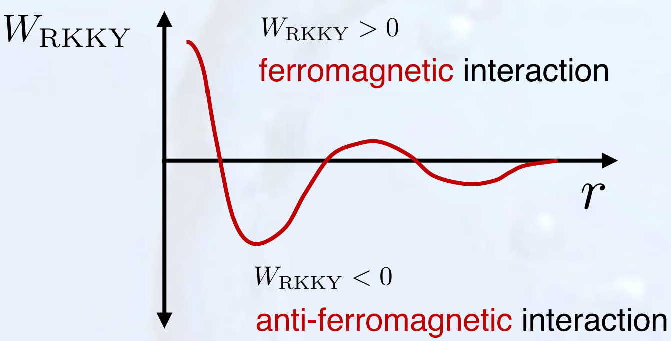
Friedel oscillation

Interaction of Conduction Electrons and Localized Magnetic Moments

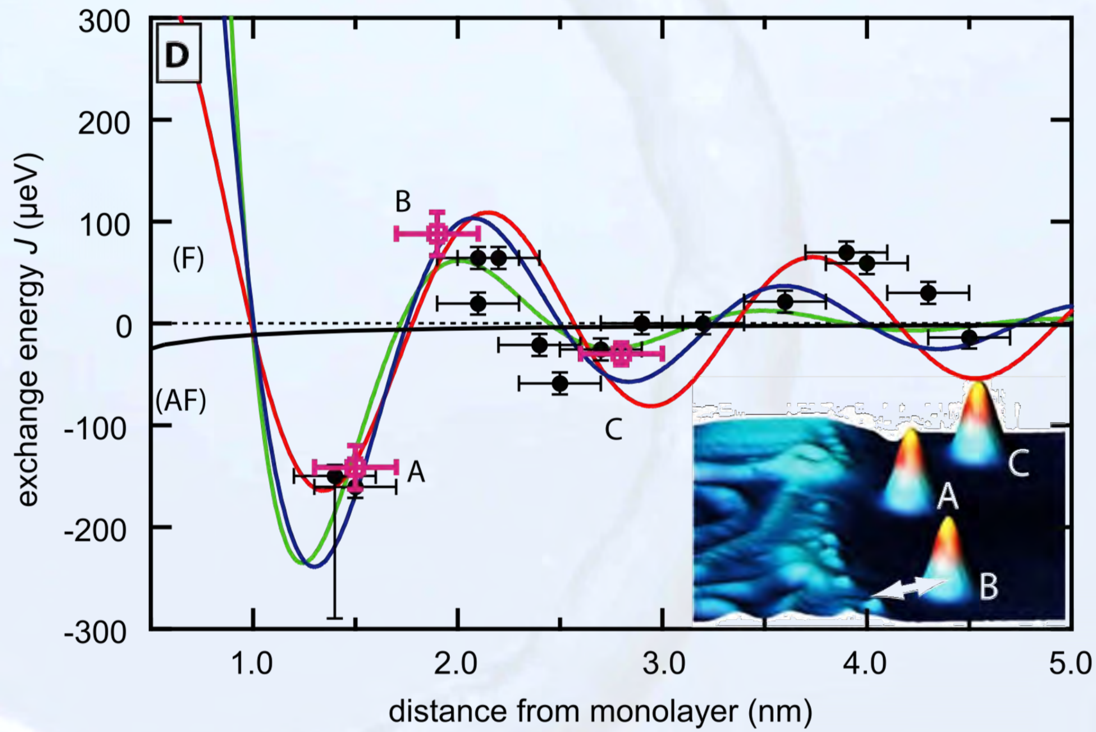
Indirect exchange interaction (d-d) – RKKY interaction

