6.4 Influence of Normal Processes in Heat Transport

a) Poiseuille flow

N-processes are visible in heat transport experiments under special circumstances

→ ultralow temperatures (no umklapp processes) and low defect scattering

→ phonon flow in a thin crystal can be described like viscose flow of atoms in capillaries

condition:

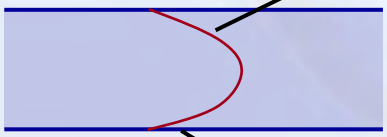
$$\ell_{\mathrm{N}}^{-1} = \ell_{\mathrm{U}}^{-1} + \ell_{\mathrm{D}}^{-1}$$

$$\ell_{\mathrm{N}} \ll d \ll \ell_{\mathrm{R}}$$

classical gas: Hagen-Poiseuille law

$$-\frac{\dot{m}}{\pi r^2}\,\frac{1}{|\nabla p|} = \frac{\varrho}{8}\,\frac{r^2}{\eta} = \frac{3}{8}\,\frac{1}{\overline{v}_{\mathrm{th}}}\,\frac{r^2}{\ell_{\mathrm{g}}}$$

$$\eta = \tfrac{1}{3}\varrho\,\overline{v}_{\mathrm{th}}\,\ell_{\mathrm{g}}$$



parabolic velocity profile

$\ell_g < r$

scattering with walls

analog equation for phonon transport (phonon gas)

$$-\frac{\dot{Q}}{\pi r^2}\,\frac{1}{|\nabla T|} = \Lambda = \frac{1}{3} C_V\, v\, \ell_{\mathrm{eff}}$$

$$\ell_{\mathrm{eff}} \approx r^2/\ell_{\mathrm{N}}$$

$\dot{Q} \mathrel{\widehat{=}} \dot{m}$

$\Delta T \mathrel{\widehat{=}} \Delta p$

effective mean free path for N-processes for heat transport

6.4 Influence of Normal Processes in Heat Transport

Interpretation: heat resistance by scattering at the surface, but each phonon has to travel the statistical path $l_{\text{eff}} \approx r^2/\ell_{\text{N}}$ because of very frequent N-processes before it reaches the surface

⟶ the observed mean free path in transport measurements are longer than the sample diameter

exact calculation

$$\Lambda = \frac{1}{3} C_V v \frac{5\,d^2}{32\,\ell_{\text{N}}} \longrightarrow \boxed{\Lambda \propto T^8}$$

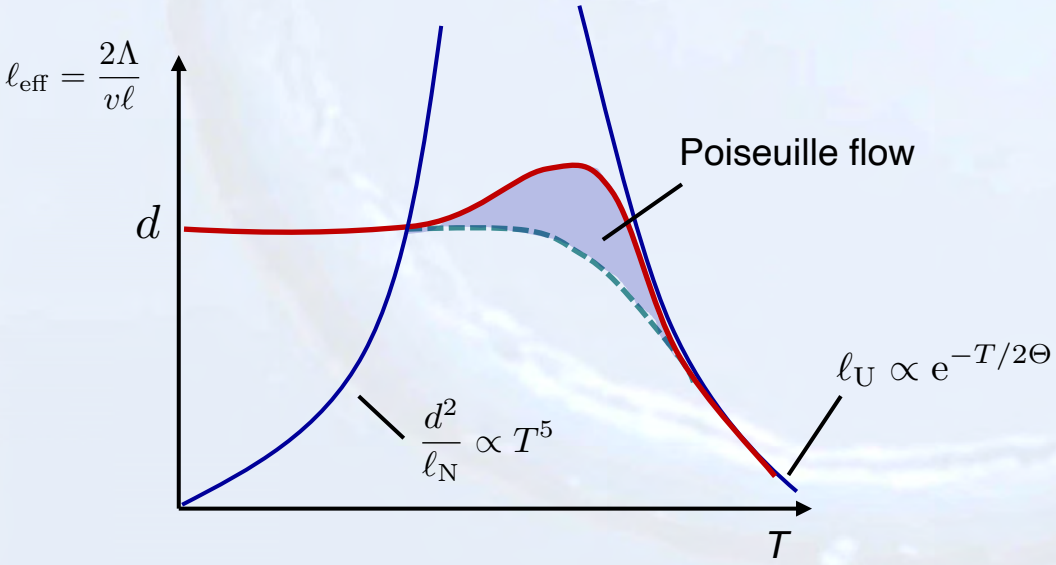
$C_V \propto T^3$, $\ell_{\text{N}} \propto T^{-5}$

3 phonon process

$\sigma \propto \prod_i \omega_i \propto \omega_1\omega_2\omega_3 \propto T^3$

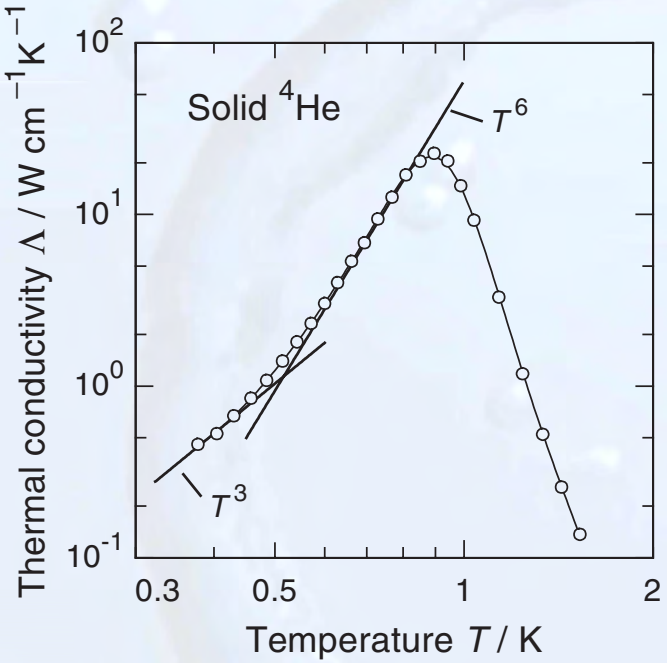
$D(\omega) \propto \omega^2 \propto T^2$

