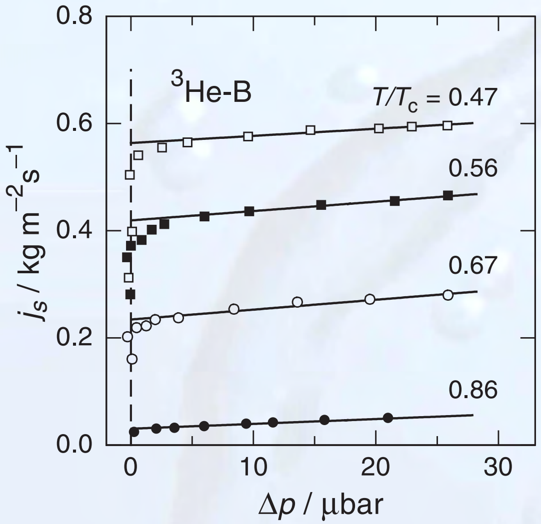
4.2 Relevance of the Two-Fluid Model

a) Flow through thin capillaries



example: ^{3}He-B: flow through 1000 parallel channels, diameter $0.8\,\mu\mathrm{m}$, length $10\,\mu\mathrm{m}$

- significant flow without pressure
- j_s depends only weakly on pressure (as for He-II)
- j_s increases with decreasing temperature

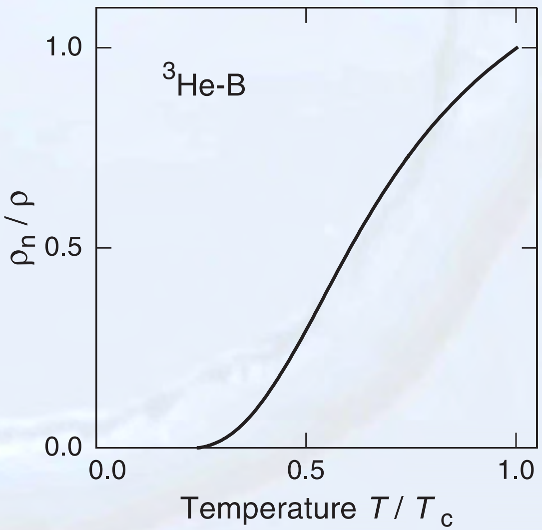
→ ϱ_s/ϱ rises with decreasing temperature (as for He-II)

→ temperature dependence of the critical velocity $v_c(T)$

b) Normalfluid density

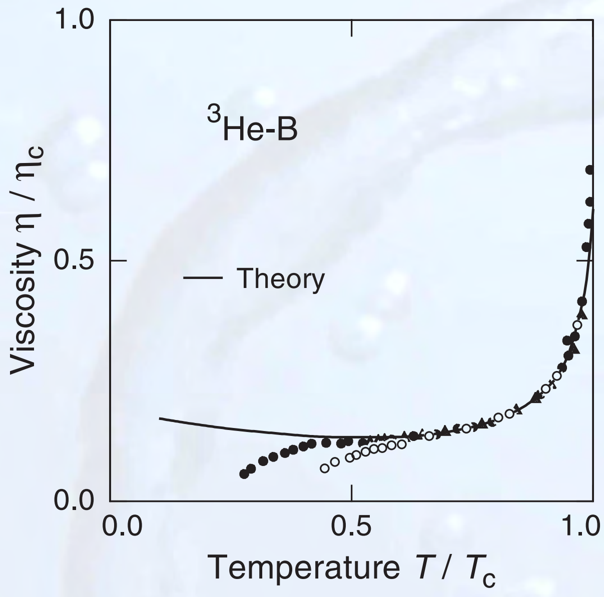
Andronikashvili-type experiment ^{3}He-B

- ϱ_n/ϱ increases with temperature (as for He-II)
- detailed temperature dependence different than for He-II

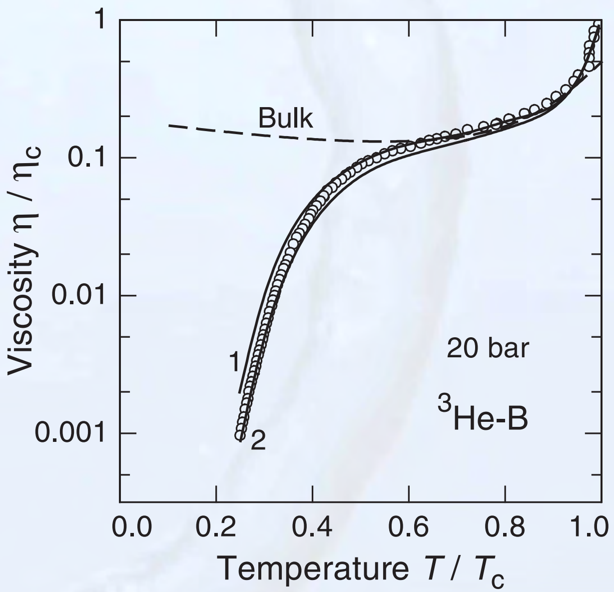


4.2 Relevance of the Two-Fluid Model

c) Viscosity



- theory for bulk ³He-B fits well above 0.5 T/T_c
- deviations below 0.5 T/T_c



- --- theory for bulk ³He-B
- (1) diffusive scattering
- (2) diffusive scattering and Andreev reflection

