2.3 Properties of He-II described using the two-fluid model

Ansatz: $\varrho = \varrho_0 + \varrho' \, \mathrm{e}^{\mathrm{i}\omega(t-x/v)}\,,$

$$S = S_0 + S' \, \mathrm{e}^{\mathrm{i}\omega(t-x/v)}$$

frequency of wave

velocity in x direction

Insertion and differentiation leads to 2 linear equations in ϱ' and S'

$$\left[\left(\frac{v}{v_1}\right)^2 - 1\right] \varrho' + \left(\frac{\partial p}{\partial S}\right)_\varrho \left(\frac{\partial \varrho}{\partial p}\right)_S S' = 0\,, \qquad \text{(i)}$$

$$\left(\frac{\partial T}{\partial \varrho}\right)_S \left(\frac{\partial S}{\partial T}\right)_\varrho \varrho' + \left[\left(\frac{v}{v_2}\right)^2 - 1\right] S' = 0 \qquad \text{(ii)}$$

with

$$v_1^2 = \left(\frac{\partial p}{\partial \varrho}\right)_S \quad \text{and} \quad v_2^2 = \frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}} S^2 \left(\frac{\partial T}{\partial S}\right)_\varrho$$

2.3 Properties of He-II described using the two-fluid model

the constrains equation for the coefficients is

$$\left[\left(\frac{v}{v_1}\right)^2-1\right]\left[\left(\frac{v}{v_2}\right)^2-1\right]=\underbrace{\left(\frac{\partial p}{\partial S}\right)_\varrho\left(\frac{\partial \varrho}{\partial p}\right)_S\left(\frac{\partial T}{\partial \varrho}\right)_S\left(\frac{\partial S}{\partial T}\right)_\varrho}$$

here standard thermodynamic relations are used

$$\boxed{\left[\left(\frac{v}{v_1}\right)^2-1\right]\left[\left(\frac{v}{v_2}\right)^2-1\right]=\frac{C_p-C_V}{C_p}}$$

for liquid helium $C_p \approx C_V$

$$\left[\left(\frac{v}{v_1}\right)^2-1\right]\left[\left(\frac{v}{v_2}\right)^2-1\right]\approx 0 \qquad \text{(iii)}$$

interpretation: two wave $\left.\begin{matrix} \nearrow v_1 \\ \searrow v_2 \end{matrix}\right\}$ weakly coupled via $\dfrac{C_p-C_V}{C_p}$

2.3 Properties of He-II described using the two-fluid model

(i) First sound

$$v = v_1 = \sqrt{\left(\frac{\partial p}{\partial \varrho}\right)_S}$$

with (i) and (iii)

$$\left(\frac{v}{v_1}\right)^2 - 1 = 0$$

$$\varrho' \neq 0 \quad S' = 0$$

$$\longrightarrow \operatorname{grad} T = 0$$

as usual for ordinary (first) sound

insert (4) into (6)

$$\varrho_\mathrm{n} \frac{\partial}{\partial t} \underbrace{(\boldsymbol{v}_\mathrm{n} - \boldsymbol{v}_\mathrm{s})}_{\boldsymbol{v}_\mathrm{n} = \boldsymbol{v}_\mathrm{s}} = \varrho S \operatorname{grad} T = 0$$

$\boldsymbol{v}_\mathrm{n} = \boldsymbol{v}_\mathrm{s}$ ➡ superfluid and normalfluid component are in phase

(4) in (6)

$$\frac{\partial \vec{v}_s}{\partial t} = S \operatorname{grad} T + \frac{1}{\varrho} \frac{\partial \vec{j}}{\partial t}$$