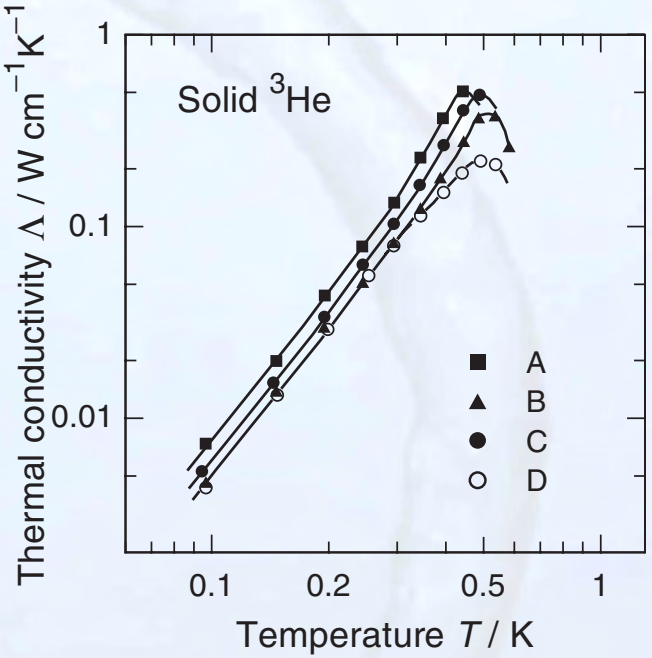
$\longrightarrow \ell_{\text{N}} \propto T^{-5}$



6.4 Influence of Normal Processes in Heat Transport

experimental evidence



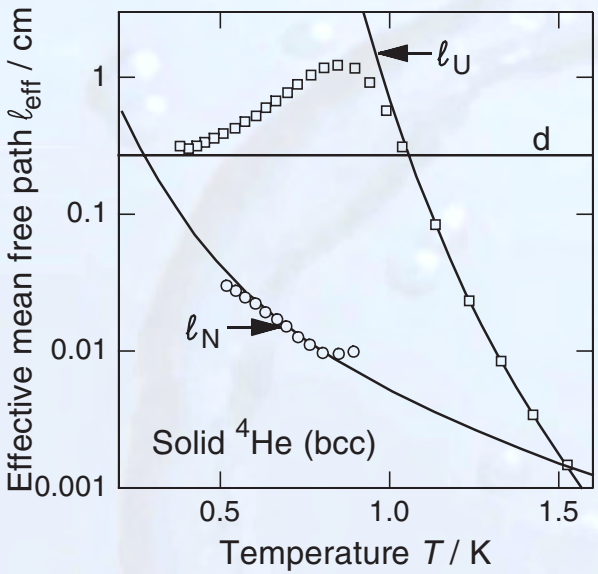


- steeper than T^3 temperature dependence observed
- temperature dependence is T^6 instead of T^8
- only small temperature range

- A,B,C crystals with different densities
- D contains 100 ppm ^{4}He
- slightly steeper temperature dependence as T^3

6.4 Influence of Normal Processes in Heat Transport

analysis of ^{4}He data



- separation of ℓ_N and ℓ_U
- $\ell_N \propto T^{-3}$ instead of $\ell_N \propto T^{-5}$
- possible explanation: influence of dislocations reduces the temperature dependence

Why is the Poiseuille flow not always observed?

$\ell_N < d$ ⟶ means high temperature, can be fulfilled easily

in addition: momentum must be carried to wall ⟶ random walk $r \approx \sqrt{p}\,\ell_N$

crystal radius

number of scattering processes necessary to reach surface

6.4 Influence of Normal Processes in Heat Transport

at the same time no scattering processes leading to heat resistance should occur

$\ell_R > p\,\ell_N$ (i)

$\ell_R^{-1} = \ell_U^{-1} + \ell_D^{-1}$

and $\ell_R > \ell_{eff} \approx r^2/\ell_N$ $\Rightarrow$ $\ell_R > r^2/\ell_N$ (ii)

mean free path for Poiseuille flow

conditions (i) and (ii) are difficult to fulfill at the same time

b) Second sound

second sound in “phonon gas” $\widehat{=}$ first sound in classical gas

density wave in classical gases $\ell < \lambda_{sound}$: $v_s \approx \frac{1}{\sqrt{3}}\,\overline{v}_{th}$ sound velocity

analog for phonon gas: $v_2 \approx \frac{1}{\sqrt{3}}\,v_s$ velocity of second sound

6.4 Influence of Normal Processes in Heat Transport

conditions:

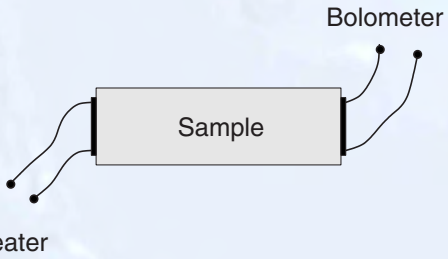
- no umklapp processes, no defect scattering
- $\ell_N \ll v_s t_p \ll \ell_R$ ⟷ $\tau_N \ll t_p \ll \tau_R$

t_p: pulse length

→ can be fulfilled more easily than condition of Poiseuille flow if $\ell_R \gg \ell_N$;

pulse length can be adjusted

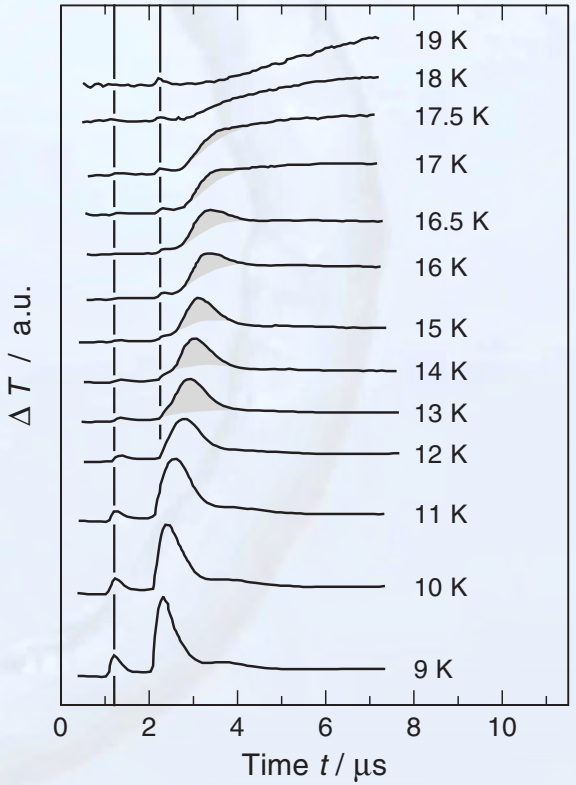
experiment



- T = 9 K → ballistic propagation: τ_N too long
- $T >$ 17 K → diffusive phonon scattering
- 9 K < T < 17 K → indications for second sound