⟶ interaction with wall dominates

³He-A: much more complicated behavior:
influence of magnetic fields, vessel geometry, textures, velocity fields, …

4.3 Quantum States of Superfluid ^{3}He

Cooper pairs: $S = 0$

^{3}He pairs: strong magnetic exchange interaction (pairs of quasi particles)

$$\underbrace{\boxed{S = 1}}_{\text{symmetric}} \longrightarrow \underbrace{\boxed{L = 1}, 3, \ \ldots}_{\text{anti-symmetric}}$$

spin-triplet pairing $S = 1 \quad S_z = 0, \pm 1$

$$|S, S_z\rangle \longrightarrow \begin{aligned} |1,+1\rangle &= |\uparrow\uparrow\rangle\,, \\ |1,\ 0\,\rangle &= \frac{1}{\sqrt{2}}\Big[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big] \\ |1,-1\rangle &= |\downarrow\downarrow\rangle\,. \end{aligned}$$

analog for orbital momentum $L = 1 \quad L_z = 0, \pm 1$

general wave function: linear combinations $\longrightarrow$ $3 \times 3 = 9$ terms each with amplitude and phase

$\longrightarrow$ $2(2S+1)(2L+1) = 18$ real components

$\longrightarrow$ order parameter: 3×3 matrix with complex values

4.3 Quantum States of Superfluid ^{3}He

probability to find pairs $(\boldsymbol{k}\sigma_1, -\boldsymbol{k}\sigma_2)$ $\longrightarrow$ $\begin{pmatrix} \Delta_{\uparrow\uparrow}(\boldsymbol{k}) & \Delta_{\uparrow\downarrow}(\boldsymbol{k}) \\ \Delta_{\downarrow\uparrow}(\boldsymbol{k}) & \Delta_{\downarrow\downarrow}(\boldsymbol{k}) \end{pmatrix}$

usual representation in case of ^{3}He (a bit hard to see through)

order parameter representation by vector $\boldsymbol{d}(\widehat{\boldsymbol{k}})$ represents the pair amplitude in direction $\widehat{\boldsymbol{k}}$

$$\begin{pmatrix} \Delta_{\uparrow\uparrow}(\boldsymbol{k}) & \Delta_{\uparrow\downarrow}(\boldsymbol{k}) \\ \Delta_{\downarrow\uparrow}(\boldsymbol{k}) & \Delta_{\downarrow\downarrow}(\boldsymbol{k}) \end{pmatrix} = \begin{pmatrix} -d_x(\boldsymbol{k}) + \mathrm{i}d_y(\boldsymbol{k}) & d_z(\boldsymbol{k}) \\ d_z(\boldsymbol{k}) & d_x(\boldsymbol{k}) + \mathrm{i}d_y(\boldsymbol{k}) \end{pmatrix}$$

special properties of $\boldsymbol{d}$:

- $d_x,\ d_y$ and d_z are described by complex-valued parameters and transform like vectors under rotation
- pair amplitude: $|\boldsymbol{d}| = \sqrt{d_x^2 + d_y^2 + d_z^2} = \sqrt{n_\mathrm{s}}$
- spatial orientation of spin wave function: $\widehat{\boldsymbol{d}} = \boldsymbol{d}/|\boldsymbol{d}|$
- direction of $\boldsymbol{d}$ $\longrightarrow$ $\boldsymbol{d} \cdot \boldsymbol{S} = 0$ at any point of Fermi surface
- if $\boldsymbol{d}$ has only real components $\longrightarrow$ $\langle \boldsymbol{S} \rangle = 0$

4.3 Quantum States of Superfluid ^{3}He

general spin wave function expressed using $\boldsymbol{d}$:

$$|\Psi\rangle = d_x\Big[|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle\Big] + \mathrm{i}d_y\Big[|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle\Big] + d_z\Big[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big]$$

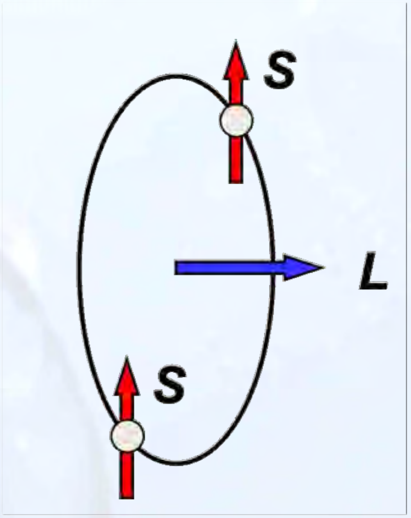
$$= -\,(d_x - \mathrm{i}d_y)\,\underbrace{|\uparrow\uparrow\rangle}_{|S_z=+1\rangle} + (d_x + \mathrm{i}d_y)\,\underbrace{|\downarrow\downarrow\rangle}_{|S_z=-1\rangle} + d_z\underbrace{\Big[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big]}_{|S_z=0\rangle}$$