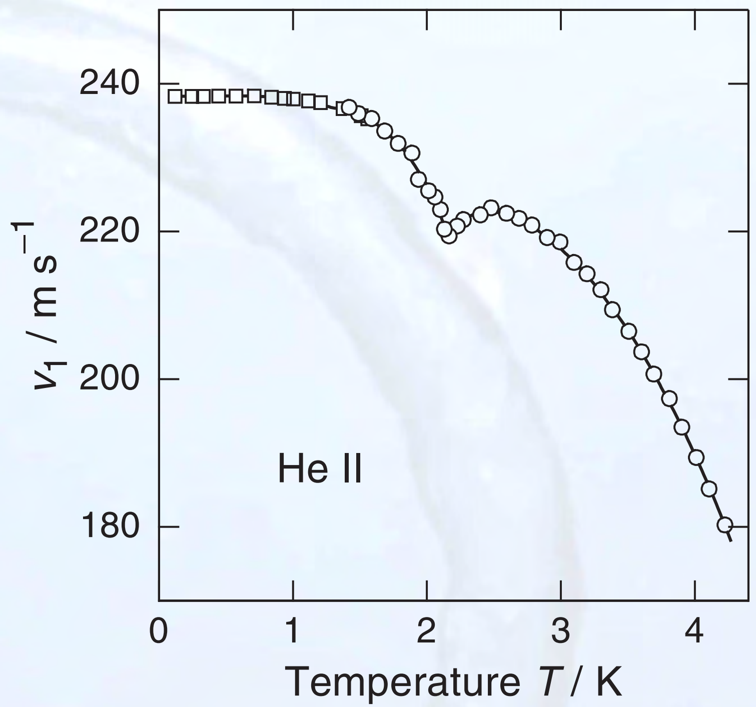
insert (2) $\times \varrho$

$$\curvearrowright \varrho \left(\frac{\partial \vec{v}_s}{\partial t}\right) = \varrho S \operatorname{grad} T + \varrho_n \frac{\partial \vec{v}_n}{\partial t} + \varrho_s \frac{\partial \vec{v}_s}{\partial t}$$

$$\curvearrowright \varrho_n \frac{\partial}{\partial t} (\vec{v}_s - \vec{v}_n) = \varrho S \operatorname{grad} T = 0$$

2.3 Properties of He-II described using the two-fluid model

(i) First sound



- for $T \rightarrow 0$: $v_1 \approx 238\,\mathrm{m\,s^{-1}}$.

  ➡ only density variation ➡ almost ordinary sound

- for $T \rightarrow T_\lambda$: corrections become important

2.3 Properties of He-II described using the two-fluid model

(ii) Second sound

with (ii) and (iii) we find

$$v = v_2 = \sqrt{\frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}} S^2 \left(\frac{\partial T}{\partial S}\right)_\varrho}$$

$$\left(\frac{v}{v_2}\right)^2 - 1 = 0$$

$$S' \neq 0 \ , \quad \varrho' = 0$$

$$\rightarrow \quad \mathrm{grad}\, p = 0$$

with (4)

$$\frac{\partial \boldsymbol{j}}{\partial t} = -\mathrm{grad}\, p \overset{!}{=} 0 \quad \Rightarrow \quad \frac{\partial \varrho_\mathrm{n} \boldsymbol{v}_\mathrm{n}}{\partial t} + \frac{\partial \varrho_\mathrm{s} \boldsymbol{v}_\mathrm{s}}{\partial t} = 0$$

$$\varrho_\mathrm{n} \boldsymbol{v}_\mathrm{n} + \varrho_\mathrm{s} \boldsymbol{v}_\mathrm{s} = 0$$

no mass flow in closed vessel

$\curvearrowright \quad \varrho_\mathrm{n} \uparrow \ , \quad \varrho_\mathrm{s} \downarrow$ counter flow and no density variation

$\Rightarrow$ temperature wave

2.3 Properties of He-II described using the two-fluid model

$$\boxed{v_2 = \sqrt{\frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}} S^2 \left(\frac{\partial T}{\partial S}\right)_\varrho} = \sqrt{\frac{\varrho_\mathrm{s}}{\varrho_\mathrm{n}} \frac{T\,S^2}{C_p}}}$$

density variation in phonon gas

possibility to determine $\varrho_\mathrm{s}/\varrho_\mathrm{n}$

ultra-low temperatures:

excitations at $T \to 0$ are only longitudinal phonons

Landau

Debye model $A = 2\pi^2 k_\mathrm{B}^4/(45\hbar^3 v_1^3 \varrho)$

$$C_p = 3AT^3$$
$$S = AT^3$$

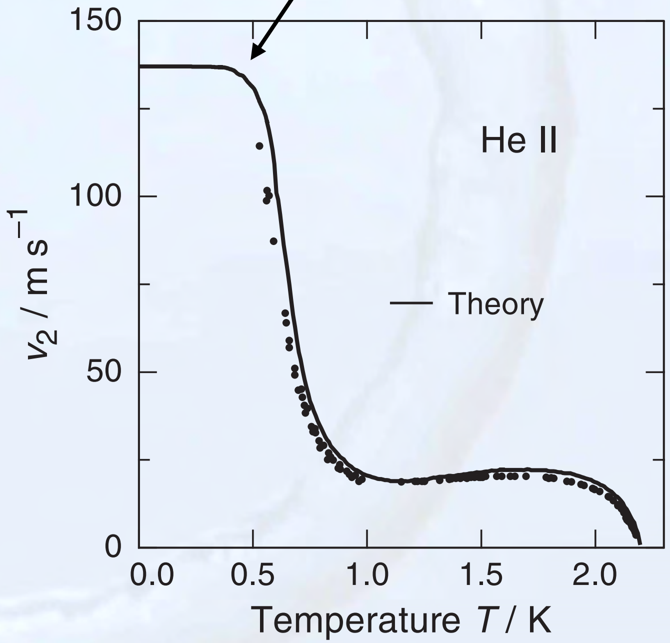
in addition

$$\varrho_\mathrm{s} \approx \varrho$$
$$\varrho_\mathrm{n} = A\varrho T^4/v_1^2$$

for $T \to 0$:

$$v_2 \to v_1/\sqrt{3} \approx 137\,\mathrm{m\,s^{-1}}$$

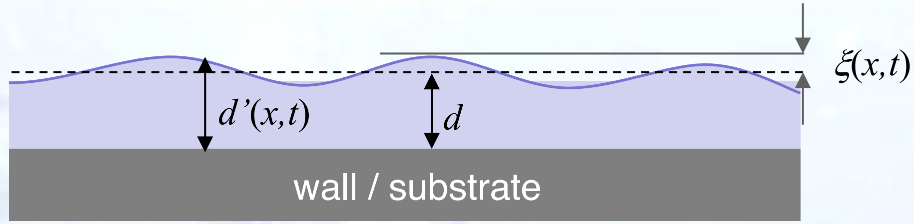
for $T \to 0$ second sound difficult determine since $\varrho_\mathrm{n} \to 0$



2.3 Properties of He-II described using the two-fluid model

(iii) Third sound

sound propagating in thin films



$d'(x,t) = d + \xi(x,t)$

mean film thickness

assumptions: thin films $v_n = 0$

$\lambda \gg d \longrightarrow$ motion parallel to substrate in x-direction $(v_y = v_z = 0)$

$\mathrm{grad}\, T \approx 0$ (questionable ?)

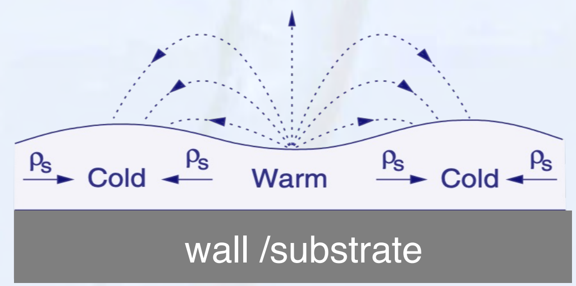
$\Rightarrow \quad \dfrac{\partial^2 \xi}{\partial t^2} = f d \dfrac{\varrho_s}{\varrho} \dfrac{\partial^2 \xi}{\partial x^2} \quad \Rightarrow \quad \boxed{v_3^2 = \dfrac{\varrho_s}{\varrho} 3 g z}$

