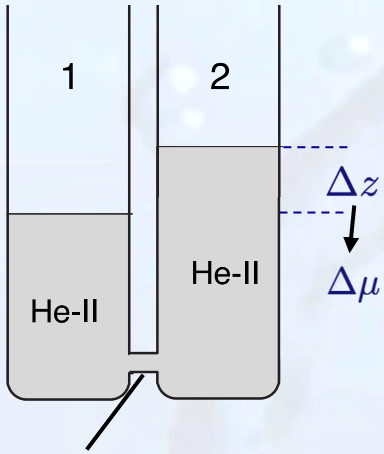
2.5 Macroscopic Quantum State

Josephson Effects



$\Delta\mu = \mu_2 - \mu_1 = m_4 g \Delta z$

healing length

$d \approx \xi = 1 \ldots 2\,\text{Å}$

$$\xi_4 = \frac{0.3\,\text{nm}}{(1 - T/T_\lambda)^{2/3}}$$

diverges for $T \to T_\lambda$

Schrödinger Eq.

$$i\hbar\dot{\Psi}_1 = \mu_1\Psi_1 + \mathcal{K}\Psi_2$$

$$i\hbar\dot{\Psi}_2 = \mu_2\Psi_2 + \mathcal{K}\Psi_1$$

with $\Psi_1 = \sqrt{\varrho_s}e^{i\varphi_1}$ and $\Psi_2 = \sqrt{\varrho_s}e^{i\varphi_2}$

$$\frac{\partial\varrho_s}{\partial t} = \frac{2\mathcal{K}}{\hbar}\,\varrho_s \sin(\varphi_2 - \varphi_1)$$

$T = 0$

$$\frac{\partial}{\partial t}(\varphi_2 - \varphi_1) = -\frac{1}{\hbar}(\mu_2 - \mu_1) = -\frac{1}{\hbar} m_4 g \Delta z$$

$\Delta\mu = 0$ phase difference constant → Josephson dc effect

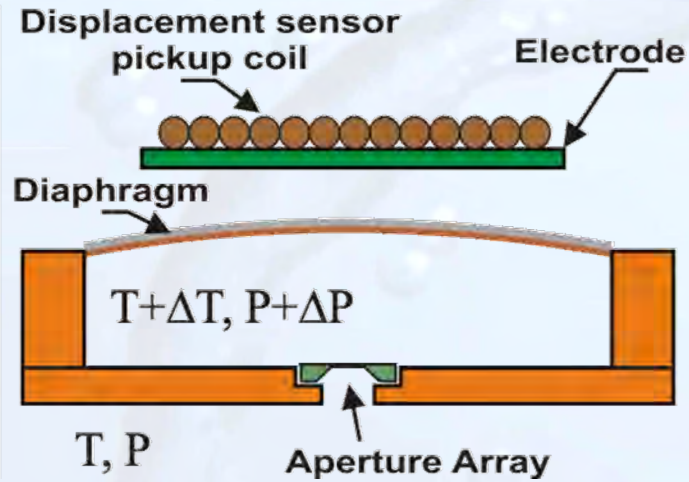
$\Delta\mu \neq 0$ phase difference changes → Josephson ac effect

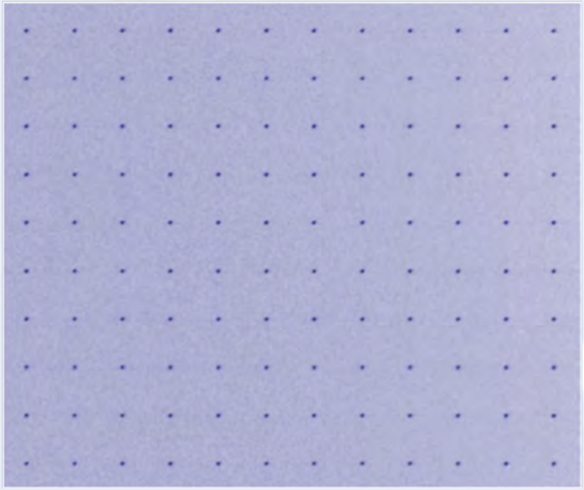
$$T = 0 \longrightarrow \omega_J = \frac{\Delta\mu}{\hbar} = \frac{m_4\Delta p}{\hbar\varrho} \quad \text{with } \Delta p = \varrho g \Delta z$$