Ruderman
Kittel
Kusuya
Yosida

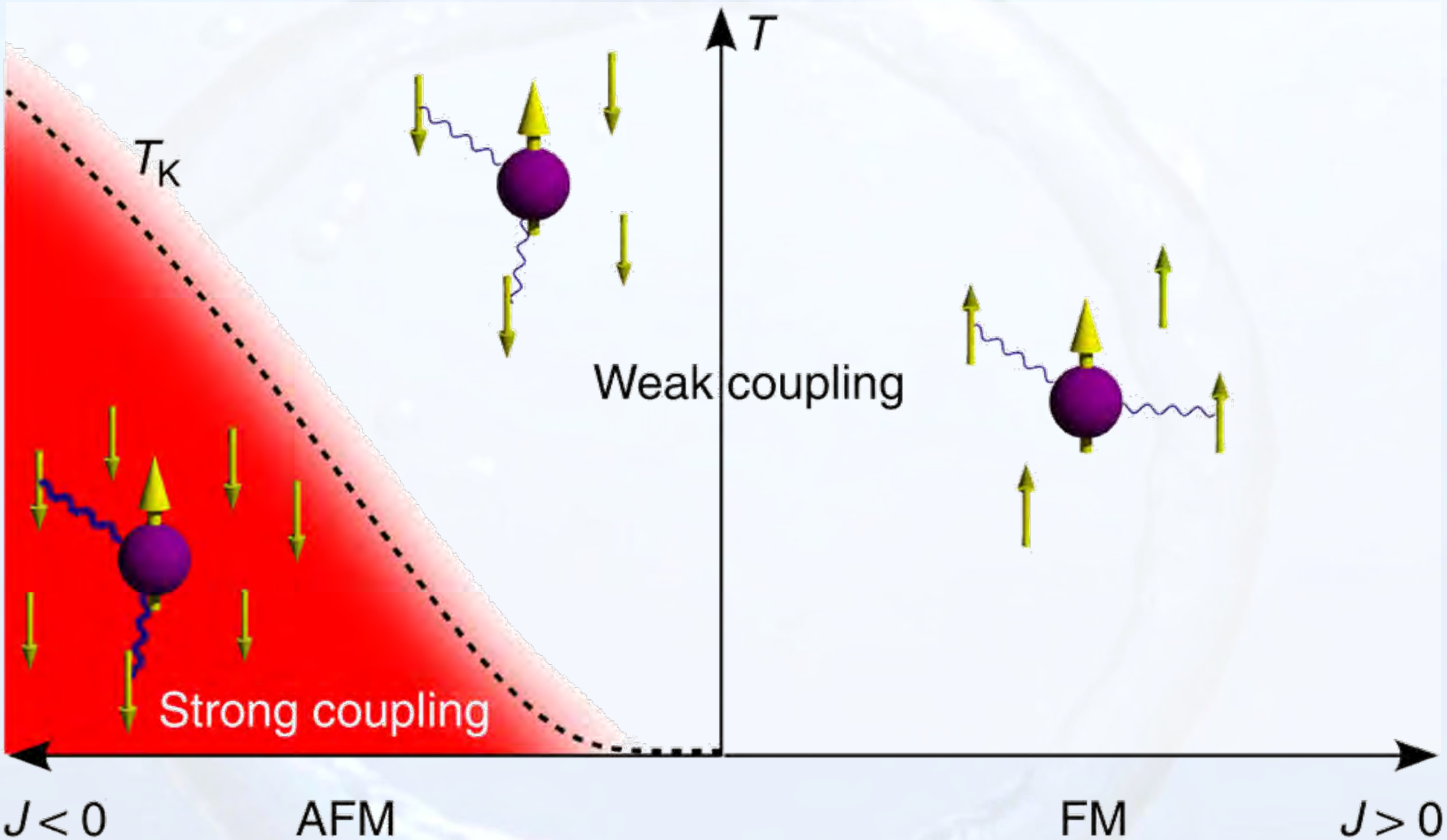
$$W_{\mathrm{RKKY}} \propto J^2_{\mathrm{RKKY}} D(E_{\mathrm{F}}) \frac{\cos(2k_{\mathrm{F}}r)}{r^3}$$



