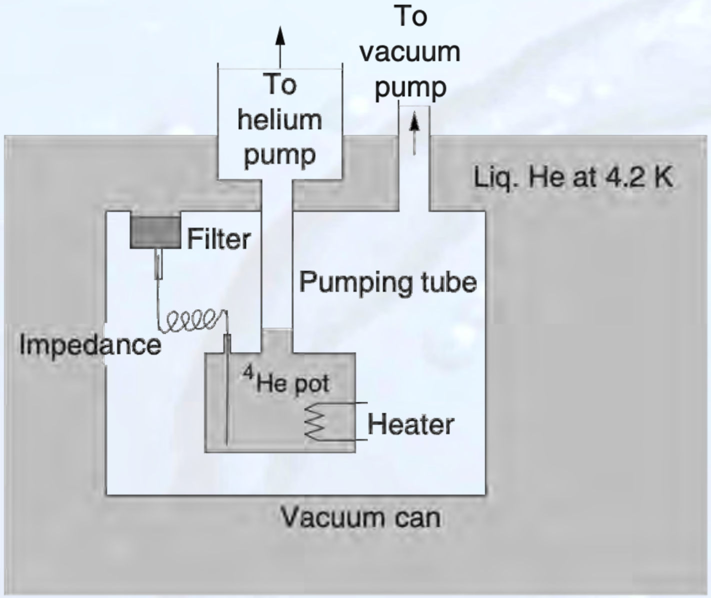
11.1 Bath Cryostats

Cryostats with 1-K-Pot



$$^4\mathrm{He} \quad L = 90\,\mathrm{J\,mol^{-1}}$$

$$^3\mathrm{He} \quad L = 40\,\mathrm{J\,mol^{-1}}$$

Vapor pressure curve of various cryogenic liquids

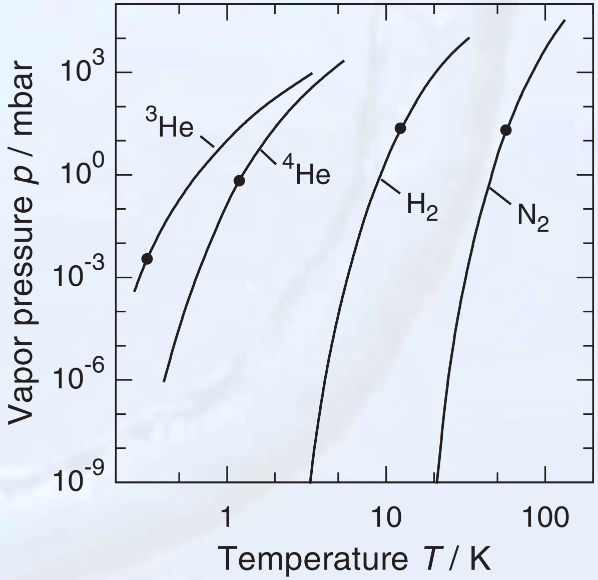
Clausius-Clapeyron equation

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{\Delta V\, T}$$

$$\Delta V = V_\mathrm{g} - V_\ell \approx V_\mathrm{g} \qquad pV_\mathrm{g} = RT$$

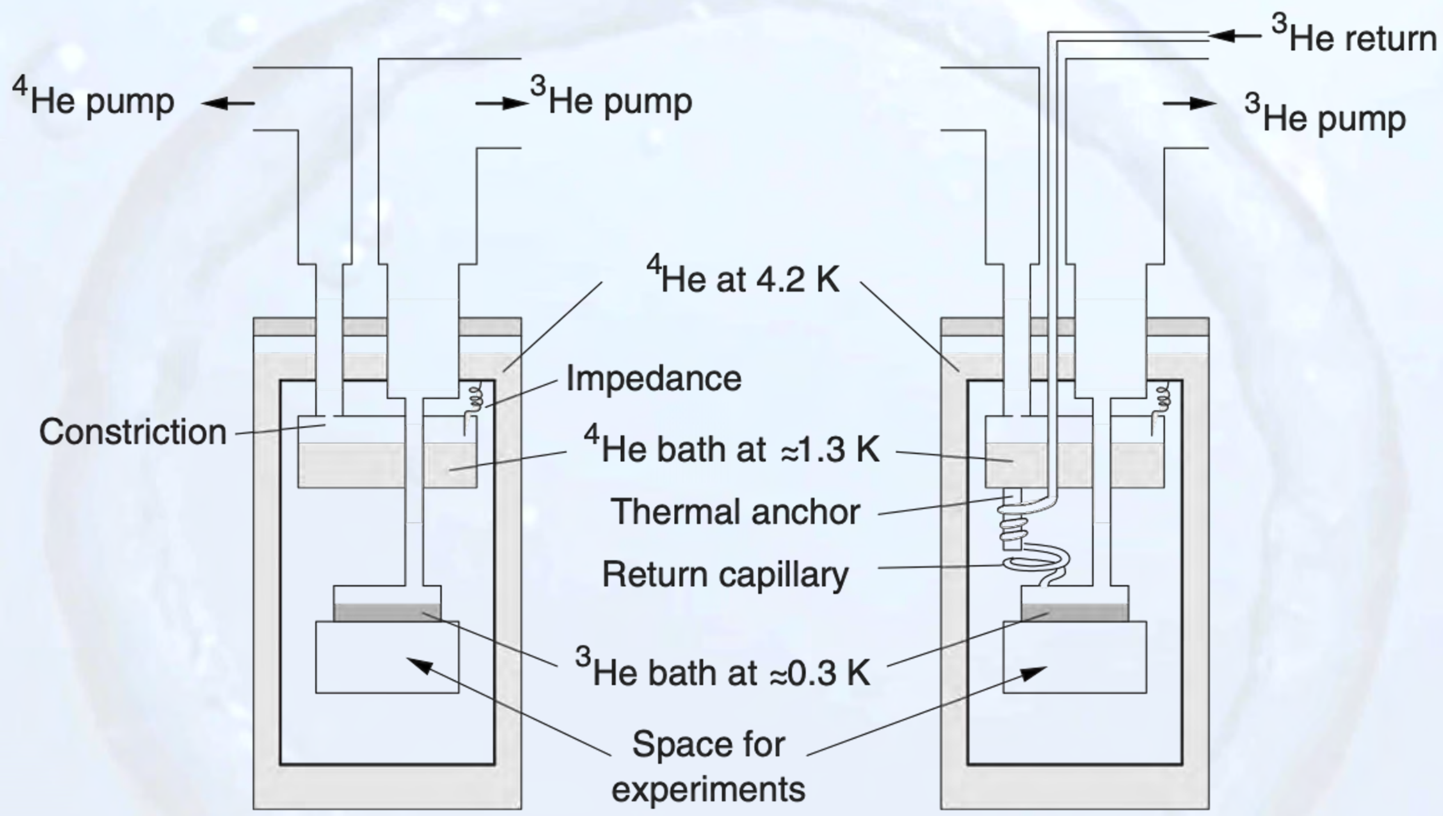
$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{R\,T^2}\,p \quad \Rightarrow \quad \boxed{p(T) = p_0\,\mathrm{e}^{-L/RT}}$$

vapor pressure curve



11.1 Bath Cryostats

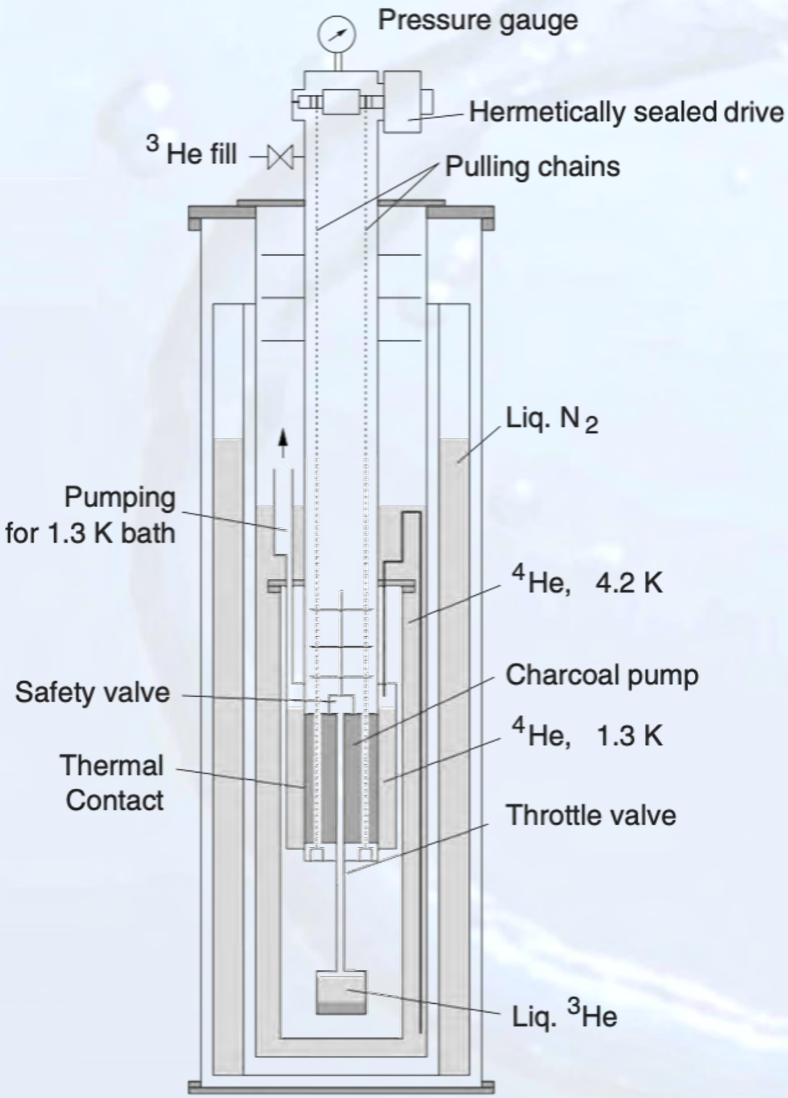
^{3}He cryostats

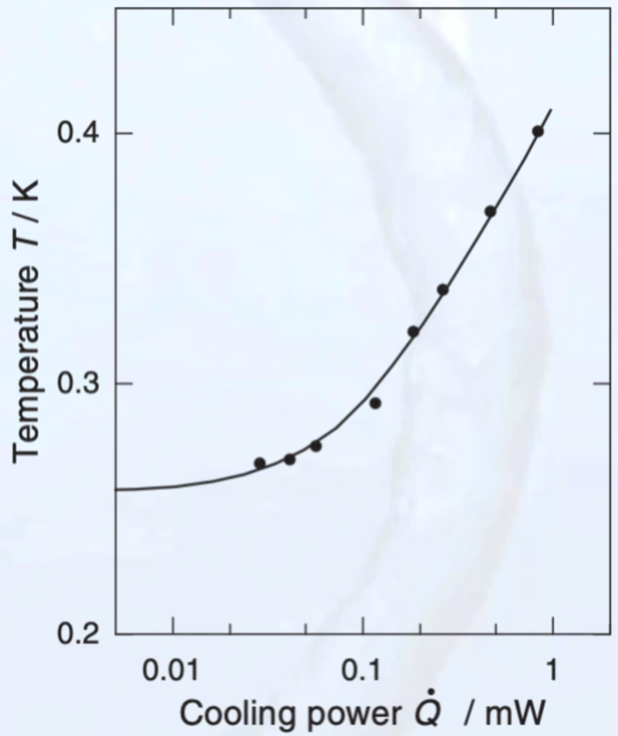


cooling power $\dot{Q} = \dot{n}_{\rm g}\, L \propto p \propto \mathrm{e}^{-L/RT}$