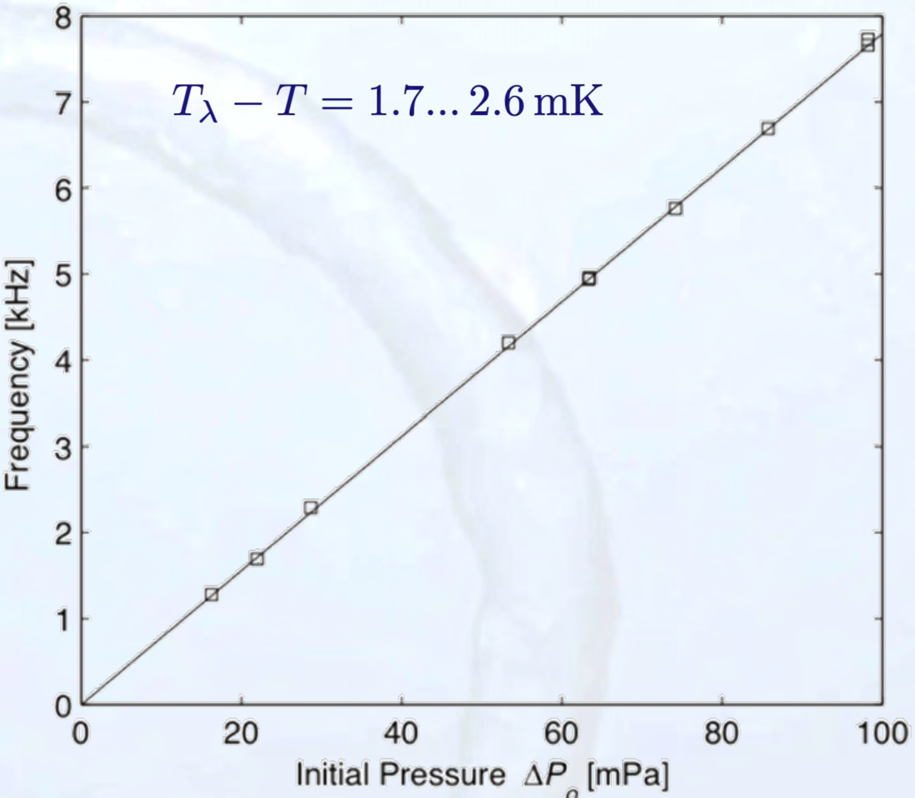
$$T \neq 0 \longrightarrow \omega_J = \frac{\Delta\mu}{\hbar} = \frac{m_4\Delta p}{\hbar\varrho} - m_4\frac{S\Delta T}{\hbar}$$

2.5 Macroscopic Quantum State

Josephson Effects







measurement directly after pressure applied

$\longrightarrow \Delta T \approx 0$

65×65 array

$70\,\mathrm{nm}$ apertures spaced $3\,\mu\mathrm{m}$

50 nm-thick silicon nitride membrane

2.6 Excitation Spectrum of He-II: Landau Model

Is the occurrence of the condensate equivalent to superfluidity ?