v. d. Waals force

height over liquid level

problem: evaporation and condensation

$\Rightarrow$ increases the amplitude and changes velocity



can be taken into account

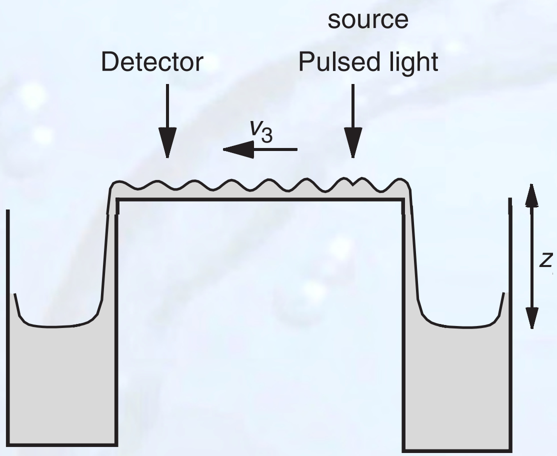
$$v_3^2 \approx \frac{\varrho_s}{\varrho} 3 g z \left(1 + \frac{TS}{L}\right)$$

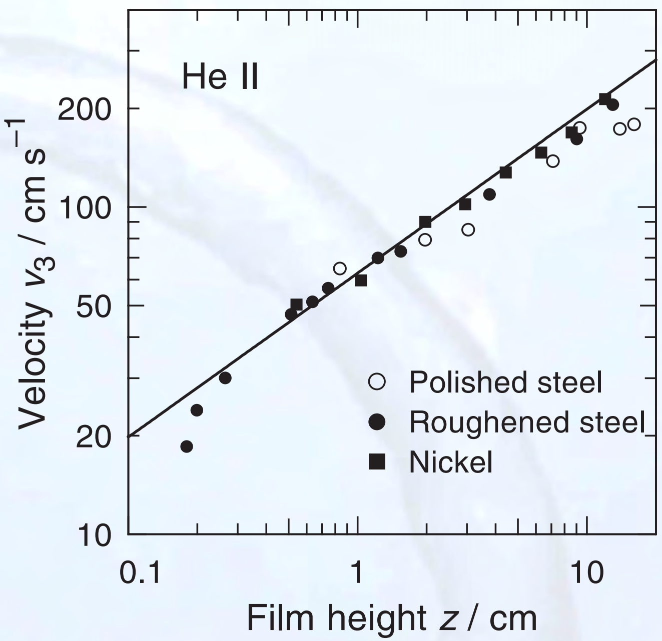
TS/L = 0.01 at 1 K
TS/L = 0.15 at T_λ

not really a problem

2.3 Properties of He-II described using the two-fluid model

3rd sound experiment





Procedure

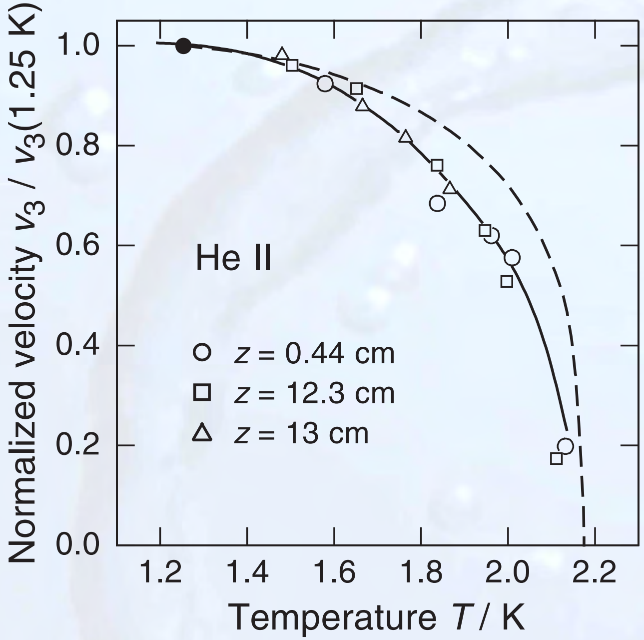
- periodic local heating
- ϱ_s flows to warm location ⟶ thickness changes
- surface wave ≙ 3rd sound
- optical detection of thickness

Measurement and results

- 3rd sound velocity vs. z (log/log plot)
- different surfaces: v_3 almost independent
- line ≙ theory $v_3 \propto \sqrt{z}$
- good agreement except for very thick films

