4.4 Orderparameter Orientation — Textures

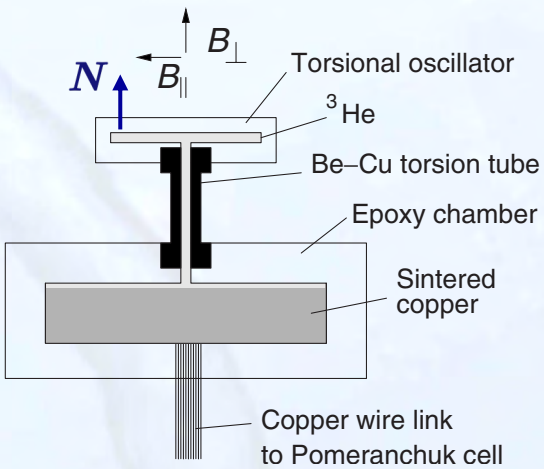
Example for influence of wall and magnetic field

Determination of ϱ_s/ϱ with with a disc like resonator

- ϱ_n is dragged with resonator because of η_n
- mass of ϱ_n adds to moment of inertia
- resonance frequency depends on ϱ_n/ϱ

$\longrightarrow \varrho_s/\varrho$

Andronikasvili-like experiment



(i) $\boldsymbol{B}$ parallel to wall $\boldsymbol{B}_\parallel \perp \boldsymbol{N}$

$$\left.\begin{array}{l} \boldsymbol{l} \parallel \boldsymbol{N} \\ \boldsymbol{S} \parallel \boldsymbol{B}_\parallel \curvearrowright \boldsymbol{d} \perp \boldsymbol{B}_\parallel \end{array}\right\} \boldsymbol{d} \parallel \boldsymbol{l}$$

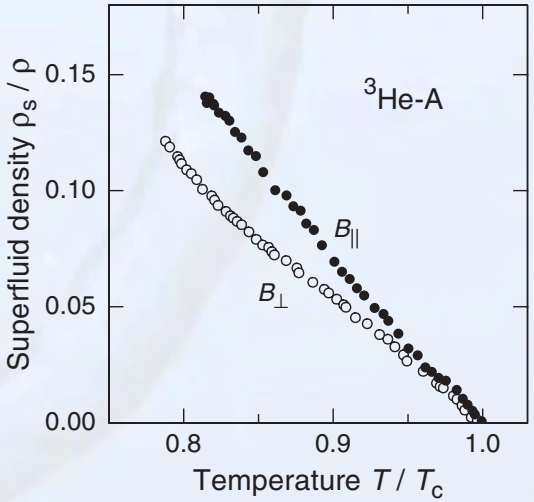
optimal even without external field

(ii) $\boldsymbol{B}$ perpendicular to wall $\boldsymbol{B}_\perp \parallel \boldsymbol{N}$

$$\left.\begin{array}{l} \boldsymbol{l} \parallel \boldsymbol{N} \\ \boldsymbol{S} \parallel \boldsymbol{B}_\perp \curvearrowright \boldsymbol{d} \perp \boldsymbol{B}_\perp \end{array}\right\} \boldsymbol{d} \perp \boldsymbol{l}$$

not optimal for dipole dipole interaction

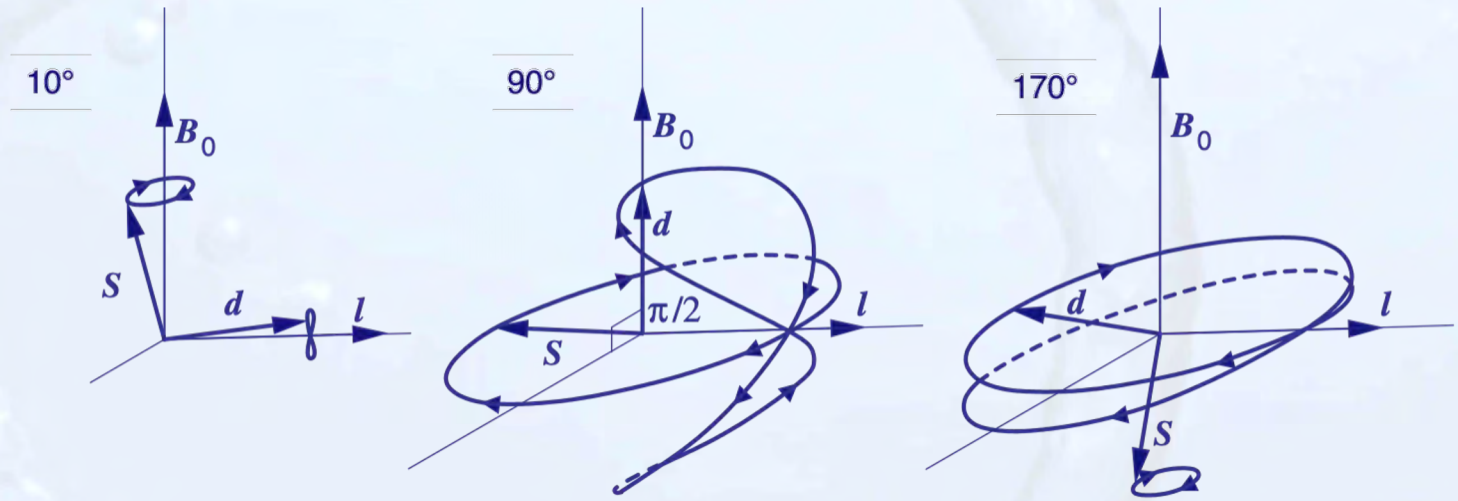
$\longrightarrow \varrho_{s\perp} < \varrho_{s\parallel}$



4.5 Spin Dynamics — NMR Experiments

- static field $\boldsymbol{B}_0 \longrightarrow \omega_{\mathrm{L}} = \gamma|\boldsymbol{B}_0|$ Lamor frequency
- rf pulse $\longrightarrow$ tipping of the magnetization $\langle \boldsymbol{S} \rangle$
- ^{3}He: coupling of $\boldsymbol{S}, \boldsymbol{d}$ $(\boldsymbol{d} \cdot \boldsymbol{S} = 0) \longrightarrow$ additional restoring force

angle between $\boldsymbol{B}_0$ and $\boldsymbol{S}$ (tipping angle)



- without external influences: state of minimal dipole-dipole energy $\boldsymbol{d} \parallel \boldsymbol{l}$
- any deviation from $\boldsymbol{d} \parallel \boldsymbol{l}$ costs energy proportional to $\sin^2(\boldsymbol{d}, \boldsymbol{l})$

  $\longrightarrow$ resonance frequency increases

4.5 Spin Dynamics — NMR Experiments

Leggett equations:

$$\boldsymbol{R}_\mathrm{d}(T) = \frac{6}{5}\, g_\mathrm{d}(T)\,(\boldsymbol{d}\times\boldsymbol{l})\,(\boldsymbol{d}\cdot\boldsymbol{l})$$ additional restoring force

$$\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \gamma\boldsymbol{S}\times\boldsymbol{B}_0 + \boldsymbol{R}_\mathrm{d}$$

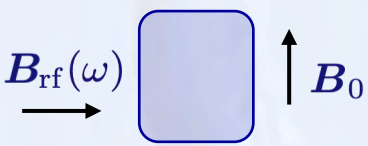
$$\frac{\mathrm{d}\boldsymbol{d}}{\mathrm{d}t} = \boldsymbol{d}\times\gamma\boldsymbol{B}_\mathrm{eff} = \boldsymbol{d}\times\gamma\left(\boldsymbol{B} - \frac{\mu_0\gamma\boldsymbol{S}}{\chi_\mathrm{N}}\right)$$

comment:

"Bloch equations" for superfluid ^{3}He-A

all predictions from these equations have precisely observed

transversal resonance

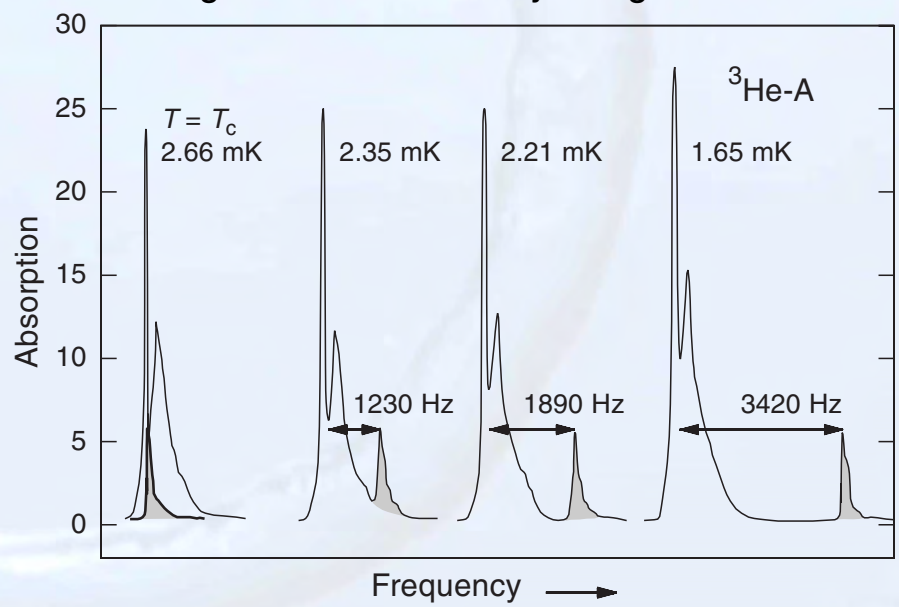


small (tipping) angle solution:

$$\omega_\mathrm{t}^2 = (\gamma B_0)^2 + \frac{\gamma^2\mu_0\langle H_\mathrm{d}\rangle}{\chi_\mathrm{N}} = \omega_\mathrm{L}^2 + \Omega_\mathrm{A}^2(T)$$

spatial mean of dipole-dipole coupling

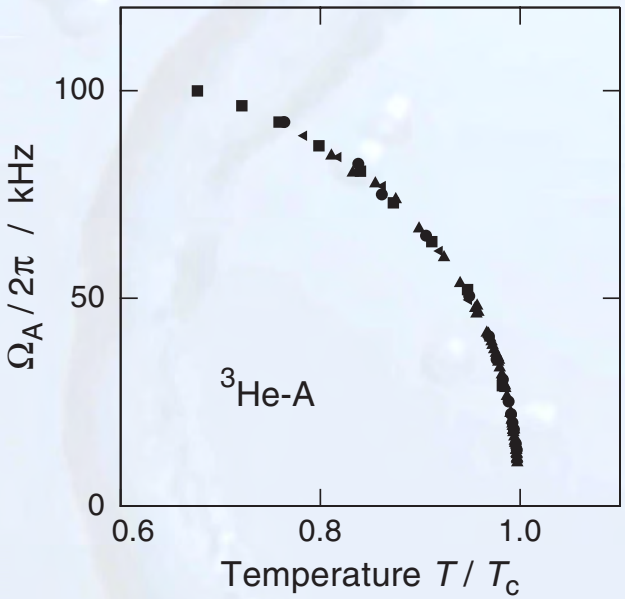
original observations by Doug Osheroff



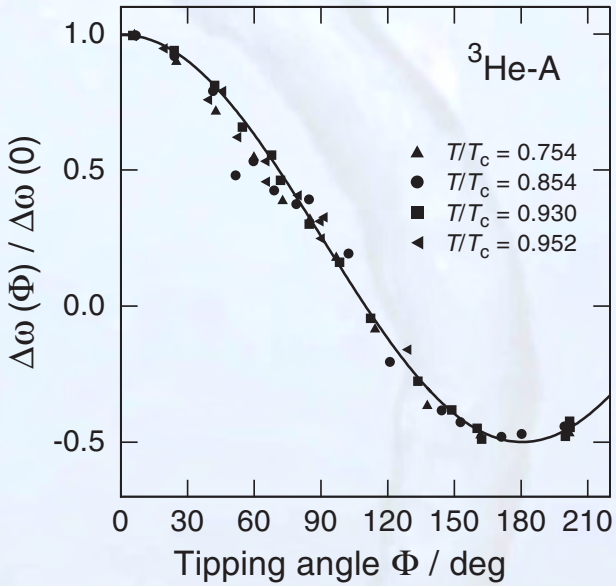
4.5 Spin Dynamics — NMR Experiments

extended NMR experiments with transvers geometry

small angle measurement



tipping angle dependence

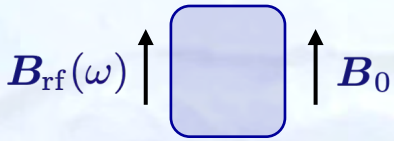


- resonance frequency increases proportional to ϱ_s
- temperature dependence of order parameter

- line shows prediction from Leggett equations
- excellent agreement with theory

4.5 Spin Dynamics — NMR Experiments

Longitudinal resonance



modulation of static field $B_0 \longrightarrow B_z = B_0 + B_{\mathrm{rf}}(\omega)$

↓

oscillation of $\boldsymbol{d}$ ⟶ resorting force

in ordinary liquids $\boldsymbol{R}_{\mathrm{d}} = 0$

because of $(\dot{\boldsymbol{S}} \times \boldsymbol{B})_z = 0 \longrightarrow \mathrm{d}S_z/\mathrm{d}t = R_{\mathrm{d,z}}$ first Leggett equation

^{3}He-A: $R_{\mathrm{d}} \neq 0 \quad |\uparrow\uparrow\rangle \longleftrightarrow |\downarrow\downarrow\rangle$

$$\Delta S_z = \frac{\chi}{\mu_0 \gamma} \Delta B_0 \, (1 - \cos \Omega_{\mathrm{A}} t)$$

^{3}He-B:

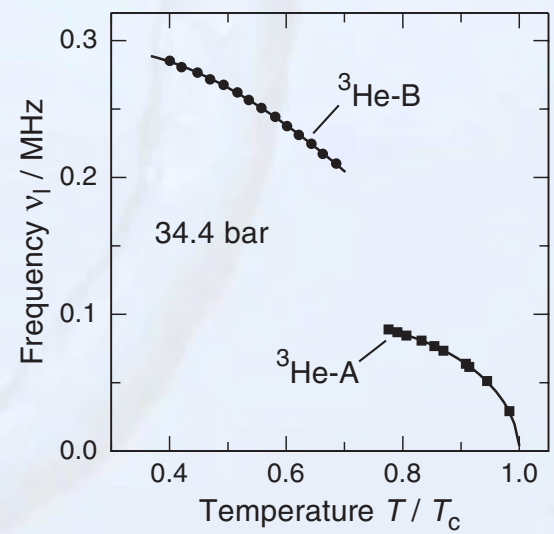
reduced, since 1/3 in $|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle$

$$\Omega_{\mathrm{B}}^2(T) = \Omega_{\mathrm{A}}^2(T) \, \frac{5\,\chi_{\mathrm{B}}}{2\,\chi_{\mathrm{A}}}$$

$\chi_{\mathrm{A}} \mathrel{\widehat{=}}$ ^{3}He-N

^{3}He-A$_1$:

no effect, since only one spin configuration $|\uparrow\uparrow\rangle$



4.6 Macroscopic Quantum Effects

macroscopic wave function

$$\Psi_{\alpha\beta}(\boldsymbol{r}) = \mathcal{A}_{\alpha\beta}(\boldsymbol{r})\,\mathrm{e}^{\mathrm{i}\varphi(\boldsymbol{r})}$$

3 x 3 matrix

18 degrees of freedom

i) quantization of circulation

→ ^{4}He circulation is quantized

→ ^{3}He behavior is more complicated

^{3}He-A: → circulation is only irrotational under ideal conditions, which means without external influences

→ if $\operatorname{curl} \boldsymbol{v}_\mathrm{s} \neq 0$ → $\boldsymbol{v}_\mathrm{s} \neq \dfrac{\hbar}{2m_3}\nabla\varphi$

- in general $\operatorname{curl} \boldsymbol{v}_\mathrm{s} = \dfrac{\hbar}{2m_3 r}\,\hat{\boldsymbol{l}} \cdot \left(\dfrac{\partial \hat{\boldsymbol{l}}}{\partial \phi} \times \dfrac{\partial \hat{\boldsymbol{l}}}{\partial r}\right)$
- phase can be adjusted by modification of $\boldsymbol{l}$
  structure of vortices depend on $\boldsymbol{l}(\boldsymbol{r})$

