10. Superconductivity

Superconductors in magnetic fields

we need: ***H***: Magnetic Field & ***B***: Magnetic Flux Density

(and patience, many use “B” and “Tesla” only...)

Type-I superconductors (pure metals like Pb, Hg, In, Al, …)

for $H_\mathrm{i} < H_\mathrm{c}$

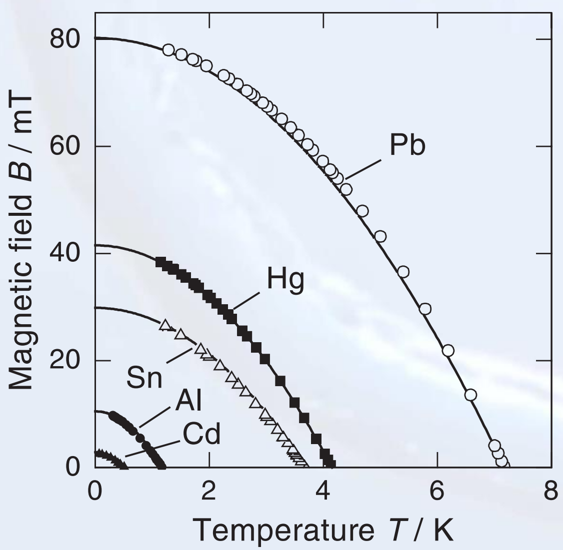
B–field is expelled $\quad B_\mathrm{i} = \mu_0(H_\mathrm{i} + M) = 0 \quad$ (Meißner phase)

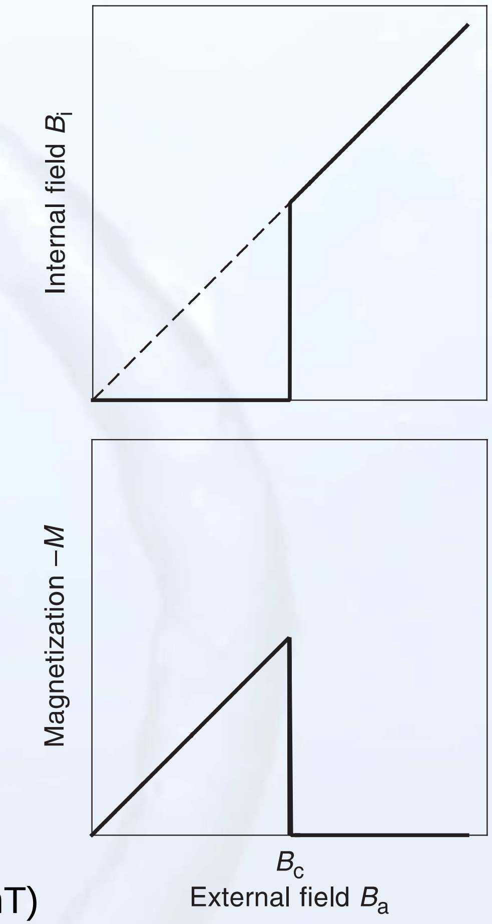
$\longrightarrow \quad M = -H_\mathrm{i}$

$\longrightarrow \quad \chi = \dfrac{M}{H_i} = -1$ ideal diamagnet

for long sample || H_0: $H_\mathrm{i} = H_0$, “text book behavior”

$B_c(T)$





- critical fields are low (mT)
- empirical relation for $B_c(T)$

$$H_\mathrm{c}(T) = H_\mathrm{c}(0)\left[1 - \frac{T^2}{T_\mathrm{c}^2}\right]$$

Intermediate State

shape of sample is important: demagnetization factor D

$$H_\mathrm{i} = H_0 - D\,M$$

D = 1/3 for sphere
1/2 for thin cylinder ⊥ to field
0 for thin cylinder || to field

for ideal diamagnet ($M = -H_i$): $$H_i = \frac{H_0}{1-D}$$

for a sphere: $H_\mathrm{i} = \frac{3}{2}\,H_0$ ⟶ already at $H_0 = \frac{2}{3}\,H_\mathrm{c}$ field can penetrate ⟶ nc