comment:

superconductivity in metals is related to an energy gap

nature of excitations is important

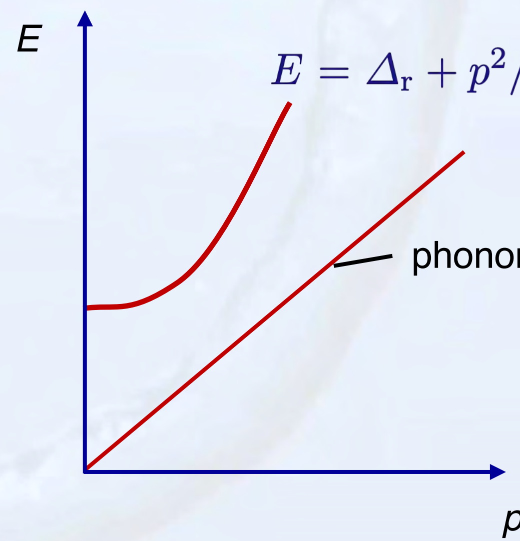
ideal Bose gas:

$$E = \frac{p^2}{2m}$$ → arbitrary energy transfer possible

➡ no superfluidity !

Idea of Landau 1941

- at low temperatures: only longitudinal phonons with linear dispersion
- at “high” temperatures: more and other kinds of excitations contribute, but with energy gap



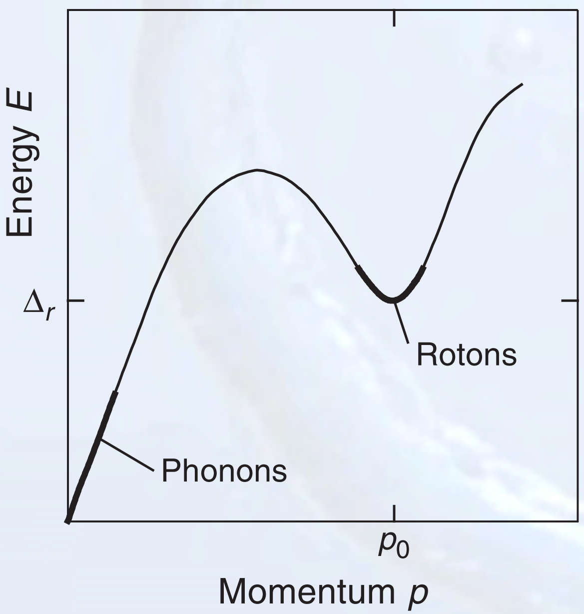
2.6 Excitation Spectrum of He-II: Landau Model

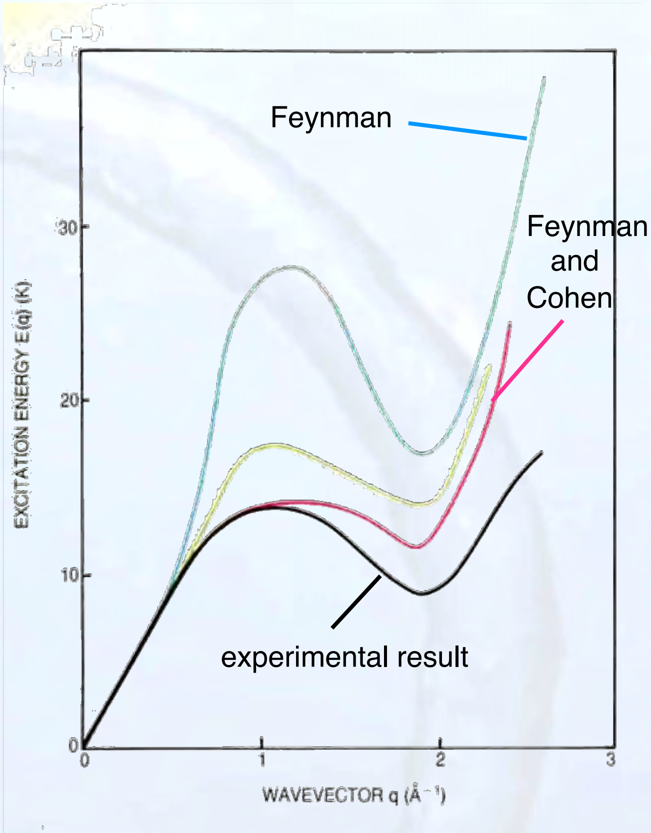
Landau's modification in 1947:

⟶ common dispersion curve

roton dispersion:

$$E = \Delta_r + \frac{(p - p_0)^2}{2m^*}$$





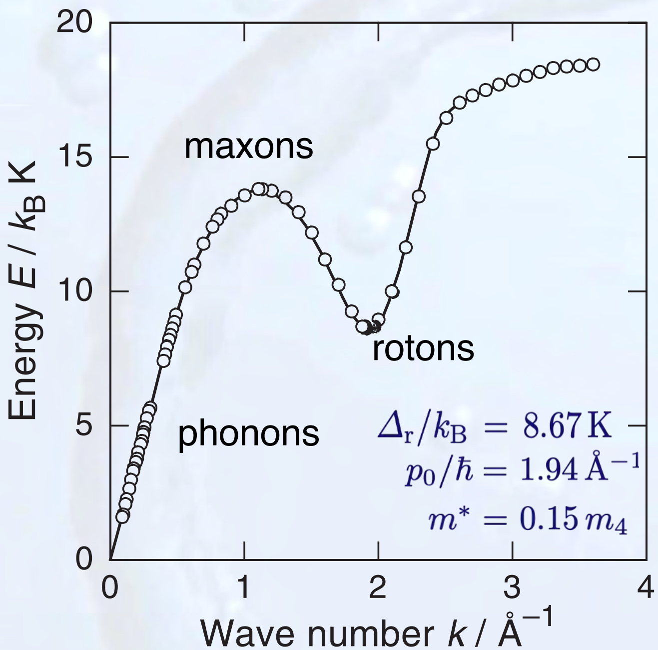
Feynman 1954:

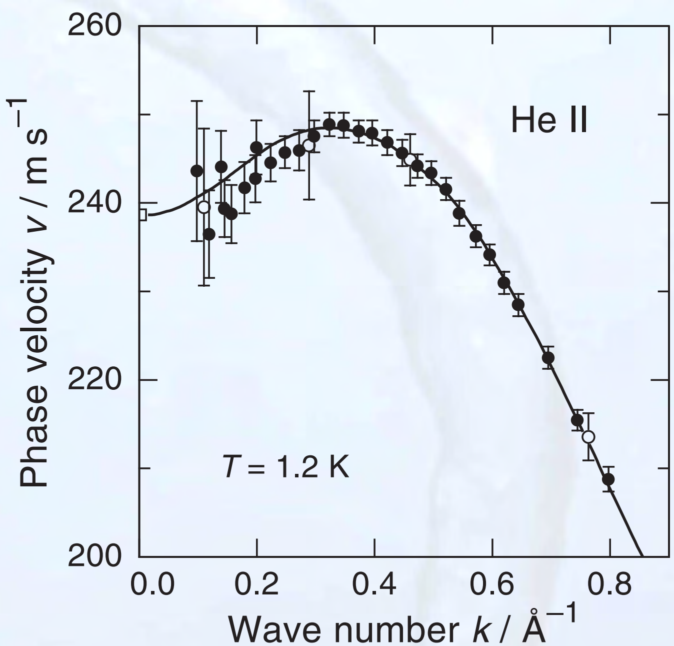
- QM calculation of dispersion curve from symmetry considerations
- improved by Feynman and Cohen in 1955

2.6 Excitation Spectrum of He-II: Landau Model

Experimental determination of the dispersion

Feynman's idea: inelastic neutron scattering





- good agreement with q_{min}, q_{max}
- linear dispersion with v = 238 m/s
- sharp excitations even at high q vectors
- single particle excitations are suppressed