11.1 Bath Cryostats

Cooling power of a ^{3}He cryostat with charcoal absorption pump





11.2 Dilution Refrigerators

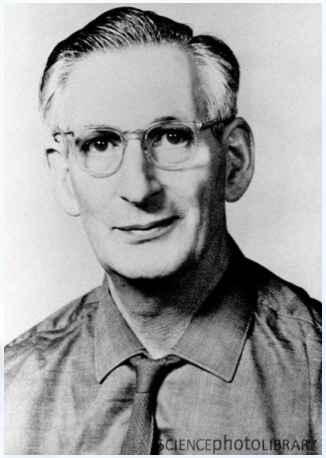
History

1951 basic idea suggested by Heinz London

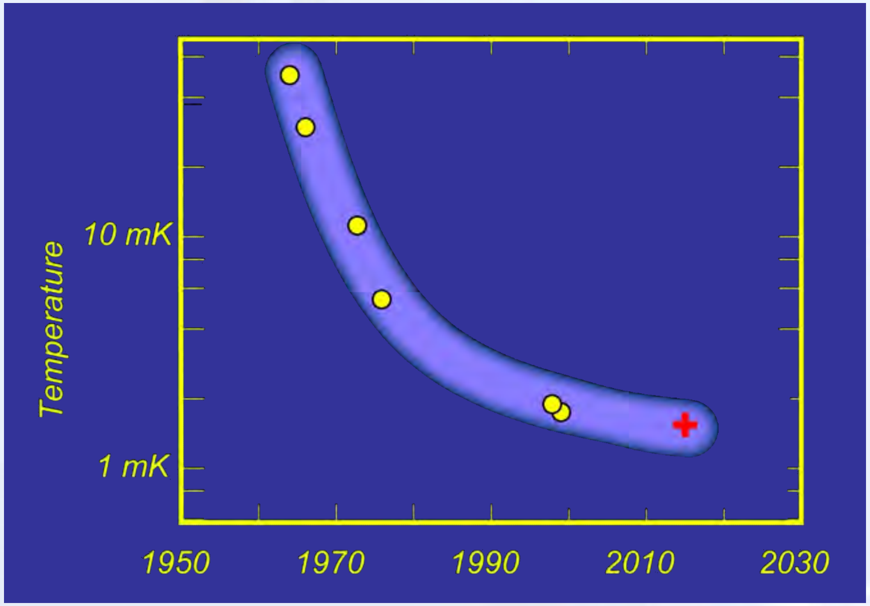
1962 detailed concept worked out by London, Clark, Mendoza

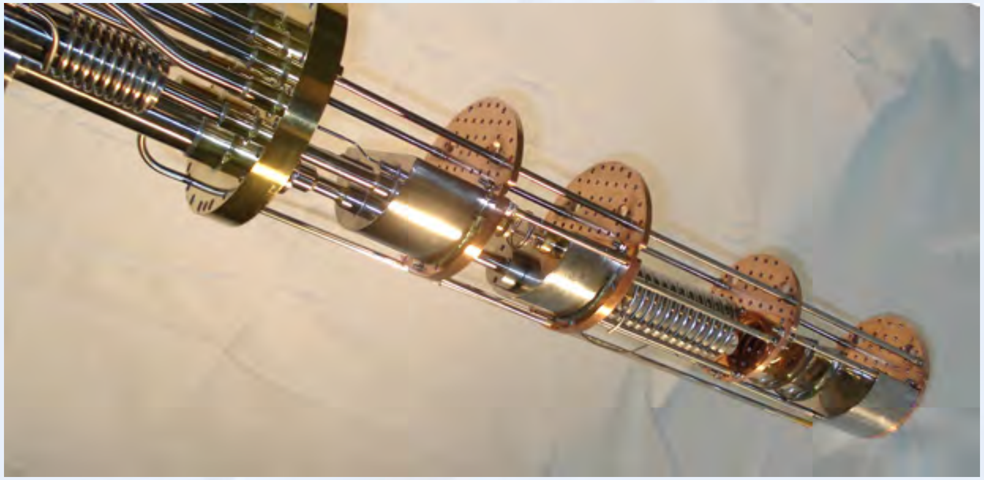
1965 first realization Das, De Bruyn Ouboter, Taconis T_{min} = 220 mK

1999 lowest temperature obtained , J.C. Cousins *et al.* T_{min} = 1.75 mK

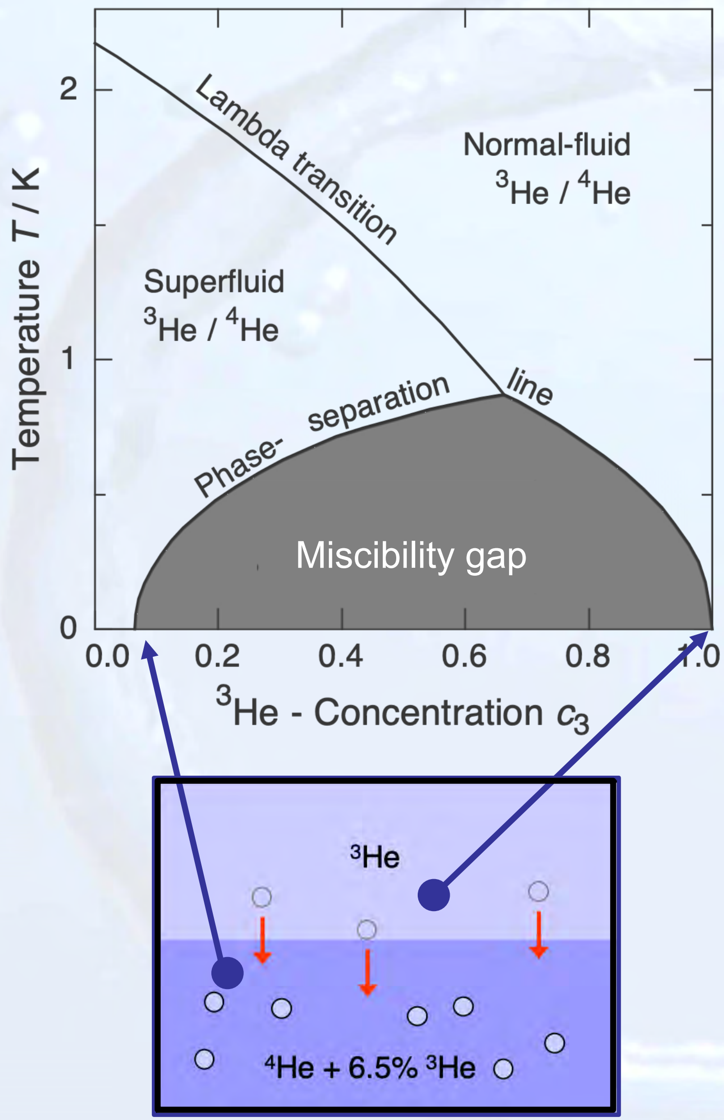


Heinz London





11.2 Dilution Refrigerators



occurrence of miscibility gap

but 6.5 % ^{3}He in ^{4}He at $T = 0$ K

reason:
zero-point motion weakens binding

^{3}He–^{4}He stronger binding

as ^{3}He–^{3}He

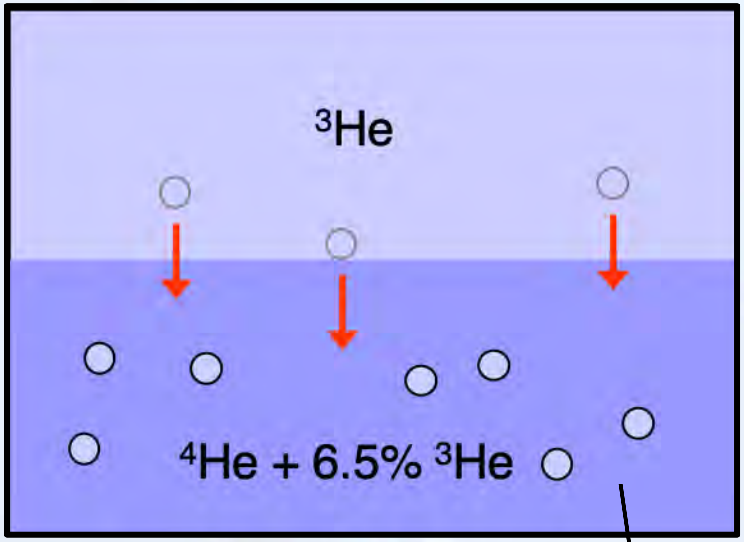
but: Fermi energy

max. 6.5% ^{3}He in ^{4}He at $T = 0$ K

11.2 Dilution Refrigerators

principal of cooling by mixing ^{3}He/^{4}He

- transition of ^{3}He into the ^{4}He rich phase
- cooling by „evaporation" of ^{3}He into ^{4}He quasi vacuum



superfluid ^{4}He

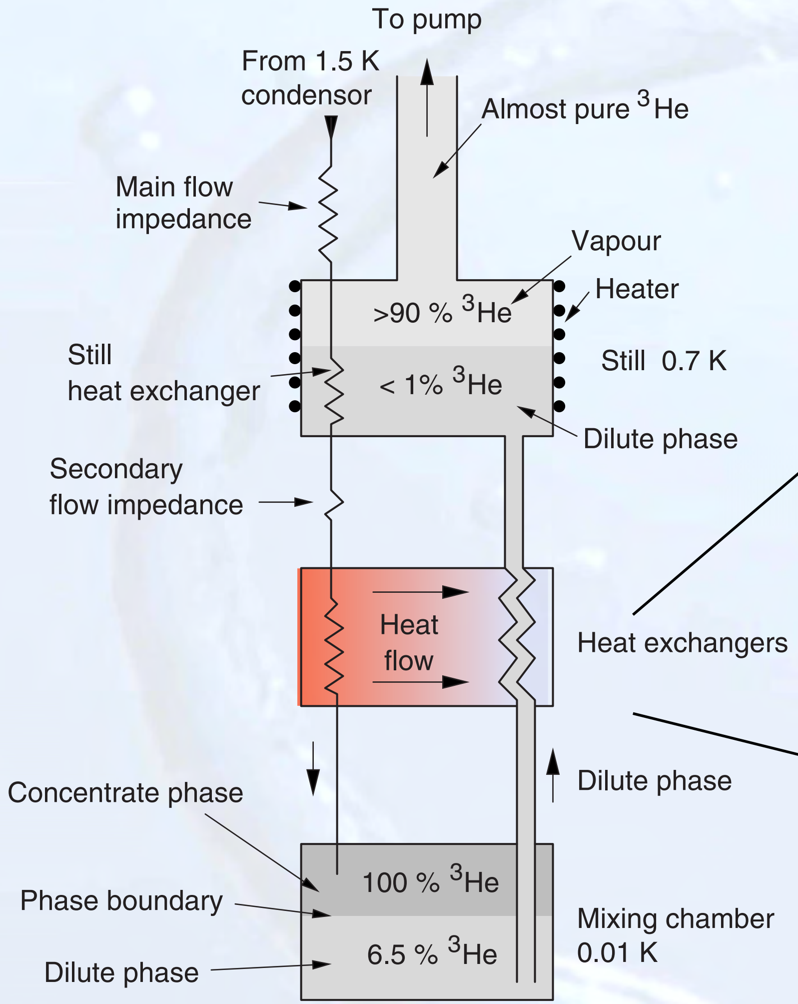
heat of solubility per Mol:

$$\Delta Q = T\Delta S = aT^2$$

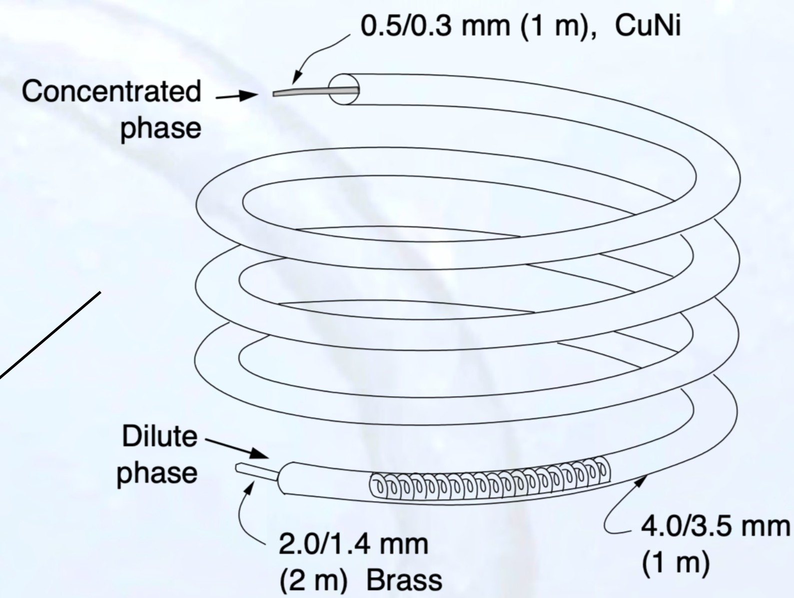
$$a = -84\,\mathrm{J/K^2}$$

11.2 Dilution Refrigerators

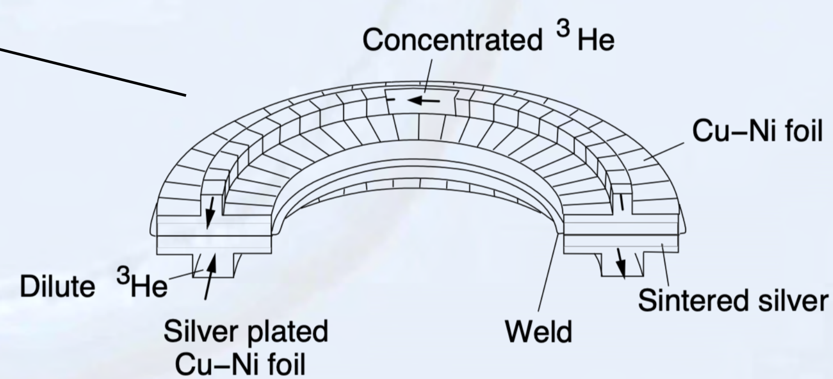
Realisation of $^3He/^4He$ cooling cycle



