2.7 Motion of Ions in He-II

Testbed for the generation of excitations and the critical velocity

type of ions:

- electrons (–) : zero-point motion ⟶ bubbles $r = 19$ Å
- $^4He^+$, H_2^+ (+) : attract He atoms ⟶ snowballs $r \approx 7$ Å
- other ions (–, +) : properties depend on wave function

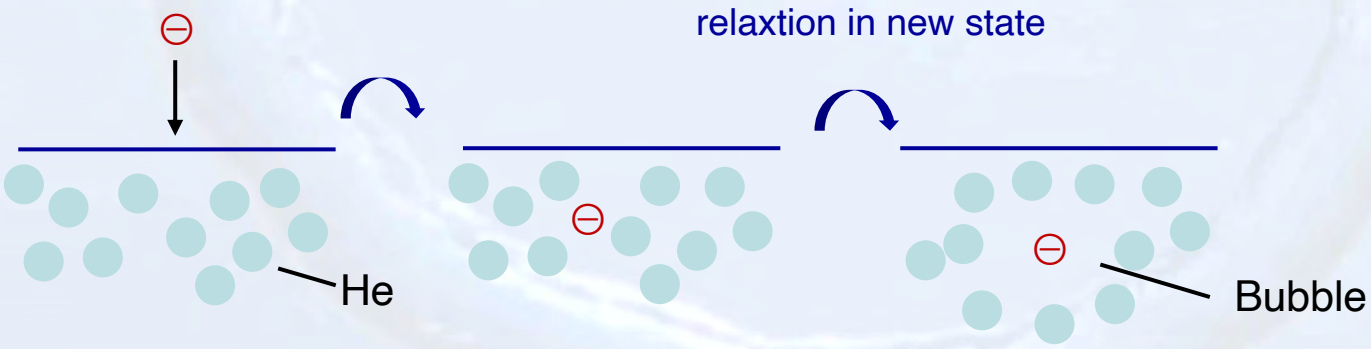
Electrons in liquid He

electrons need energy to be emerged in helium ~ 1 eV, which means they need more that 1 eV of kinetic energy to enter liquid He.

comment:

similar to work function of electrons in metals

bubble formation



2.7 Motion of Ions in He-II

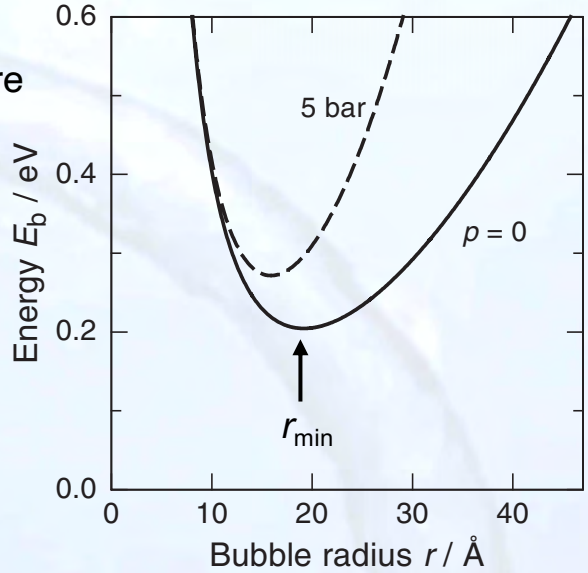
Energy of bubble

$$E_b = \frac{h^2}{8mr^2} + 4\pi r^2\alpha + \frac{4}{3}\pi r^3 p$$

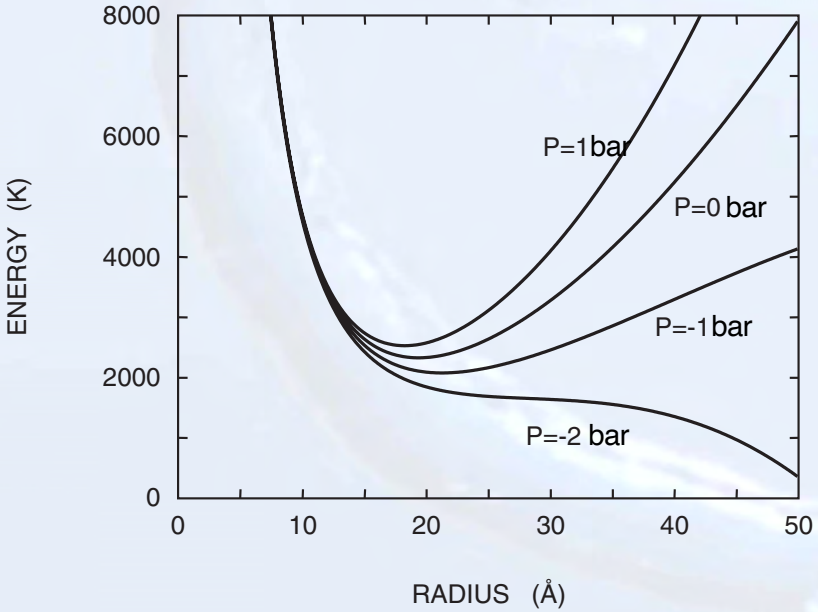
- $\frac{h^2}{8mr^2}$: zero-point energy
- $4\pi r^2\alpha$: surface tension, $\alpha = 3.41\ \mu J/cm^2$
- $\frac{4}{3}\pi r^3 p$: volume × applied pressure

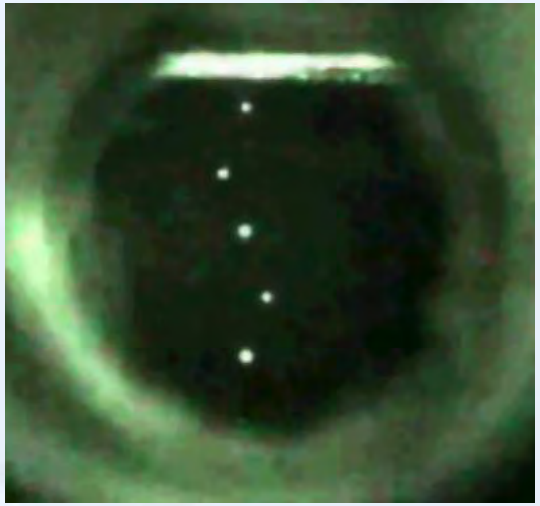
bubble size:

$$\frac{\partial E}{\partial r} = 0 \longrightarrow r_{min}(p = 0) = 19\ Å$$



size depends on pressure

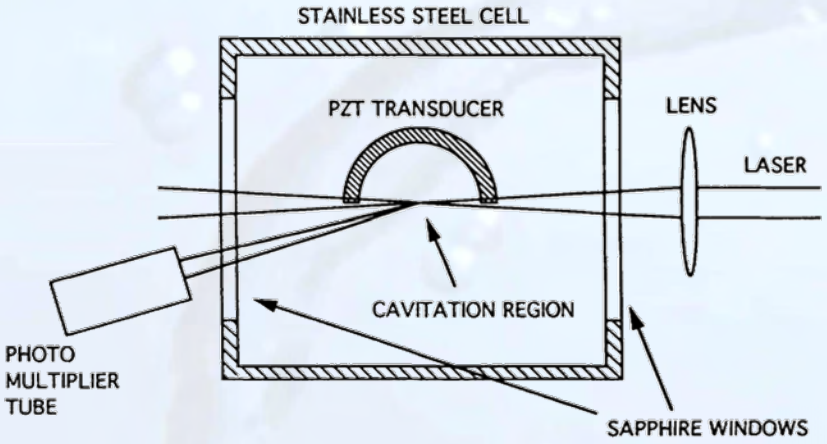


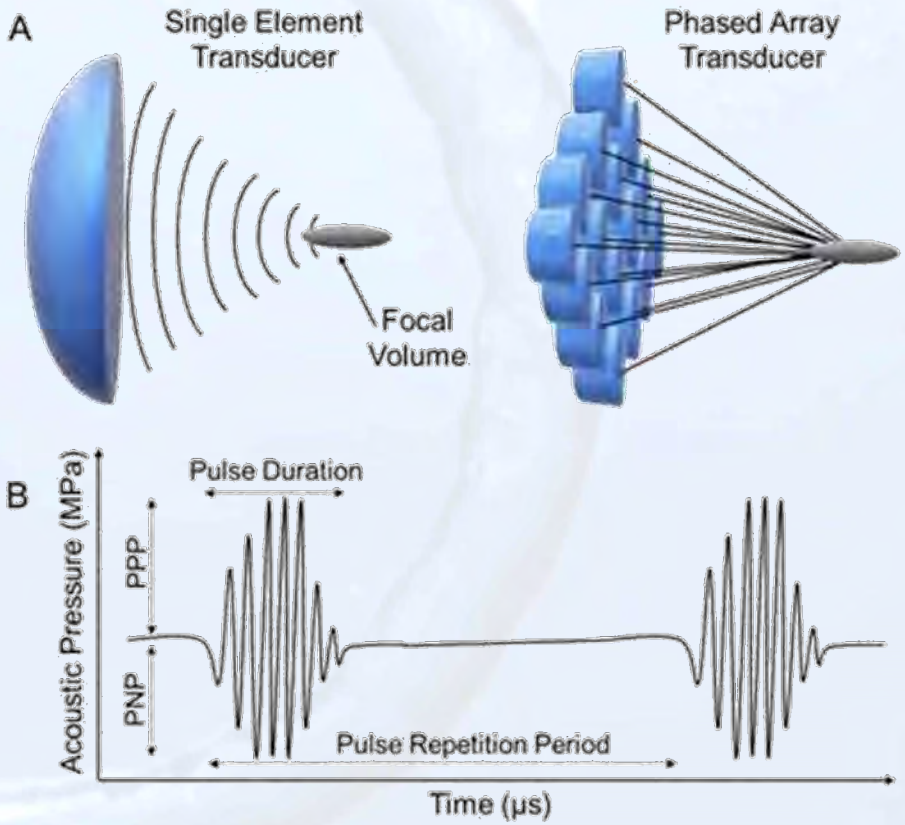


exploding bubbles at negative pressure

2.7 Motion of Ions in He-II

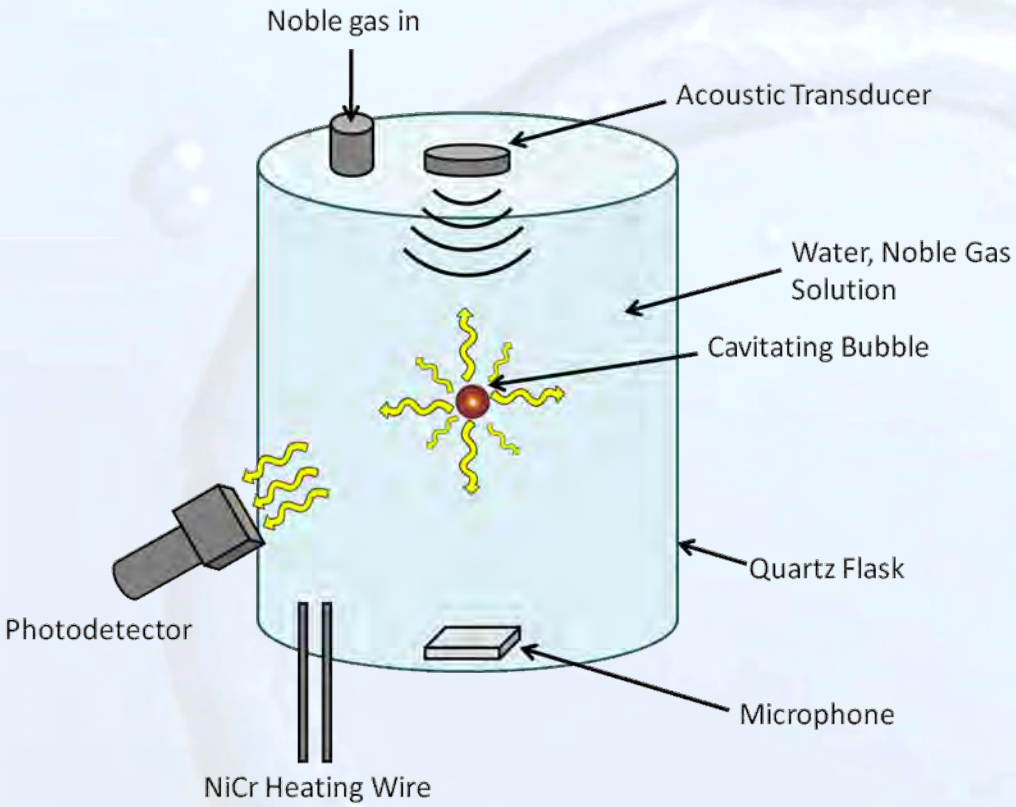
Creation of negative pressure and observation of bubbles



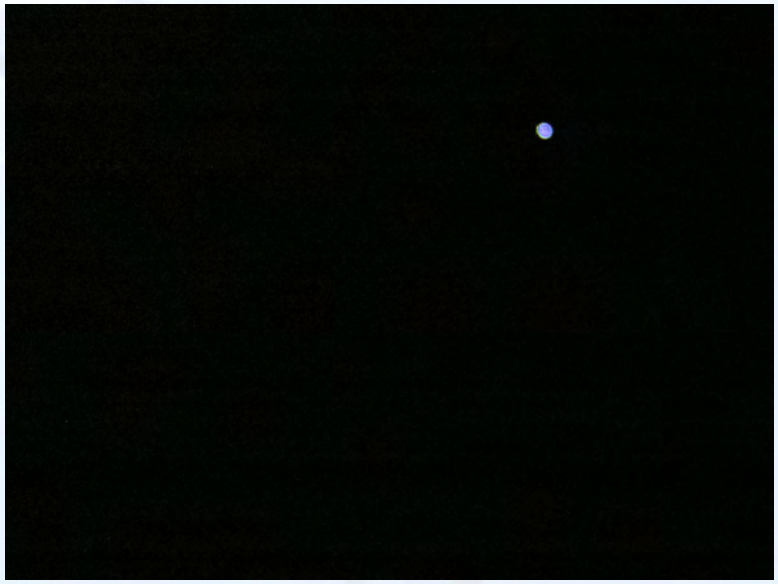


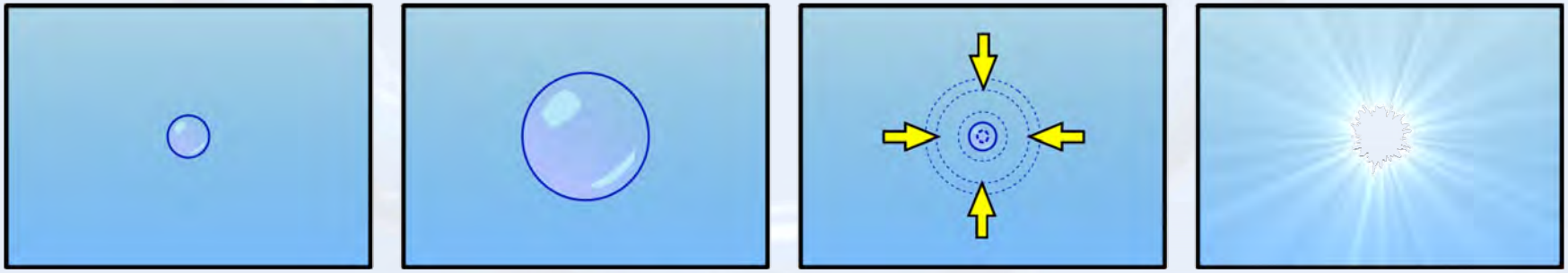
Generation of Bubbles and Cavitation Processes

