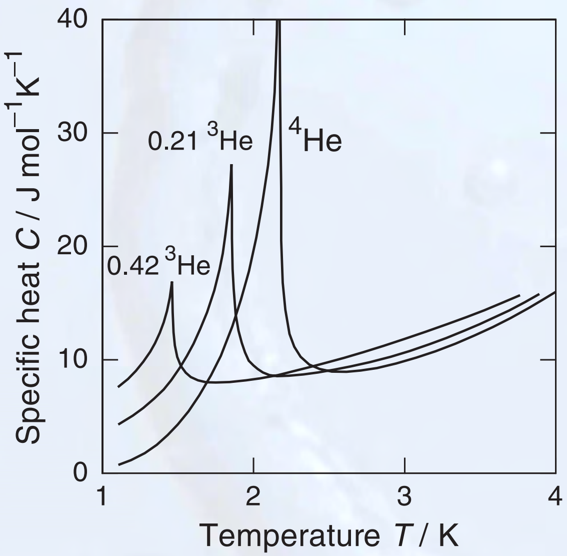
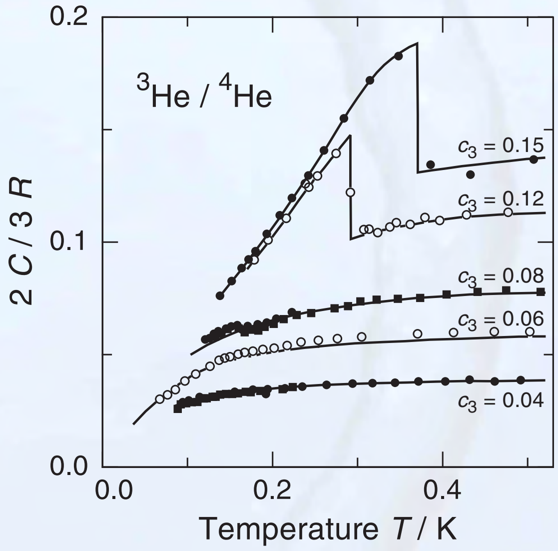
5) ^{3}He/^{4}He Mixtures

first experiments 1947
observation of second sound 1950

- → interesting for technical reasons: dilution cryostats
- → test for different theories: Fermi liquids, RGT, …

5.1 Specific heat and phase diagram





- ► lambda transition shifts to lower temperatures with $c_3 = N_3/(N_3 + N_4)$

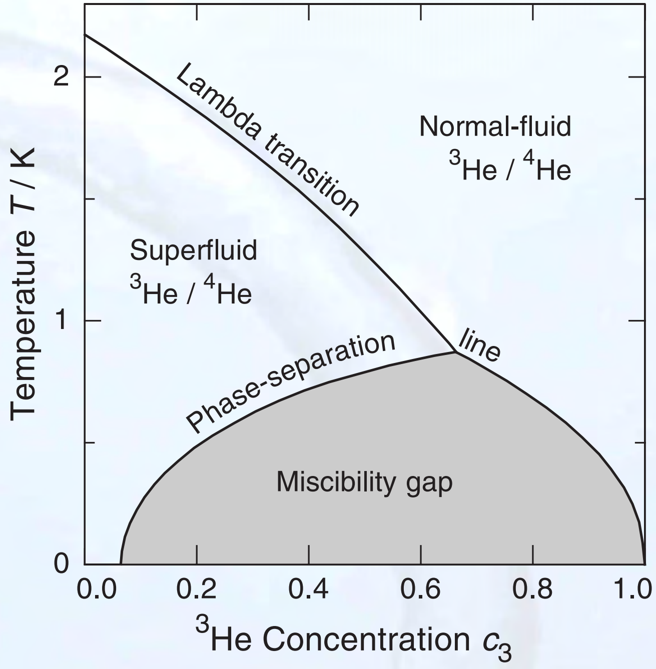
- ► low temperatures: jump in specific heat