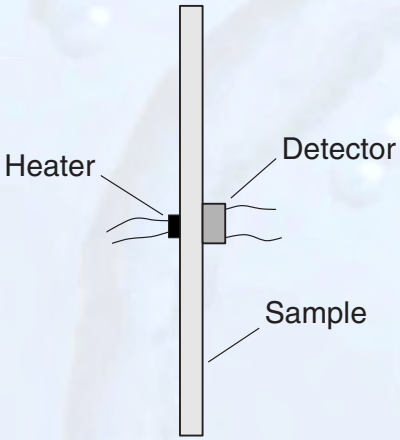
second sound so far observed in: ^{3}He, ^{4}He, NaF, Bi

second sound in NaF

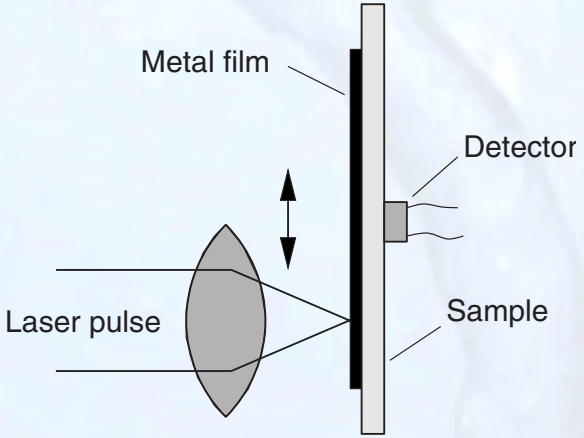


6.5 Ballistic Propagation of Phonons

Experimental techniques



time resolved measurements



position dependent measurement

$$t_i = d/v_i$$

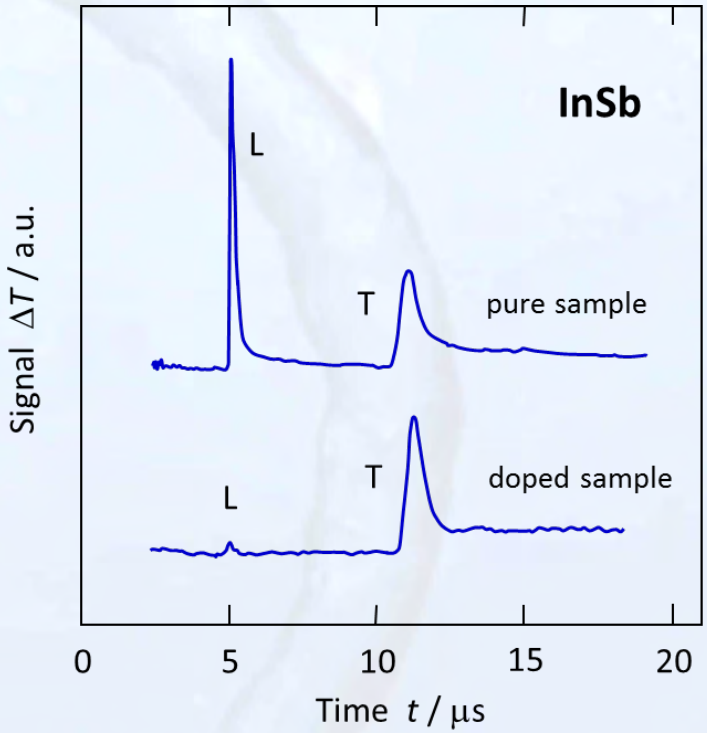
\

time of phonon with polarization i
from heater to detector

6.5 Ballistic Propagation of Phonons

a) time resolved measurements

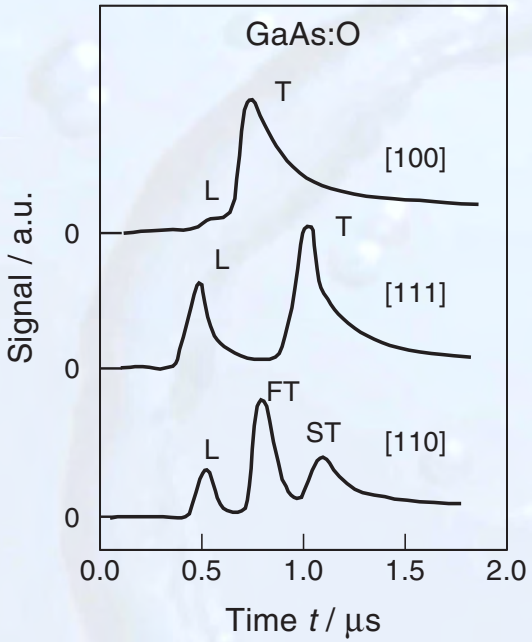
Example: InSb, investigation of electron-phonon coupling