continuous heat exchanger

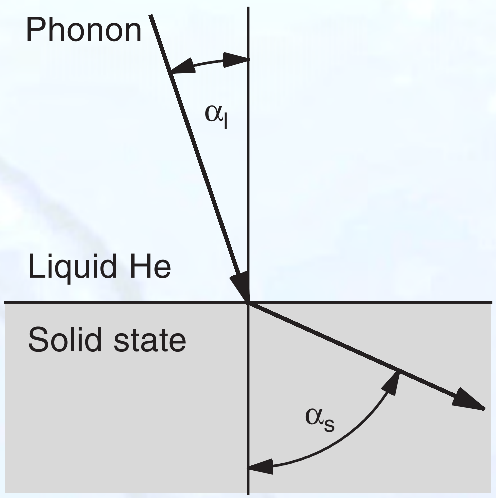


step heat exchanger



11.2 Dilution Refrigerators

Kapitza Resistance — thermal boundary resistance



Snell's law of refraction $$\frac{\sin\alpha_\ell}{\sin\alpha_s} = \frac{v_\ell}{v_s}$$

critical angle of total reflection $$\alpha_\ell^c = \arcsin\left(\frac{v_\ell}{v_s}\right)$$

for liquid helium and copper $$\alpha_\ell^c \approx 4^\circ$$

fraction of phonons incident within critical angle

$$f = \frac{1}{2}\sin^2\alpha_\ell^c = \frac{1}{2}\left(\frac{v_\ell}{v_s}\right)^2 < 10^{-2}$$

transmission coefficient

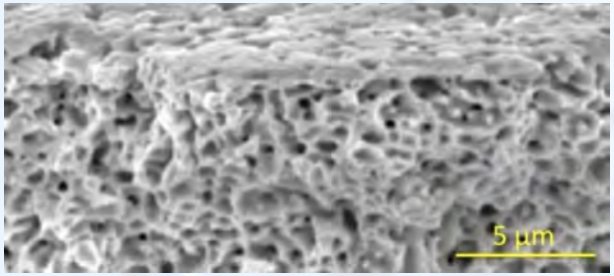
$$t = \frac{4Z_\ell Z_s}{(Z_\ell + Z_s)^2} \approx \frac{4Z_\ell}{Z_s} = \frac{4\varrho_\ell v_\ell}{\varrho_s v_s}$$

$Z_\ell = \varrho_\ell v_\ell$ $\quad$ $Z_s = \varrho_s v_s$ acoustic impedances

fraction of phonons crossing the interface $$\boxed{ft = \frac{2\varrho_\ell v_\ell^3}{\varrho_s v_s^3}}$$

- Kaptiza resistance occurs at any solid-solid, liquid-solid interface
- particular problematic for liquid helium because of the low sound velocity
- helium-copper $ft < 10^{-5}$

11.2 Dilution Refrigerators



silver sinter
SEM image

heat flow from liquid to solid (using Debye model)

$$\dot{Q} = \frac{1}{2} f t u v_\ell A = \frac{\pi^2 k_B^4 \varrho_\ell v_\ell}{30\hbar^3 \varrho_s v_s^3} A T^4$$

$$u = U/V = \pi^2 k_B^4 T^4 / (30\hbar^3 v_\ell^3)$$

in equilibrium identical heat flow from solid to liquid

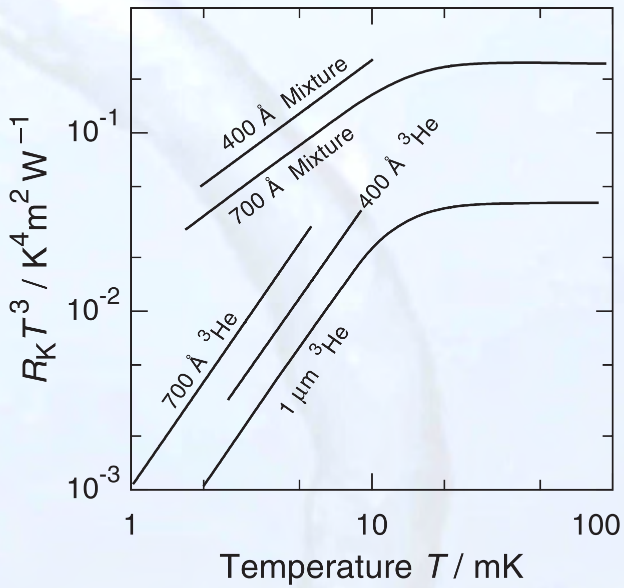
net flow in non-equilibrium (ΔT)

$$\dot{Q} = \frac{\mathrm{d}\dot{Q}}{\mathrm{d}T} \Delta T = \frac{2\pi^2 k_B^4 \varrho_\ell v_\ell}{15\hbar^3 \varrho_s v_s^3} A T^3 \Delta T$$

Kapitza resistance

$$\boxed{R_K = \frac{A \Delta T}{\dot{Q}} = \frac{15\hbar^3 \varrho_s v_s^3}{2\pi^2 {k_B}^4 \varrho_\ell v_\ell} \frac{1}{T^3}}$$

Kapitza resistance between pure ^{3}He and ^{3}He/^{4}He mixtures and silver sinters of different grain sizes



- $20\,\mathrm{mK} < T < 100\,\mathrm{mK}$ good agreement with Debye model $R_K \propto T^{-3}$
- below 20 mK $R_K \propto T^{-2}$ or $R_K \propto T^{-1}$

⟶ anomalous Kapitza resistance

origin: TLS, coupling to zero and second sound modes, phonon wavelength larger than sinter grains

11.2 Dilution Refrigerators

Cooling power

assuming 100% ^{3}He circulation one finds in equilibrium:

$$\dot{Q}_{\mathrm{mc}} + \dot{N}_3\,[H_3(T_{\mathrm{ex}}) - H_3(T_{\mathrm{mc}})] = \dot{N}_3\,[H_{3,\mathrm{d}}(T_{\mathrm{mc}}) - H_3(T_{\mathrm{mc}})]$$

circulation rate

heat leak and/or
available cooling power

enthalpy of ^{3}He-dilute phase

enthalpy of ^{3}He-rich phase

mixing chamber temperature

temperature after last heat exchanger

enthalpy

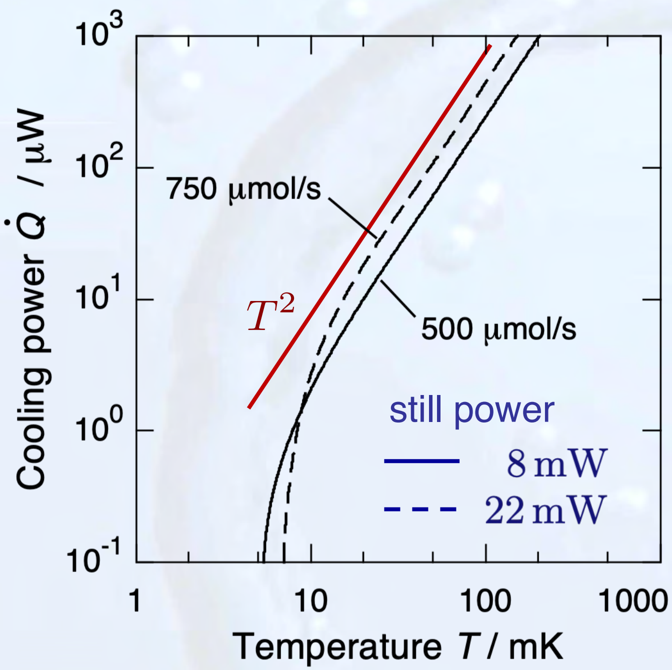
$H = U + pV$

inserting the enthalpies

$$\boxed{\begin{aligned}\dot{Q}_{\mathrm{mc}} &= \dot{N}_3\,[H_{3,\mathrm{d}}(T_{\mathrm{mc}}) - H_3(T_{\mathrm{ex}})] \\ &= \dot{N}_3\,(95\,T_{\mathrm{mc}}^2 - 11\,T_{\mathrm{ex}}^2)\,\left(\frac{\mathrm{J}}{\mathrm{mol\ K^2}}\right)\end{aligned}}$$

11.2 Dilution Refrigerators

Temperature and circulation rate dependence of the cooling power



limiting case of vanishing cooling power: $\dot{Q}_{\mathrm{mc}} = 0$

$$95\, T_{\mathrm{mc}}^2 - 11\, T_{\mathrm{ex}}^2 = 0$$

$$\boxed{\frac{T_{\mathrm{ex}}}{T_{\mathrm{mc}}} = 2.8}$$

→ this underlines the importance of the heat exchanger quality

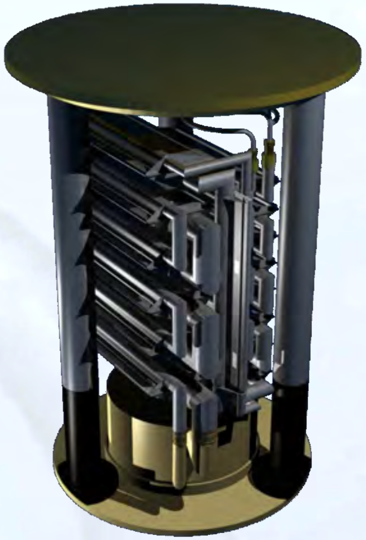
- for $\dot{Q} \gg \dot{Q}_{\text{heat leak}} \longrightarrow \dot{Q} \propto T^2 \;, \quad \dot{Q} \propto \dot{N}_3$

  circulation rate
- heat leak determines lowest temperature

11.2 Dilution Refrigerators

Minimum temperature

- there is no principle limit … it is determined by the heat leak!
- unavoidable heat leak: viscous friction of ^{3}He



pressure difference along the heat exchanger:

$$\Delta p = G\eta\dot{V}$$ Hagen-Poiseuille law

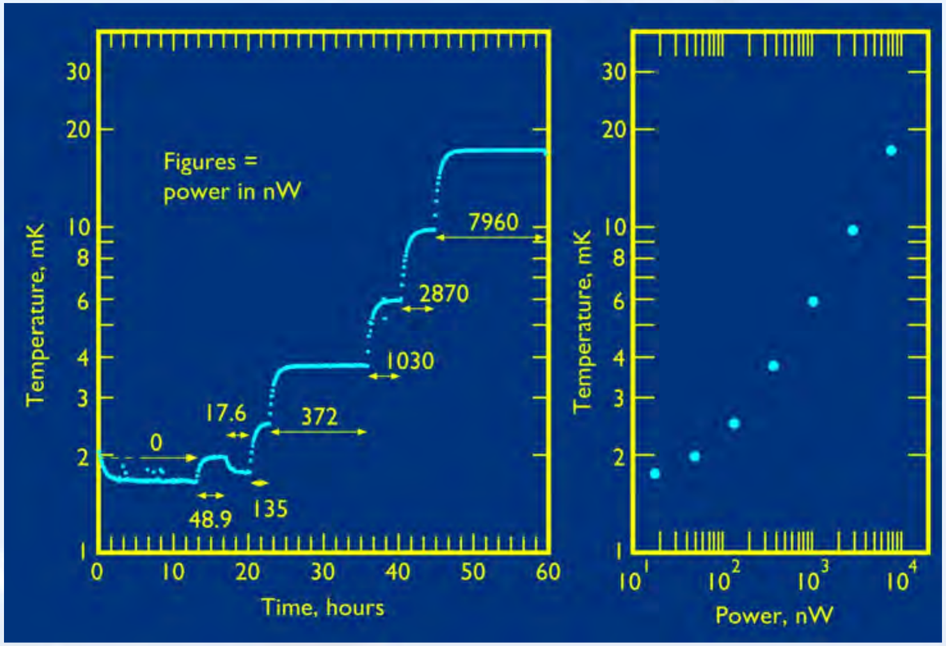
$$G = 8L/(\pi r^4)$$

heat leak due to viscous friction

$$\dot{Q}_{\mathrm{visc}} = \dot{V}\Delta p = G\eta\dot{V}^2$$

single shot minimum temperature

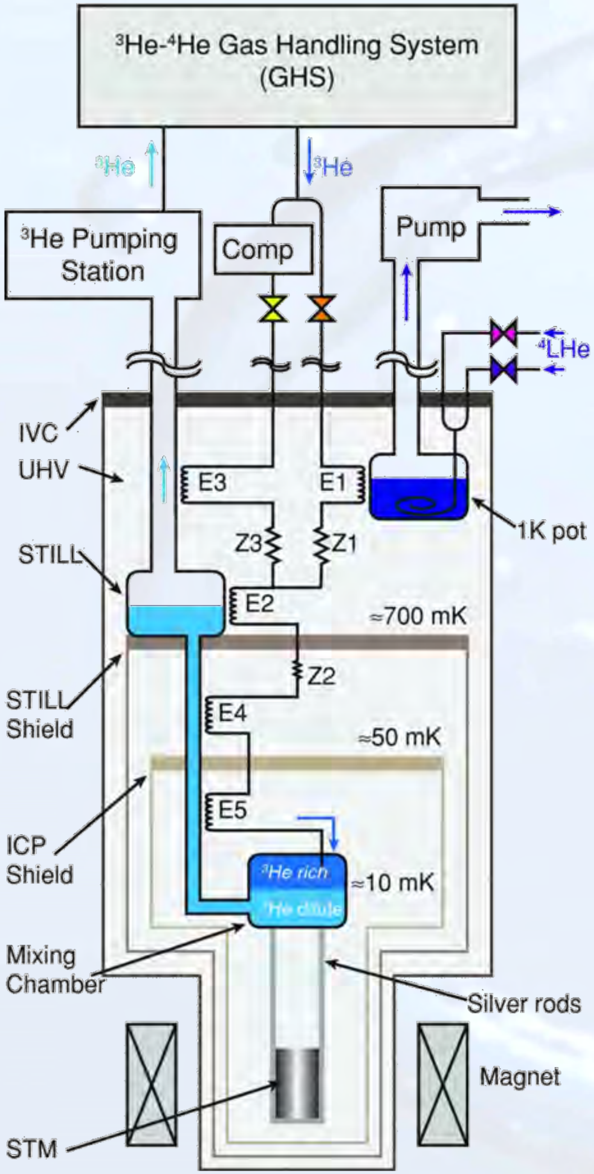
$$T_{\mathrm{min.}} = \frac{4}{\sqrt[3]{d}}\,\mathrm{mK(mm)}^{1/3}$$