⟶ 1st order phase transition

⟶ de-mixing of ^{3}He and ^{4}He

5.1 Specific heat and phase diagram

Phase diagram



- tri-critical point $T = 0.87$ K, $c_3 = 0.67$
- miscibility gap is observed

$a = 0.85\,\mathrm{K}^{-3/2}$

$b = 0.56\,\mathrm{K}$

light phase (^{3}He rich): $c_4 = (1 - c_3) = a\,\sqrt{T^3}\,\mathrm{e}^{-b/T}$

heavy phase (^{4}He rich): $c_3 = c_{3,0}\,(1 + \tilde{a}\,T^2 + \tilde{b}\,T^3)$

$\tilde{b} = 9.4\,\mathrm{K}^{-3}$

$\tilde{a} = 8.4\,\mathrm{K}^{-2}$

$c_{3,0} = 0.0648$

5.1 Specific heat and phase diagram

dilute solutions of ³He in He-II ($c_3 < 0.15$, $T < 0.5$ K)

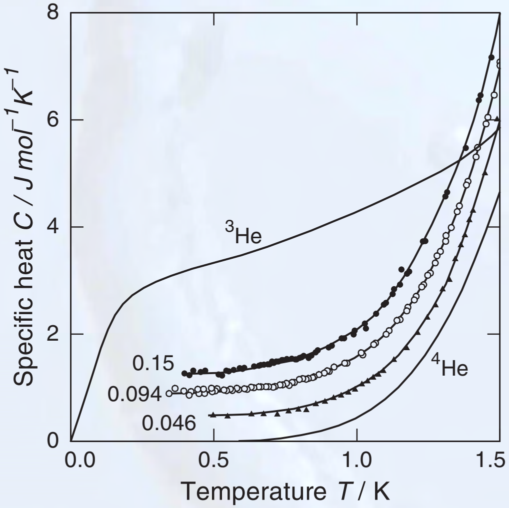
→ ⁴He: passive background fluid

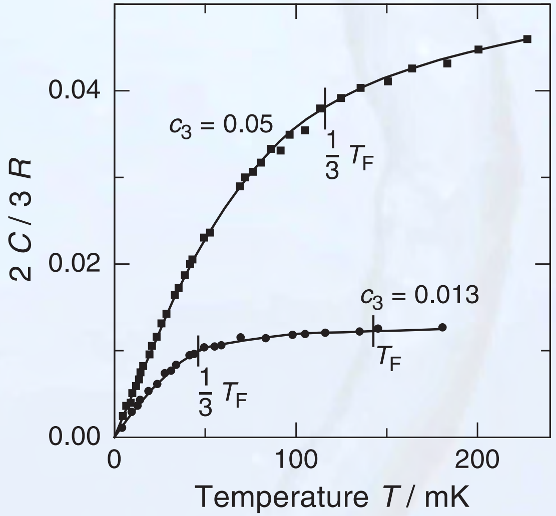
→ ³He: “free” atoms in a quasi vacuum and effective mass $m_3^* = 2.4\, m_3$

Fermi gas $$T_\mathrm{F} = \frac{\hbar^2}{2m_3^* k_\mathrm{B}} \left(3\pi^2 n_3\right)^{2/3} \propto c_3^{2/3}$$

$C \rightarrow T > T_\mathrm{F}, \quad C \propto c_3 T^0 \quad (\widehat{=} \frac{3}{2}R)$ high T

$C \rightarrow T < \frac{1}{3} T_\mathrm{F}, \quad C \propto T$ low T





- T_λ depends on c_3
- pure ³He: transition Fermi gas → Fermi liquid
- high T, dilute solution: classical gas with m^*

- low T: transition classical gas → Fermi gas
- lines correspond to theory

5.1 Specific heat and phase diagram

Finite solubility of ^{3}He in liquid ^{4}He at $T = 0$

reason: difference in zero-point motion of ^{3}He and ^{4}He

v. Waals interaction identical for ^{3}He-^{3}He and ^{3}He-^{4}He

but: larger zero-point motion of ^{3}He weakens the bonding

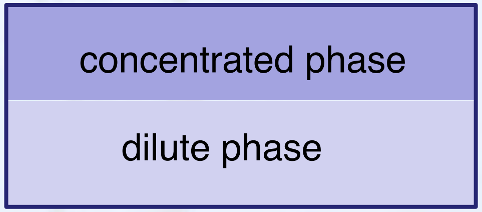
} stronger effective binding for ^{3}He–^{4}He