2.3 Properties of He-II described using the two-fluid model

3$^{\mathrm{rd}}$ sound experiment: temperature dependence



Measurement and results

- 3$^{\mathrm{rd}}$ sound velocity vs T
- points at T = 1.25 K normalized to (●)
- v_3 is rising with decreasing T
- $T \rightarrow 0$: v_3 = 1.5 m/s (very slow)
- dashed line $\triangleq$ theory $v_3 \propto \sqrt{\varrho_s}$
- systematic deviations: origin unknow, but likely due to generation process

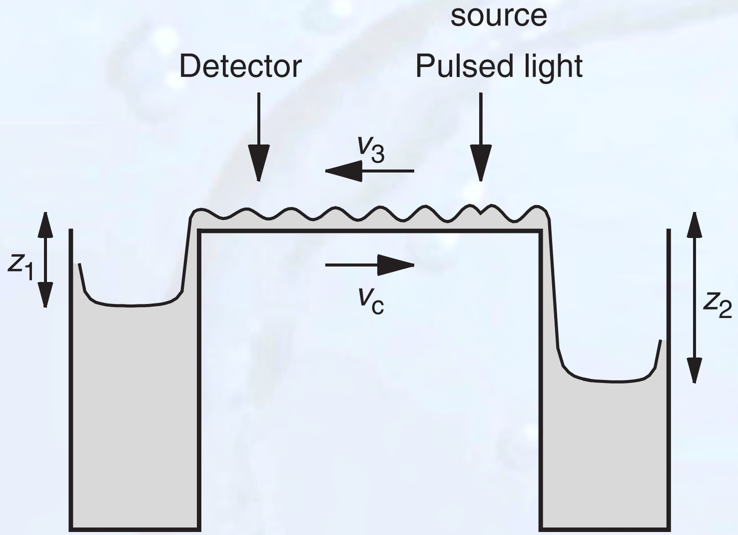
3$^{\mathrm{rd}}$ sound in very thin films:

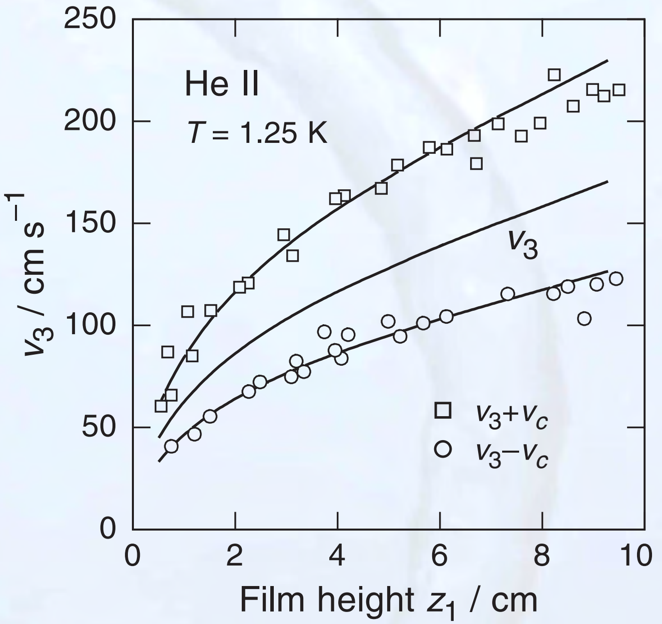
3$^{\mathrm{rd}}$ sound propagation can be observed down to 2.1 monolayers

➡ onset of superfluidity

2.3 Properties of He-II described using the two-fluid model

3rd sound in moving films:





3rd sound propagation in moving films → Doppler effect $v_3 \pm v_{\mathrm{f}}$

v_c critical velocity

2.3 Properties of He-II described using the two-fluid model

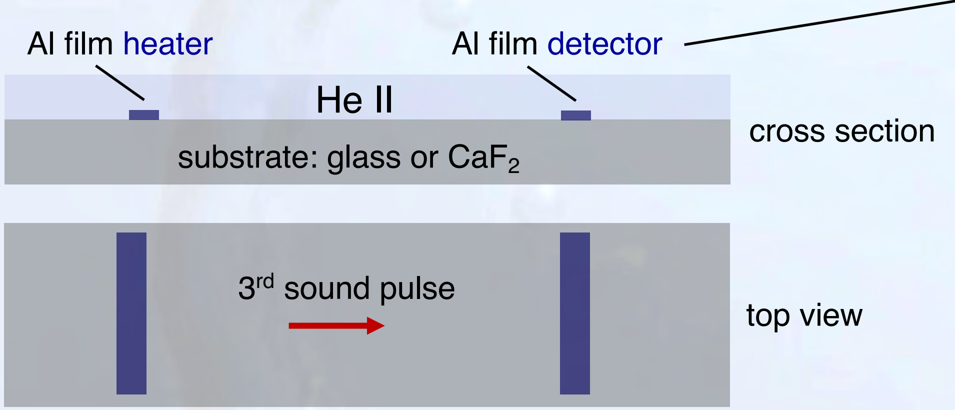
Detection of 3rd sound experiment in ultralow films:

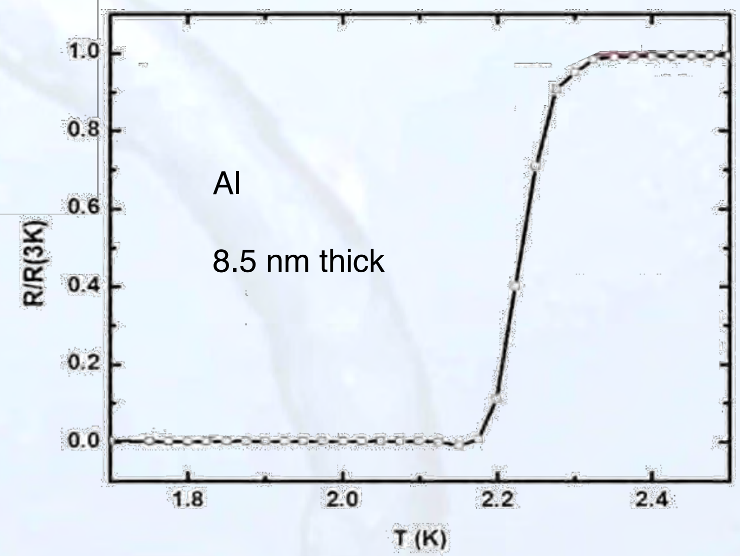
Third Sound and the Healing Length of He II in Films as Thin as 2.1 Atomic Layers*

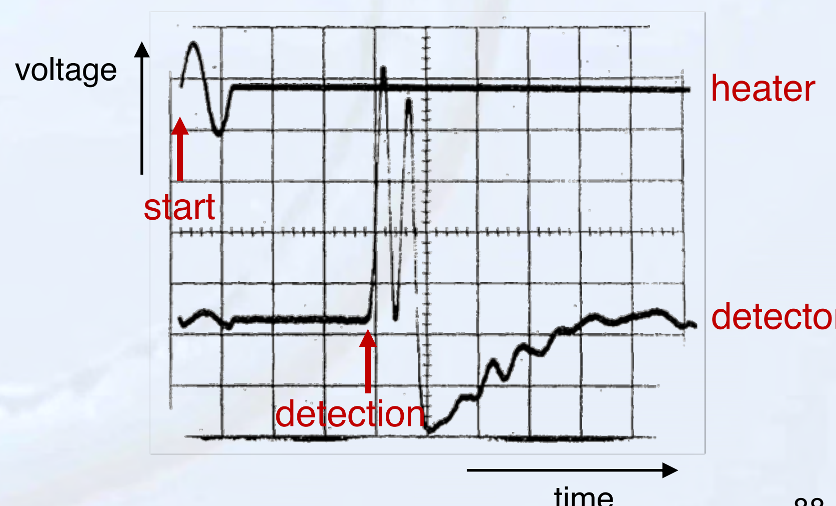
J. H. Scholtz, E. O. McLean,† and I. Rudnick
University of California, Los Angeles, California 90024
(Received 23 August 1973)

Measurements of the velocity of third sound on films as thin as 2.1 atomic layers yield the healing length of superfluid He II down to temperatures of 0.1 K. It is argued that these films are two-dimensional superfluids.