4.3 Quantum States of Superfluid ^{3}He

association with superfluid phases

^{3}He-A$_1$ (exist only in magnetic fields)

spins align parallel to applied magnetic field $(S_z = +1)$ $\longrightarrow$ only pairs $|\uparrow\uparrow\rangle$



$\longrightarrow$ $d_x + \mathrm{i}d_y = 0 \qquad d_z = 0$

$\longrightarrow$ $\boxed{|\Psi_{\mathrm{A}_1}\rangle = -2\,d_x\,|\uparrow\uparrow\rangle}$

^{3}He-A

$S_z = \pm 1 \longrightarrow d_z = 0$

$\longrightarrow$ $\boxed{|\Psi_\mathrm{A}\rangle = -(d_x - \mathrm{i}d_y)\,|\uparrow\uparrow\rangle + (d_x + \mathrm{i}d_y)\,|\downarrow\downarrow\rangle}$ ABM state

Anderson, Brinkman, Morel 1961, 1963

^{3}He-B

$\longrightarrow$ general expression of wave function $\longrightarrow$ quasi isotropic! total momentum: $J = L + S = 0$

$\longrightarrow$ $\boxed{|\Psi_\mathrm{B}\rangle = d_x\Big[|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle\Big] + \mathrm{i}d_y\Big[|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle\Big] + d_z\Big[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big]}$ BW state

Bailian, Werthammer 1963

4.3 Quantum States of Superfluid ^{3}He

at the phase transition 3 symmetries are (partially) broken at once

gauge (phase) $\longleftrightarrow$ superfluidity

spin momentum $\longleftrightarrow$ ferromagnets

orbital momentum $\longleftrightarrow$ liquid crystals

group theory: symmetry of ^{3}He

$$G = SO(3)_L \times SO(3)_S \times U(1)_\varphi$$

special orthogonal non-Ablian rotational group

unitary Ablian rotational group

example: ferromagnet $SO(3)_S$ magnetization

above T_c : isotropic (paramagnet)

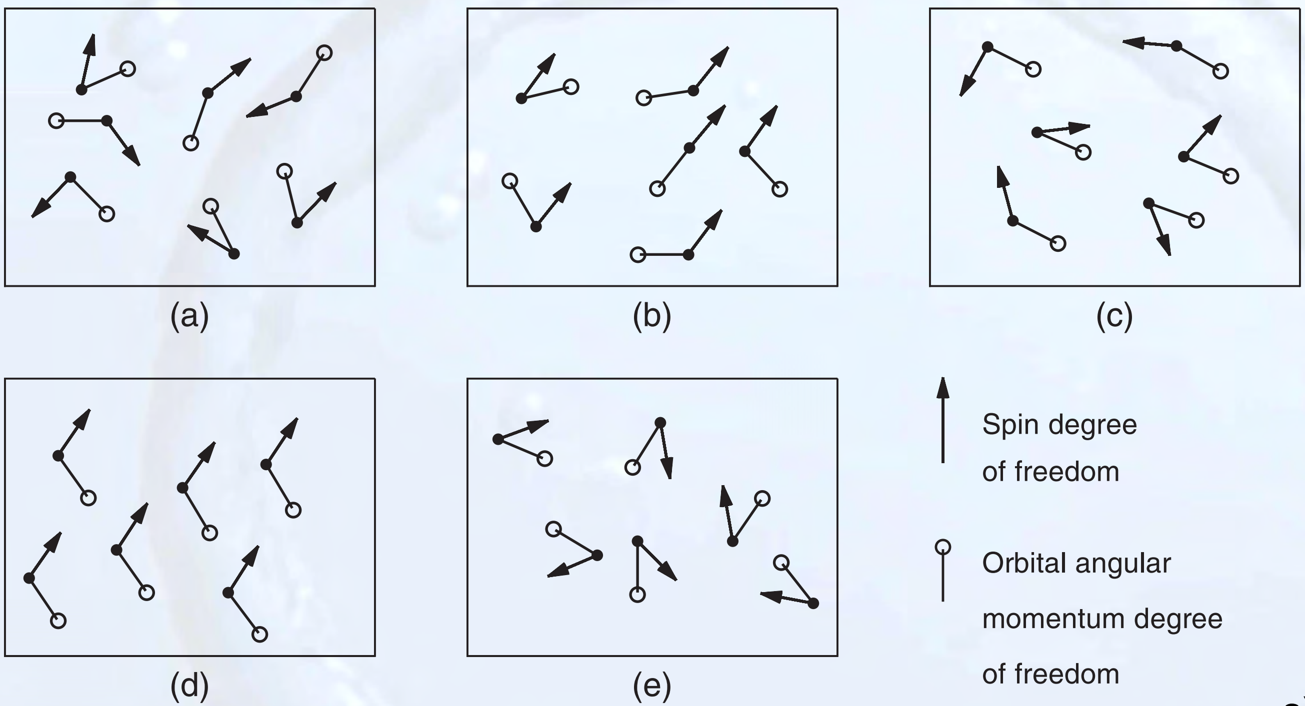
below T_c : one direction selected, but still rotational symmetry about axis of magnetization