MFM measurement using magnetic moments on a noble metal surface



Interaction of Conduction Electrons and Localized Magnetic Moments



Scattering of Conduction Electrons on Localized Magnetic Moments

Jun Kondo

electrical resistance → inelastic scattering

electrical resistance → elastic scattering ← Kondo effect

consider (★) in "*N* representation" | $\mathcal{H}_{sd} = -J\,\boldsymbol{S}\cdot\boldsymbol{s}\;\delta(\boldsymbol{r}-\boldsymbol{R})$

$$\mathcal{H}_{sd} = -J \sum_{\boldsymbol{k}\boldsymbol{k}'} S_z\left(c^{+}_{\boldsymbol{k}'\uparrow}\,c_{\boldsymbol{k}\uparrow} - c^{+}_{\boldsymbol{k}'\downarrow}\,c_{\boldsymbol{k}\downarrow}\right) + S_+ c^{+}_{\boldsymbol{k}'\downarrow}\,c_{\boldsymbol{k}\uparrow} + S_- c^{+}_{\boldsymbol{k}'\uparrow}\,c_{\boldsymbol{k}\downarrow}$$

J: const. < 0!

$c^{+}_{\boldsymbol{k}}$ creation operator
$c_{\boldsymbol{k}}$ annihilation operator
} act on conduction electrons

$S_+ = S_x + \mathrm{i}S_y$
$S_- = S_x - \mathrm{i}S_y$
} spin states

$\boldsymbol{k}$ wave vector of conduction electrons

Harmonic Oscillator

$H = \hbar\omega\left(a^+a + \frac{1}{2}\right) \quad \Big| \quad H = \frac{P^2}{2m} + \frac{m\omega^2 Q^2}{2}$

$Q = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^+)$

$P = \frac{1}{i}\sqrt{\frac{m\omega\hbar}{2}}\,(a - a^+)$

$a^+ = \sqrt{\frac{m\omega}{2\hbar}}\left(Q - \frac{i}{m\omega}P\right)$

$a = \sqrt{\frac{m\omega}{2\hbar}}\left(Q + \frac{i}{m\omega}P\right)$

$a^+a = N$

$a^+|\psi_n\rangle = \sqrt{n+1}\,|\psi_{n+1}\rangle$

Scattering of Conduction Electrons on Localized Magnetic Moments

(i) first-order perturbation theory $|\boldsymbol{k}\uparrow\rangle \longrightarrow |\boldsymbol{k}'\uparrow\rangle$

$$t^{(1)} = \langle \boldsymbol{k}'\uparrow|\mathcal{H}_{sd}|\boldsymbol{k}\uparrow\rangle = -J\ S_z$$

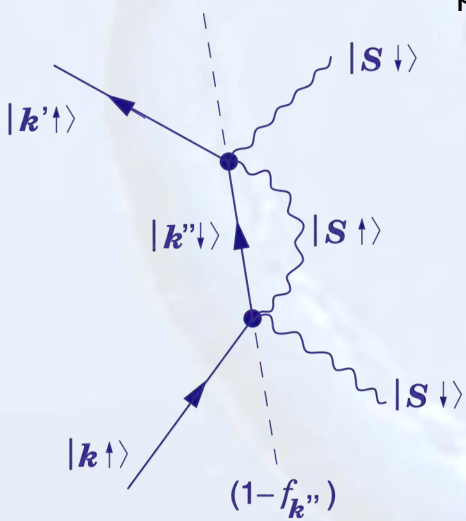
temperature independent constant

scattering amplitude

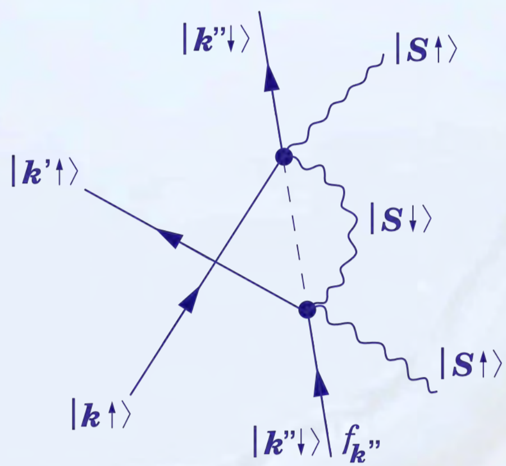
(ii) second-order perturbation theory: two possible process