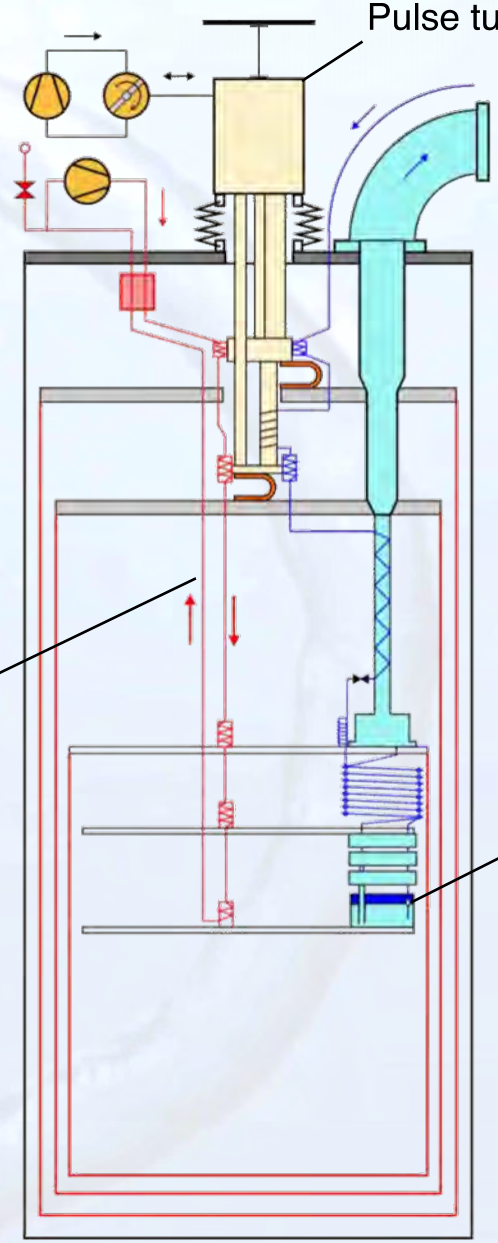
11.2 Dilution Refrigerators

Realizations

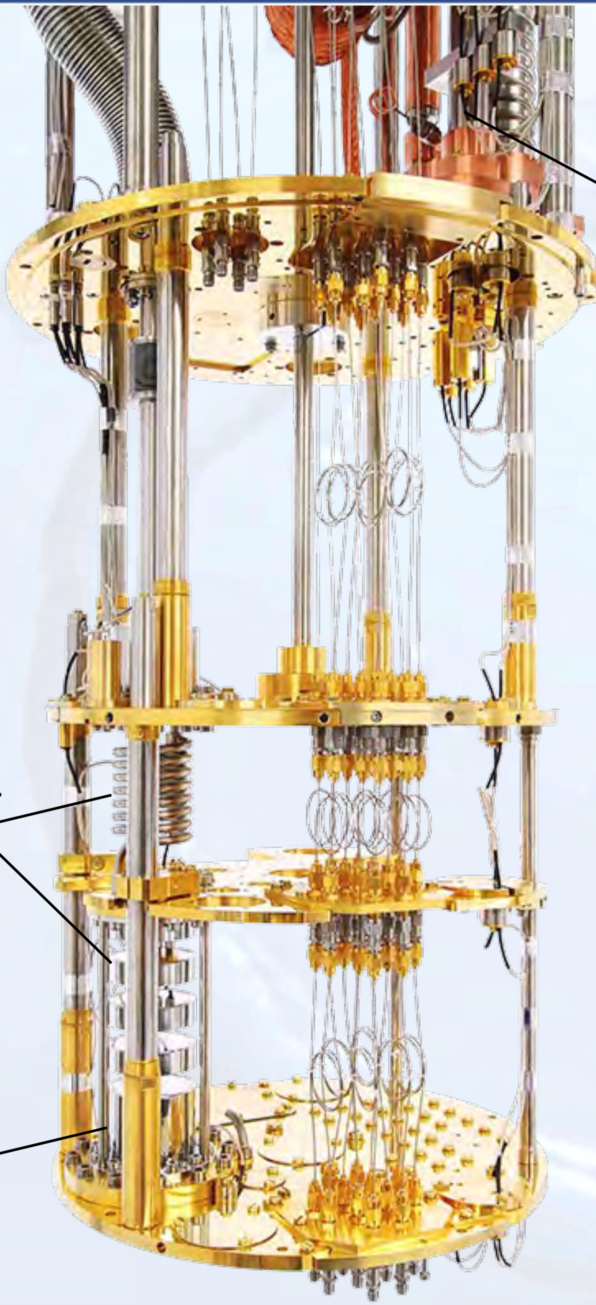
Wet



Dry



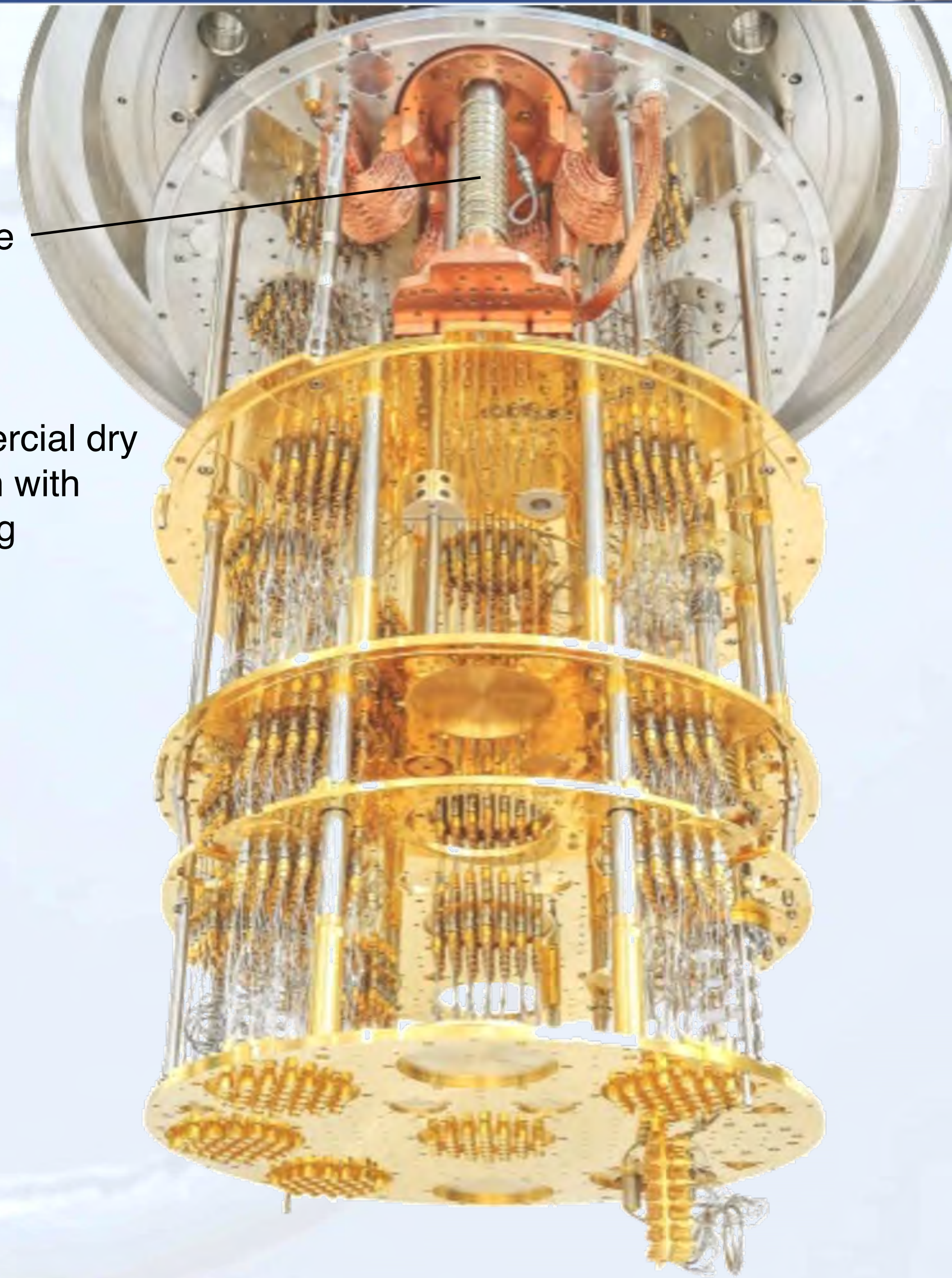
11.2 Dilution Refrigerators



pulse tube

heat exchanger

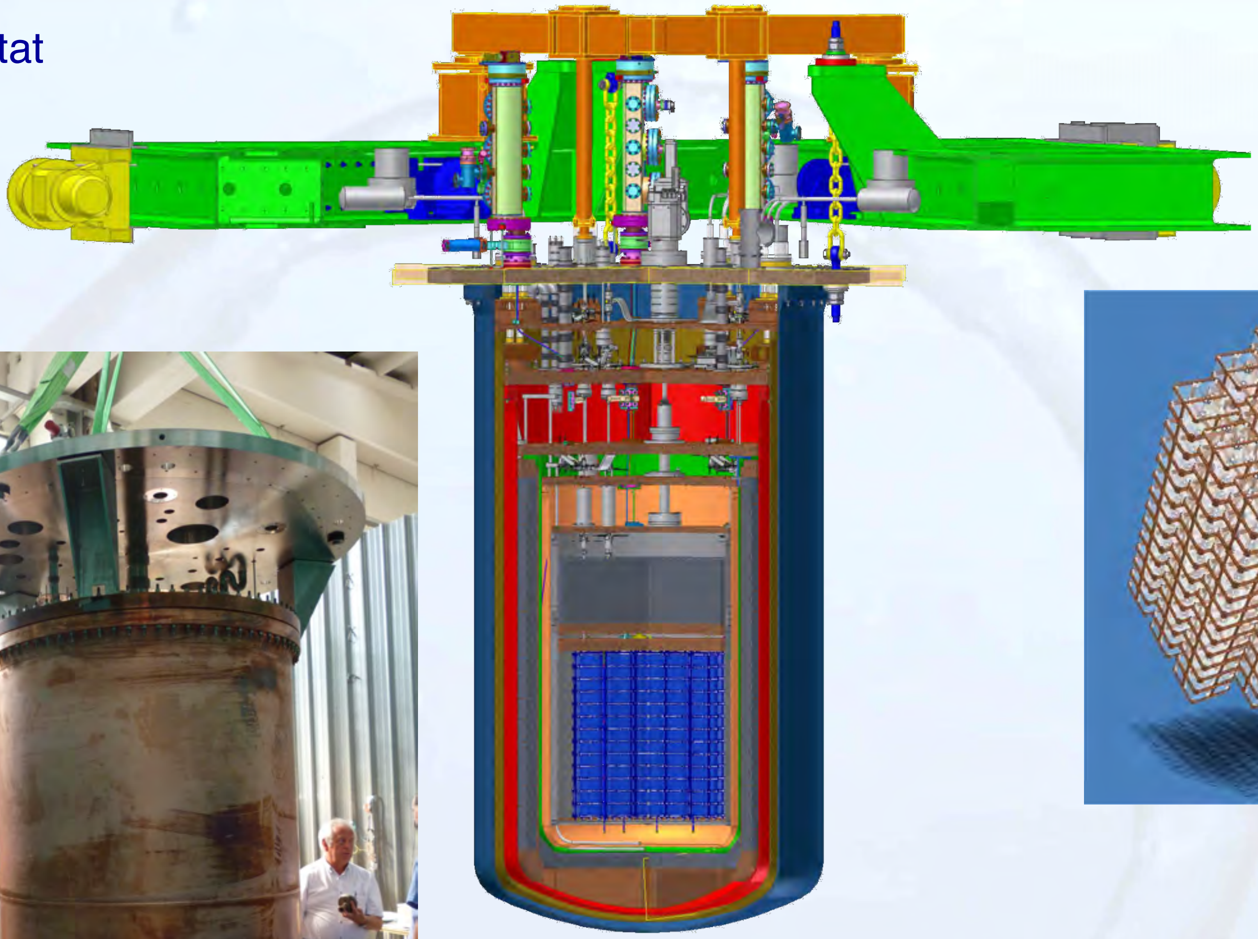
mixing chamber

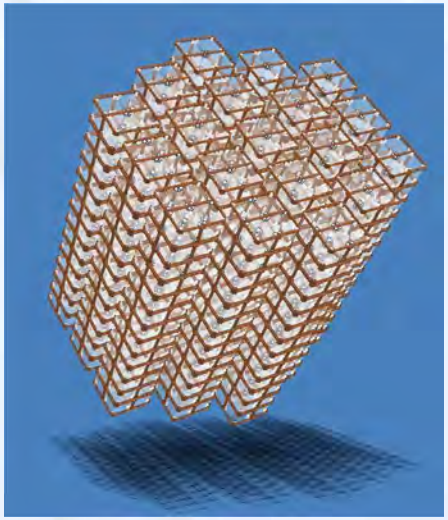


commercial dry system with rf wiring

11.2 Dilution Refrigerators

Cuore Cryostat



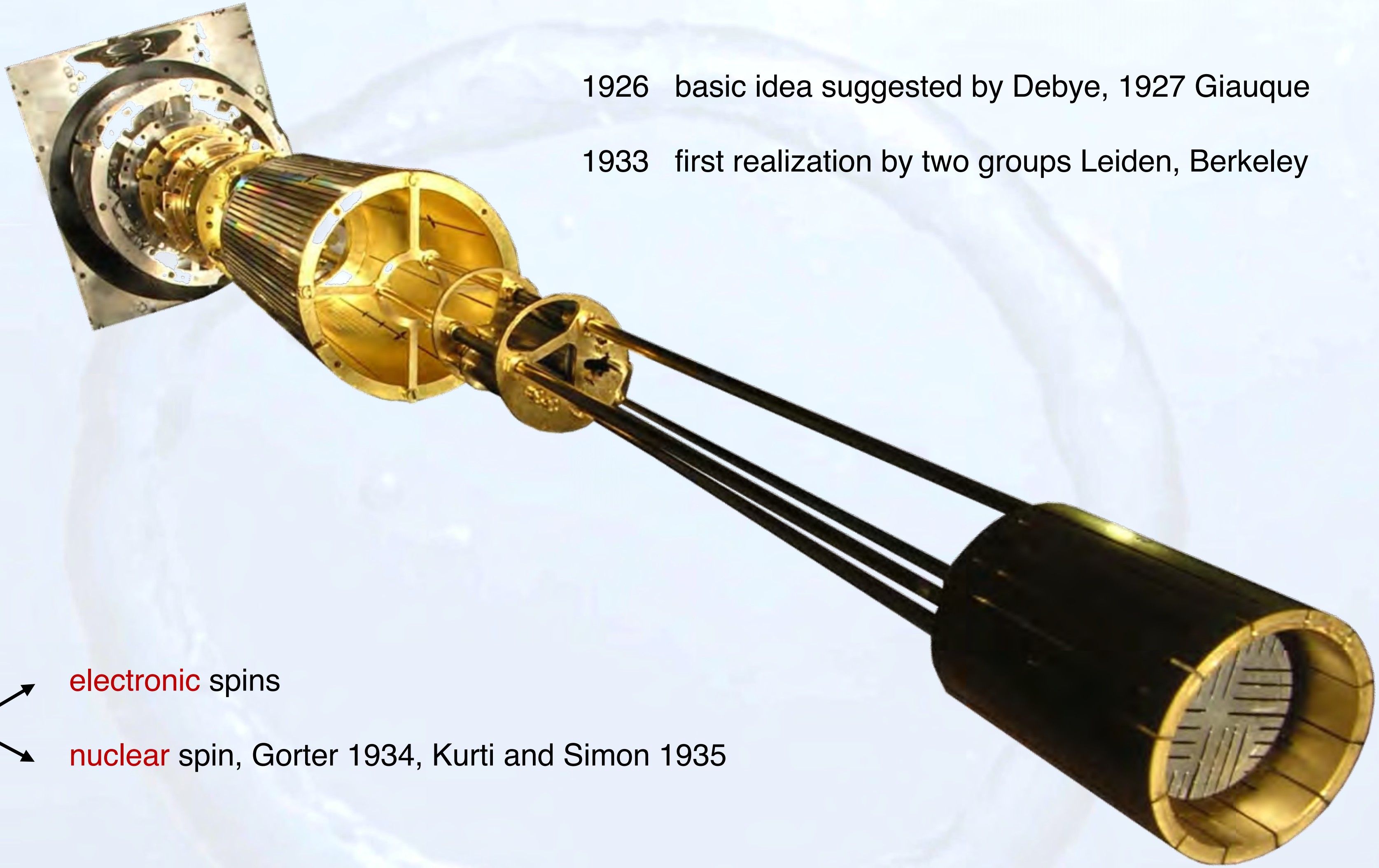


- 750 kg of TeO_2 cooled by the mixing chamber to 10 mK
- made of materials with low level radioactivity materials
- two cold shields made of ancient roman lead 10 cm thick
- 5 Pulse Tube Coolers

11.2 Dilution Refrigerators