PRL **32** 147 (1974)



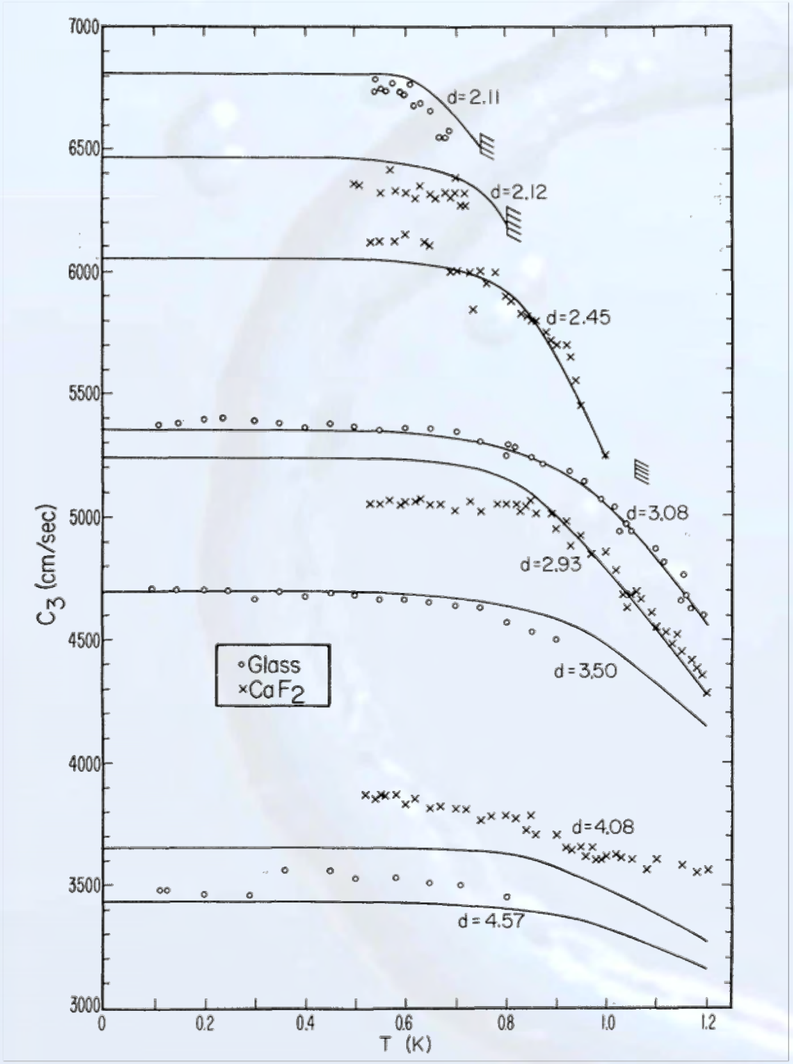




- time of flight detection of 3rd sound pulse
- thin film Al heater and detector
- detector operated at the transition temperature
- detection of heat pulse associated with 3rd sound pulse
- very sensitive because of steep transition curve

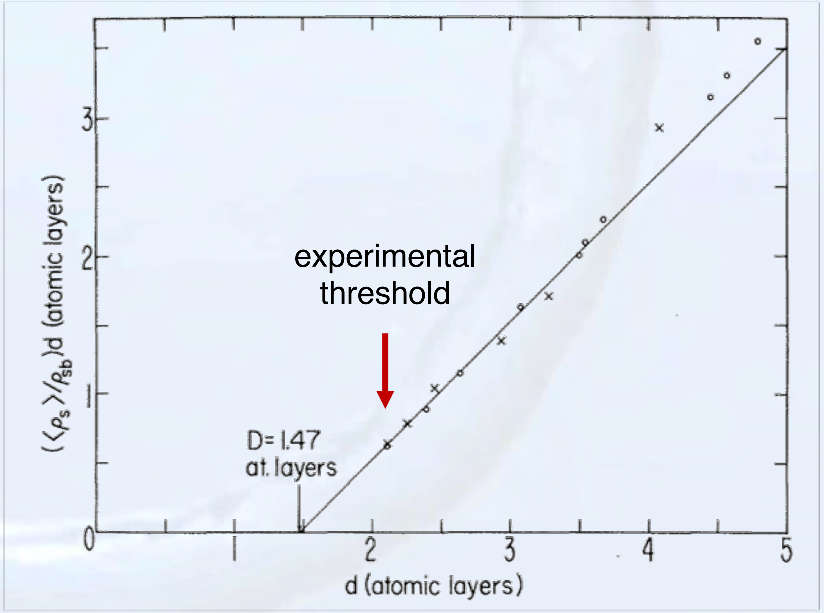
2.3 Properties of He-II described using the two-fluid model