- direct process (normal order)
- exchange process (reverse order)

processes without spin flip



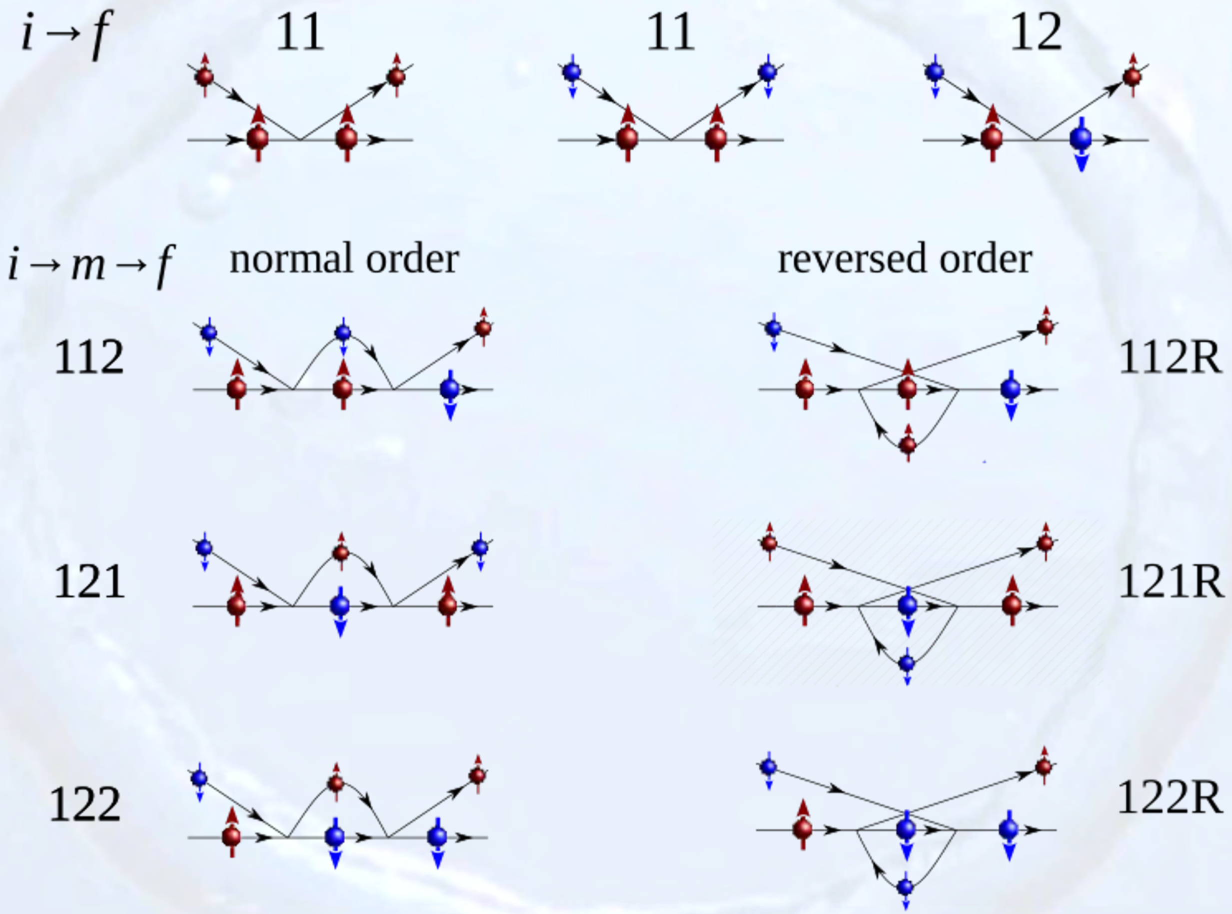
direct process



exchange process

Scattering of Conduction Electrons on Localized Magnetic Moments

all first and second order processes



Scattering of Conduction Electrons on Localized Magnetic Moments

direct and exchange processes together

$$t^{(2)} = \sum_{\boldsymbol{k}'',\sigma} \frac{1}{E(\boldsymbol{k}) - E(\boldsymbol{k}'')} \Big[(1 - f_{\boldsymbol{k}''}) \langle \boldsymbol{k}' \uparrow | \mathcal{H}_{sd} | \boldsymbol{k}'' \sigma \rangle \, \langle \boldsymbol{k}'' \sigma | \mathcal{H}_{sd} | \boldsymbol{k} \uparrow \rangle$$
$$+ f_{\boldsymbol{k}''} \, \langle \boldsymbol{k}'' \sigma | \mathcal{H}_{sd} | \boldsymbol{k} \uparrow \rangle \langle \boldsymbol{k}' \uparrow | \mathcal{H}_{sd} | \boldsymbol{k}'' \sigma \rangle \Big]$$

↗ without spin flip ⟶ small temperature independent contribution (only terms with S_z contribute)

↘ with spin flip ⟶ Kondo effect

Kondo resistance

algebraic calculation leads to $t^{(2)} = J^2 \, S_z \sum_{\boldsymbol{k}''} \dfrac{2 f_{\boldsymbol{k}''} - 1}{E(\boldsymbol{k}) - E(\boldsymbol{k}'')}$