Cuore Cryostat

11.3 Adiabatic Demagnetization Refrigerators



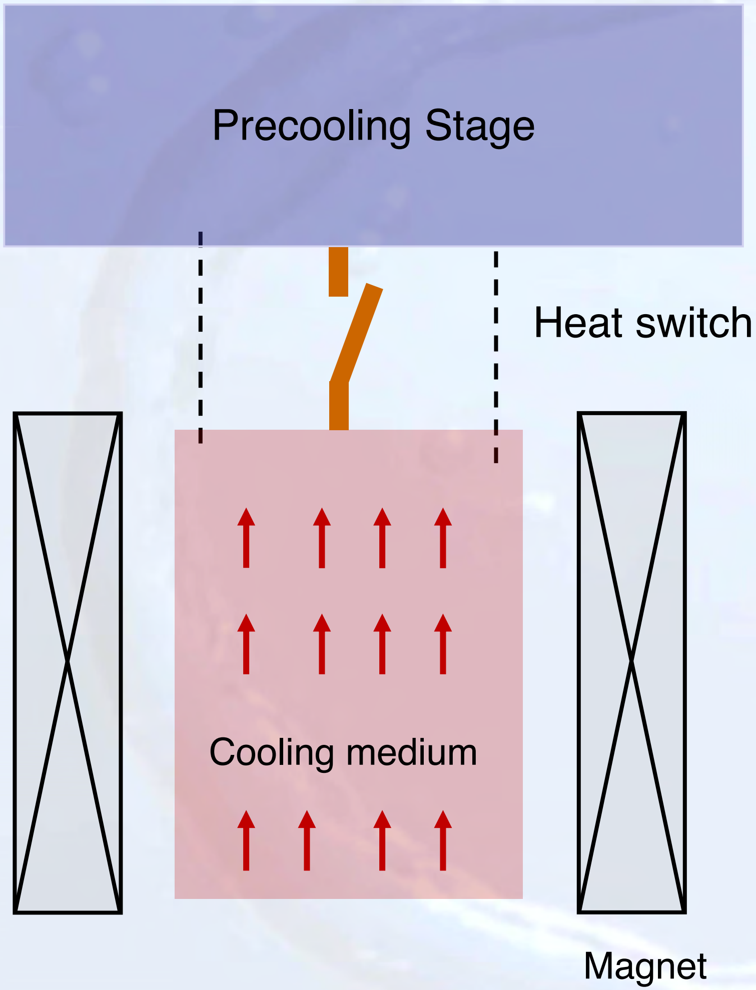
1926 basic idea suggested by Debye, 1927 Giauque

1933 first realization by two groups Leiden, Berkeley

- electronic spins
- nuclear spin, Gorter 1934, Kurti and Simon 1935

11.3 Adiabatic Demagnetization Refrigerators

General cooling principle



- precooling
- isothermal magnetisation

$$\Delta Q_{\mathrm{mag}} = -T_{\mathrm{i}}[\,S(B_{\mathrm{i}}, T_{\mathrm{i}}) - S(0, T_{\mathrm{i}})\,]$$

- thermal isolation
- heat switch opened
- adiabatic demagnetisation

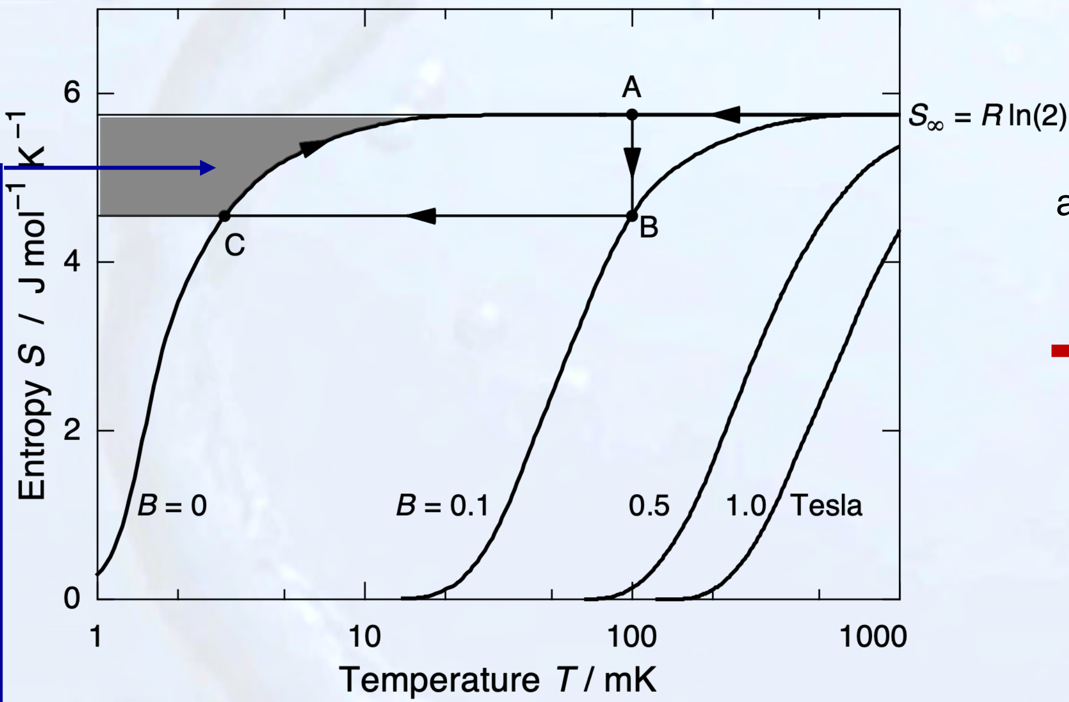
$$S = S\left(\frac{B}{T}\right) = \mathrm{const.}$$