- dispersion not perfectly linear
- anomaly at low wave vectors

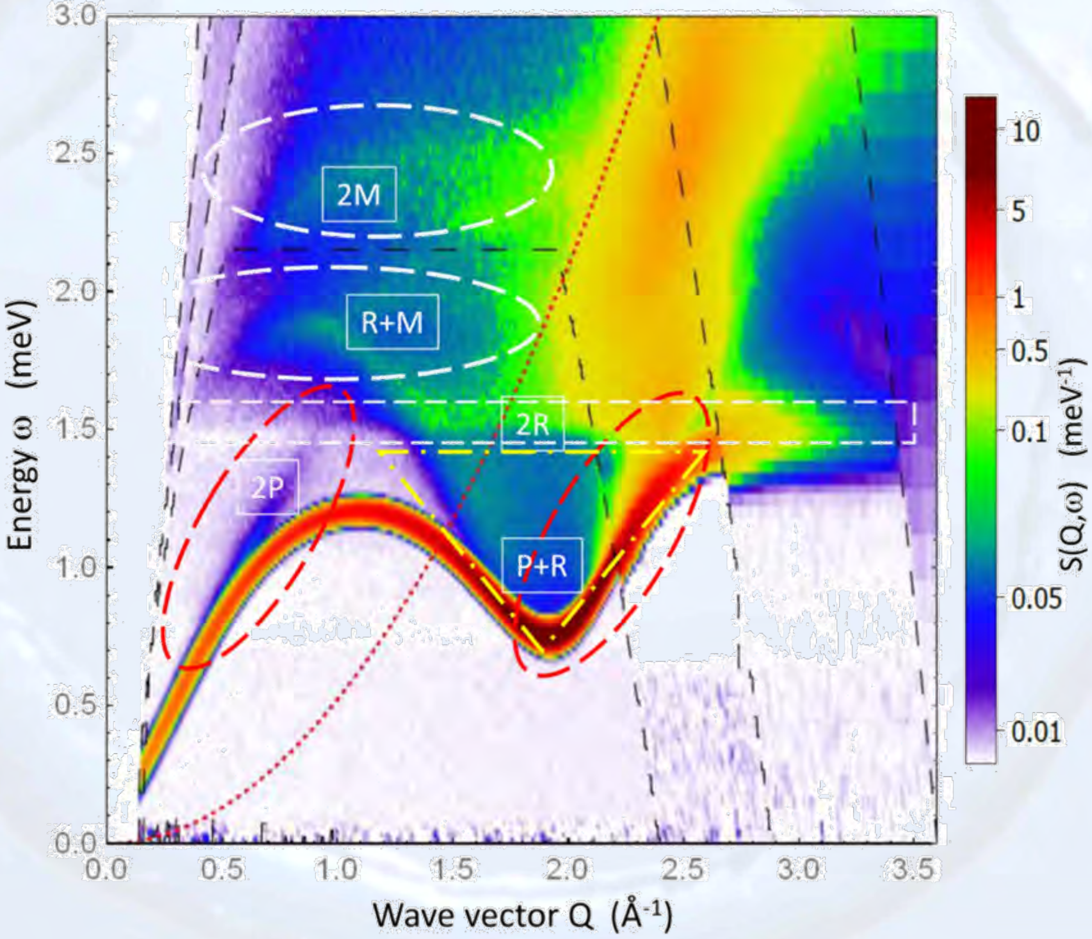
⟶ causes damping by three phonon scattering

⟶ anomaly disappears at $p > 20$ bar

2.6 Excitation Spectrum of He-II: Landau Model

Experimental Determination of the Dispersion

new high-precision measurement



2.6 Excitation Spectrum of He-II: Landau Model

Normalfluid component:

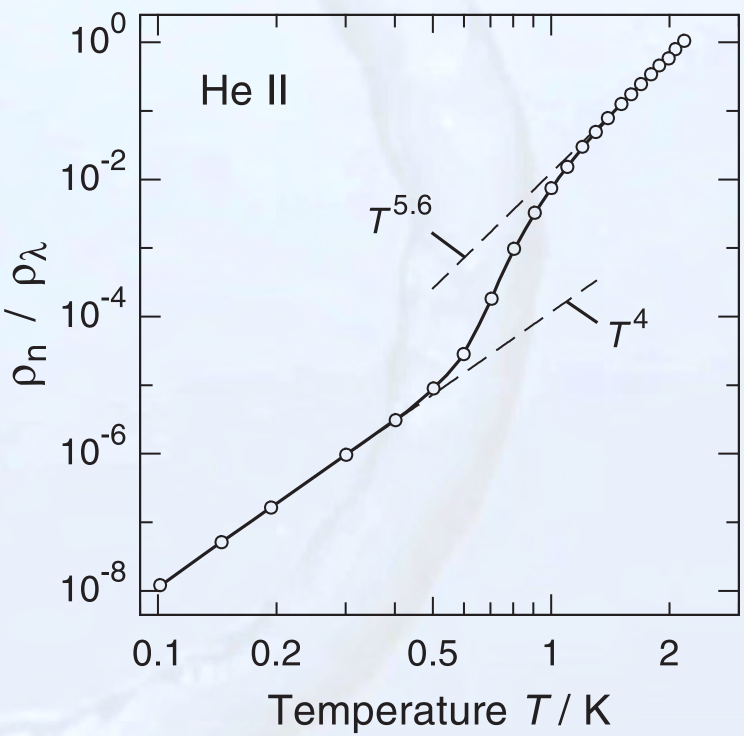
$$\varrho_{\mathrm{n,r}} = \frac{2\,p_0^4}{3\,\hbar^3}\sqrt{\frac{m^*}{(2\pi)^3 k_{\mathrm{B}} T}}\,\mathrm{e}^{-\Delta_{\mathrm{r}}/k_{\mathrm{B}}T}$$

Rotons

$$\varrho_{\mathrm{n}} = \varrho_{\mathrm{n,ph}} + \varrho_{\mathrm{n,r}}$$

$$\varrho_{\mathrm{n,ph}} = \frac{2\pi^2 k_{\mathrm{B}}^4}{45\,\hbar^3\,v_1^5}\,T^4$$

Phonons



- at low temperatures $\varrho_{\mathrm{n}} \propto T^4$ due to phonons
- rotons dominate between 0.5 K and 1.2 K
- above 1.2 K nature of excitations more complex

2.6 Excitation Spectrum of He-II: Landau Model

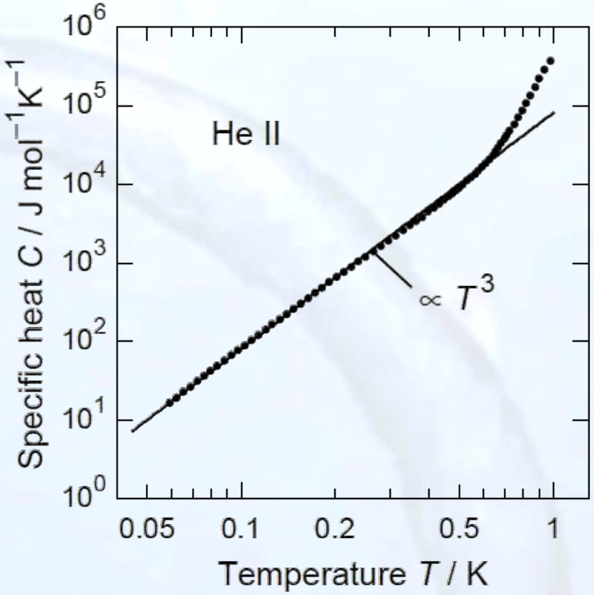
Specific heat:

a) low temperatures $T < 0.6\,\mathrm{K}$

only long wavelength phonons contribute

→ Debye model

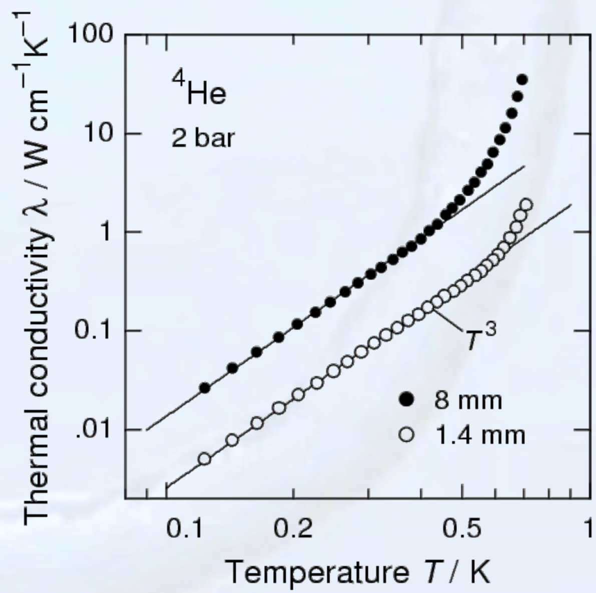
$$C_{\mathrm{ph}} = \frac{2\pi^2 k_{\mathrm{B}}^4}{15\varrho\hbar^3 v_1^3}\,T^3$$



measurement of thermal conductivity

Casimir regime $\ell = d$ — capillary cross section

→ $\Lambda = \frac{1}{3} C_{\mathrm{ph}}\, v\, d \propto T^3$



2.6 Excitation Spectrum of He-II: Landau Model

b) intermediate temperatures $\mathbf{0.6 < T < 1.2\,K}$

free energy $F_\mathrm{r} = -k_\mathrm{B} T n_\mathrm{r}$

$$n_\mathrm{r} = \frac{2p_0^2}{3\varrho\hbar^3}\sqrt{\frac{m^* k_\mathrm{B} T}{(2\pi)^3}}\, \mathrm{e}^{-\Delta_\mathrm{r}/k_\mathrm{B}T}$$

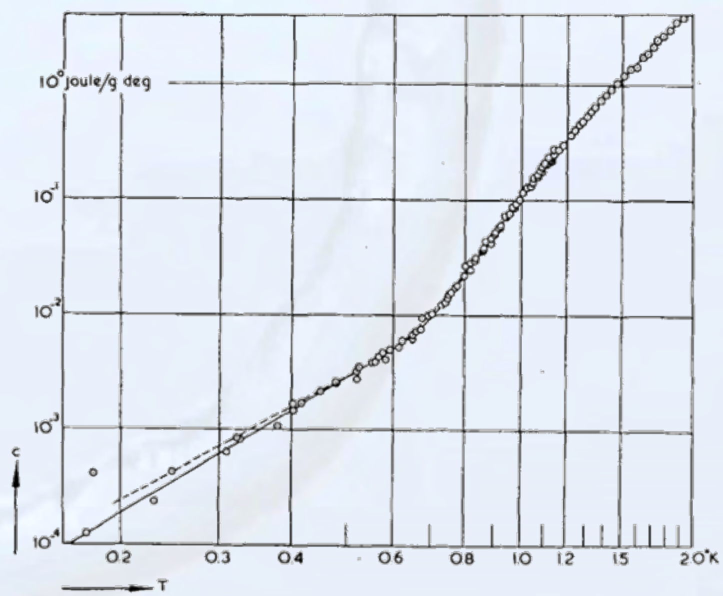
number density of rotons

$S_\mathrm{r} = -\partial F_\mathrm{r}/\partial T$

↓

$C_\mathrm{r} = T\partial S_\mathrm{r}/\partial T$

$$C_\mathrm{r} = \frac{2k_\mathrm{B}p_0^2}{3\varrho\hbar^3}\sqrt{\frac{m^* k_\mathrm{B} T}{(2\pi)^3}}\left\{\frac{3}{4} + \frac{\Delta_\mathrm{r}}{k_\mathrm{B}T} + \left(\frac{\Delta_\mathrm{r}}{k_\mathrm{B}T}\right)^2\right\}\mathrm{e}^{-\Delta_\mathrm{r}/k_\mathrm{B}T}$$



c) high temperatures $\mathbf{1.2\,K < T < T_\lambda}$

additional excitations contribute: maxons
lifetime of rotons becomes very short

⟶ excitations are not well-defined

2.6 Excitation Spectrum of He-II: Landau Model

Landau's concept of critical velocity

superconductors ⟶ energy gap

superfluid He-II ⟶ no energy gap, but velocity gap!