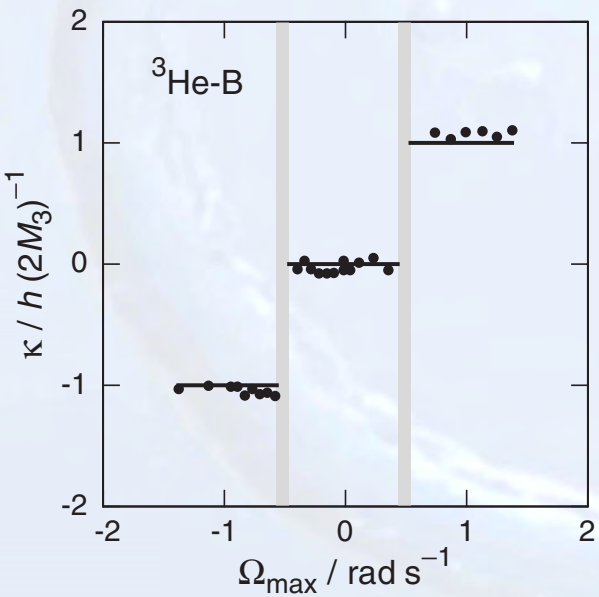
4.6 Macroscopic Quantum Effects

^{3}He-B: ⟶ circulation is quantized

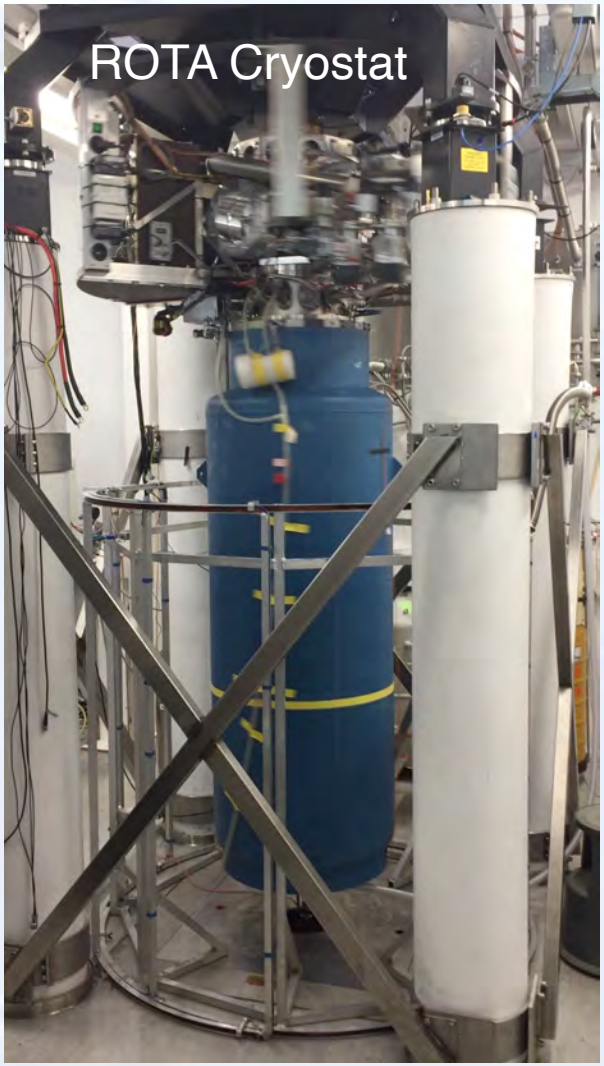
⟶ $$\boldsymbol{v}_\mathrm{s} = \frac{\hbar}{2m_3}\nabla\varphi$$

⟶ $$\boldsymbol{\kappa}_3 = \frac{h}{2m_3}$$

- Vinen-type experiment
- 1 rad/s = 0.16 revolutions /s

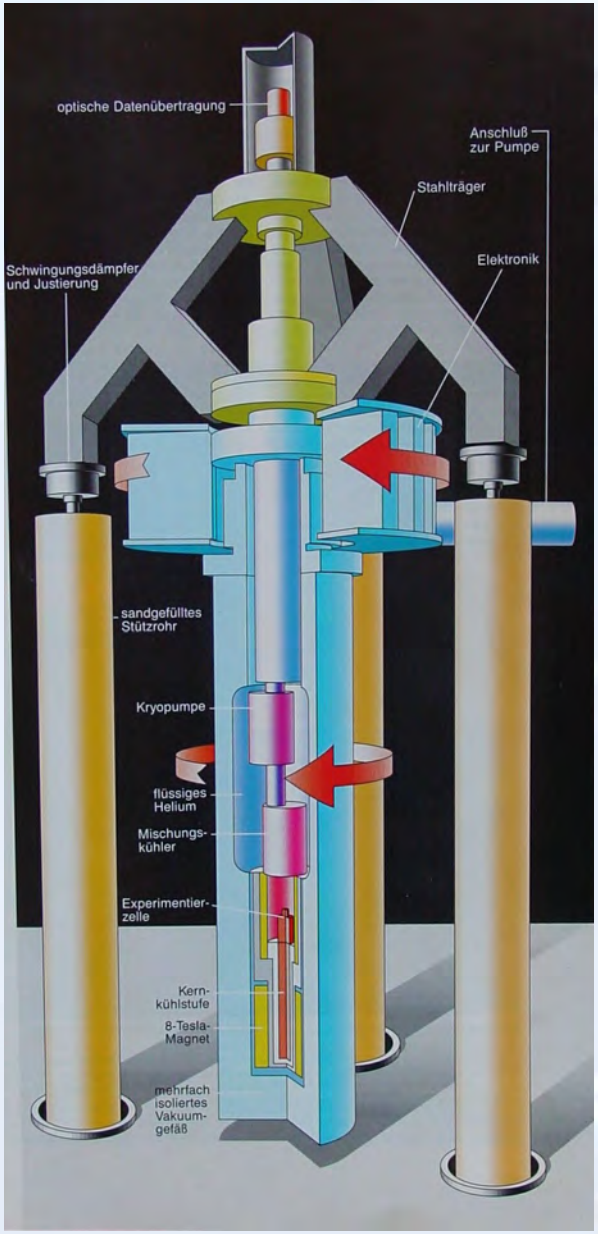


experimental problem:
rotation at very low temperatures



up to 3 revolutions / s

4.6 Macroscopic Quantum Effects



4.6 Macroscopic Quantum Effects

Quantized Vortices (structure much more complicated as in He-II)

^{3}He-A:

a) with uniform texture and orbital field $\boldsymbol{l}$ ⟶ vortices with normal-fluid hard core $\xi_0 \approx 100\,\mathrm{nm}$

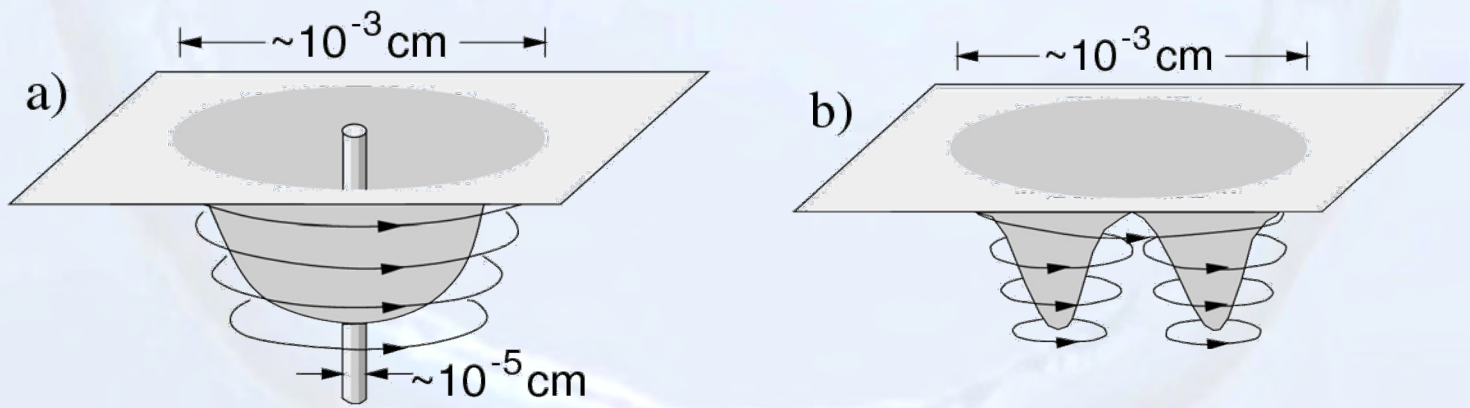
coherence length

extended soft region $\xi_\mathrm{d} \approx 6\,\mu\mathrm{m}$ $n = 1$

dipole healing length it describes over which distance $\boldsymbol{d} \parallel \boldsymbol{l}$ recovers