in addition: $D(E) \approx D(E_\mathrm{F})$ $\quad f_{\boldsymbol{k}''} \longrightarrow$ step function ($T = 0$)

$\delta E < |E_\mathrm{F} \pm \mathcal{D}|$ $\quad \sum \rightarrow \int$

$$t^{(1)} + t^{(2)} = -J S_z \left[1 - 2\, J \, D(E_\mathrm{F}) \, \ln \frac{\mathcal{D}}{|E_\mathrm{F} - E(\boldsymbol{k})|} \right]$$

Scattering of Conduction Electrons on Localized Magnetic Moments

scattering probability $w(\boldsymbol{k}\uparrow,\boldsymbol{k}'\uparrow) = t_{\mathrm{K}}^2$

$$\longrightarrow \quad w(\boldsymbol{k}\uparrow,\boldsymbol{k}'\uparrow) \propto J^2\,S_z^2\left[1 - 4\,J\,D(E_{\mathrm{F}})\;\ln\frac{\mathcal{D}}{|E_{\mathrm{F}} - E(\boldsymbol{k})|} + \quad \cdots \quad\right]$$

terms of the order of $\mathrm{O}(J^4)$ are omitted, integration over all vectors and energies with $E(\boldsymbol{k}) \approx E_{\mathrm{F}} \pm k_{\mathrm{B}}T$

$$\longrightarrow \quad \varrho(T) \propto \varrho_0\left[1 - 4J\,D(E_{\mathrm{F}})\;\ln\frac{\mathcal{D}}{k_{\mathrm{B}}T}\right]$$

$J < 0 \longrightarrow$ increase with decreasing temperature

adding the lattice contribution $\varrho_{\mathrm{ph}} = aT^5 \longrightarrow$ total resistance