Sonoluminescence



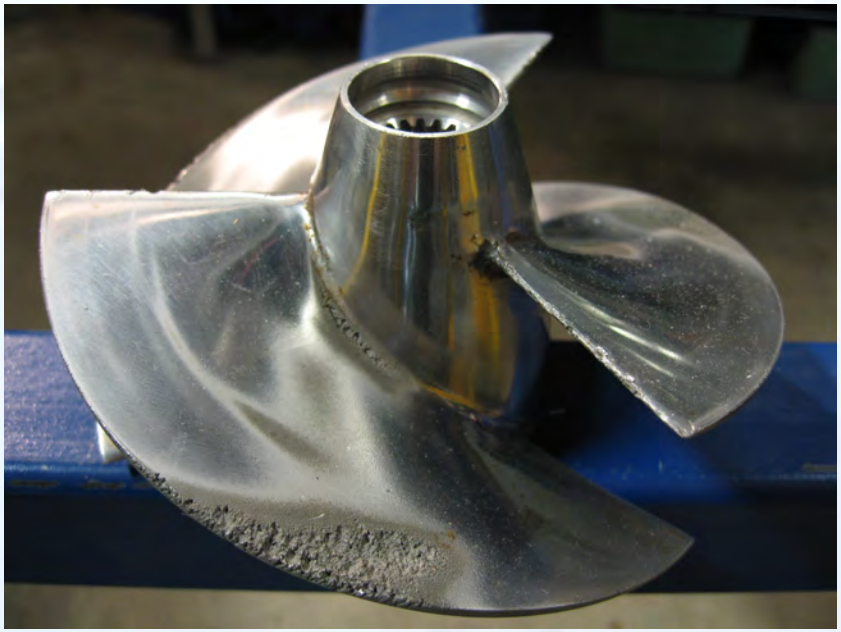
Sonoluminescence in water





Generation of Bubbles and Cavitation Processes

Collapsing bubbles are of great technical importance



Extracorporeal shockwave therapy using cavitation processes

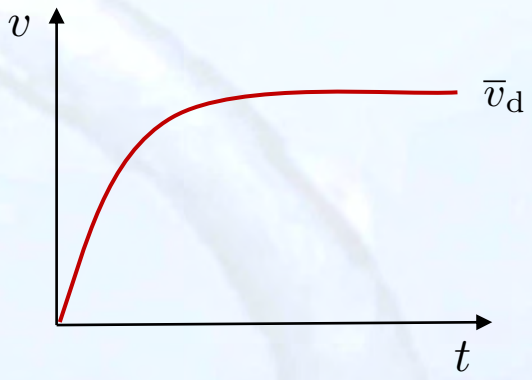
2.7 Motion of Ions in He-II

Acceleration of ions in constant field

constant drift velocity is reached $\quad \overline{v}_\mathrm{d} = \dfrac{q\mathcal{E}}{6\pi\eta r}$

Stokes law of viscos friction

constant electrical field $\mathcal{E}$



mobility:

$$\mu = \frac{\overline{v}_\mathrm{d}}{\mathcal{E}} = \frac{q}{6\pi\eta r}$$

snowball (electrons 4π)

collision partners: phonons, rotons, ^{3}He, …

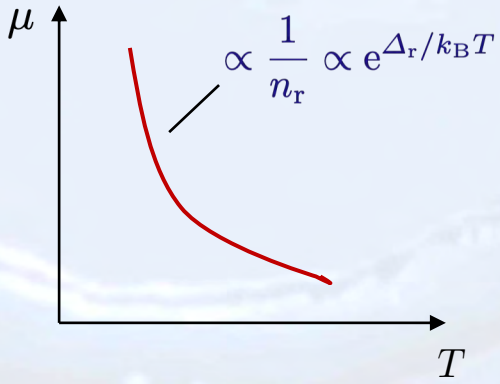
impurities, which at some level are always present

0.7 K < *T* < 1.8 K: rotons should dominate however, difficult to observe because of other excitations / impurities

mobility for roton scattering

$$\mu \propto \frac{1}{\eta} \propto \frac{1}{\tau} \propto \frac{1}{n_\mathrm{r}}$$

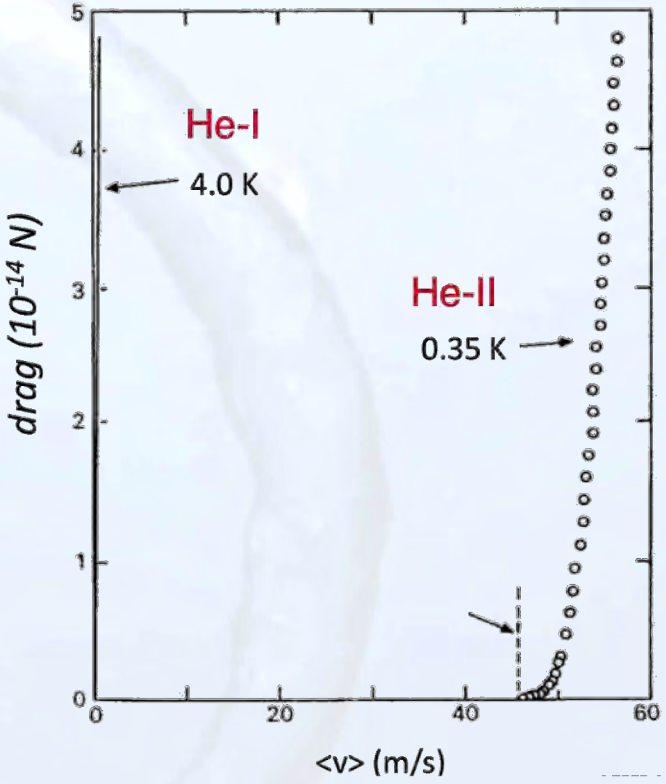
$$\eta = \frac{1}{3}\varrho v^2 \tau = \frac{1}{3}\varrho v \ell$$



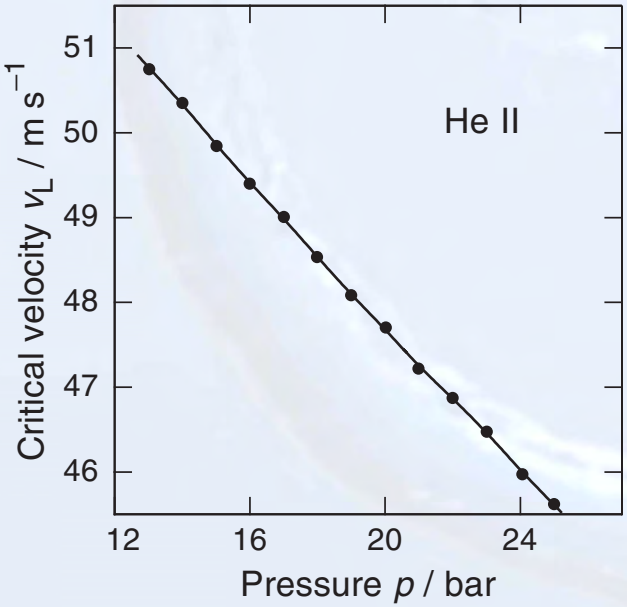
2.7 Motion of Ions in He-II

in ultra-pure He-II under pressure ions can be accelerated up to Landau velocity

- negative ions accelerated in electric field under high pressure
- drag is measured by time-of-flight method
- in He-I: drag proportional to velocity
- in He-II: drag is not observable until critical velocity is reached



pressure dependence of v_c



- $v_L \mathrel{\widehat{=}} v_c$ Landau velocity
- roton pair production
- $p \uparrow \longrightarrow v_L \downarrow$ since $\Delta_r(p)$ decreases with pressure