$SO(3)_S$ only partially broken $\longrightarrow$ $R = U(1)$

residual symmetry

4.3 Quantum States of Superfluid ^{3}He

Two-dimensional model $G = U(1)_L \times U(1)_S$

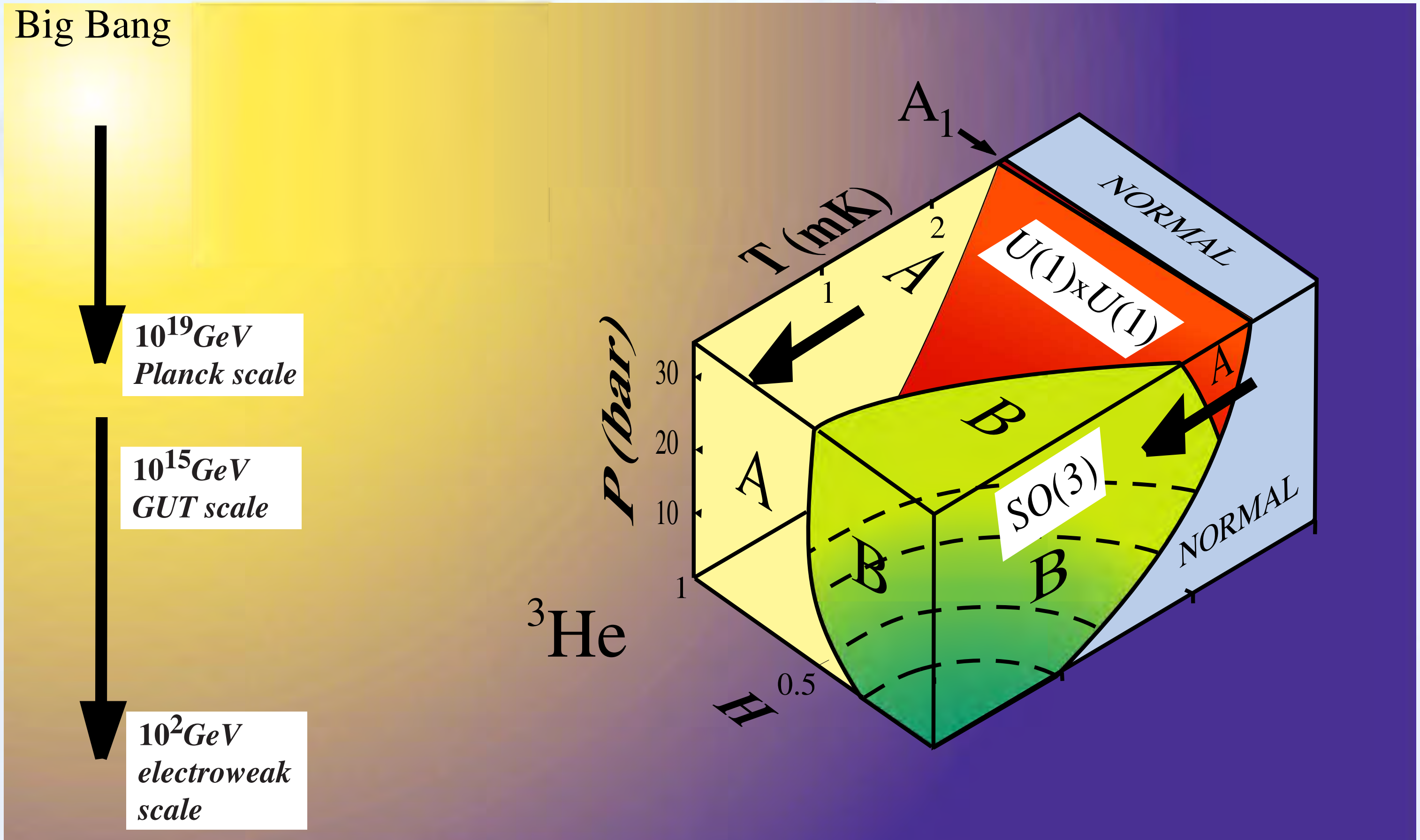


a) isotope paramagnetic fluid
b) liquid ferromagnet
c) nematic liquid crystal
d) ^{3}He-A, ^{3}He-A$_1$
e) ^{3}He-B