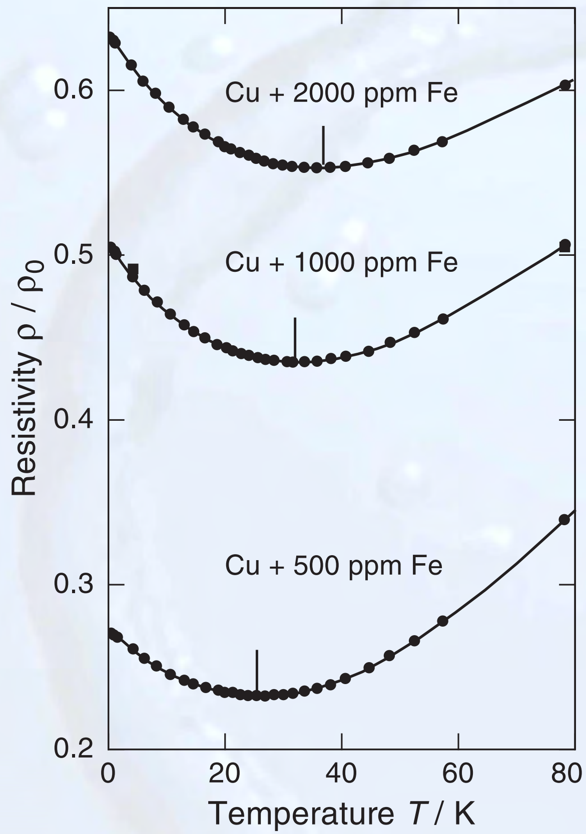
$$\longrightarrow \quad \varrho = aT^5 + c\varrho_0 + c\varrho_1\;\ln\frac{\mathcal{D}}{k_{\mathrm{B}}T}$$

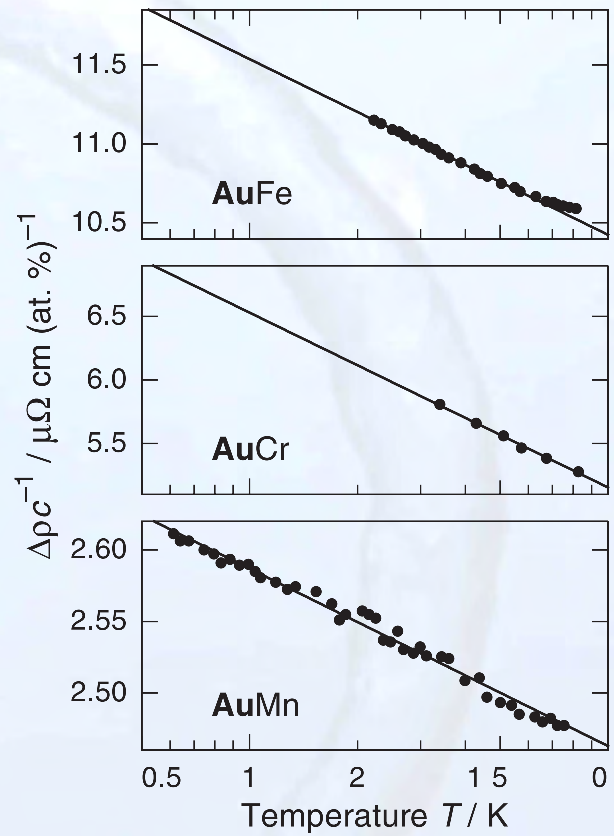
concentration of magnetic impurities

minimum expected at $T_{\mathrm{min}} = \left(\frac{c\varrho_1}{5a}\right)^{1/5}$

Scattering of Conduction Electrons on Localized Magnetic Moments

experimental observations:





► concentration-dependent minimum

► logarithmic temperature dependence

Scattering of Conduction Electrons on Localized Magnetic Moments

dependence for $T \rightarrow 0$? ⟶ logarithmic divergence is nonphysical

↗ $T > T_{\mathrm{K}}$ weak coupling regime $(-\ln T)$

↘ $T < T_{\mathrm{K}}$ strong coupling regime

Kondo temperature: $T_{\mathrm{K}} \approx T_{\mathrm{F}}\, \mathrm{e}^{-1/JD(E_{\mathrm{F}})}$

strong coupling regime

- strong screening through surrounding conduction electrons
  - ⟶ spin-compensated singlet ground state
  - ⟶ coherent Kondo state
- transition from a magnetic to a non-magnetic system
- energy necessary to form the spin-compensated cloud: $k_{\mathrm{B}}T_{\mathrm{K}}$
- maximum in specific heat of Kondo systems at T_{K}
- magnetic moment disappears below T_{K}

T_{K} ↗ **CuMn** 1 mK
T_{K} ↘ **AlMn** 1000 K