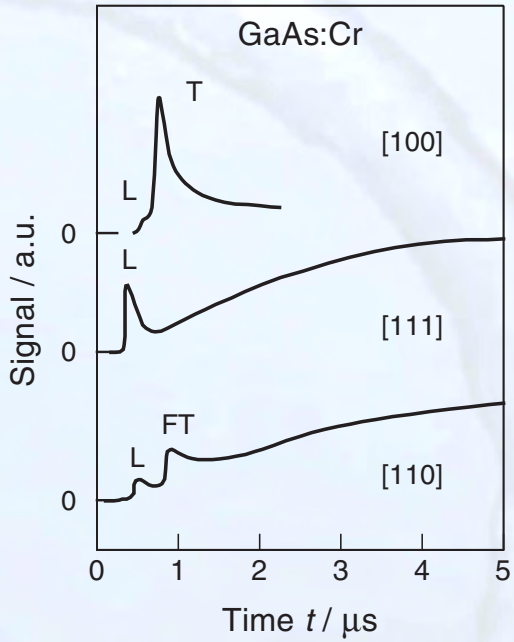


- samples with different doping level
- electrons interact with phonons via density variation
- longitudinal phonons: density variation → strong coupling
- transverse phonons: no density variation → no coupling

6.5 Ballistic Propagation of Phonons

2nd example: GaAs:O, GaAs:Cr, phonon-defect coupling is investigated
free electrons are unimportant





- [110] 3 branches L, FT, ST
- [100], [111], FT, ST degenerate

- [100] no change
- [111] FT, ST disappear
- [110] ST disappears, FT reduced
- diffused phonons appear in [110], [111]
- reason: resonant scattering with Cr defects

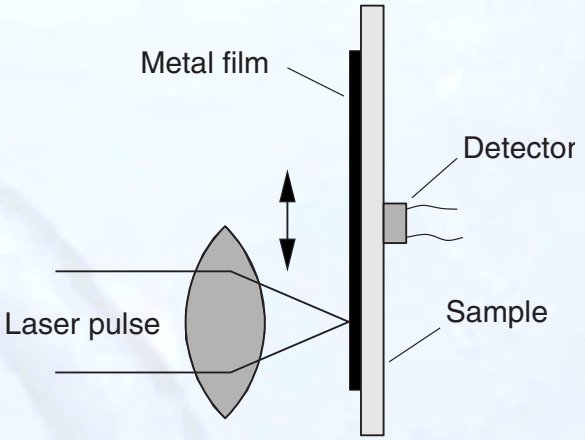
6.5 Ballistic Propagation of Phonons

b) position depended measurements

single crystals are elastically anisotropic

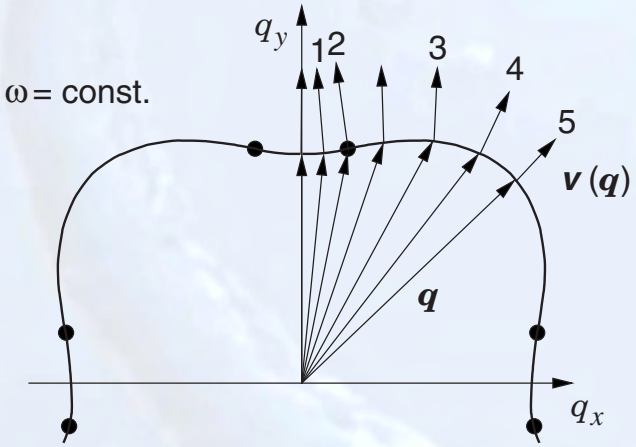
→ sound waves are propagating not always in direction of wave vector, but in direction of the group velocity

$$\boldsymbol{v} = \mathrm{d}\omega(\boldsymbol{q})/\mathrm{d}\boldsymbol{q}$$

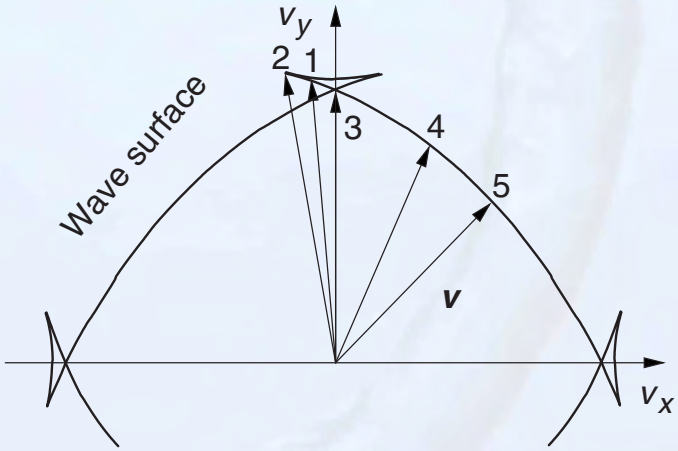


Construction of wave front

constant ω curve in *k* space



wave front reconstruction



→ propagation in preferred directions

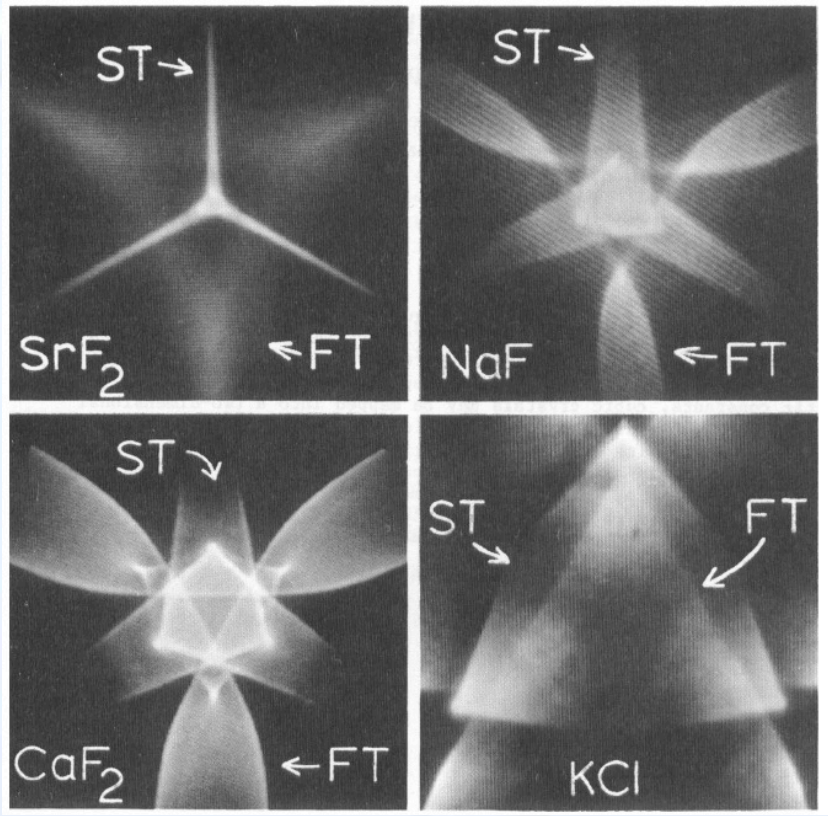
→ phonon focusing

Phonon Focussing on Cubic Crystals in ⟨111⟩ direction

example:

four cubic crystals, [111] direction

→ 3-fold symmetry



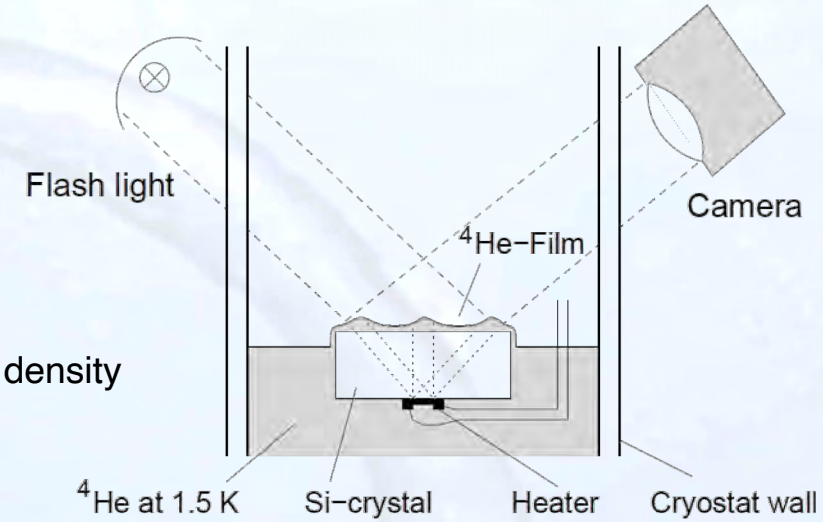
Phonon Focusing made visible by ^{4}He Films