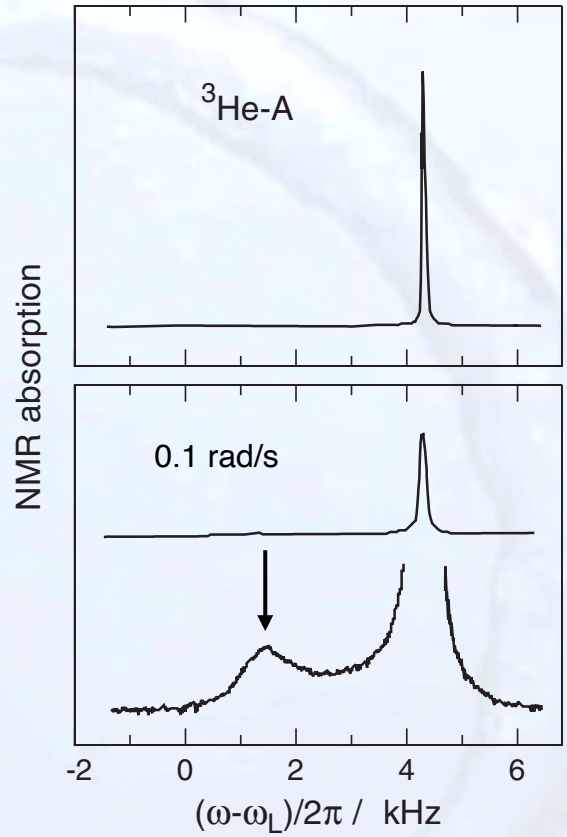
b) if $\boldsymbol{l}$ can adjust freely one finds continuous vortices with $n = 2$ without singularity (no hard core)

continuous velocity field



4.6 Macroscopic Quantum Effects

Investigation of vortices in ^{3}He-A with NMR



cryostat at rest

cryostat rotating

frequency shift because of localized spin waves in core!

container diameter 2.5 mm

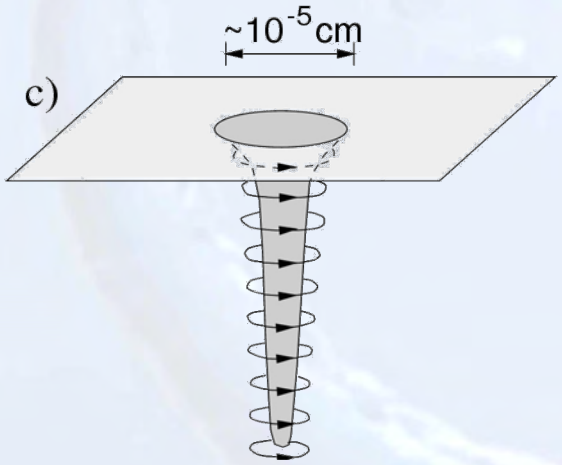
4.6 Macroscopic Quantum Effects

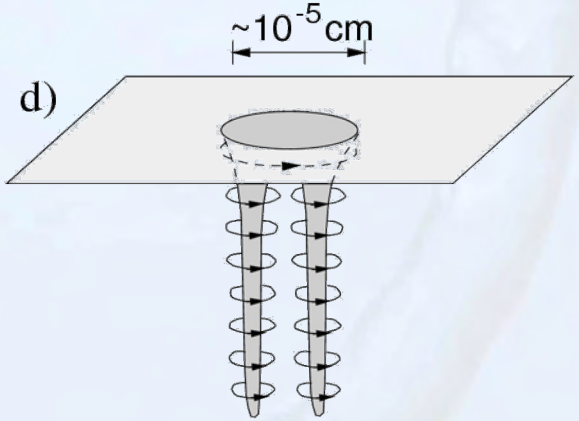
^{3}He-B: only vortices with hard core $\xi_0 \approx 10\ldots\ 100\,\mathrm{nm}$

depends on pressure

c) single vortices with A phase in core

d) double vortices with two half-quantum of circulation and normal-fluid core

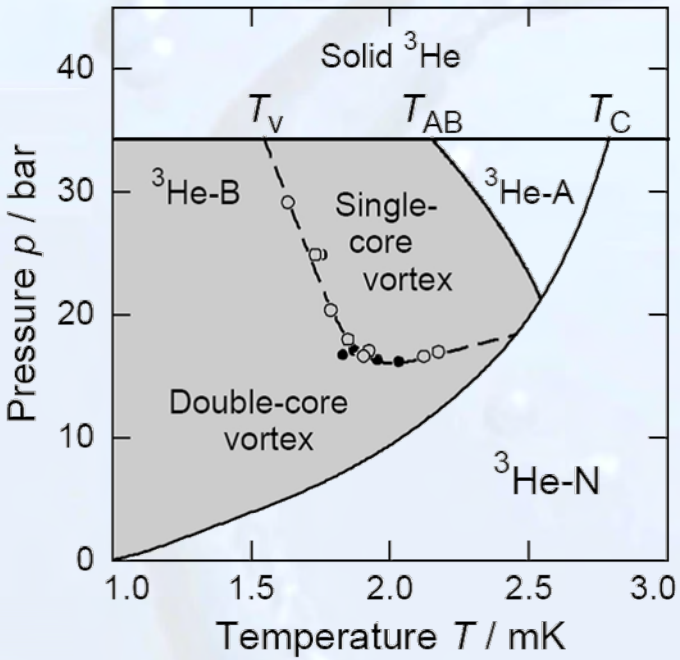




these vortices exist in distinct parts of the phase diagram

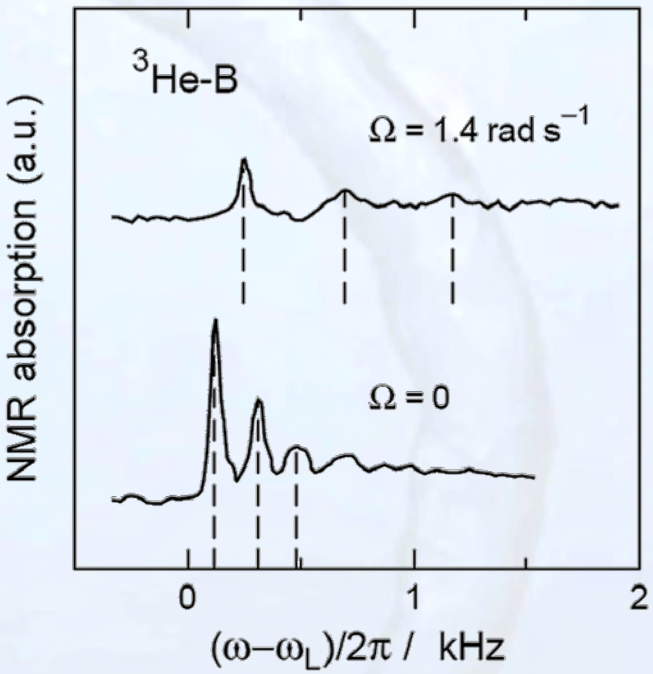
4.6 Macroscopic Quantum Effects

^{3}He-B: phase diagram under rotation



first order phase transition

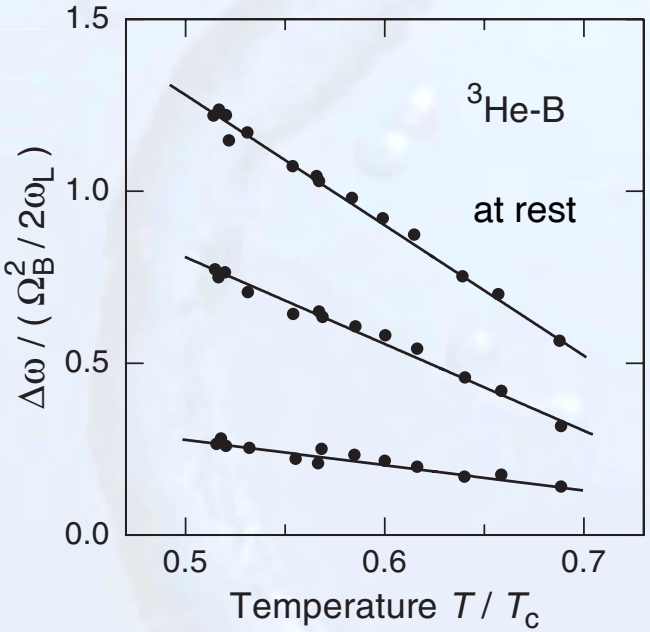
spin waves resonances (collision-less)