4.3 Quantum States of Superfluid ^{3}He

Suprafluid ^{3}He – a Model System for „all“ Physics

Grigory E. Volovik

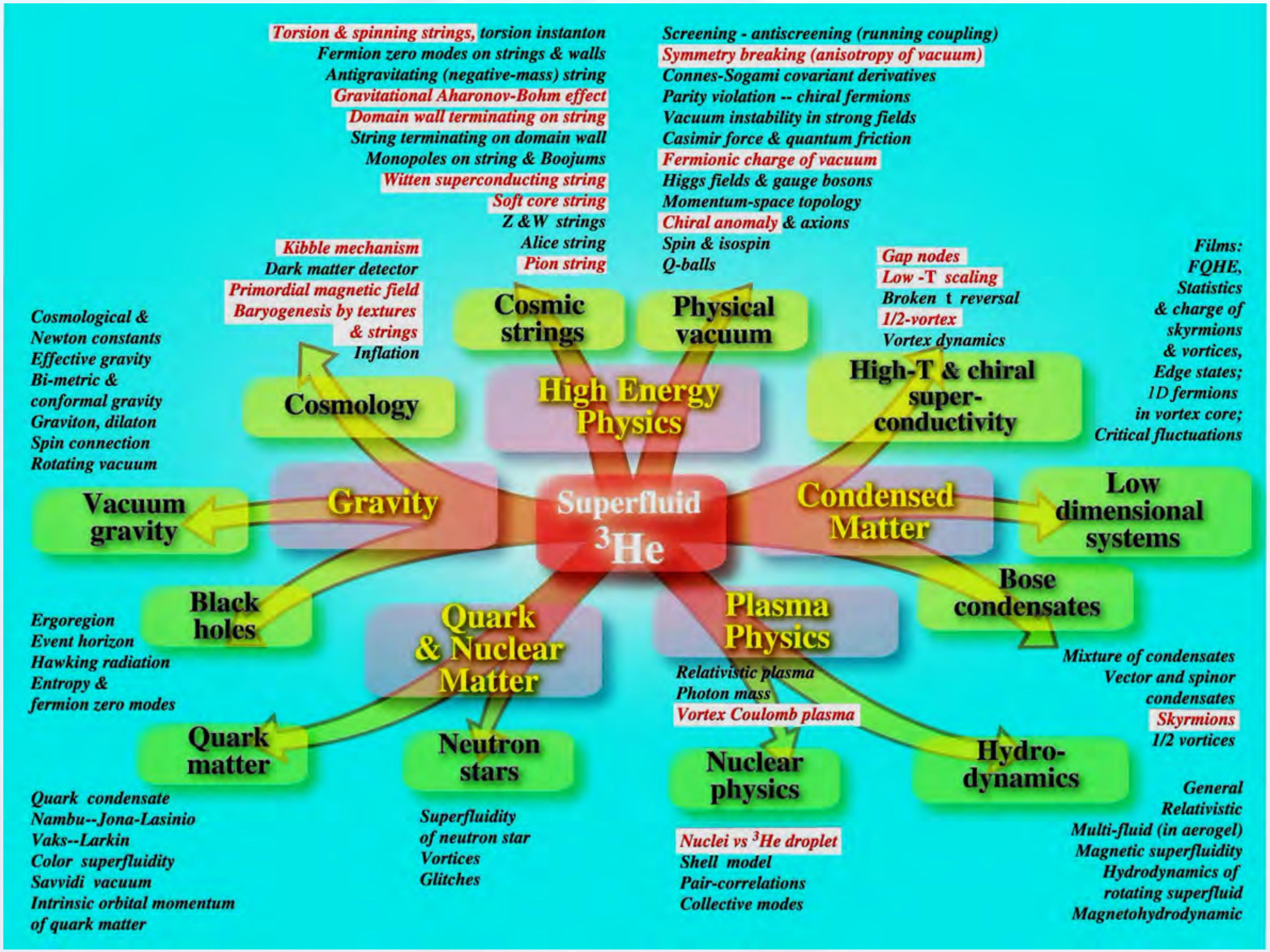


GUT in Standard Model *symmetry breaking phase transitions*	GUT in superfluid ^{3}He *symmetry breaking phase transitions*
$SO(10) \rightarrow SU(3)\times SU(2)\times U(1) \rightarrow SU(3)\times U(1)$	$SO(3)\times SO(3)\times U(1) \rightarrow U(1)\times U(1) \rightarrow SO(3)$

4.3 Quantum States of Superfluid ^{3}He

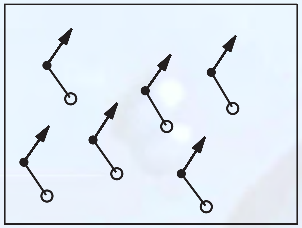
Suprafluides ^{3}He – a Model System for „all“ Physics

Grigory E. Volovik

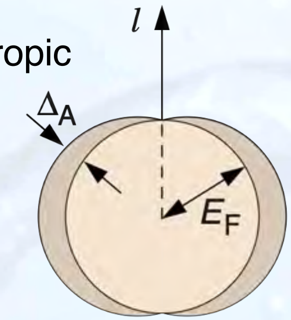


4.3 Quantum States of Superfluid ^{3}He

Energy gap

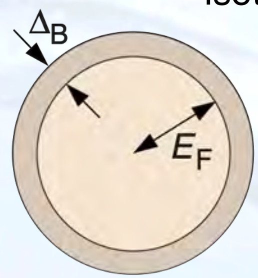


non-isotropic

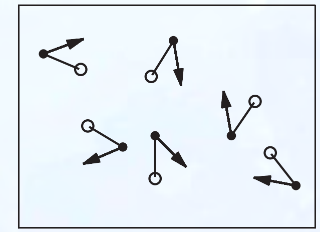


^{3}He–A

isotropic



^{3}He–B



$$\boldsymbol{d}(\boldsymbol{k}) = \sqrt{3/2}\,\Delta_\mathrm{m}(T)\,\sin(\widehat{\boldsymbol{k}},\widehat{\boldsymbol{l}})\,\widehat{\boldsymbol{d}} = \Delta_\mathrm{A}(\widehat{\boldsymbol{k}},T)\,\widehat{\boldsymbol{d}}$$