compared to ^{3}He–^{3}He

in equilibrium on finds $$\mu_{3,\mathrm{d}}(T, c_{3,\mathrm{d}}) = \mu_{3,\mathrm{c}}(T, c_{3,\mathrm{c}})$$

dilute

concentrated



concentrated phase

dilute phase

- ► $T = 0 \longrightarrow$ $c_3 = 1$ for concentrated phase (pure ^{3}He)
- ► necessary energy to bring one ^{3}He atom into “vacuum” $L_3(T = 0)$

latent heat

$$\longrightarrow \quad \mu_{3,\mathrm{c}}(0,1) = \mu_3(0) = -L_3(0) = -2.473\,\mathrm{K}$$

- ► dilute phase: $E_3 = -\mu_{3,\mathrm{d}}(0,0) \longrightarrow$ binding energy

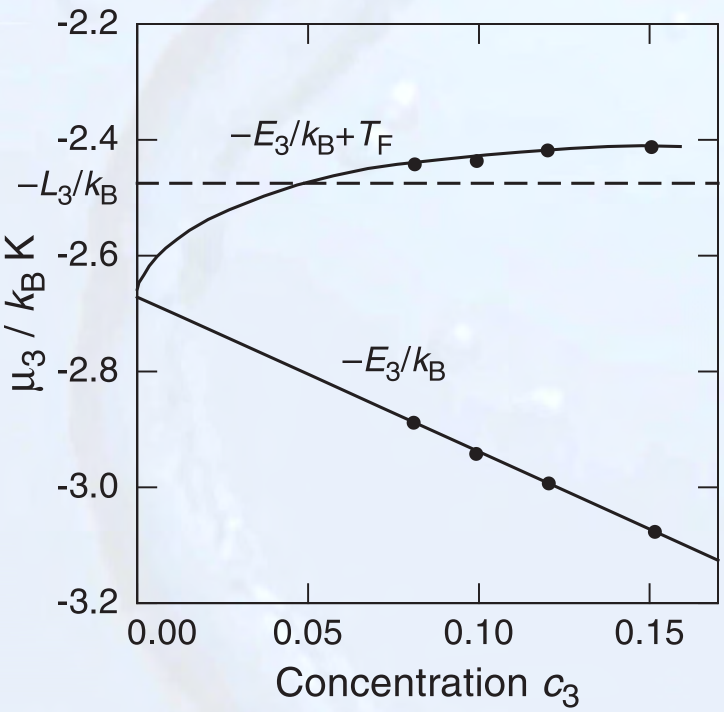
$$c_{3,\mathrm{d}} \rightarrow 0$$

- ► with increasing concentration, the effective binding energy for ^{3}He is reduced because of the Pauli principle $\longrightarrow$ Fermi gas: $E_\mathrm{F} = k_\mathrm{B} T_\mathrm{F}(c_3)$