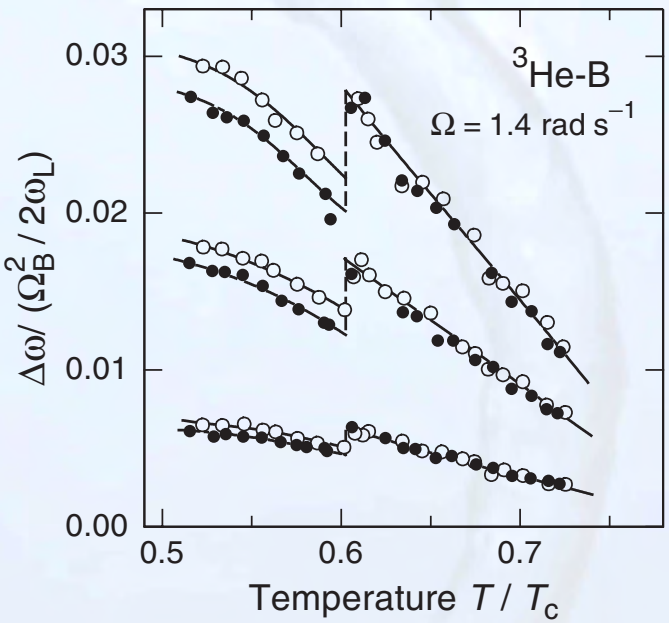
under rotation ⟶ larger spacing
because additional term in free energy

4.6 Macroscopic Quantum Effects

^{3}He-B: phase diagram under rotation



spin waves resonances (collision-less)