maximal gap — orbital momentum

$$\Delta_\mathrm{m}(0) = 2.029\,k_\mathrm{B}T_\mathrm{c}$$

BCS theory

$$T \to 0 \quad \Delta_\mathrm{B} \approx 1.76\,k_\mathrm{B}\,T_\mathrm{c}$$

$$\boldsymbol{d}(\boldsymbol{k}) = \Delta_\mathrm{B}(T)\,\widehat{\boldsymbol{d}}$$

$$\widehat{\boldsymbol{d}} = \overleftrightarrow{\mathbf{R}}\,(\widehat{\boldsymbol{n}},\Theta)\widehat{\boldsymbol{k}}$$

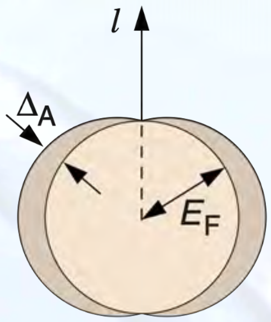
rotation of spin coordinates
relative to $\boldsymbol{k}$ vector of pair

- in A phase pairs can be broken at arbitrarily small energy …. still it is a superfluid!
  $\longrightarrow$ metastable persistent flow
- but: massive objects cannot be moved without friction in ^{3}He-A

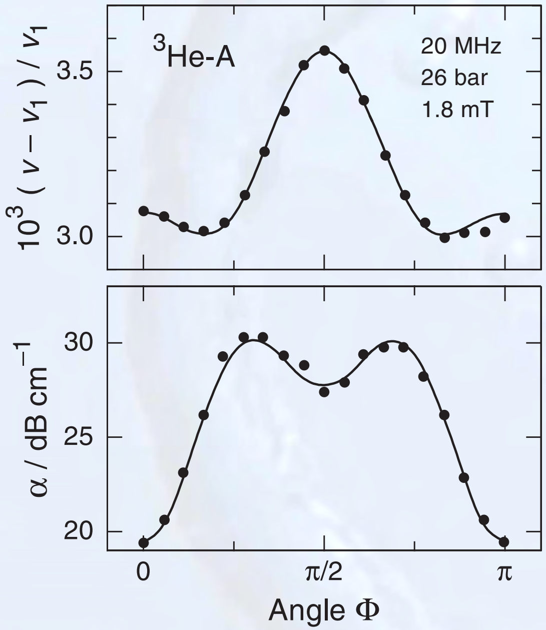
- like He-II $\longrightarrow$ stable persistent flow
- massive objects can be moved without friction in ^{3}He-B for $v < v_\mathrm{c}$

4.3 Quantum States of Superfluid ^{3}He

experimental determination of anisotropy of gap of ^{3}He-A



propagation of longitudinal zero sound



- ► in this experiment $\boldsymbol{l}$ is oriented by a small magnetic field 1.8 mT
- ► ϕ is the angle between $\boldsymbol{B}$ and $\boldsymbol{q}$

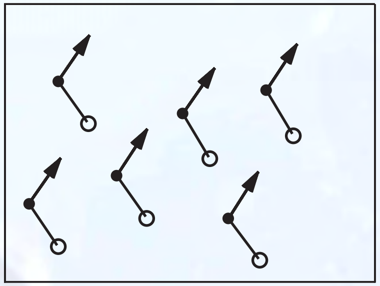
wave vector of sound wave

- ► expected anisotropy is clearly observed

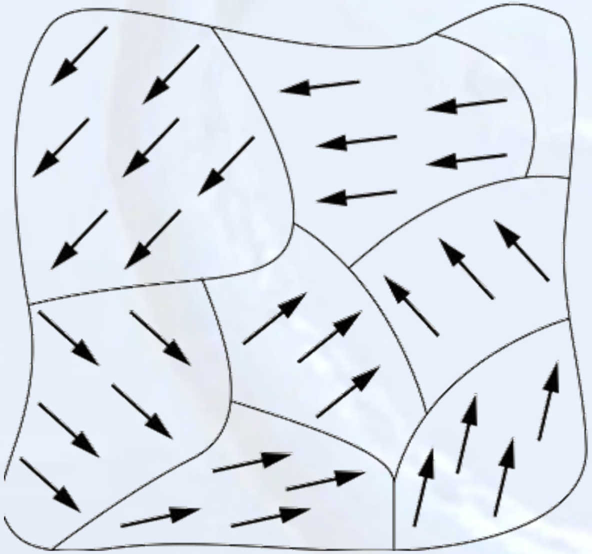
4.4 Orderparameter Orientation — Textures

Textures:

- this term was introduced by de Gennes (similar to liquid crystals)
- denotes orientational effects of $\boldsymbol{l}$ and $\boldsymbol{d}$
- texture depends on many things: dipole-dipole interaction, magnetic and electric fields, geometry, …