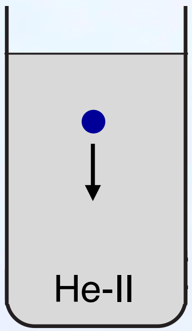
comment:

phonons can be excited at arbitrary small energies

Landau's Gedankenexperiment: dropping a massive sphere in He-II at $T = 0$

let's assume that sphere generates one excitation with energy $\mathcal{E}$ and momentum $\boldsymbol{p}$

How fast must this sphere fall in He-II to generate dissipation ?



energy conservation

$$\frac{1}{2}\mathcal{M}v^2 = \frac{1}{2}\mathcal{M}v'^2 + \mathcal{E} \qquad (1)$$

momentum conservation

$$\mathcal{M}\boldsymbol{v} - \boldsymbol{p} = \mathcal{M}\boldsymbol{v}' \qquad (2)$$

not all combinations of $\mathcal{E}$ and $\boldsymbol{p}$ fulfill both conservation law's at the same time, even if the direction of the excitation is not fixed

2.6 Excitation Spectrum of He-II: Landau Model

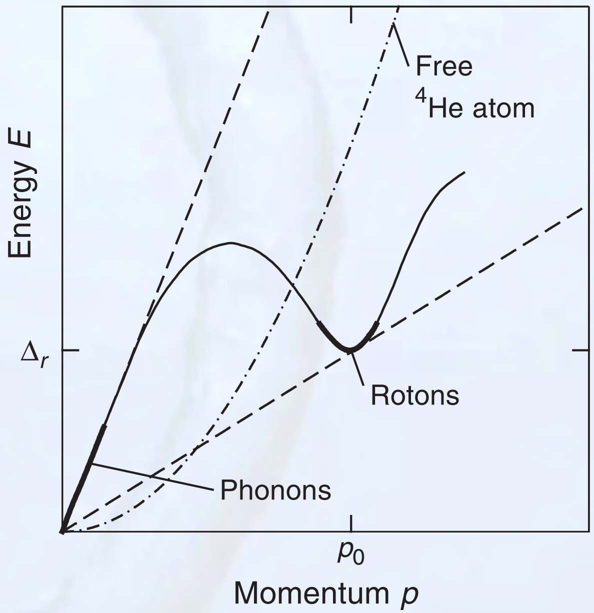
let's test this: square (2) and divide by $2\mathcal{M}$ $\longrightarrow$ $\frac{1}{2}\mathcal{M}v^2 - \boldsymbol{v}\cdot\boldsymbol{p} + \frac{1}{2\mathcal{M}}p^2 = \frac{1}{2}\mathcal{M}v'^2$

comparison with (1) results in: $\boldsymbol{v}\cdot\boldsymbol{p} - \frac{1}{2\mathcal{M}}p^2 = \mathcal{E}$

with $\boldsymbol{v} \parallel \boldsymbol{p}$

mass of sphere is large

$\Rightarrow$ $v_c = \frac{\mathcal{E}}{p}$ independent of nature of excitation



phonons: $v_c = 238\,\mathrm{m\,s^{-1}}$

rotons: $v_c \approx 60\,\mathrm{m\,s^{-1}}$

free ^{4}He atom: $v_c = 0$ $\longleftarrow$ do not exist in He-II

- no excitation for $v < v_c$
- no dissipation $\longrightarrow$ superfluidity
- for $v \geq v_c$ sudden onset of dissipation, laminar $\longrightarrow$ turbulent flow