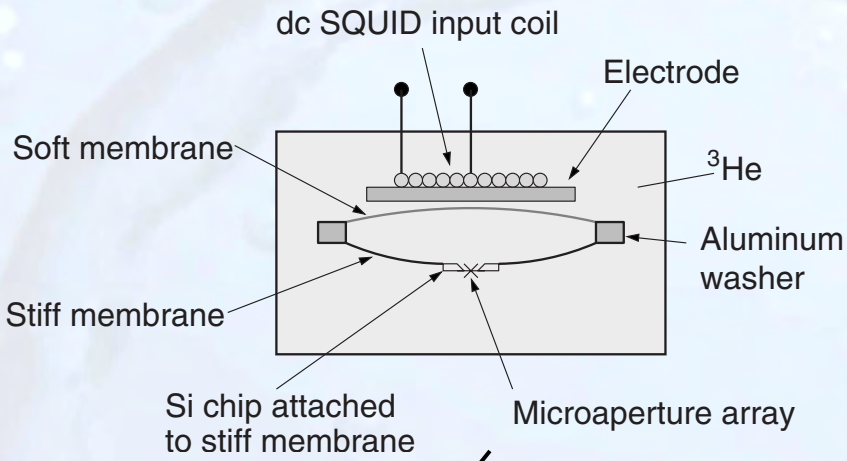


hysteresis is observed

⟶ 1st order transition

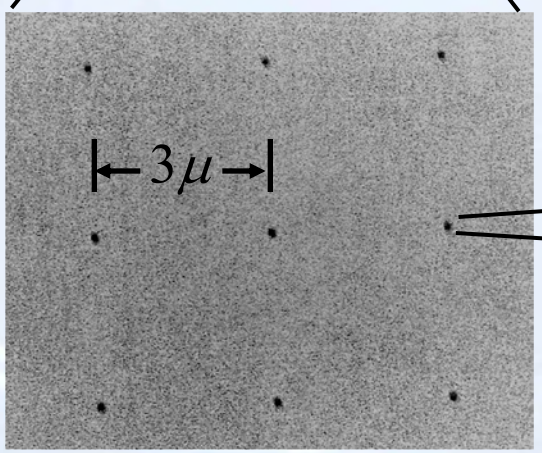
4.6 Macroscopic Quantum Effects

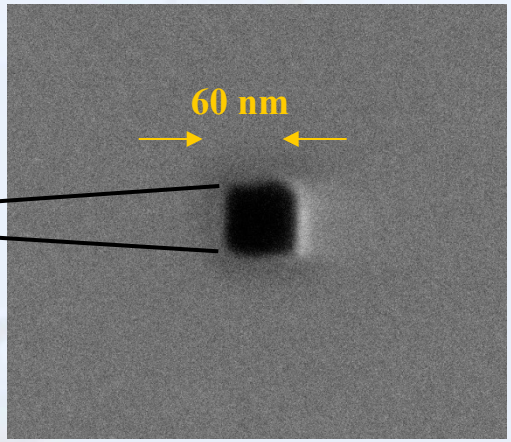
ii) Josephson effects



4225 holes in

50 nm thick menbrane

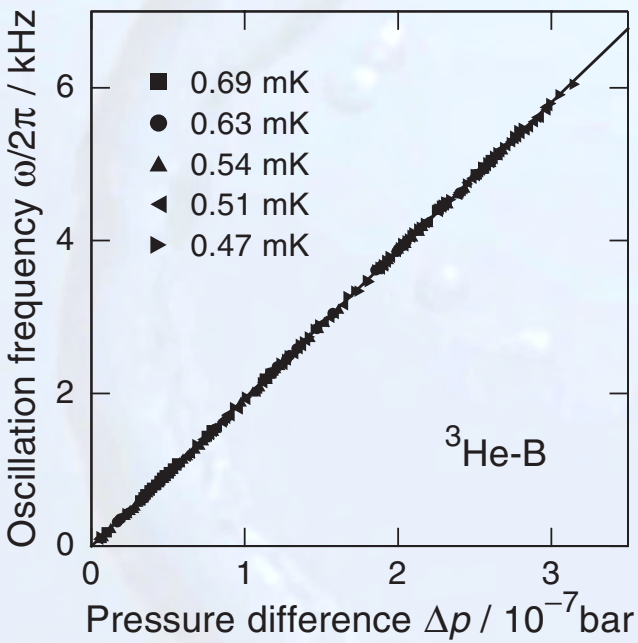




4.6 Macroscopic Quantum Effects

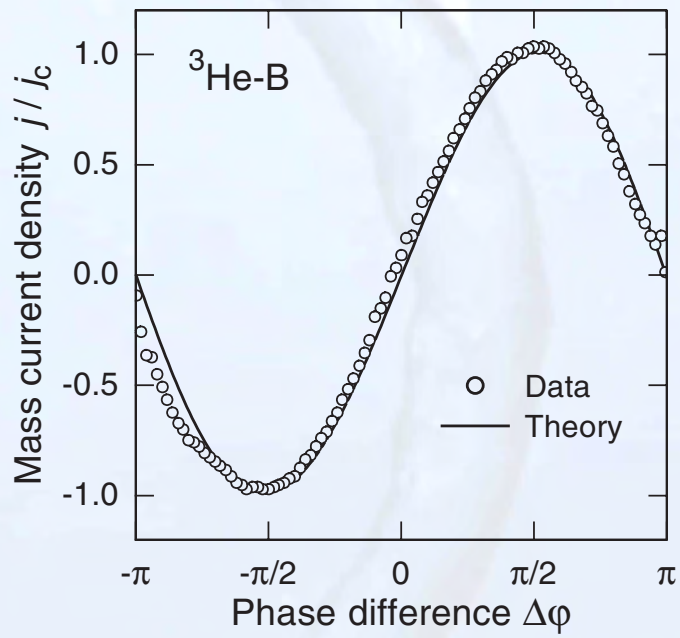
Josephson frequency

$$\omega_J = m_3 \, \Delta p / (\varrho \hbar)$$



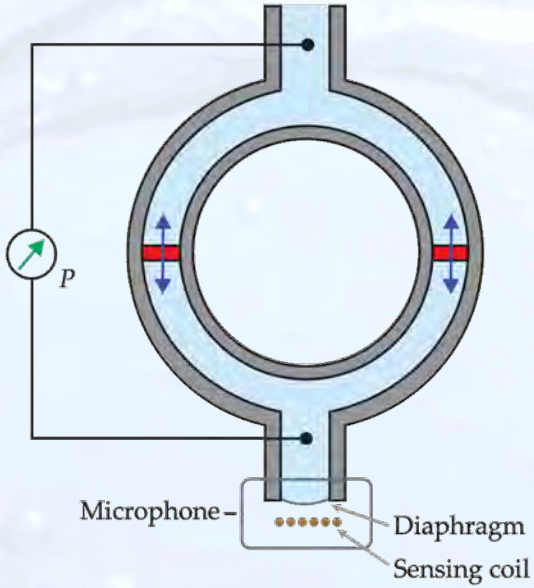
Josephson dc current

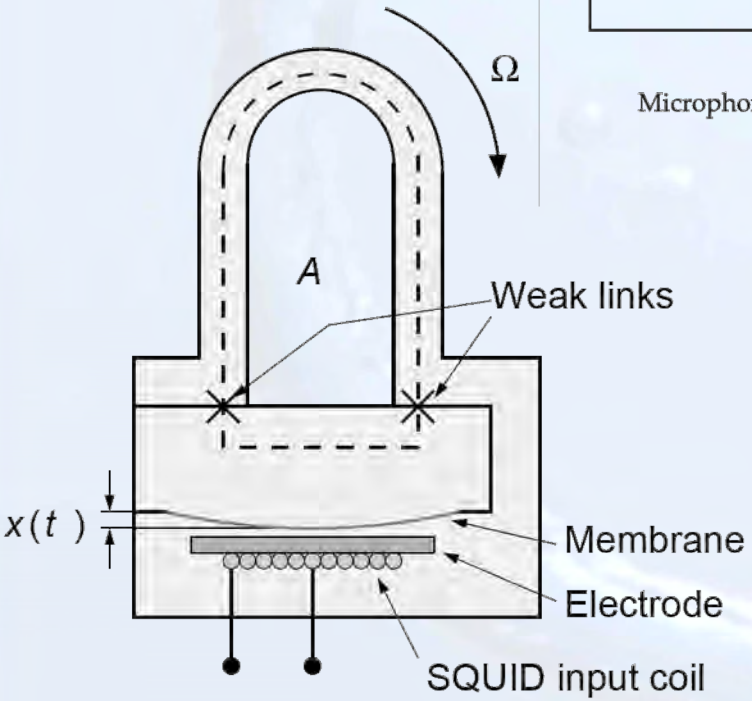
$$j = j_c \sin(\Delta\varphi)$$



4.6 Macroscopic Quantum Effects

DC-SHeQUID: Superfluid He QUantum Interference Devices

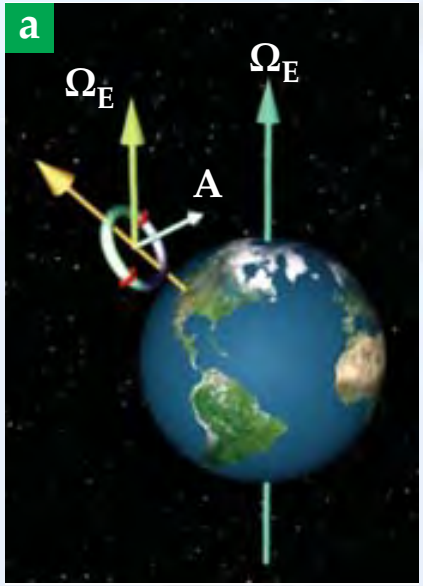




actual device

4.6 Macroscopic Quantum Effects

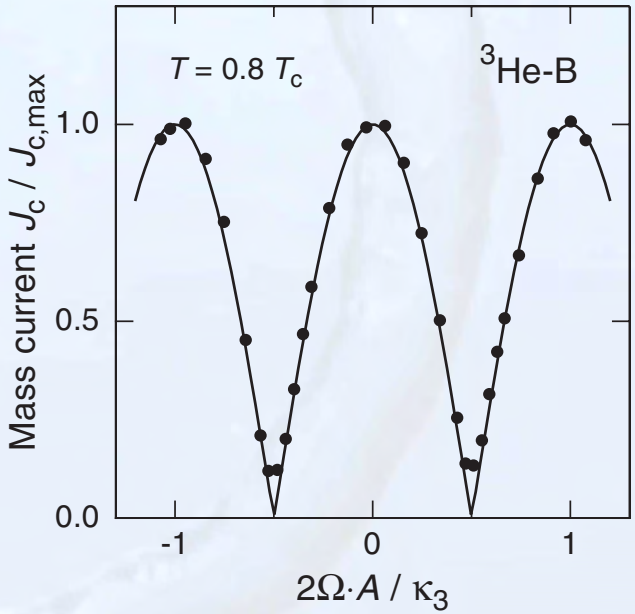
DC-SHeQUID in Earth rotation



angular velocity of rotating system (earth)

normal vector of loop

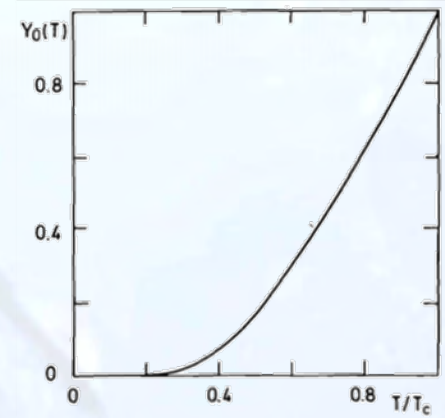
$$j_c = 2j_c^0 \left| \cos\left(\pi \frac{2\boldsymbol{\Omega}\cdot\boldsymbol{A}}{\kappa_3} \right) \right|$$



→ perfect agreement with theoretical expectations

4.7 Normalfluid Density

normalfluid density ⟷ thermal excitations of quasi-particles



General expression from Landau's Fermi liquid theory

$$\overleftrightarrow{\varrho}_{\mathrm{n}} = \frac{m^*}{m}\left(\overleftrightarrow{\mathbf{1}} + \frac{1}{3}F_1\frac{\overleftrightarrow{\varrho}_{\mathrm{n},0}}{\varrho}\right)^{-1}\overleftrightarrow{\varrho}_{\mathrm{n},0}$$

normalfluid density without Fermi liquid correction

⟶ temperature dependence given by Yosida function $Y_0(\widehat{\boldsymbol{k}}, T)$

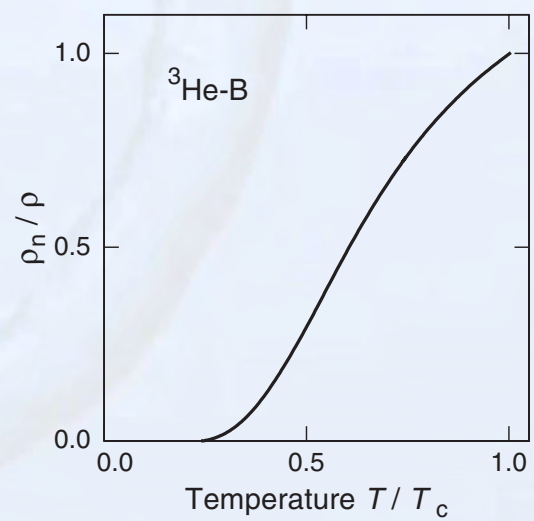
^{3}He-B

isotropic ⟶ independent of $\widehat{\boldsymbol{k}}$ ⟶ $Y_0(\widehat{\boldsymbol{k}}, T) = Y_0$

Andronikasvili-like experiment

$$\varrho_{\mathrm{n}} = \varrho\frac{(1+F_1/3)\,Y_0}{1+F_1Y_0/3}$$

temperature dependent scalar!



- monotonic increase for $T \to T_c$
- disappears exponentially for $T \to 0$

4.7 Normalfluid Density

^{3}He-A much more complicated situation!

close to T_c ϱ_n can approximated by:

$$\varrho_{n,\perp} - \varrho = 2\varrho_{n,\parallel} = -\frac{7}{5}\zeta(3)\frac{m}{m^*}\varrho\left(\frac{\Delta_m}{\pi k_B T_c}\right)^2$$

parallel to orbital momentum

perpendicular to orbital momentum

very low temperatures:

$$\varrho_{n,\parallel} = \pi^2 \frac{m}{m^*}\varrho\left(\frac{k_B T}{\Delta_m}\right)^2 \qquad \propto T^2$$

$$\varrho_{n,\perp} = \frac{7}{15}\pi^4 \frac{m}{m^*}\varrho\left(\frac{k_B T}{\Delta_m}\right)^4 \qquad \propto T^4$$

Specific heat

^{3}He-B

$$C_B(T) = \sqrt{2\pi}\, D(E_F)\, k_B \Delta_B \left(\frac{\Delta_B}{k_B T}\right)^{3/2} e^{-\Delta_B/(k_B T)}$$