2.7 Motion of Ions in He-II

$T < 0.3$ K

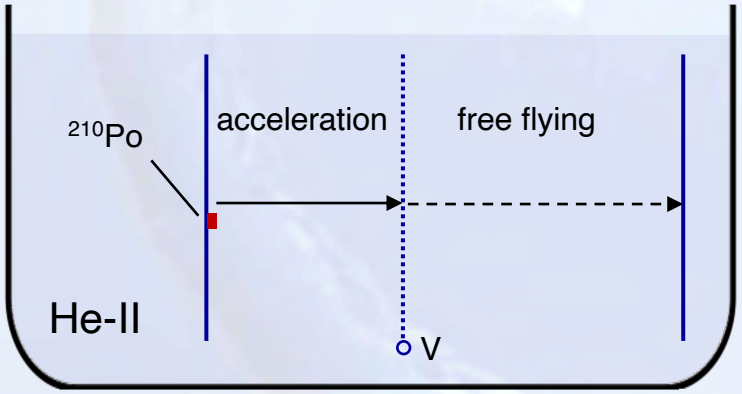
no thermal rotons are excited

phonons mean free path becomes very large ⟶ several cm! $v_c \to 238 \frac{\mathrm{m}}{\mathrm{s}}$?

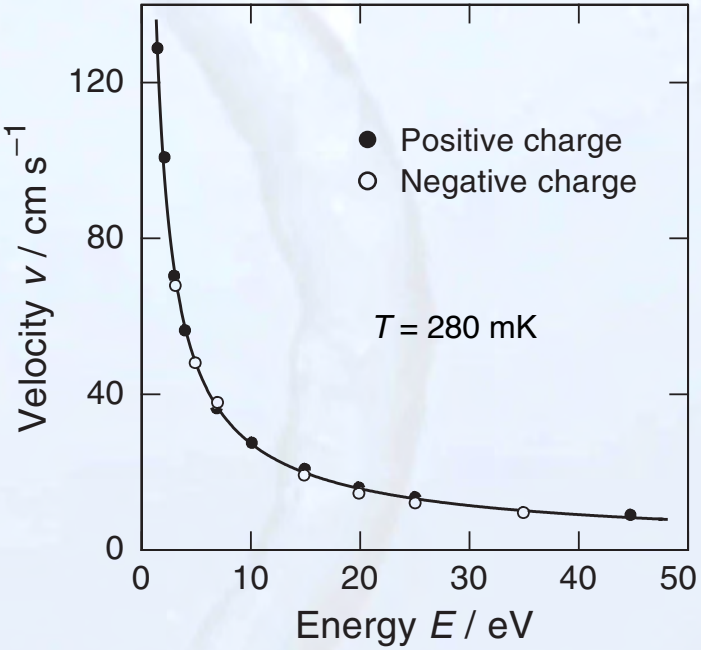
experimental answer: no! $\overline{v}_\mathrm{d}$ = 10 … 100 cm/s

in addition: $\overline{v}_\mathrm{d}$ decreases with energy of ions, which means it decreases with accelerating field

Experiment by Rayfield and Reif 1964



⟶ measurement of ion velocity by time of flight



explanation:

- creation of vortex rings and trapping of ions
- experiment observes motion of vortex rings

2.7 Motion of Ions in He-II

vortex rings

kinetic energy of vortex ring: He-II $\varrho \rightarrow \varrho_s$

$$E_{vr} = \int \frac{1}{2}\varrho_s v_s^2 \mathrm{d}V = \frac{1}{2}\varrho_s \kappa^2 r \left[\ln\left(\frac{8r}{a_0}\right) - \frac{7}{4}\right] \quad \propto r$$

momentum of vortex ring $p_{vr} = \pi \varrho_s \kappa r^2$

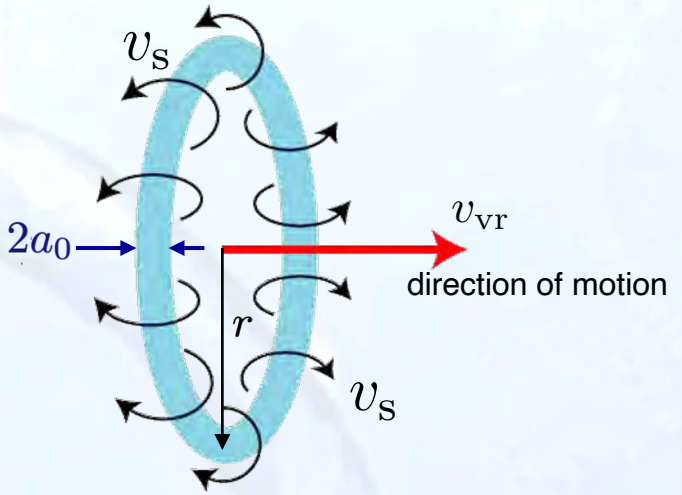
$$\rightarrow \quad v_{vr} = \frac{\partial E}{\partial p_{vr}} = \frac{\kappa}{4\pi r}\left[\ln\left(\frac{8r}{a_0}\right) - \frac{1}{4}\right]$$

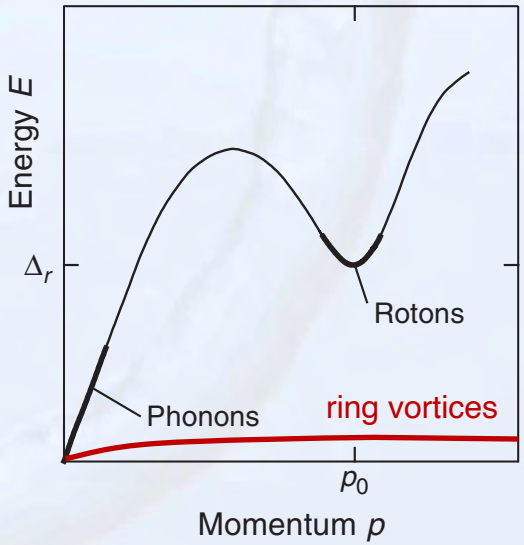
$$\rightarrow \quad p_{vr} \propto r^2 \propto E_{vr}^2 \quad \text{and} \quad v_{vr} \propto 1/E$$

$$\rightarrow \quad \boxed{E_{vr} \propto \sqrt{p_{vr}}}$$

as observed

dispersion of vortex ring





2.7 Motion of Ions in He-II

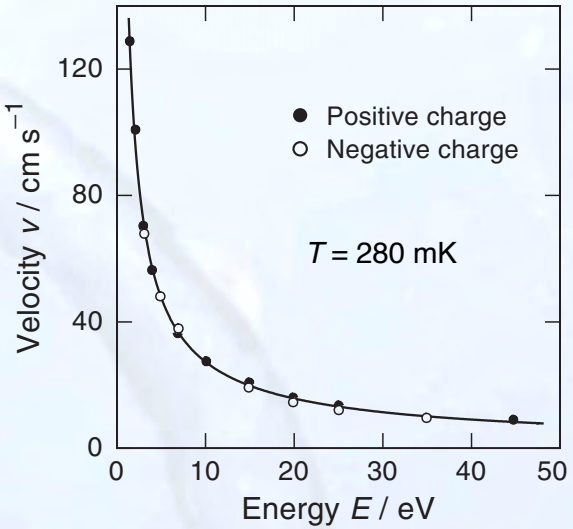
2.7 Motion of Ions in He-II

Explanation of the experiment by Rayfield and Reif

- generation of vortex rings
- ions are captured by vortex ring
- field increases kinetic energy of vortex ring