11.3 Adiabatic Demagnetization Refrigerators

Entropy: ideal system, no heat leaks

internal field caused by spin-spin interaction

$$S = N k_\mathrm{B} \left\{ \ln(2J+1) - \frac{g^2 J(J+1)\mu_\mathrm{B}^2}{6\,{k_\mathrm{B}}^2} \frac{B^2 + B_\mathrm{int}^2}{T^2} \right\}$$



adiabatic demagnetization $S_\mathrm{B} = S_\mathrm{c}$

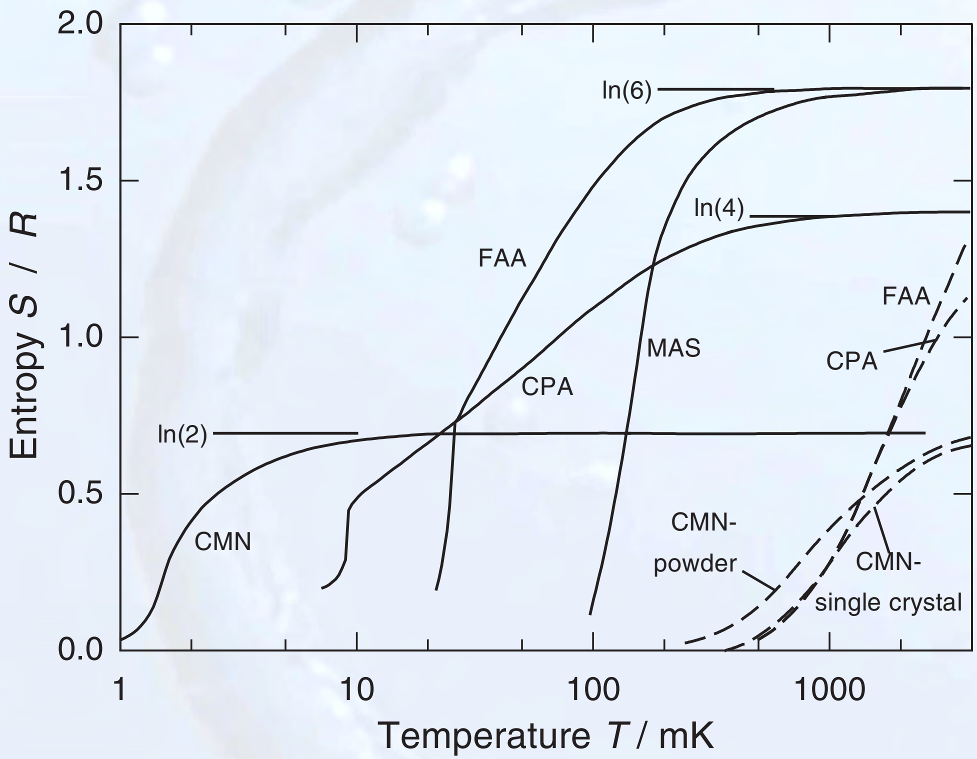
$$T_\mathrm{f} = T_\mathrm{i} \sqrt{\left(\frac{B_\mathrm{f}^2 + B_\mathrm{int}^2}{B_\mathrm{i}^2 + B_\mathrm{int}^2} \right)}$$

$$\Delta Q_\mathrm{spin}(B_\mathrm{C}) = \int_{T_\mathrm{C}}^{T_\mathrm{A}} C_\mathrm{spin}\,\mathrm{d}T = \int_{T_\mathrm{C}}^{T_\mathrm{A}} T \left(\frac{\partial S}{\partial T} \right)_{B_\mathrm{C}} \mathrm{d}T$$

11.3 Adiabatic Demagnetization Refrigerators

a) Electronic spins

entropy of different paramagnetic salts



MAS for $MnSO_4 \cdot (NH_4)_2SO_4 \cdot 6H_2O$

FAA for $Fe_2(SO_4)_3 \cdot (NH_4)_2SO_4 \cdot 24H_2O$

CPA for $Cr_2(SO_4)_3 \cdot K_2SO_4 \cdot 24H_2O$

CMN for $2Ce(NO_3)_3 \cdot 3Mg(NO_3)_2 \cdot 24H_2O$

problems with paramagnetic salts → T_c relatively high

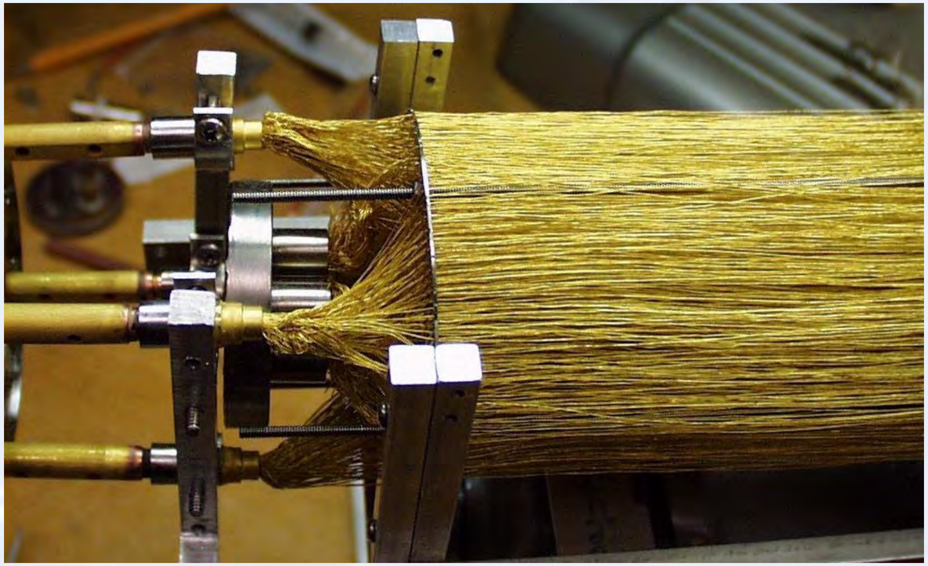
→ low thermal conductivity

11.3 Adiabatic Demagnetization Refrigerators

high conductive wires to improve
low thermal conductivity of salt pills

NASA GSFC

- FAA salt pill for space application
- 15.000 gold wires
- salt pill grown around the wires



11.3 Adiabatic Demagnetization Refrigerators

b) Nuclear spins

- metals with fast relaxation time
- nuclei with large magnetic moment
- isotopes with large natural abundance
- cubic structure to avoid quadropole contributions
- no superconductor
- pure material, easy to machine

$$\frac{\mathrm{d}T_\mathrm{n}^{-1}}{\mathrm{d}t} = -\frac{\left(T_\mathrm{n}^{-1} - T_\mathrm{e}^{-1}\right)}{\tau_1}$$

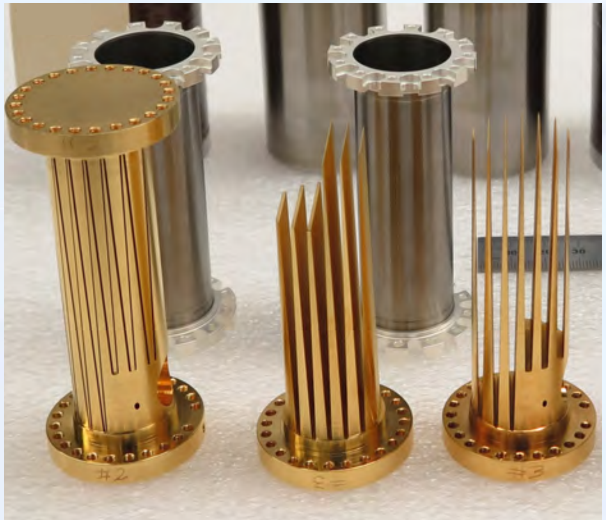
$\tau = \kappa / T_\mathrm{e}$ Korringa relation

	Structure	I	μ/μ_N	κ (K s)	Abundance (%)
^{63}Cu	fcc	3/2	2.22	1.27	69.1
^{65}Cu	fcc	3/2	2.38	1.09	30.9
^{195}Pt	fcc	1/2	0.597	0.03	33.8
^{141}PrNi$_5$	fcc	5/2	4.28	<0.001	100

van Vleck paramagnet

11.3 Adiabatic Demagnetization Refrigerators

Gas gap heat switch



exchange gas
→ pumping to open switch

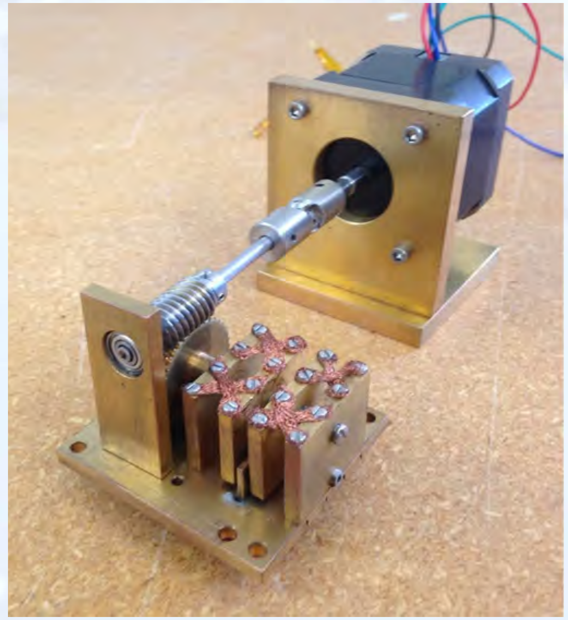
^{4}He: superfluid layer → creep

H_2: ortho-para conversion

^{3}He: no exothermic reaction
no creep
high vapor pressure

→ ideal exchange gas

Mechanical heat switch



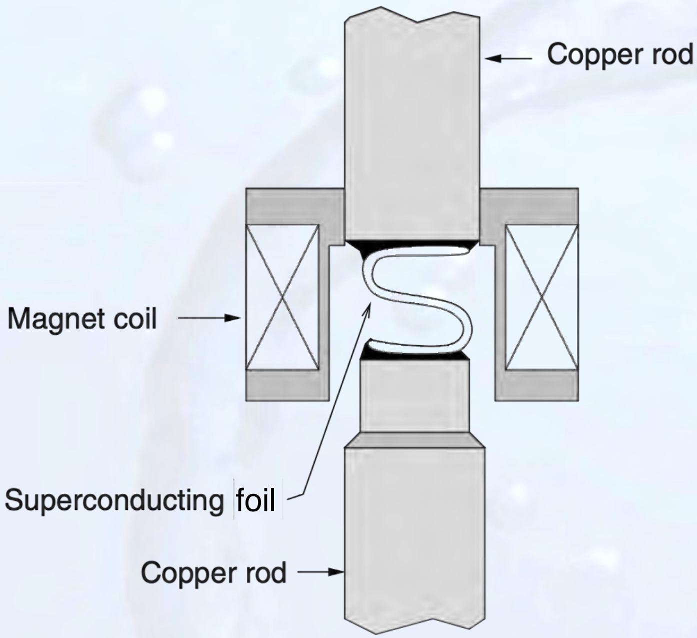
- large force needed ~ 100 N
- closed: mW/K … 1 W/K @ 15K
- problem: heating on opening

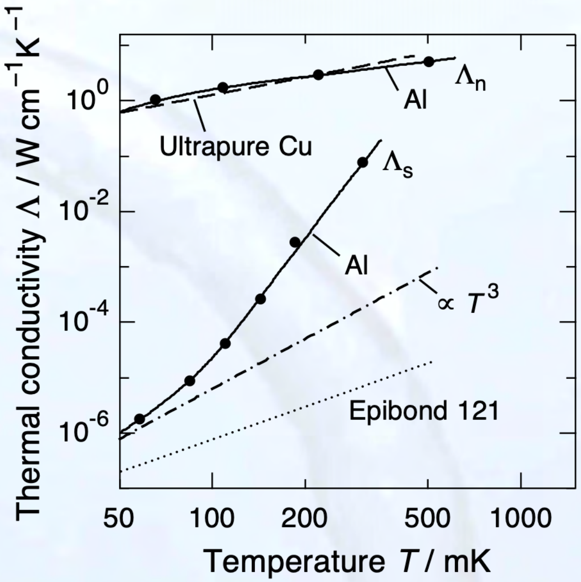
Superconducting heat switch

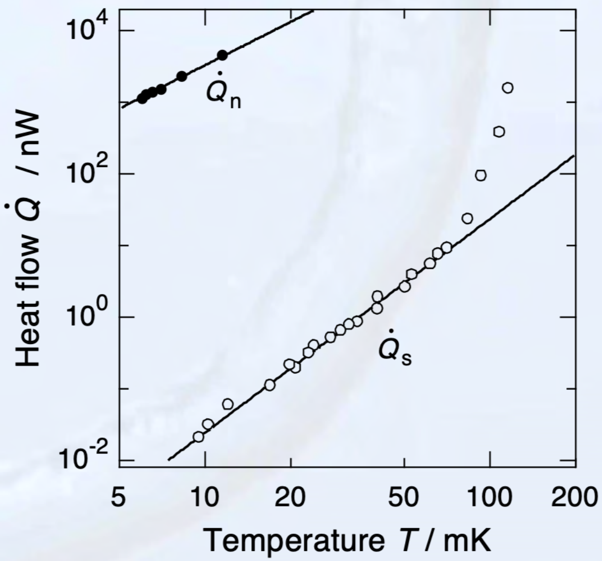
- only good well below T_c
- open means low conductivity
- problems: eddy currents
  flux trapping