^{3}He-A

$$C_A(T) = \frac{7}{5}\pi^2\left(\frac{T}{\Delta_m}\right)^2 C_N(T) \propto T^3$$

for $T \ll \Delta_m/k_B$ and $\hat{\boldsymbol{k}} = \pm\hat{\boldsymbol{l}}$

$$C = \frac{m^*}{m} C_{FG}$$

4.8 Collective Excitations — Sound Propagation

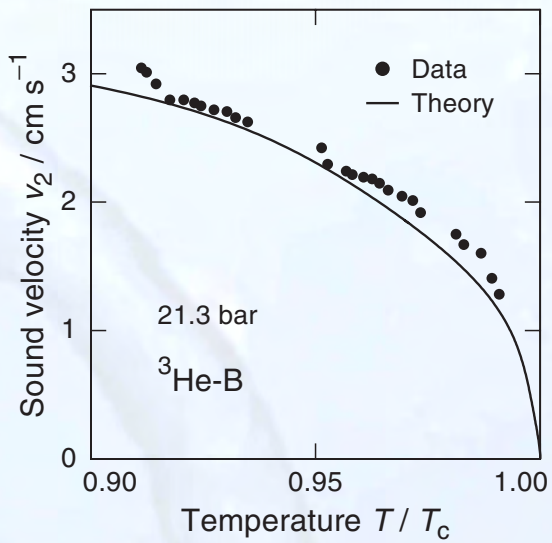
a) 2nd Sound

^{3}He-B

$$v_2 = \sqrt{\frac{\varrho_s}{\varrho_n} S^2 \left(\frac{\partial T}{\partial S}\right)_\varrho}$$

as in case of He-II

- v_2 just a few cm/s
- reduction $\propto \frac{T}{T_F}$ in S

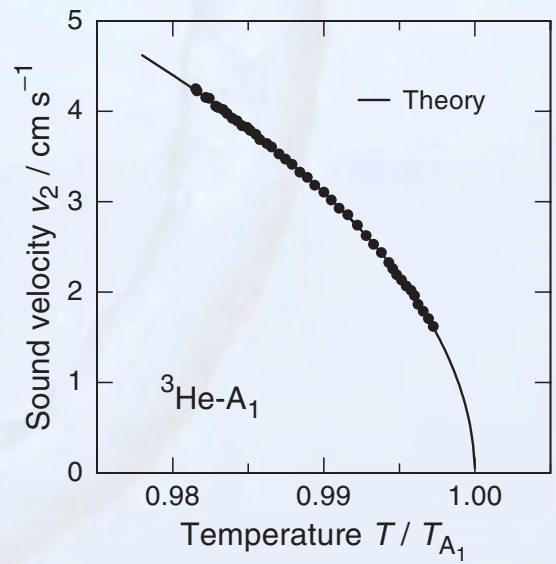


^{3}He-A$_1$

$$v_2 = \frac{\gamma\hbar}{2m^*}\sqrt{\frac{\varrho}{\chi}\frac{\varrho_{s,\perp}}{\varrho_{n,\perp}}}$$

not only entropy wave but also spin wave

→ higher velocity as in case of ^{3}He-B

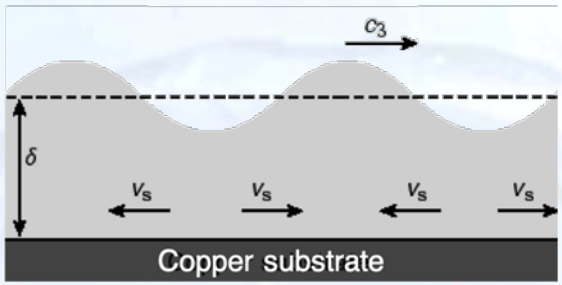


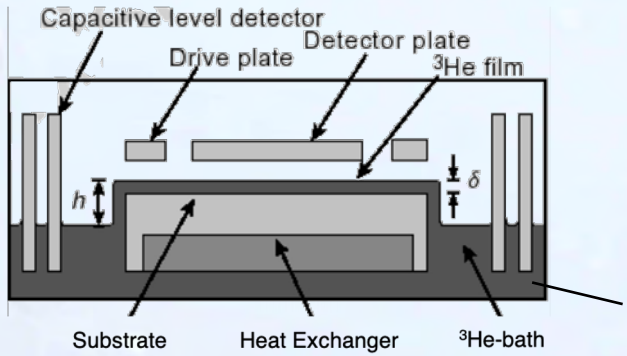
4.8 Collective Excitations — Sound Propagation

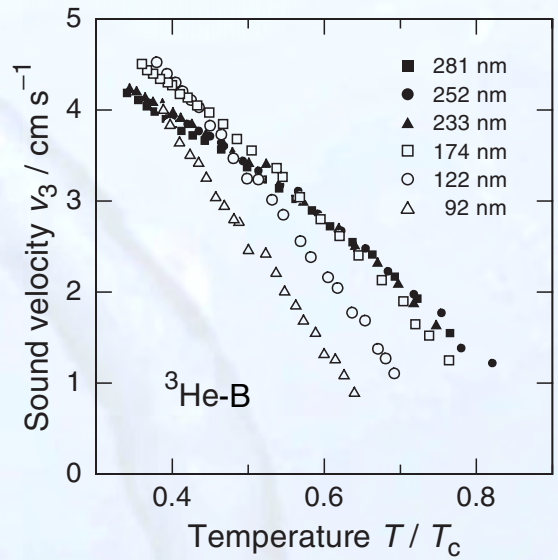
b) 3rd Sound

^{3}He-B

$$v_3 = \sqrt{\frac{\langle \varrho_s \rangle}{\varrho} \frac{3\alpha}{d^3}}$$







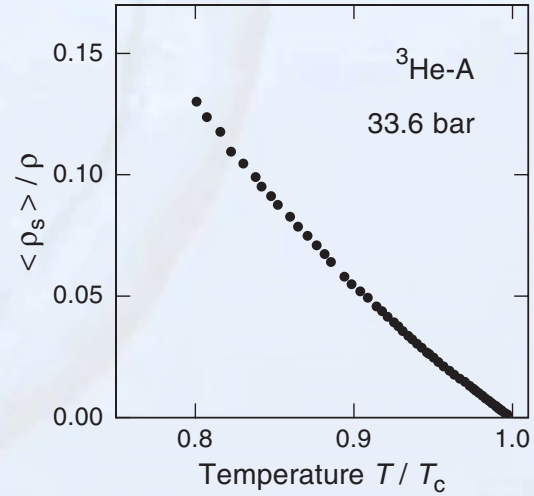
$\varepsilon = 1.0426$

c) 4th Sound

$$v_4 = v_1 \sqrt{\frac{\langle \varrho_s \rangle}{\varrho} \frac{5}{3} [2 - \cos^2 \Theta)]}$$

$$\langle \varrho_s \rangle / \varrho \propto (1 - T/T_c)$$

angle between q and l



4.8 Collective Excitations — Order parameter modes

d) order parameter modes

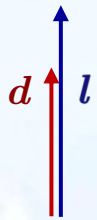
$\longrightarrow$ collective excitations of Cooper pairs $\hbar\omega < 2\Delta$

- relative motion of $\boldsymbol{d}$ and $\boldsymbol{l}$
- inner structure of Cooper pairs

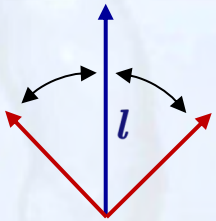
$\longrightarrow$ 18 different order parameter modes

$\hbar\omega > 2\Delta$ $\longrightarrow$ pair breaking

example $\boldsymbol{d} \parallel \boldsymbol{l}$



$\boldsymbol{d}$ splits up in two perpendicular components

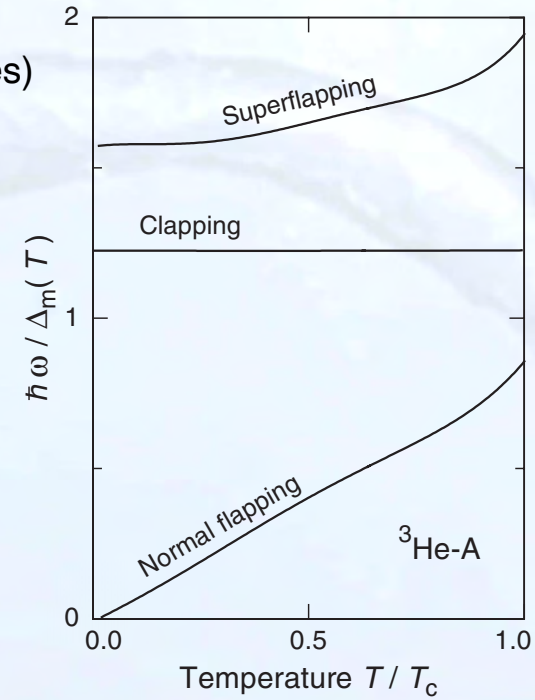


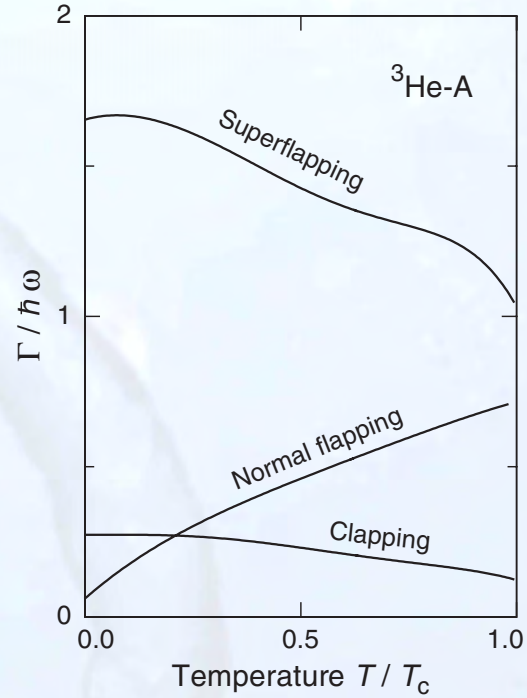
^{3}He-A (examples of order parameter modes)