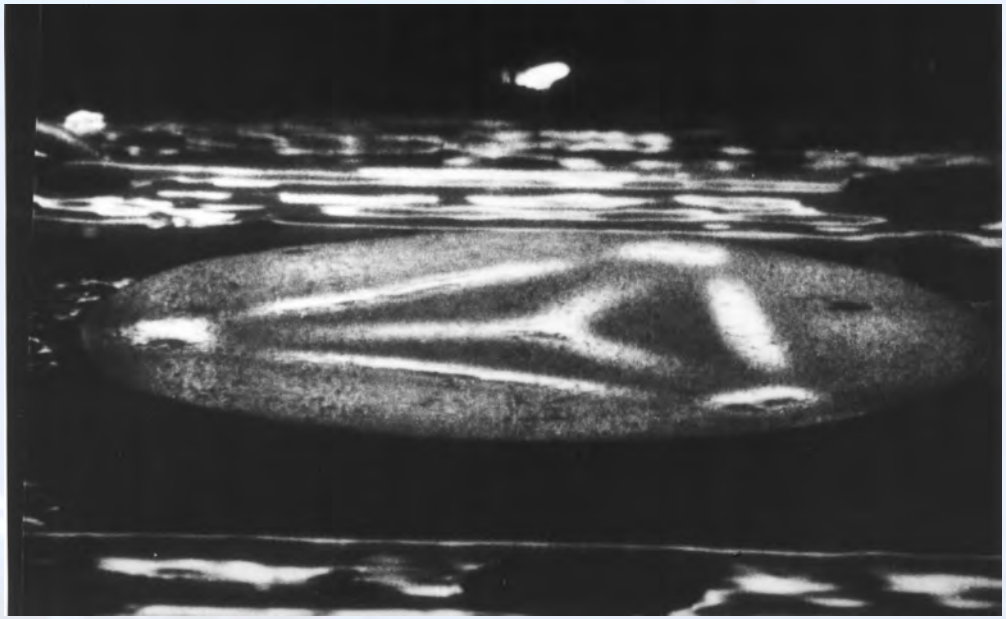
demonstration using superfluid ^{4}He

ballistic phonons ⟶ substrate locally heated

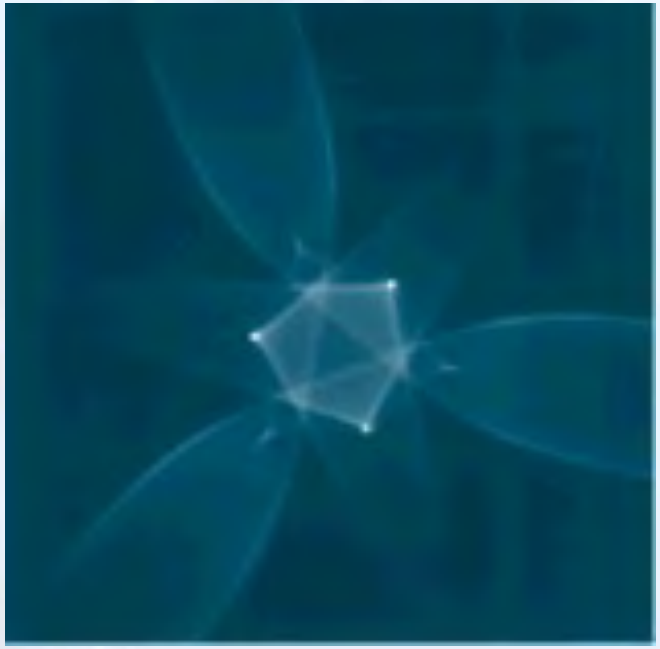
⟶ fountain effect

⟶ film thickness changes ≙ phonon density

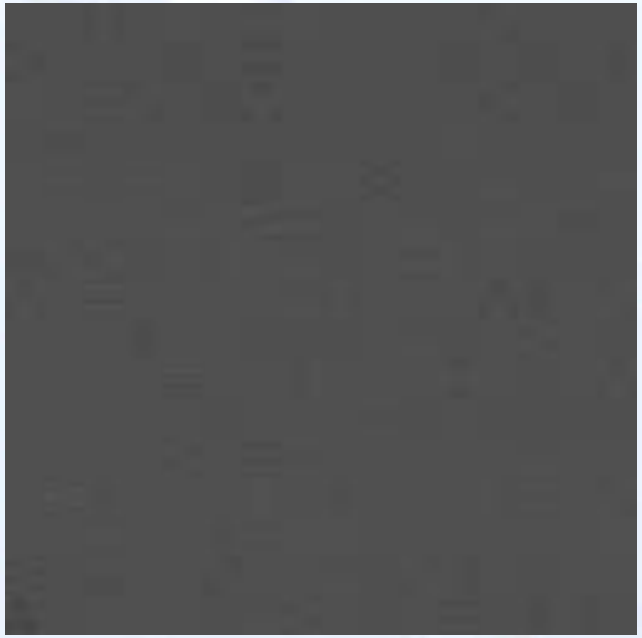




Phonon Focusing

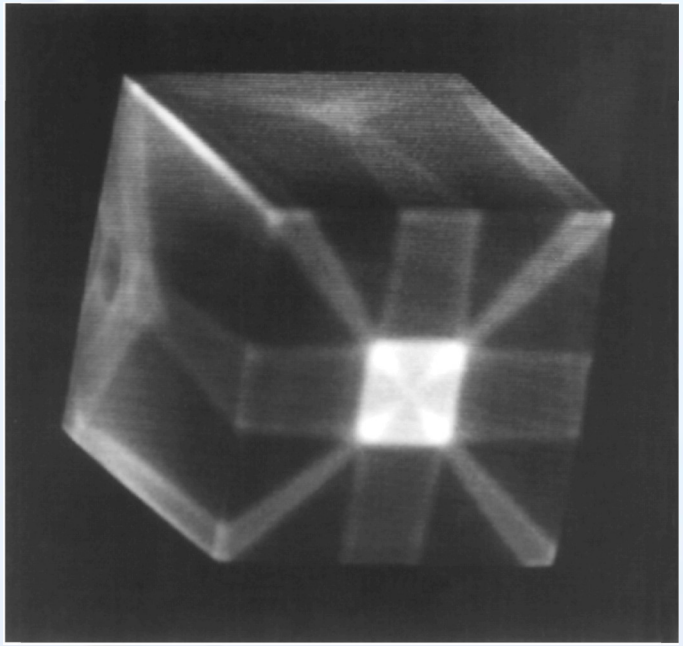


CaF_2

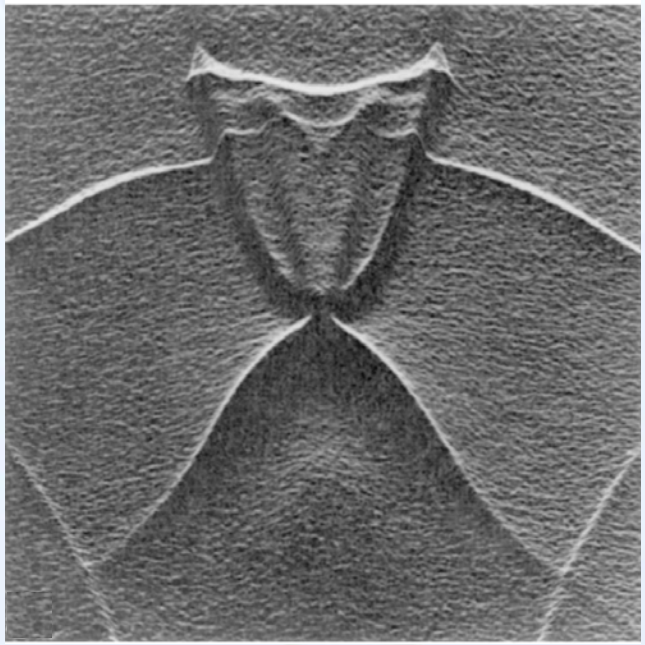


Nb at 1.8 K

Phonon Focusing



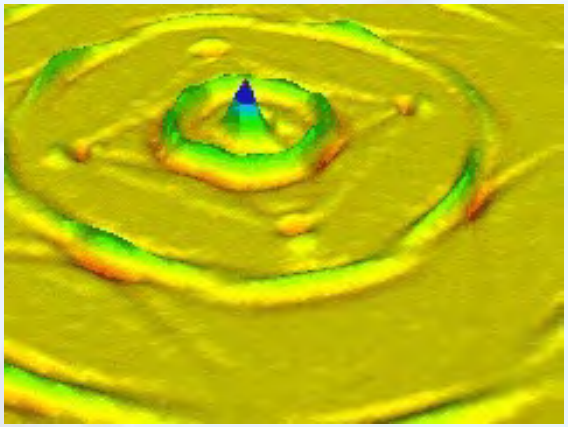
Ge
wide-angle measurement

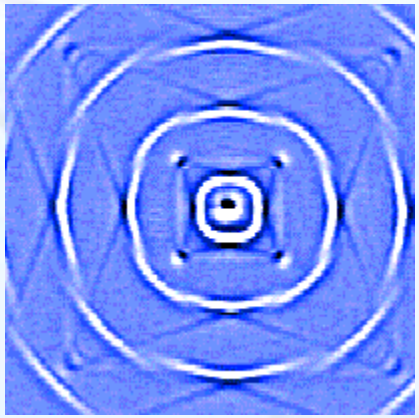


y-cut quartz
e-beam scanning

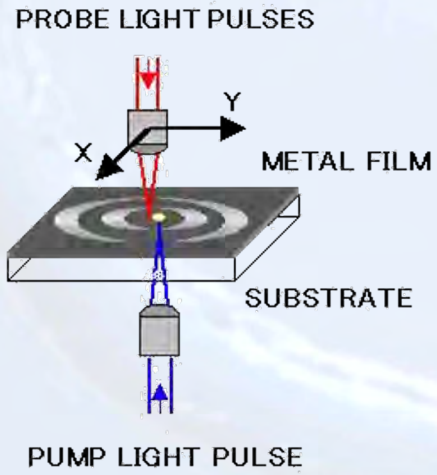
Phonon Focusing

TeO_2

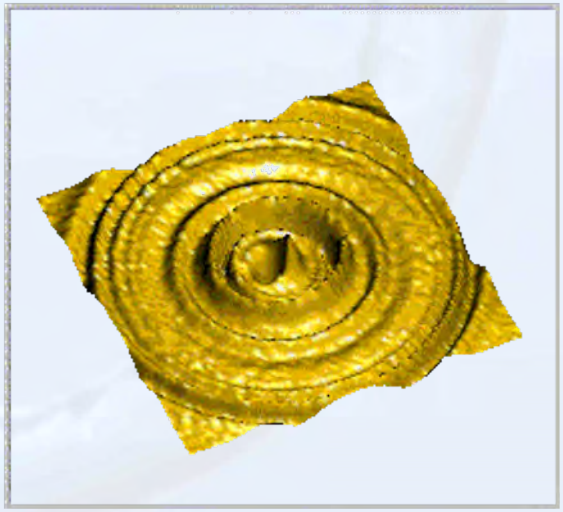




experimental technique



isotrop phonon propagation in glass



7. Conduction Electrons

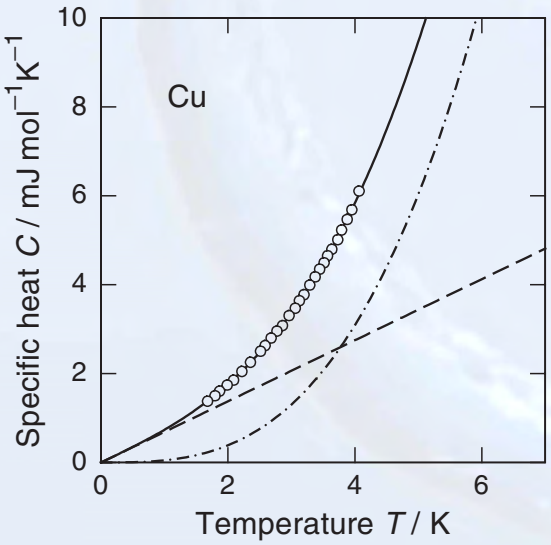
Drude (1900) → Sommerfeld (1927) → Bloch (1940)

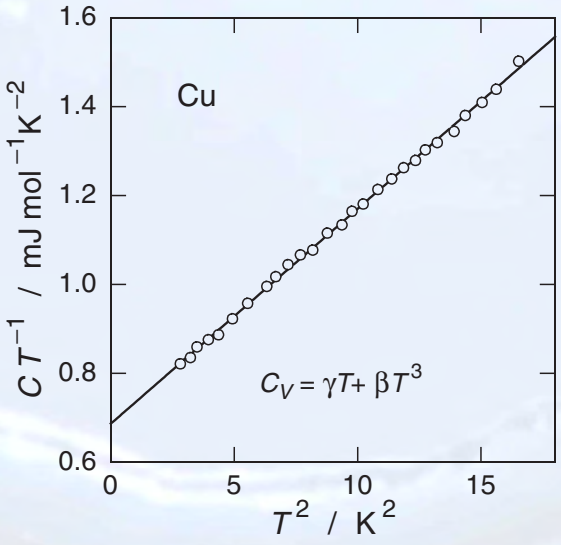
7.1 Specific heat

a) simple metals → free electrons gas