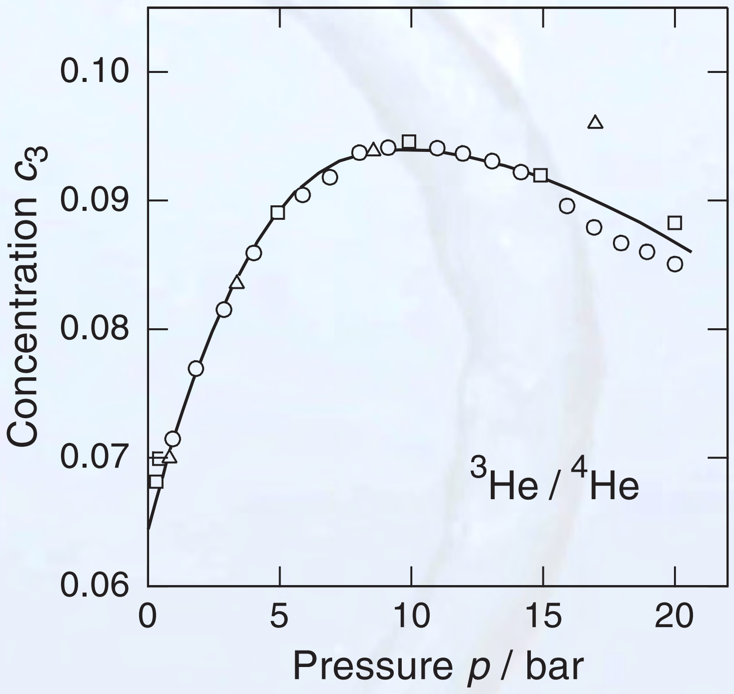
5.1 Specific heat and phase diagram

equilibrium concentration at $T = 0$

$$-L_3(0) = -E_3(0, c_3) + k_\mathrm{B} T_\mathrm{F}(c_3)$$



pressure dependence



- calculation of $E_3(0,c_3)$ is not trivial

  → Bardeen, Baym, Pines model

- maximum at 8.7 bar
- concentration $c_3 = 0.096$

5.2 Normalfluid Component

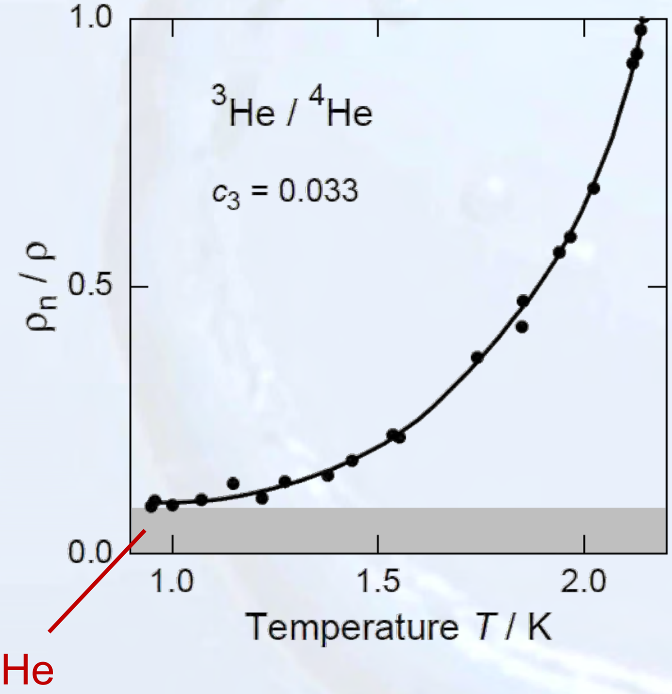
determination of ϱ_n → Andronikasvili-type experiment

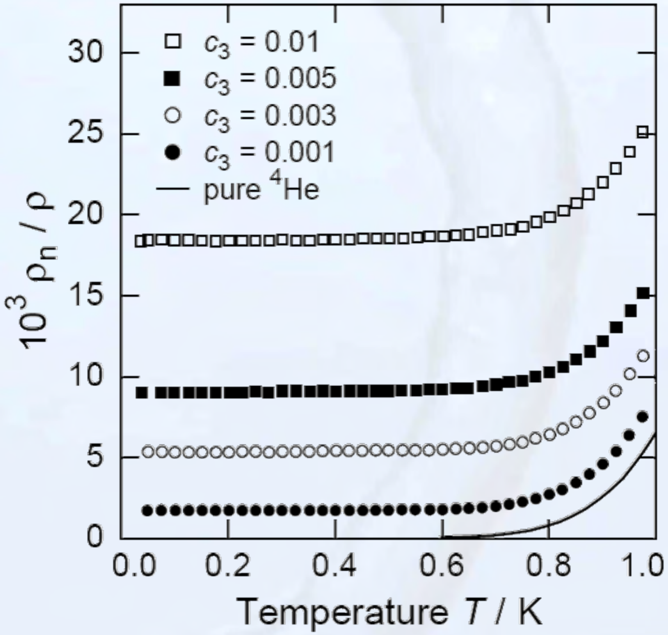
15 mica sheets
4 cm diameter
190 μm spacing

$$\varrho_\mathrm{n} = \varrho_{\mathrm{n},4} + \varrho \, \frac{m_3^*}{m_4} \, c_3$$