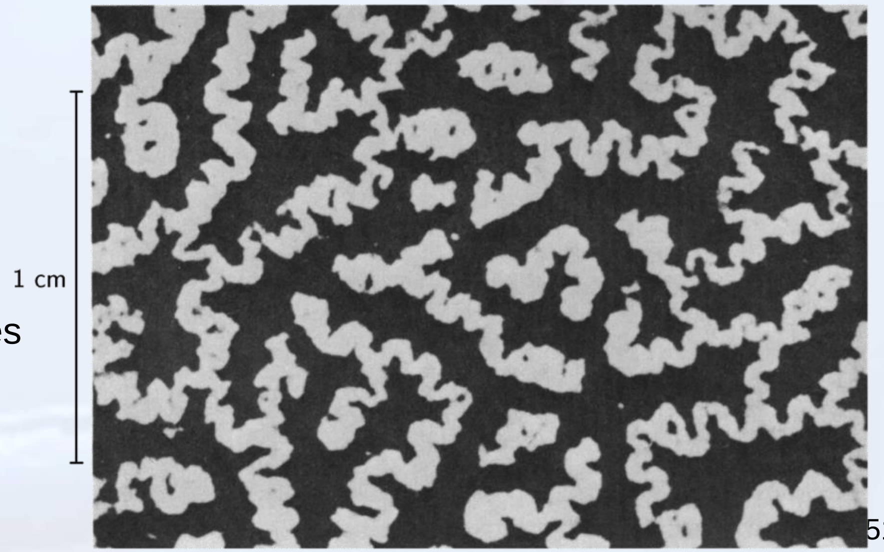
⟶ sc more slim ⟶ D decreases ⟶ H_i decreases below H_c ⟶ sc.

Further increase H_0

➡ stationary intermediate state forms

here. extreme case of thin film ⊥ H_0

- intermediate state for In at T = 1.98 K
- made visible by decoration technique with Nb spheres
  ⟶ bright regions normal conducting



10. Superconductivity

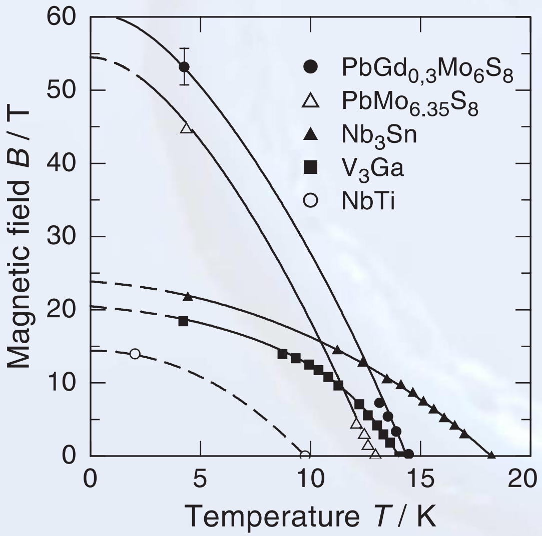
Type-II superconductors (alloys, metallic glasses, high-T_c sc, …)

$H < H_{c1}$ Meißner phase: field fully expelled

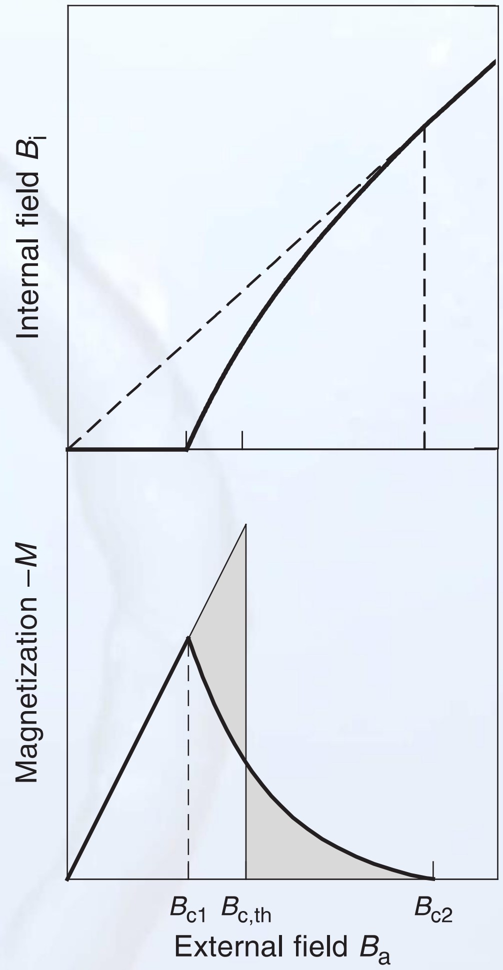
$H_{c1} < H < H_{c2}$ Subnikov phase: magnetic flux in form of vortices penetrate into sample