free electrons gas: $c_V = \gamma T + \beta T^3$ (phonons)

electrons: $\gamma = \dfrac{\pi^2 n k_B^2}{2E_F} \propto n^{1/3}\, m^*_{th}$





- electrons dominate below ~ 4 K
- very good qualitative agreement

7.1 Specific Heat

Element	γ_{exp}	γ_{theo}	m^*_{th}/m	Element	γ_{exp}	γ_{theo}	m^*_{th}/m
Ag	0.64	0.64	1.00	Cu	0.69	0.50	1.37
Al	1.35	0.91	1.48	Ga	0.60	1.02	0.59
Au	0.69	0.64	1.08	In	1.66	1.26	1.31
Ba	2.70	1.95	1.38	K	2.08	1.75	1.19
Be	0.17	0.49	0.35	Li	1.65	0.75	2.19
Ca	2.73	1.52	1.80	Mg	1.26	1.00	1.26
Cd	0.69	0.95	0.73	Na	1.38	1.3	1.22
Cs	3.97	2.73	1.46	Pb	2.99	1.50	1.99

→ good qualitative agreement for simple metals

→ $\gamma_{exp}/\gamma_{theo} = m^*_{th}/m$ for quantitative agreement

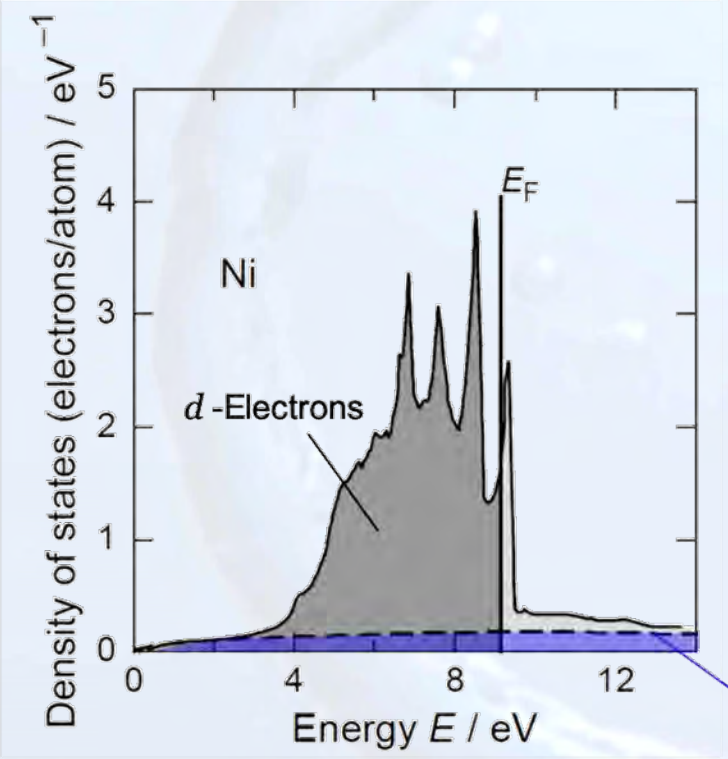
7.1 Specific Heat

but: transition series metals

example nickel: $m^*_{\mathrm{th}} \approx 15\,m$ → reason is *d*-electrons contribute, which are not (completely) free