pure He-II

const

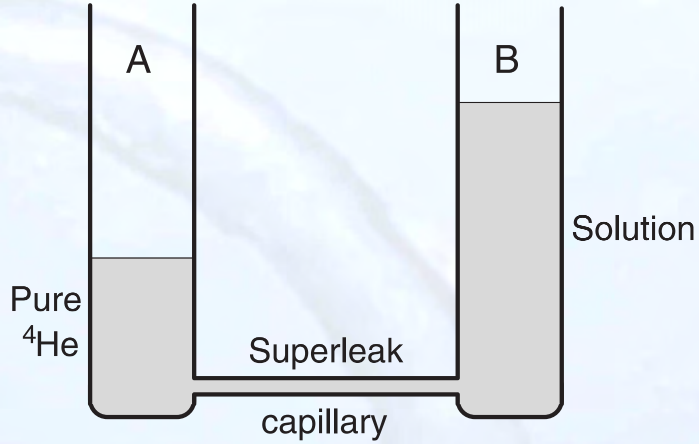




$$\Longrightarrow \quad \varrho_\mathrm{n}(T \to 0) = \mathrm{const} \propto c_3$$

5.2 Normalfluid Component

Osmotic pressure



- ^{4}He flows to solution to thin the ^{3}He concentration
- ^{3}He is blocked

→ osmotic pressure

van't Hofft law ($T \gg T_\mathrm{F}$, classical regime)

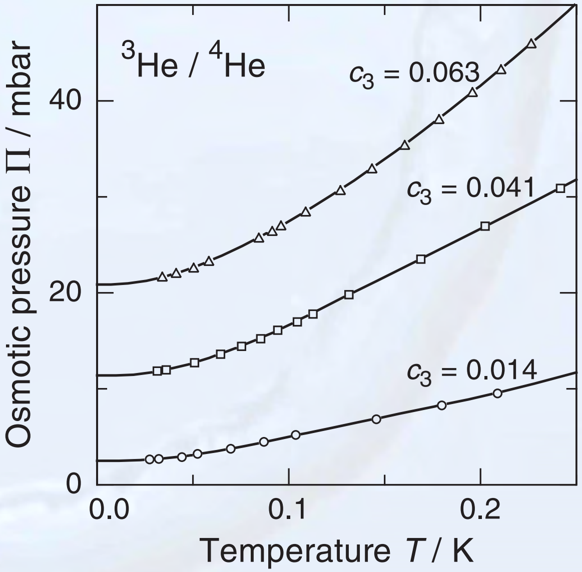
$$\Pi = n_3 k_\mathrm{B} T \;\propto\; c_3 T$$

$T \ll T_\mathrm{F}$, degenerate Fermi gas

$$\Pi = \frac{2}{5} n_3 k_\mathrm{B} T_\mathrm{F} \propto c_3^{5/3} = \text{const}$$