11.3 Adiabatic Demagnetization Refrigerators

Performance of superconducting heat switch







- switching ratio 10^6 at 10 mK
- heat leak in open state 10 pW

11.3 Adiabatic Demagnetization Refrigerators

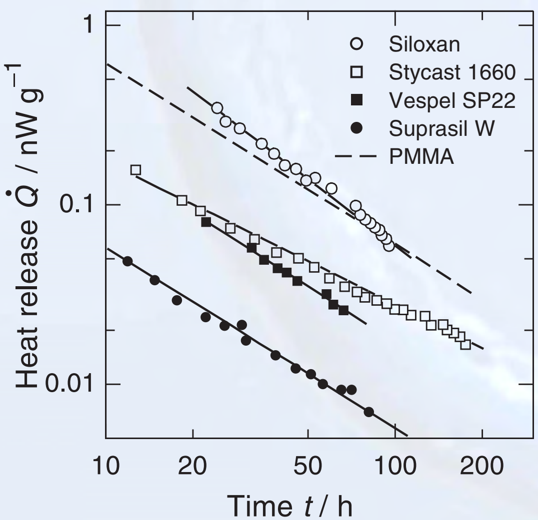
Heat leaks

- eddy current heating $\dot{Q}_{\text{eddy}} = f \frac{V \dot{B}^2}{\varrho}$
- em fields and vibrations
- ortho-para conversion
- radioactive impurities
- tunneling systems

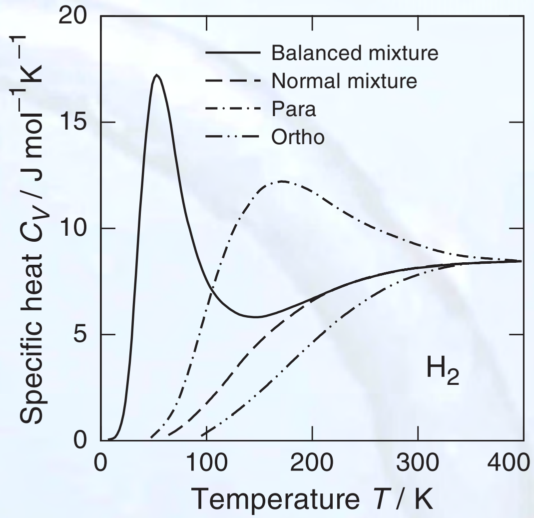
(ortho-para conversion, radioactive impurities, tunneling systems: time dependent heat leaks)

atomic tunneling systems

$$\dot{Q} = \frac{\pi^2 k_B^2}{24} P_0 \left(T_1^2 - T_0^2\right) \frac{1}{t}$$



specific heat of H_2



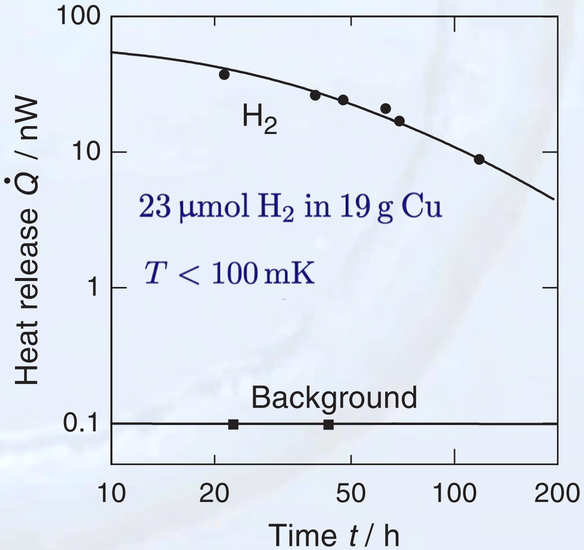
singlet state

para H_2 $j = 0$

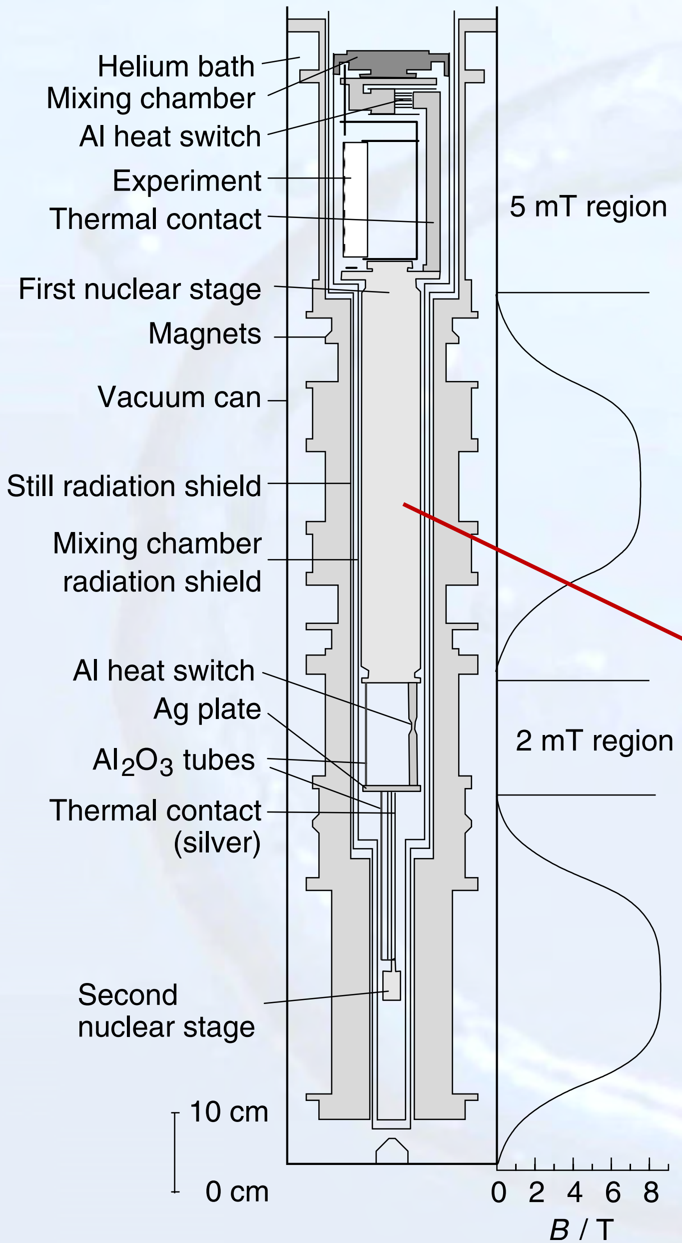
triplet state

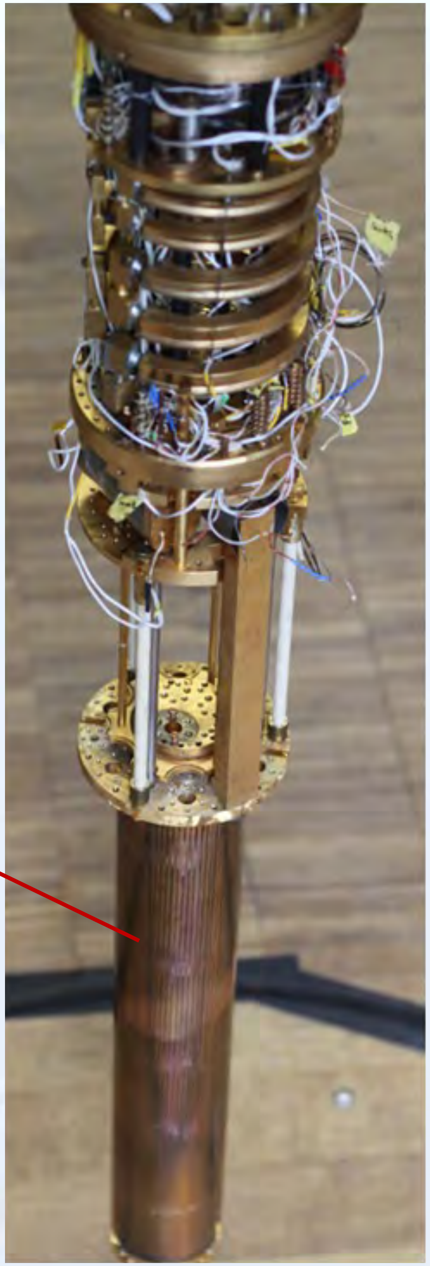
ortho H_2 $j = 1$

H_2 ortho-para conversion



11.3 Adiabatic Demagnetization Refrigerators





11.3 Adiabatic Demagnetization Refrigerators

Cooling process

- precooling to T_A and isothermal magnetization

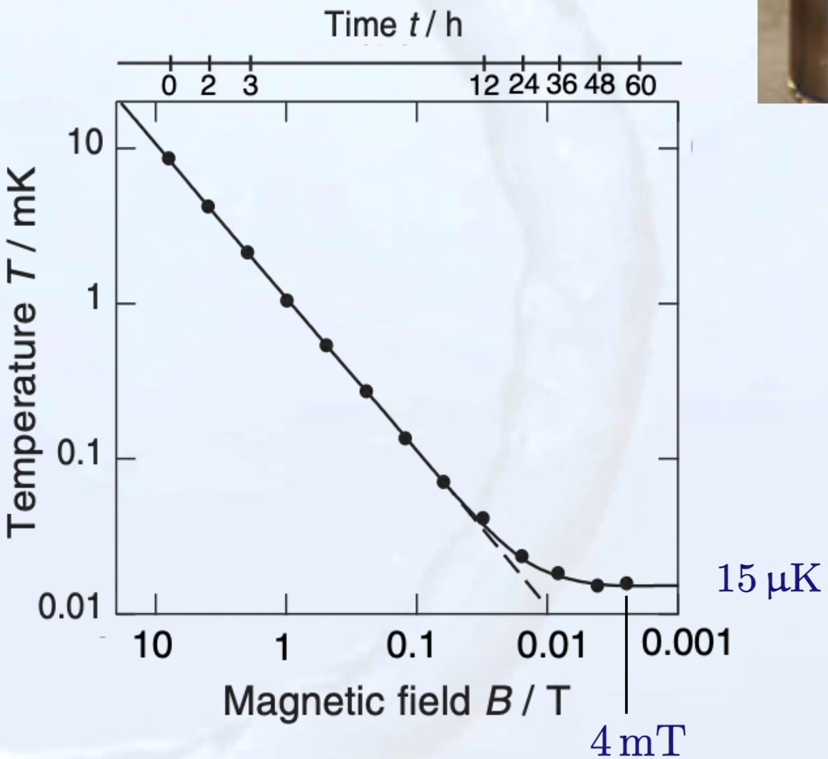
nuclear Curie constant $$\lambda_n = \frac{nI(I+1)\mu_0\mu_n^2 g_n^2}{3k_B}$$

$$\longrightarrow \quad Q = nT_A\Delta S = -\frac{\lambda_n B_i^2}{2\mu_0 T_A}$$

- reducing B in steps to optimal final field

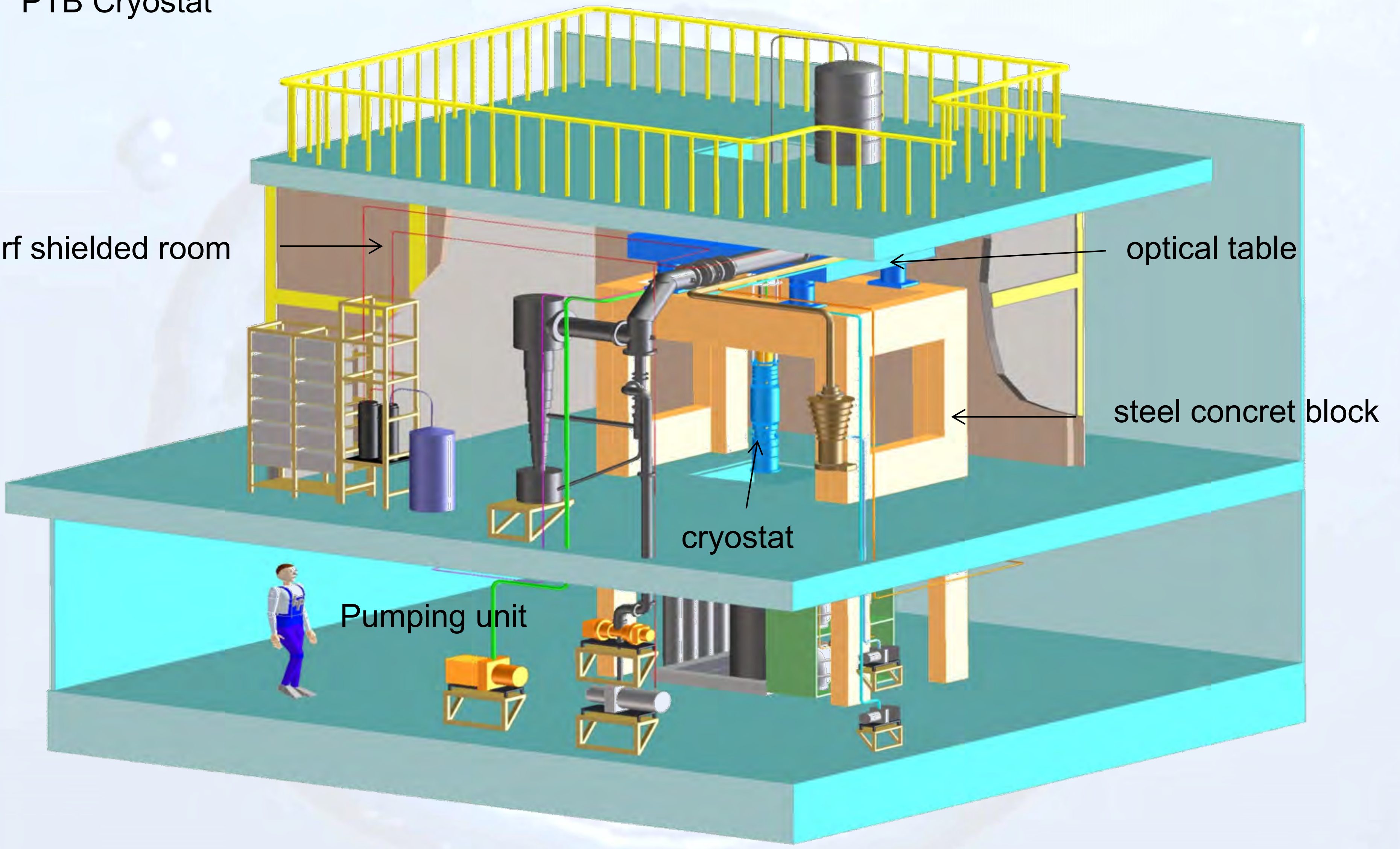
$$B_{f,opt} = \sqrt{\frac{3k_B\kappa\dot{Q}}{ng_n^2 I(I+1)\mu_n^2}}$$