involved in covalent bond, highly oriented

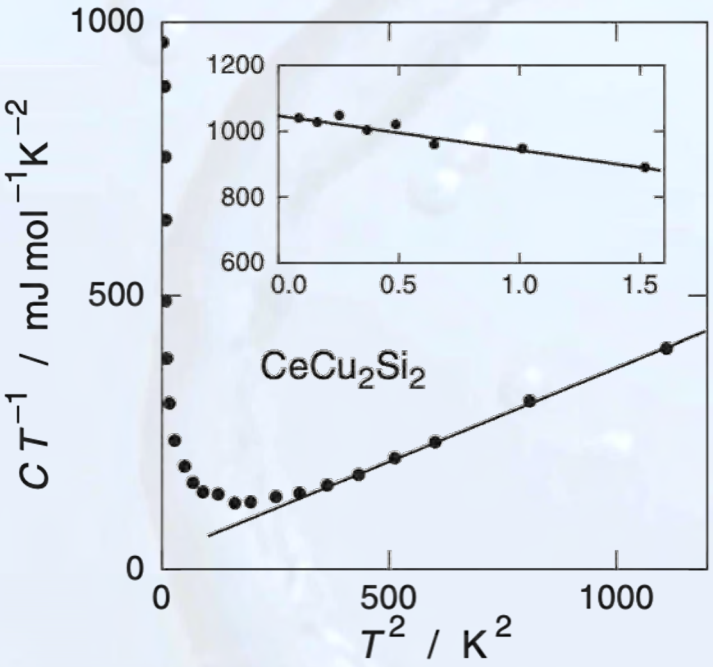
→ no spherical Fermi surface



- d-electrons with large density of state dominate at E_F
- d-electrons are localized

b) metal with heavy electrons

examples: $CeCu_2Si_2$ cer electronic configuration [Xe] $5d^1\ 4f^1\ 6s^2$



→ γ not constant below 15 K

$\gamma = C_{el}/T$

$\gamma \approx 30\,\mathrm{mJ\,(mol\,K^2)^{-1}}$ extrapolated from high T

$\gamma \approx 1050\,\mathrm{mJ\,(mol\,K^2)^{-1}}$ at low temperatures $T \rightarrow 0$

→ $4f$ electrons are localized at high T and form a conduction band at low T

→ effective mass: $m^* \approx 100\,m_e$

- $T > 15$ K, $D(E)$ and m^* are constant
- $T < 15$ K, C/T increase strongly with decreasing temperature

7.1 Specific Heat

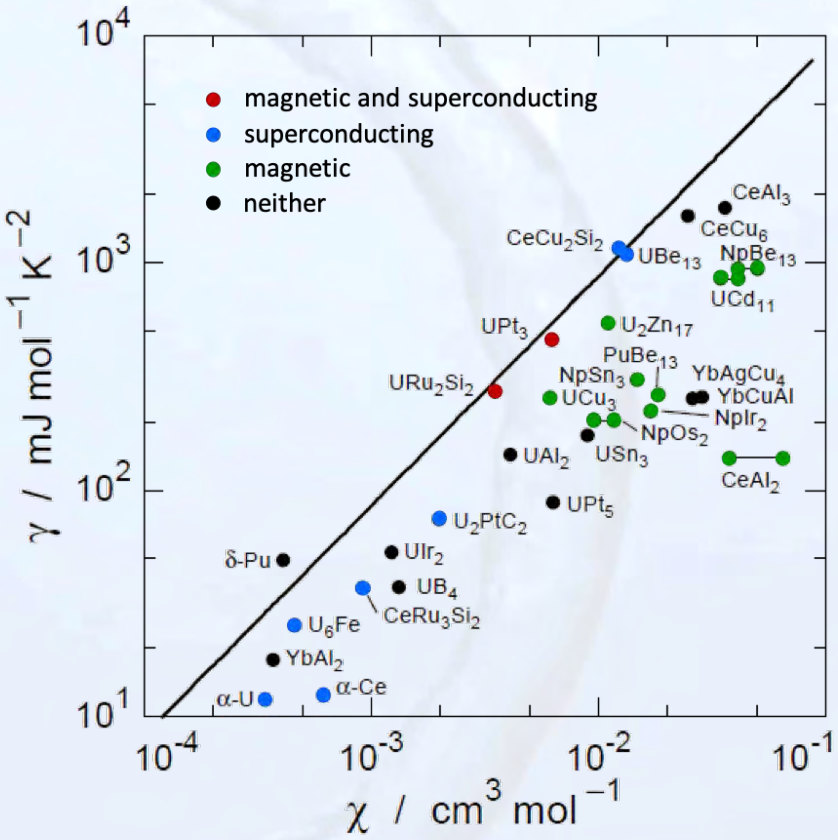
Heavy fermion systems

- interesting class of solids with strongly correlated electrons
- effective masses m^* up to 2000 m_e observed
- origin: interaction with localized spins

$$\chi = \mu_0 \mu_B^2 D(E_F) \propto n^{1/3} m^*$$

Wilson ratio: $$R = \frac{\chi}{\gamma} \frac{\pi^2 k_B^2}{\mu_0 \mu_{eff}^2}$$

$$\gamma \propto n^{1/3} m_{th}^*$$



important: Fermi liquid theory

$$C = \frac{m^*}{m} C_{FG} = \left(1 + \frac{1}{3} F_1\right) C_{FG}$$

analogy to ^{3}He reaches even further

⟶ some heavy fermion systems show unconventional superconductivity (S ≠ 0) : UPt_3, URu_2Si_2 …