- often no uniform texture $\longrightarrow$ texture domains



4.4 Orderparameter Orientation — Textures

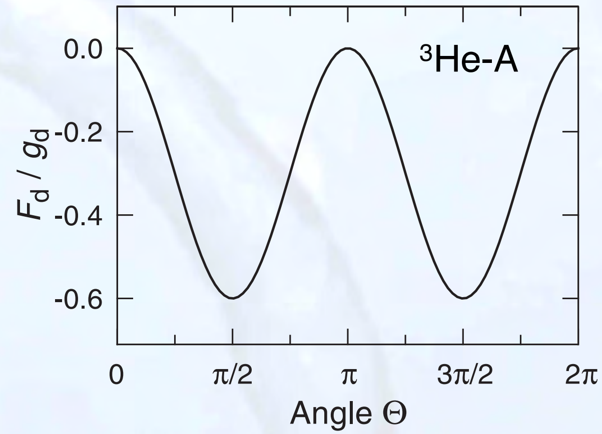
a) orientation of $\boldsymbol{l}, \boldsymbol{d}$ without external field

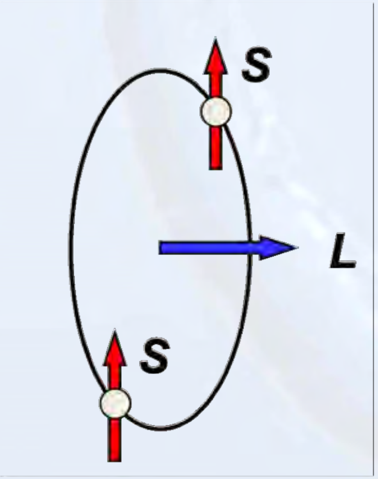
^{3}He-A: macroscopic orientation
dipole-dipole energy is minimal, if $\boldsymbol{l} \parallel \boldsymbol{d} \;\widehat{=}\; \boldsymbol{l} \perp \boldsymbol{S}$

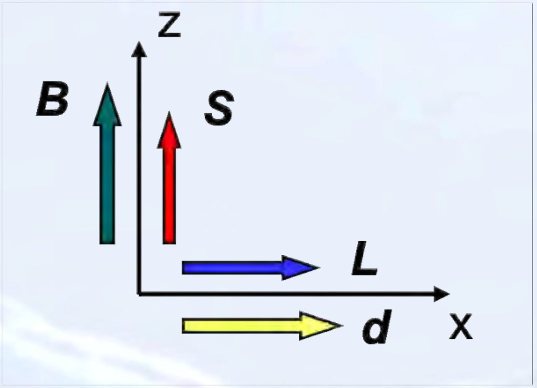
free energy: dipole-dipole interaction

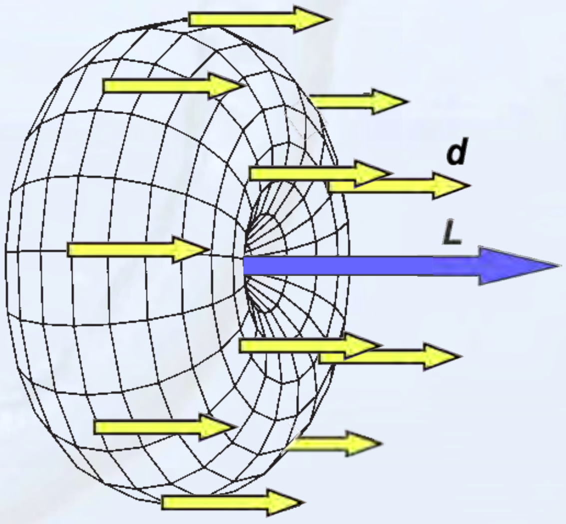
$$F_\mathrm{d} = -\frac{3}{5}\, g_\mathrm{d}(T) \left[1 - (\widehat{\boldsymbol{d}} \cdot \widehat{\boldsymbol{l}})^2\right] = -\frac{3}{5}\, g_\mathrm{d}(T)\, \sin^2 \Theta$$

$$g_\mathrm{d} \approx 10^{-10}(1 - T/T_\mathrm{c})\,\mathrm{J\,cm^{-3}} \propto \varrho_\mathrm{s}$$









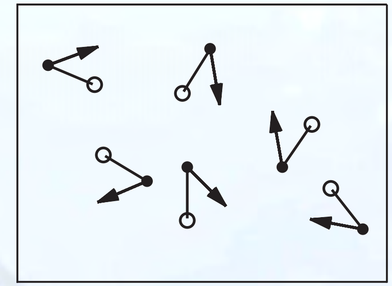
4.4 Orderparameter Orientation — Textures

^{3}He-B: isotropic regarding spin and orbital momentum

⟶ no macroscopic orientation

but: dipole-dipole interaction leads to a relative orientation of $\boldsymbol{l}$, $\boldsymbol{d}$ locally for each point on the Fermi surface described by a rotation about $\hat{\boldsymbol{n}}$ described by $\hat{\boldsymbol{d}} = \overleftrightarrow{\mathbf{R}}\,(\hat{\boldsymbol{n}}, \Theta)\hat{\boldsymbol{k}}$