Important: H_{c2} can be much higher than H_{c1}

temperature dependence of critical field

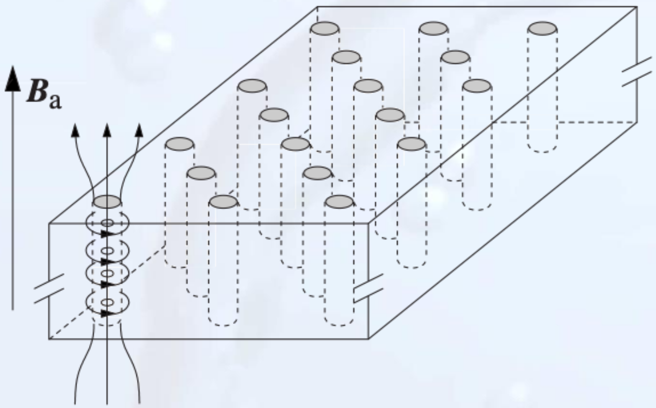


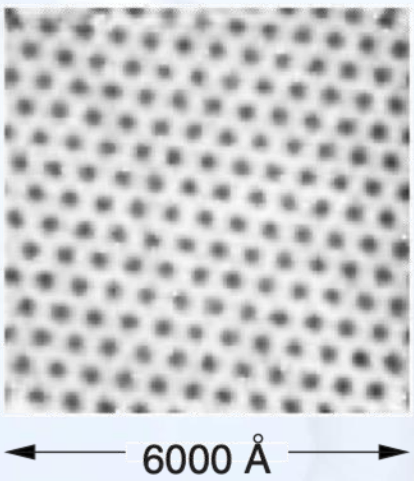
critical field H_{c2} at $T = 0$ ⟶ above 10 T



Abrikosov lattice

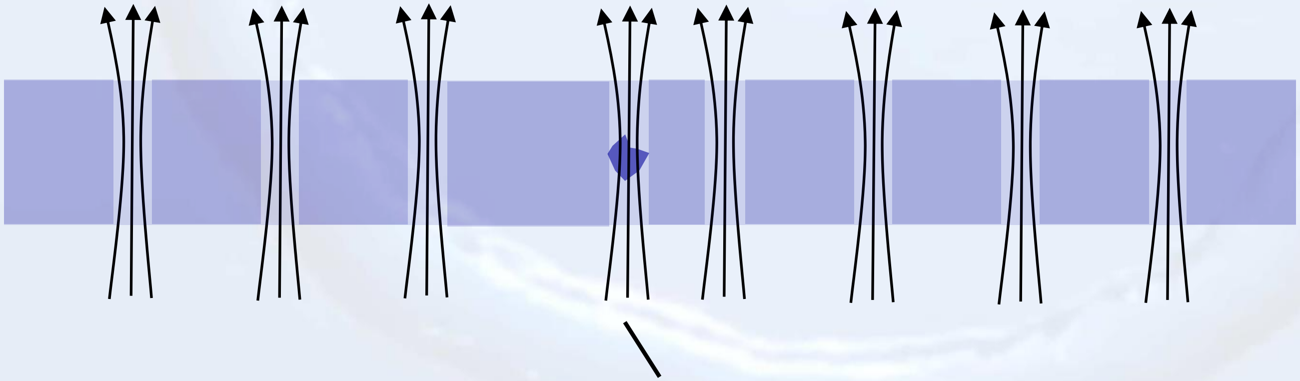
in perfect crystals (free of inclusions and crystal defects) formation of a regular lattice (pattern)