heat leak



11.3 Adiabatic Demagnetization Refrigerators

PTB Cryostat



11.3 Adiabatic Demagnetization Refrigerators

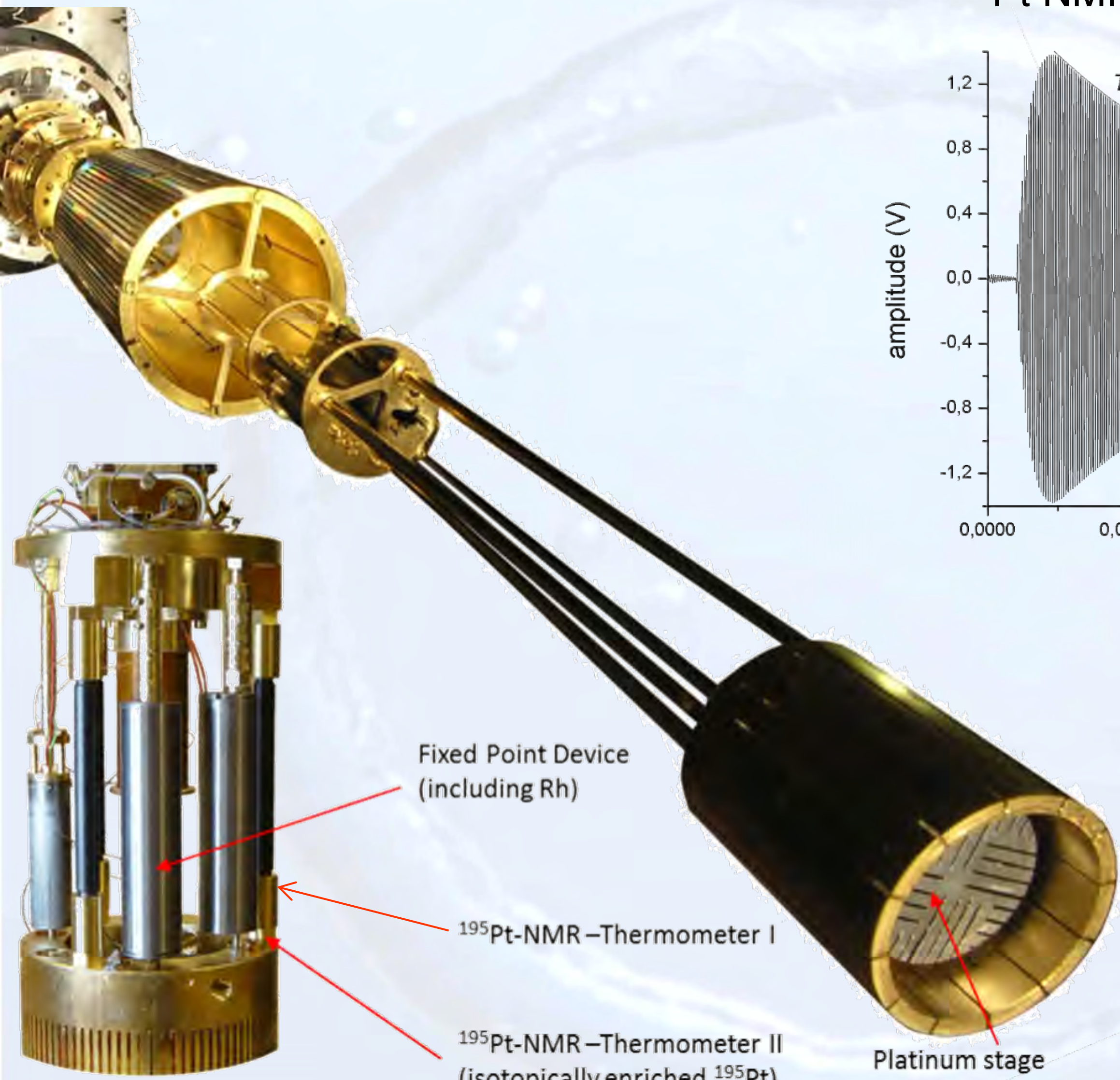
heat switch

Cu stage

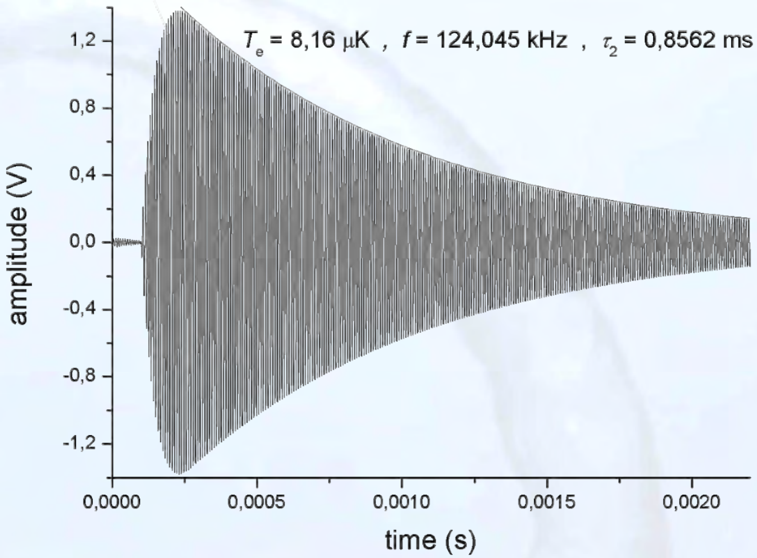
heat switch

Pt stage

11.3 Adiabatic Demagnetization Refrigerators



^{195}Pt NMR thermometer signal



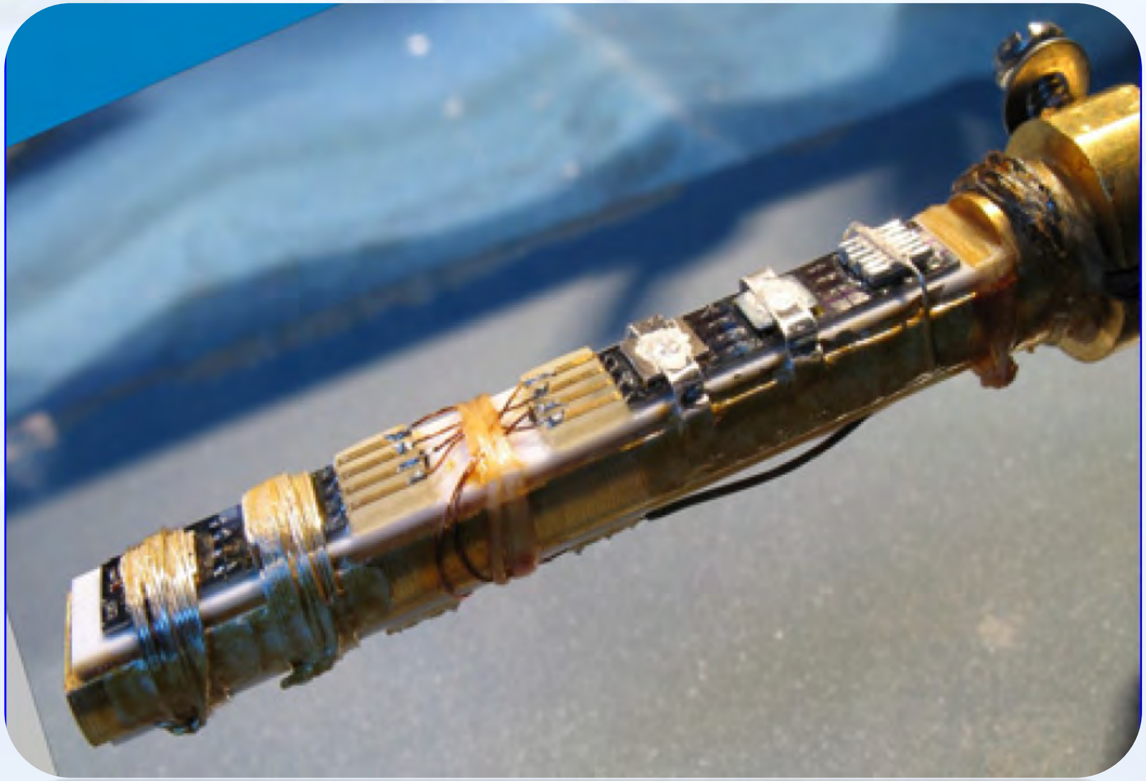
Lowest temperature at Pt stage

T_{min} = 800 nK

12. Thermometry at Low Temperature

Primary thermometers
Superconducting fixpoints
Current/flux noise
^{195}Pt NMR
Coulomb blockade
Nuclear orientation
^{3}He melting curve
....

Secondary thermometers
Resistance
Capacitance
Magnetic susceptibility
.....



12. Thermometry at Low Temperature

Temperature is a thermodynamic property of state

It can be defined by a reversible cycle, like a carnot cycle $\oint T^{-1}\mathrm{d}Q = 0$

not practical

primary thermometers: can be used without any prior calibration

secondary thermometers: must be calibrated against an other thermometer

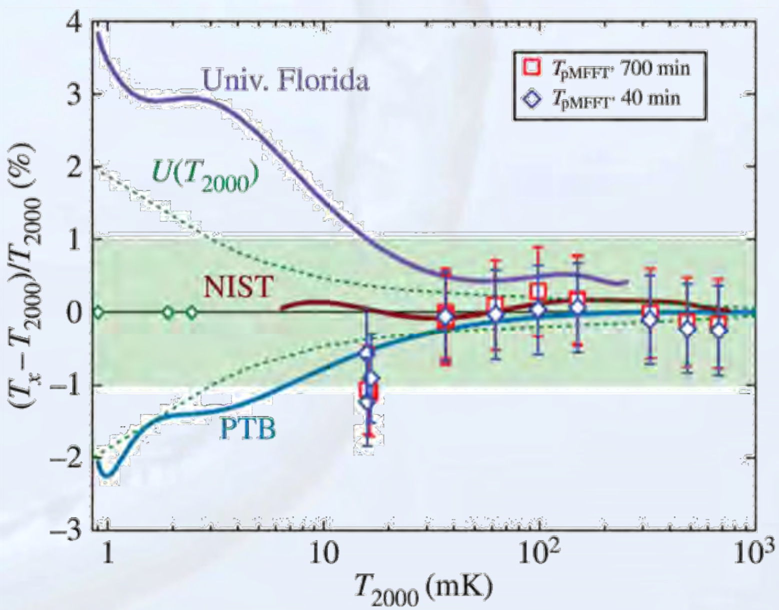
distinction is often somewhat arbitrary …

Temperature scales

defined by *Comité International des Poids et Messures*

based on fixpoints like the triple point of water
and interpolation like Pt-100 resistance thermometry
or gas thermometry

ITS-90	0.65 K	to 1358 K
PLTS-2000	0.9 mK	to 1358 K



12. Thermometry at Low Temperature

Thermometer types and ranges