$$v_{\mathrm{vr}} \propto \frac{1}{r} \propto \frac{1}{E_{\mathrm{vr}}}$$

- theory line with $a_0 = 1.2$ Å



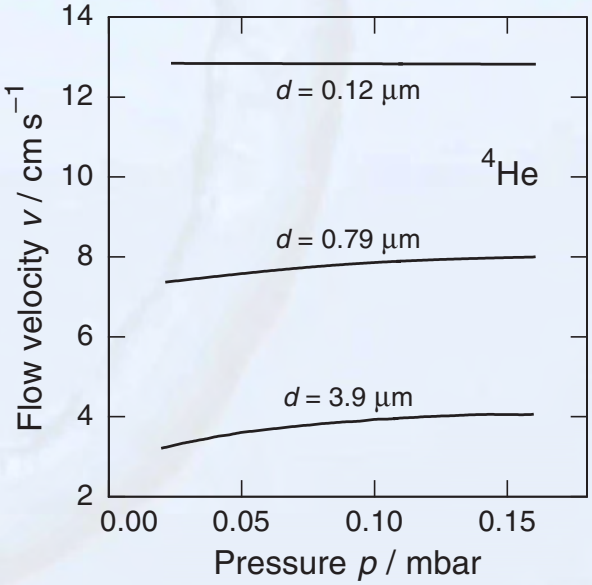
let's revisit the flow experiments through capillaries

because of $E_{\mathrm{vr}} \propto \sqrt{p_{\mathrm{vr}}}$, largest possible vortex is has minimal critical velocity

for capillary with diameter *d*

$$v_{\mathrm{c,vr}} = \frac{\hbar}{m_4 d}\left[\ln\left(\frac{4d}{a_0}\right) - \frac{1}{4}\right] \propto \frac{1}{d}$$

➡ qualitative agreement with flow experiments in capillaries

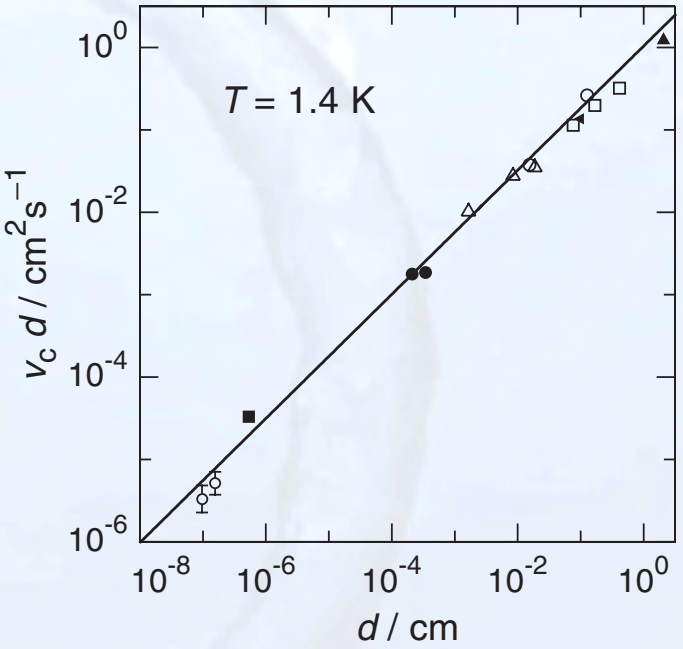


2.7 Motion of Ions in He-II

flow experiments to determine the critical velocity

how does the critical velocity depend on d ?

- potted is: $v_c d$ vs d
- critical velocity $v_c \propto d^{-1/4}$
- expected $v_c \propto d^{-1}$
- reason is unknown



2.7 Critical Behaviour of He-II at T_λ

Properties near T_c are determined by quantities that go to zero like the order parameter and quantities that diverge like susceptibilities

Landau theory of continuous phase transitions (1937, 1965)

- idea: expansion of free energy in T in terms of the order parameter
- near T_c one should find the following laws with the reduced temperature $t = (T - T_c)/T_c$

Quantity	Power Law	Critical Exponent
specific heat	$C_V \propto \lvert t\rvert^{\alpha}$	$\alpha = 0$
order parameter	$\Phi \propto \lvert t\rvert^{\beta}$	$\beta = 1/2$
susceptibility	$\chi \propto \lvert t\rvert^{-\gamma}$	$\gamma = 1$
correlation length	$\xi \propto \lvert t\rvert^{-\nu}$	$\nu = 1/2$

Landau type theories: – van der Waals theory for liquid – gas transition
– Curie-Weiss theory of ferromagnetism
– Ginzburg-Landau theory of superconductivity

2.7 Critical Behaviour of He-II at T_λ

Problem: fluctuations are not included, but they are increasingly important towards T_c

➡ every Landau-type theory breaks down near T_c

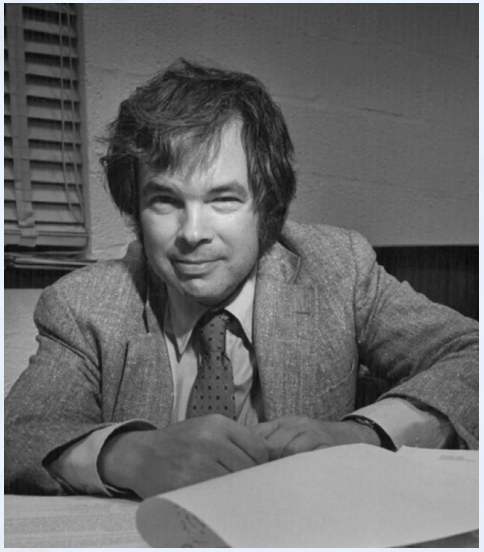
Ginzburg criterion

The condition under which a Landau-type theory holds is that fluctuations
of the order parameter are small in comparison of the mean value of the order parameter

for He-II: coherence length is very small ⟶ Ginzburg criterion is ”always” violated

Renormalization group

Despite of the short-comings of the Landau universal theory of phase transitions, it was realized that it is possible to assign different physical systems to universality classes, characterized by a set of critical exponents



Kenneth G. Wilson

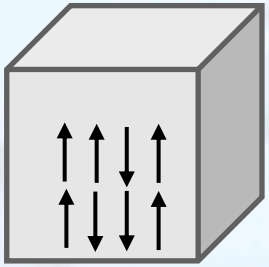
The larger framework is: renormalization group and quantum field theory

different classes are defined by: dimension of system d,
degrees of freedom of order parameter n,
interaction length compared to coherence length

2.7 Critical Behaviour of He-II at T_λ

a few examples:

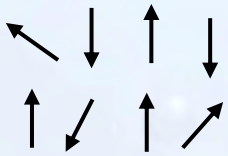
Ising 3 D



$d = 3$
$n = 1$

in this universality class liquid-solid transition fall as well

Heisenberg 2 D



$d = 2$
$n = 3$

at each lattice point each spin can point in 3 direction

x–y 3 D

He-II superconductors

$d = 3$
$n = 2$

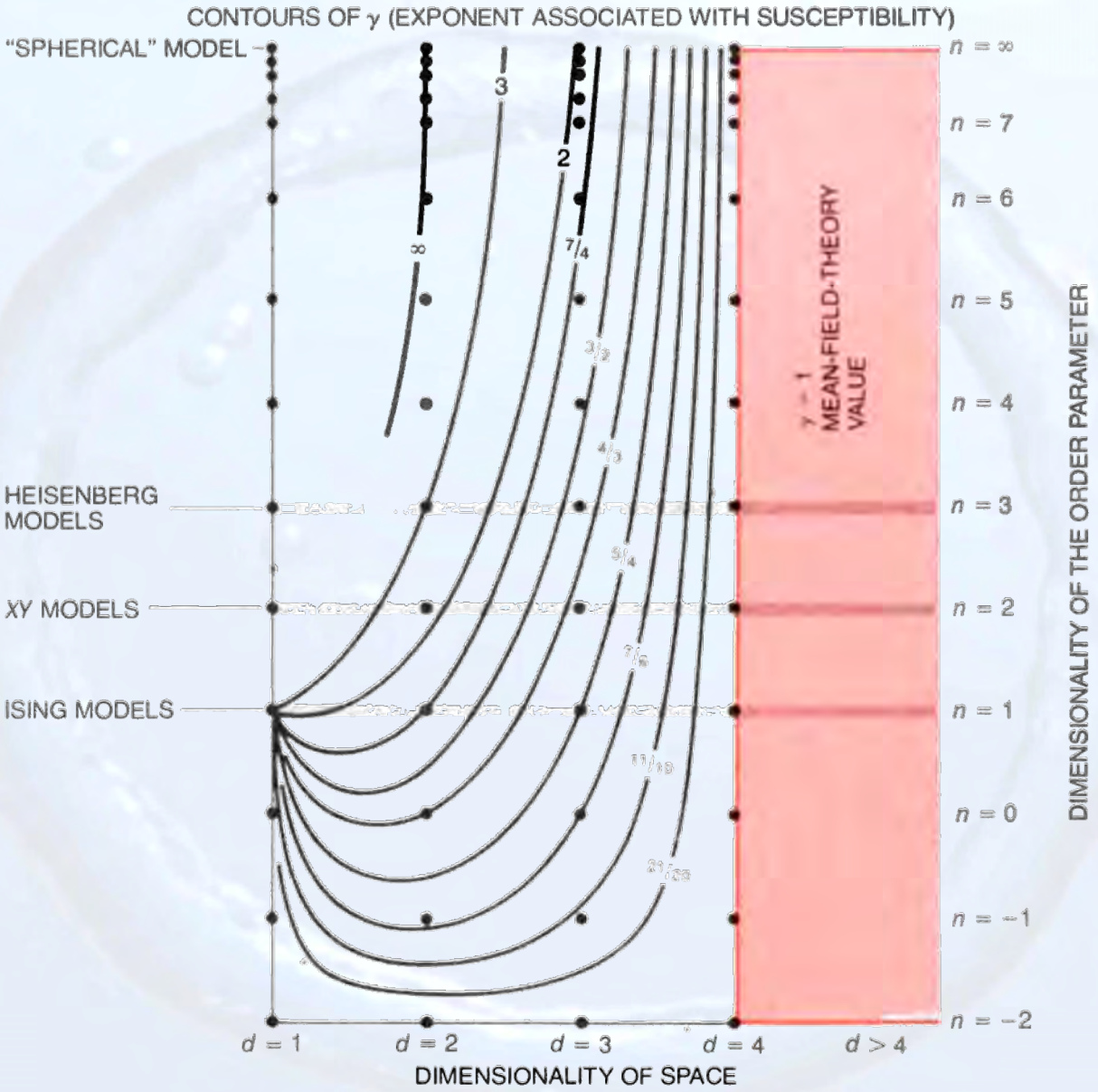
magnitude and phase of wave function

each universality class is described by a set of critical exponents and are connected by sum rules like $\alpha + 2\beta + \gamma = 2$

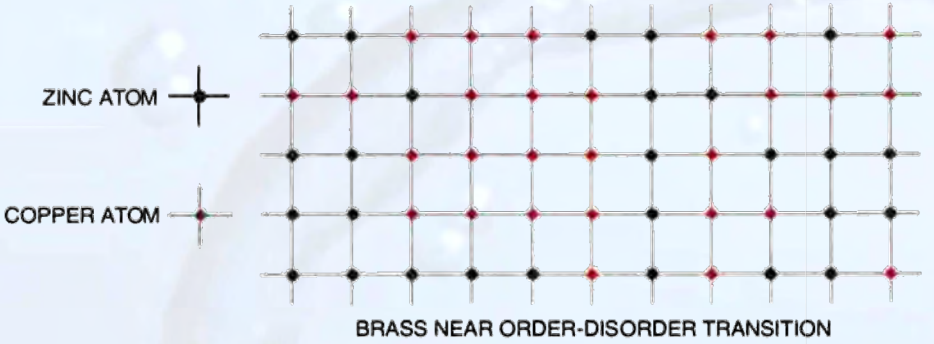
2.7 Critical Behaviour of He-II at T_λ

UNIVERSALITY CLASS		THEORETICAL MODEL	PHYSICAL SYSTEM	ORDER PARAMETER
$d = 2$	$n = 1$	Ising model in two dimensions	Adsorbed films	Surface density
	$n = 2$	*XY* model in two dimensions	Helium-4 films	Amplitude of superfluid phase
	$n = 3$	Heisenberg model in two dimensions		Magnetization
$d > 2$	$n = \infty$	"Spherical" model	None	
$d = 3$	$n = 0$	Self-avoiding random walk	Conformation of long-chain polymers	Density of chain ends
	$n = 1$	Ising model in three dimensions	Uniaxial ferromagnet	Magnetization
			Fluid near a critical point	Density difference between phases
			Mixture of liquids near consolute point	Concentration difference
			Alloy near order-disorder transition	Concentration difference
	$n = 2$	*XY* model in three dimensions	Planar ferromagnet	Magnetization
			Helium 4 near superfluid transition	Amplitude of superfluid phase
	$n = 3$	Heisenberg model in three dimensions	Isotropic ferromagnet	Magnetization
$d \leq 4$	$n = -2$		None	
	$n = 32$	Quantum chromodynamics	Quarks bound in protons, neutrons, etc.	

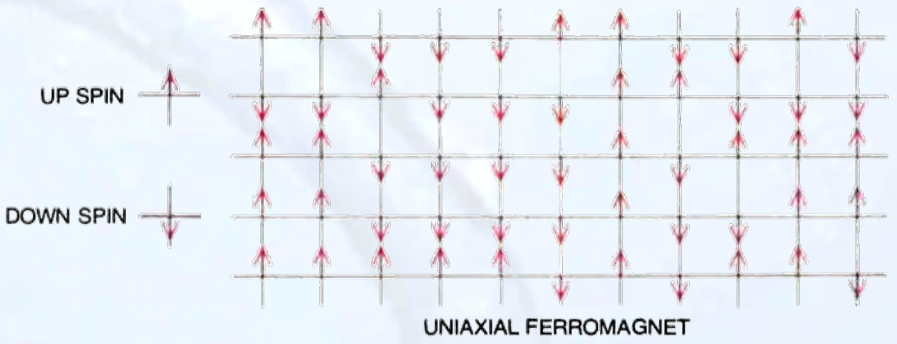
2.7 Critical Behaviour of He-II at T_λ



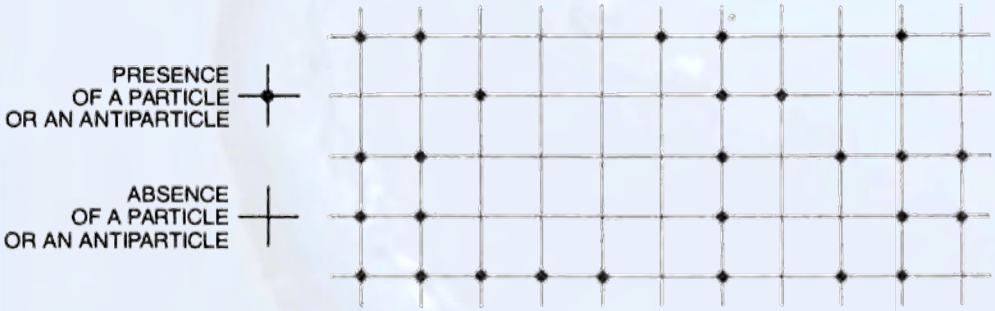
2.7 Critical Behaviour of He-II at T_λ



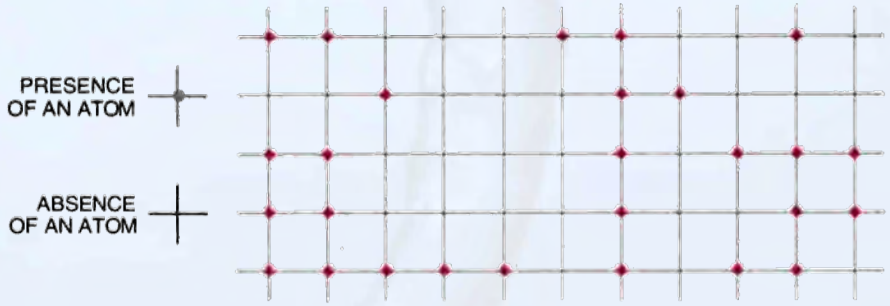
BRASS NEAR ORDER-DISORDER TRANSITION



UNIAXIAL FERROMAGNET



QUANTUM FIELD THEORY



FLUID NEAR CRITICAL POINT

2.7 Critical Behaviour of He-II at T_λ

critical exponents expected for X-Y 3D model:

$\alpha = -0.0146(8)$ ⟵

$\beta = 0.3485(2)$ ⟵

$\gamma = 1.3177(5)$

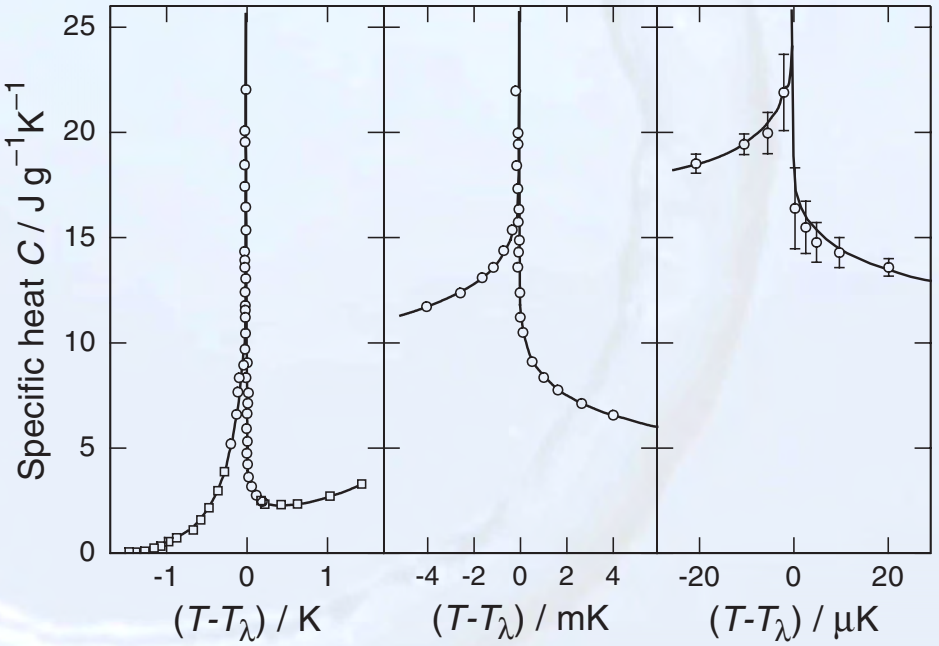
$\delta = 4.780(2)$

$\nu = 0.67155(27)$ ⟵

$\eta = 0.0380(4)$

Experiments near T_λ

a) specific heat



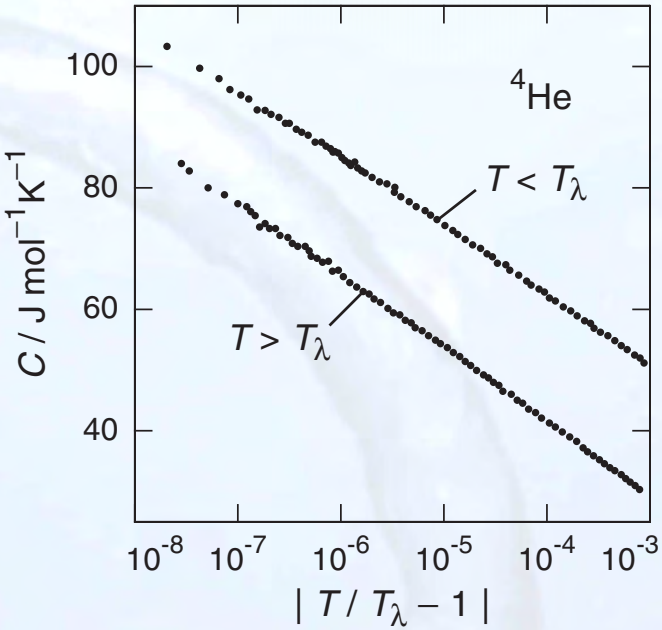
scale going from K to μK

2.7 Critical Behaviour of He-II at T_λ

power law in the vicinity of T_λ ?

data plotted a C_V vs $\log t = \log |T/T_\lambda - 1|$

data can be approximated by $C_V \propto \log t$



logarithmic divergences?

comparison with RGT

expected scaling for He-II

$$C = B + A\,\frac{t^{-\alpha}}{\alpha}\left(1 - D\sqrt{t}\right)$$ $A,\ B$ and D are constants

with critical exponent expected $\alpha = -0.146(8)$

➡ expansion in α $\quad t^{-\alpha} = \mathrm{e}^{-\alpha \ln t} \approx 1 - \alpha \ln t$ expansion justified because of small α

experimental result $\alpha \approx -0.013 \pm 0.003$

2.7 Critical Behaviour of He-II at T_λ

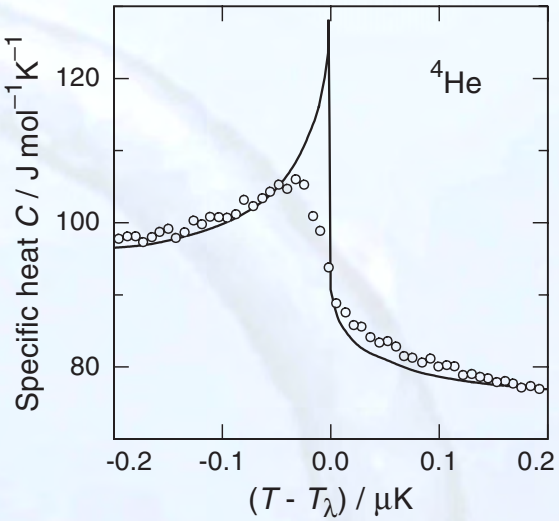
Higher precision experiments near T_λ are needed

measurement on earth

Problems:

gravitation → level height dependence

walls of vessel → first layer solid and healing length diverges with diverges near T_λ with $\xi = \xi_0 t^{-\nu}$ with $\nu = 0.67155(27)$



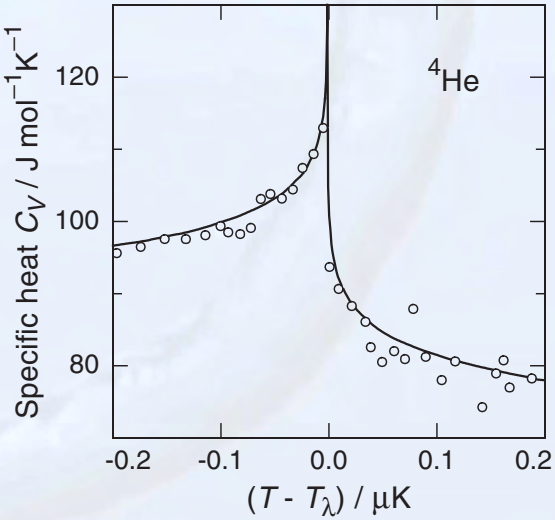
measurement on space shuttle

Problems:

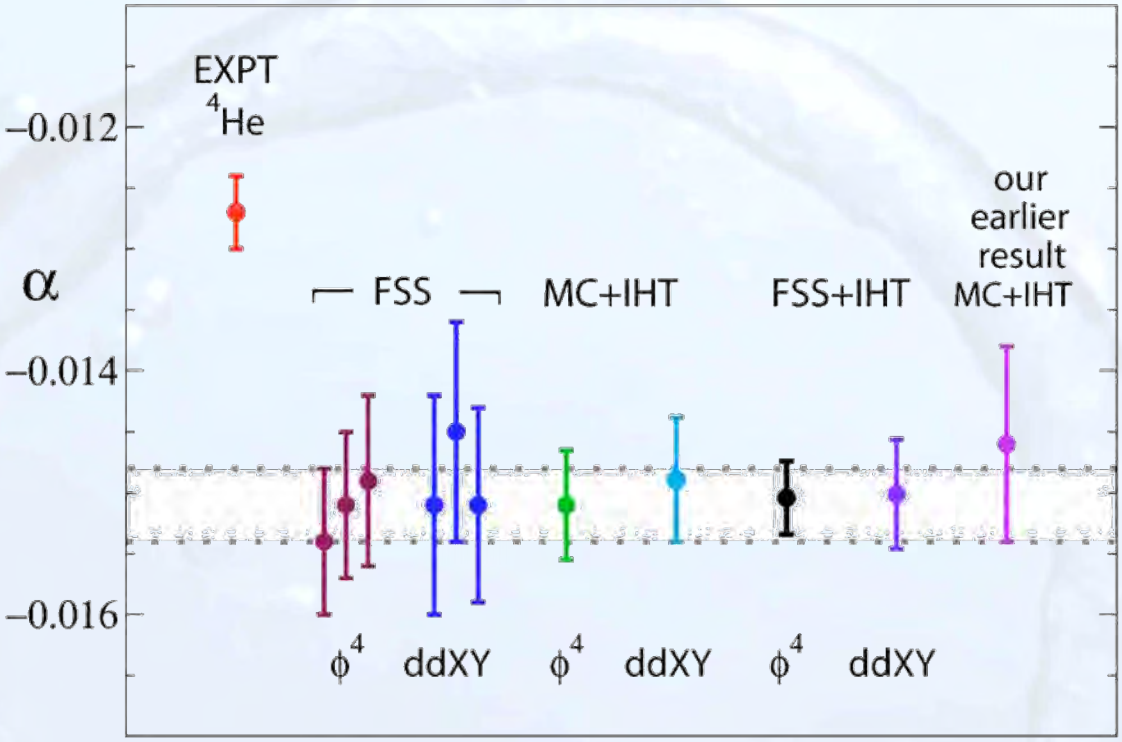
cosmic rays → time varying background (heating of thermometer)

Data shown, after sophisticated analysis

→ still somewhat noisy



2.7 Critical Behaviour of He-II at T_λ



comparison between space shuttle data and different calculations of α

➡ discrepancy between data and theory outside error bars: reason unknown

2.7 Critical Behaviour of He-II at T_λ

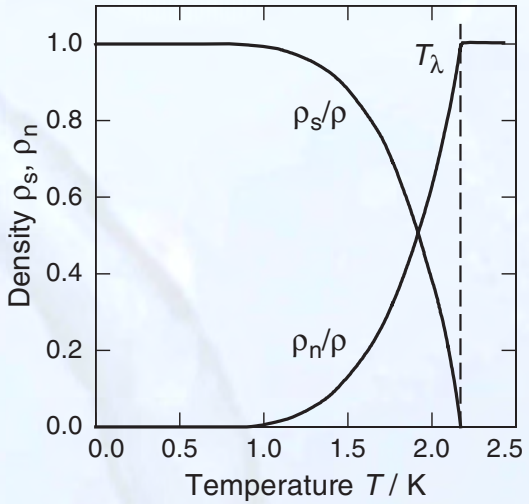
b) Order parameter

$$\psi(\boldsymbol{r}) = \psi_0 \, \mathrm{e}^{\mathrm{i}\varphi(\boldsymbol{r})} \longrightarrow \Psi_0 = \sqrt{\varrho_\mathrm{s}}$$

amplitude of wave function

expected:

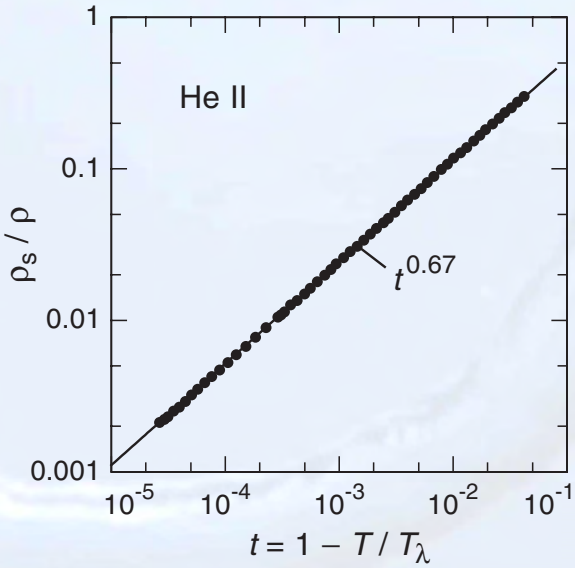
$$\varrho_\mathrm{s} = t^{2\beta} \quad \text{with} \quad \beta = 0.3485(2)$$



determined with second sound

$$\varrho_\mathrm{s} = t^{0.67}$$

➡ excellent agreement



2.7 Critical Behaviour of He-II at T_λ

c) Healing length

again, second sound measurements
and measurements on thin films

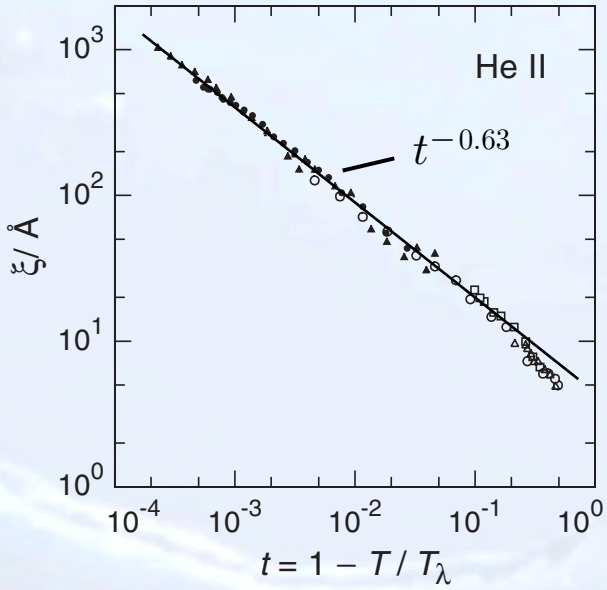
expected:

$\xi = \xi_0 \, t^{-\nu}$ with $\nu = 0.67155(27)$

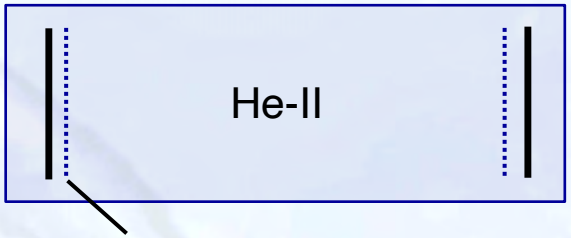
$\xi_0 = 2.8 \pm 0.5$ Å

$\nu = 0.63$

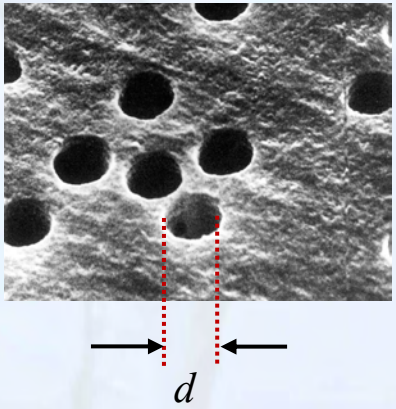
➡ excellent agreement



Helmholtz resonator



Nuclepore filters



second sound vanishes
for $\xi > d$