Experimental results:



for ultrathin films: $v_3^2 = \frac{\overline{\rho_s}}{\rho_{s,\mathrm{bulk}}} \frac{3RT}{m} \ln \frac{p_0}{p}$

- experimental threshold of 2.1 monolayers independent of substrate
- film thickness determine by amount of helium and surface area
- extrapolation suggests that 1.47 monolayers might be the onset threshold



2.3 Properties of He-II described using the two-fluid model

(iv) Fourth sound

sound propagation in fine powders / slits $\boldsymbol{v}_\mathrm{n} \approx 0$

➡ oscillations in total density, in ratio of superfluid to normalfluid density, in pressure, in temperature, in entropy

$$v_4^2 = \frac{\varrho_\mathrm{s}}{\varrho} v_1^2 \left[1 + \underbrace{\frac{2ST}{\varrho C_p} \left(\frac{\partial \varrho}{\partial T} \right)_p}_{\ll 1} \right] + \frac{\varrho_\mathrm{n}}{\varrho} v_2^2$$

$$\boxed{v_4 \approx \sqrt{\frac{\varrho_\mathrm{s}}{\varrho} v_1^2 + \frac{\varrho_\mathrm{n}}{\varrho} v_2^2}} \approx \sqrt{\frac{\varrho_\mathrm{s}}{\varrho} v_1^2}$$

5th sound

4th sound generation like for 1st sound, but $\boldsymbol{v}_\mathrm{n} \approx 0$

2.3 Properties of He-II described using the two-fluid model

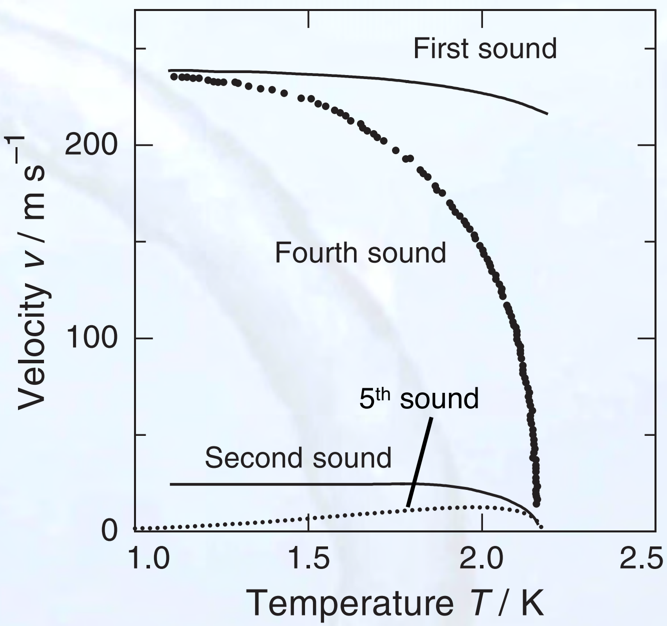
4th sound experiments

4th sound generation like for 1st sound, but $\boldsymbol{v}_\mathrm{n} \approx 0$

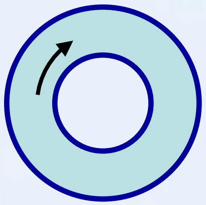
$T \to 0 \quad v_4 = v_1 \approx 238\,\mathrm{m/s}$, since $\varrho_\mathrm{s} = \varrho$

$T = T_\lambda \quad v_4 = 0$

$$v_4 \approx \sqrt{\frac{\varrho_\mathrm{s}}{\varrho}\, v_1^2 + \frac{\varrho_\mathrm{n}}{\varrho}\, v_2^2}$$



Persistent flow and 4th sound



persistent flow velocity

$$v_4 \approx v_{4,0} \pm \frac{\varrho_\mathrm{s}}{\varrho}\, v_\mathrm{D}$$

coupling of a compression wave to second sound