⟶ leads to weak texture effects

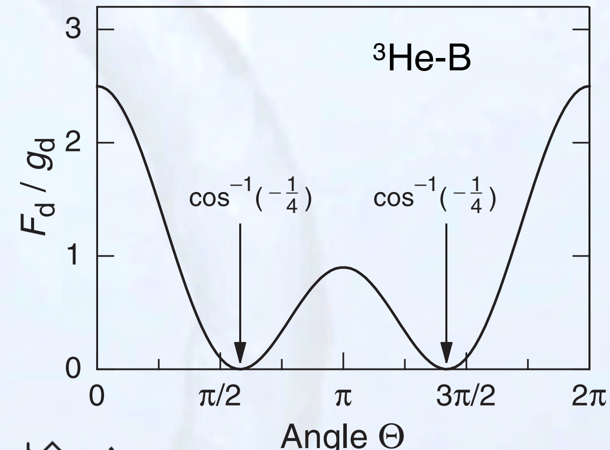


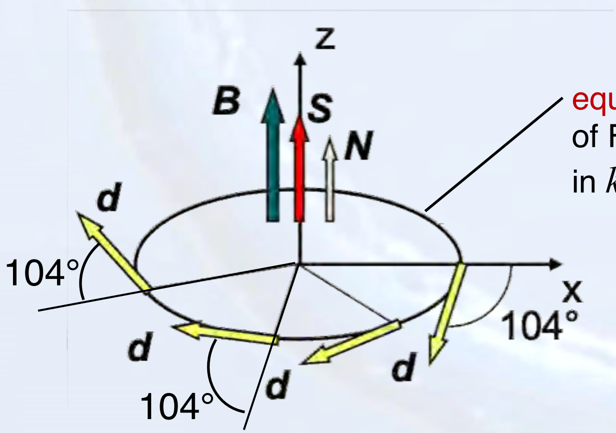
free energy: dipole-dipole interaction

$$F_{\mathrm{d}} = \frac{8}{5}\, g_{\mathrm{d}}(T) \left(\cos\Theta + \frac{1}{4}\right)^2$$

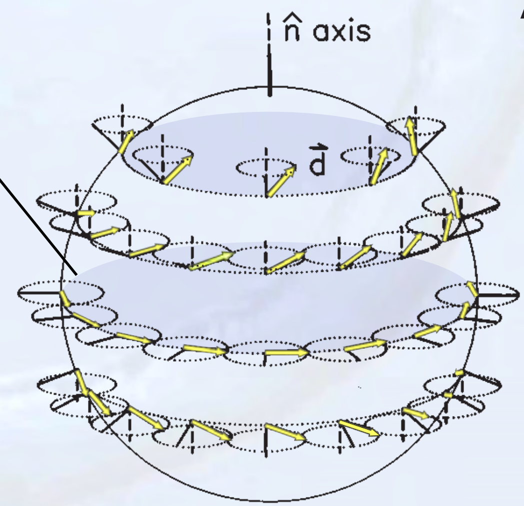
Leggett angle

dipole-dipole energy is minimal, if $\Theta = \arccos\left(-1/4\right) \approx 104°$





equatorial plane of Fermi sphere in k-space



4.4 Orderparameter Orientation — Textures

b) external influences on the orientation of $\boldsymbol{l}$, $\boldsymbol{d}$

$\longrightarrow$ changes of the texture

textures in ³He-A

preferred alignment and relative strength of different influences

	Preferred Alignment	$\Delta E/(1-T/T_c)$ $(\mathrm{J\,m^{-3}})$
magnetic dipole interaction	$\boldsymbol{d} \parallel \boldsymbol{l}$	$-6\times10^{-5}\,(\widehat{\boldsymbol{d}}\cdot\widehat{\boldsymbol{l}})^2$
electric field	$\boldsymbol{l} \perp \boldsymbol{\mathcal{E}}$	$2\times10^{-7}\,(\widehat{\boldsymbol{l}}\cdot\boldsymbol{\mathcal{E}})^2$
magnetic field	$\boldsymbol{d} \perp \boldsymbol{B}$	$5\,(\widehat{\boldsymbol{d}}\cdot\boldsymbol{B})^2$
mass flow	$\boldsymbol{l} \parallel \boldsymbol{v}_\mathrm{s}$	$-10\,(\widehat{\boldsymbol{l}}\cdot\boldsymbol{v}_\mathrm{s})^2$
wall alignment	$\boldsymbol{l} \parallel \boldsymbol{N}$	$-30\,(\widehat{\boldsymbol{l}}\cdot\widehat{\boldsymbol{N}})^2$

- most important are walls $\boldsymbol{l} \parallel \boldsymbol{N}$ and mass flow $\boldsymbol{l} \parallel \boldsymbol{v}_\mathrm{s}$
- strength compared to intrinsic alignment:
  $\mathcal{E} = 17\,\mathrm{V\,m^{-1}}$, $B = 3.3\,\mathrm{mT}$ and $v_\mathrm{s} = 2.4\,\mathrm{mm\,s^{-1}}$
- for in homogenies textures $\longrightarrow$ gradient energy must be considered

costs energy