depends on c_3

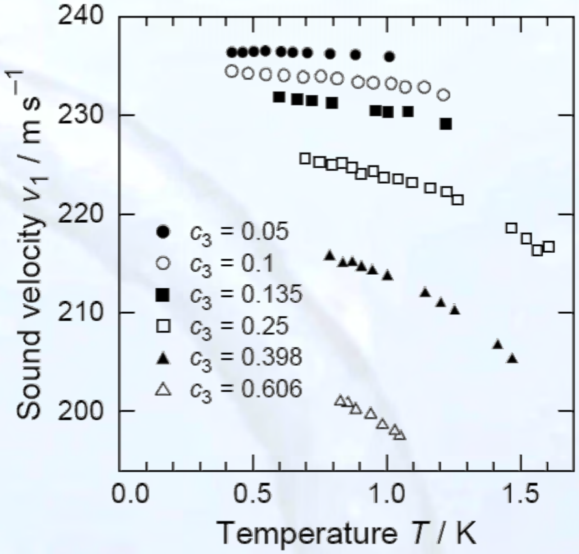
→ transition from FG to classical gas



5.3 Sound Propagation

First sound

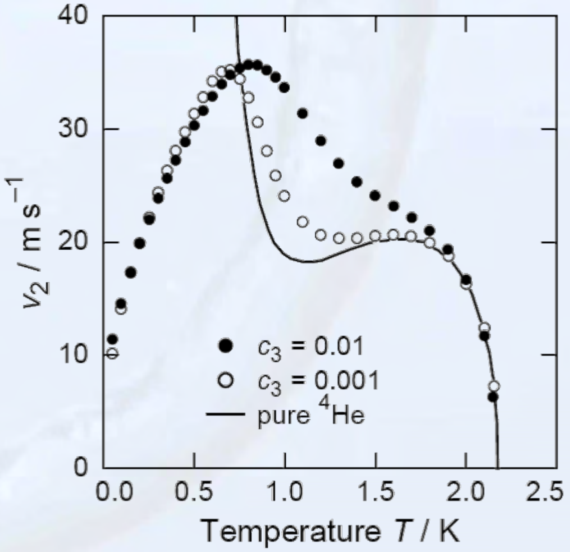
$$v_1^2 = \left(\frac{\partial p}{\partial \varrho}\right)_{S,c_3} \left[1 + \frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}} \left(\frac{\partial \varrho}{\partial c_3}\,\frac{c_3}{\varrho}\right)^2\right]$$



Second sound

$$v_2^2 = \frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}} \left[\overline{S}\left(\frac{\partial T}{\partial S}\right)_{\varrho,c_3} + c_3^2 \frac{\partial(\mu_3 - \mu_4)}{\partial c_3}\right] \left[1 + \frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}} \left(\frac{\partial \varrho}{\partial c_3}\,\frac{c_3}{\varrho}\right)^2\right]^{-1}$$

$$\overline{S} = S_{4,0} - \frac{k_\mathrm{B}}{m_4}\left[c_3 + \ln(1 - c_3)\right] + \frac{k_\mathrm{B}}{m_3} c_3$$



5.4 Transport Properties

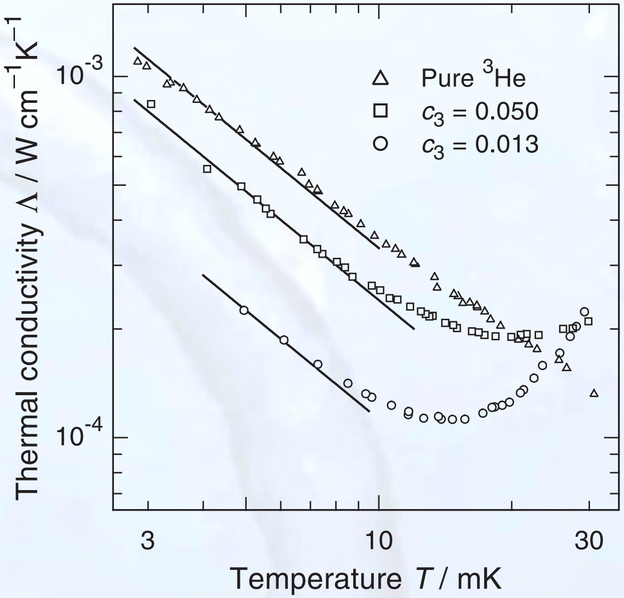
Thermal transport (rather complex)

high Temp.: ϱ_n flow leads to ^{3}He concentration gradient

→ ^{3}He atoms diffuse back

→ ^{3}He form scattering centers for ϱ_n

→ reduction of heat transport



low Temp.: ^{3}He atoms from FG

$$\Lambda = \frac{1}{3} C\, v_\mathrm{F}\, \ell \propto \frac{c_3}{T}$$

$$\ell = v_\mathrm{F}\tau \propto (T_\mathrm{F}/T)^2$$

$$v_\mathrm{F} = (\hbar/m_3^*)(3\pi^2 n_3)^{1/3},$$

$$C \propto T/T_\mathrm{F}$$

$$m_3^* = (1 + F_1/3) m_3$$

5.5 Search For a Superfluid Phase of ^{3}He in Mixture

very interesting: 3 superfluid phases in the same container ⟶ ^{4}He, ^{3}He, and dilute ^{3}He

Problem: ^{3}He/^{4}He mixtures are hard to cool to below 200 μK because of Kapitza resistance

↓

acoustic mismatch hinders cooling

new initiative:

⟶ cooling by melting of ^{4}He crystal

⟶ lowest temperature so far 90 μK