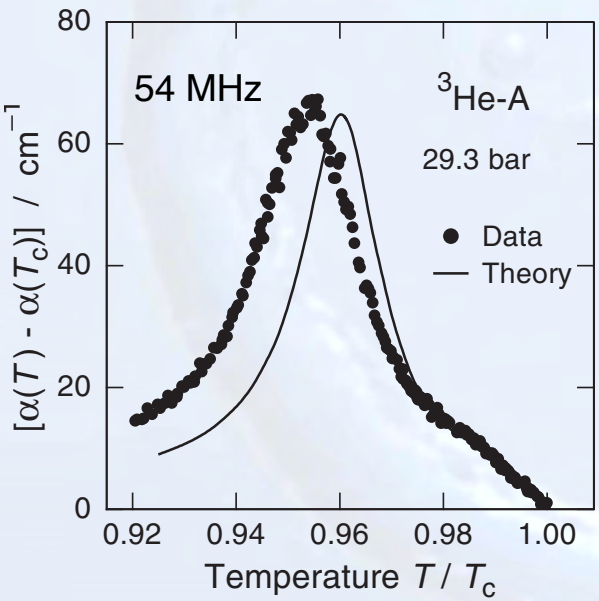
name	energy $(\hbar\omega/\Delta_{\mathrm{m}}(T))$
normal flapping	$\propto \sqrt{\frac{4}{5}\frac{T}{T_{\mathrm{c}}}}$
clapping	1.23
superflapping	1.56 for $T \to 0$
	2 for $T \to T_{\mathrm{c}}$

4.8 Collective Excitations — Order parameter modes

^{3}He-A (examples of order parameter modes)



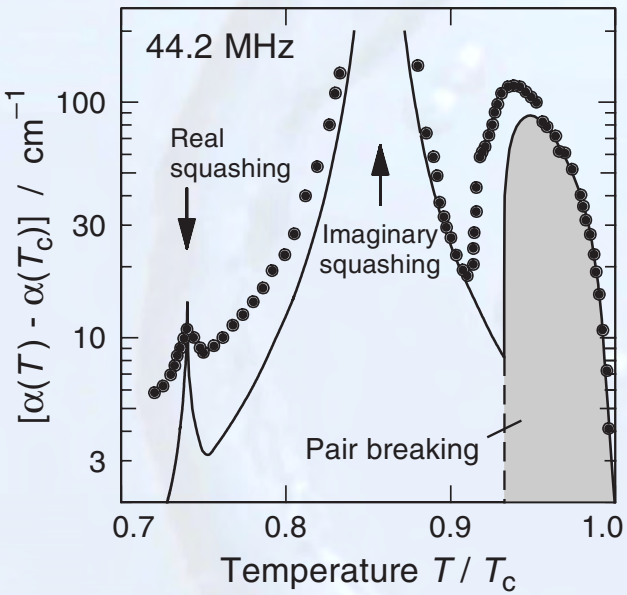




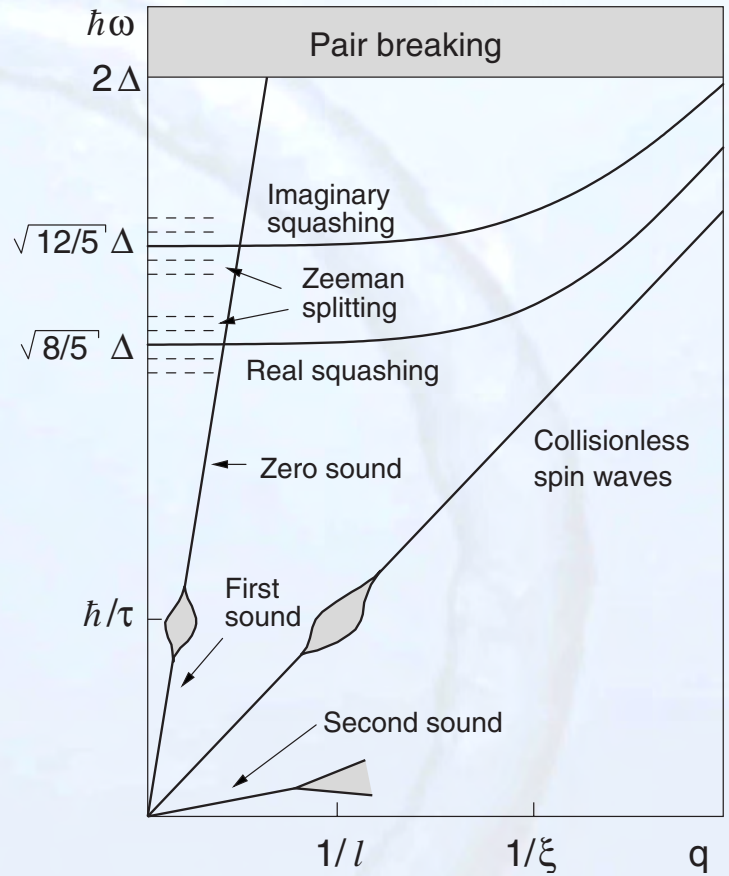
- damping of longitudinal zero sound ^{3}He-A
- clapping resonance

4.8 Collective Excitations — Order parameter modes

^{3}He-B (classification $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$, J_z)



dispersion relation



transition into collision-less regime

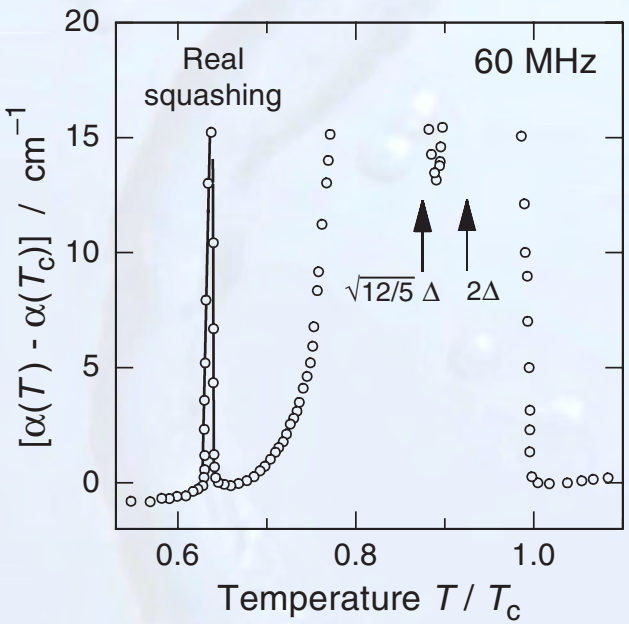
- since gap decreases with temperature

  ⟶ with fixed sound frequencies, several modes can be excited at different temperatures

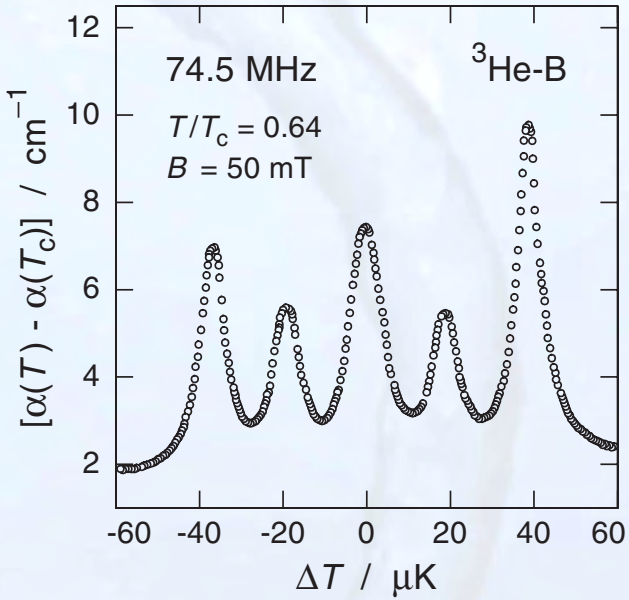
- arrows indicated expected peak position

4.8 Collective Excitations — Order parameter modes

measurement at higher frequency 60 MHz



real squashing mode in magnetic field



- pair breaking just below T_c
- extremely sharp resonances at low temperatures

- $J = 2 \longrightarrow$ multiplicity $2J+1 \longrightarrow$ 5 levels
- Zeeman splitting in magnetic field