STM image
$NbSe_2$ at 1.8 K
T_c (B = 0) = 7.2 K

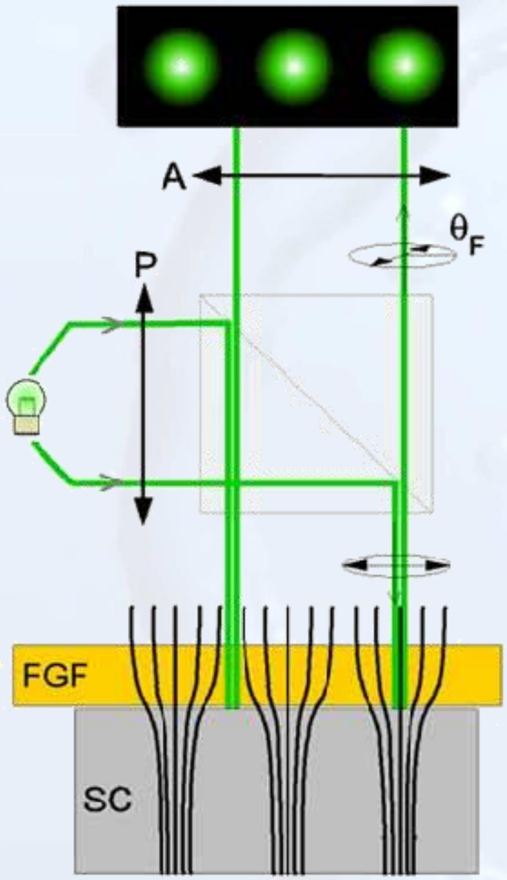
pinning effect: inclusions in crystals lead to pinning of vortex lines

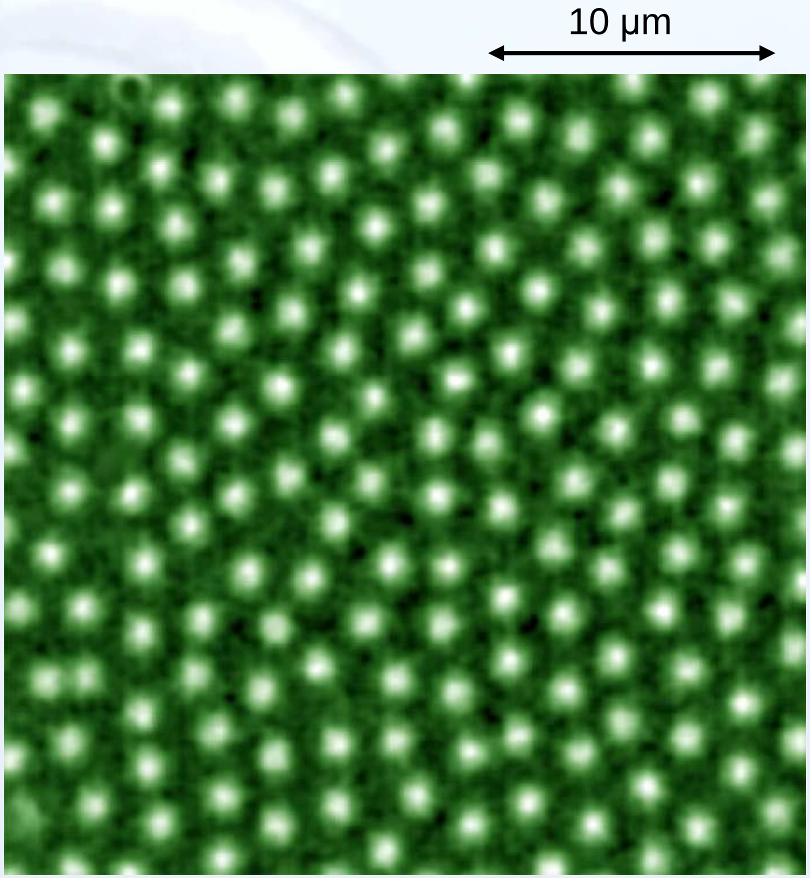


to move a pinned vortex one needs to "pay" the condensation energy

10. Superconductivity

magneto-optical image of vortex Lines

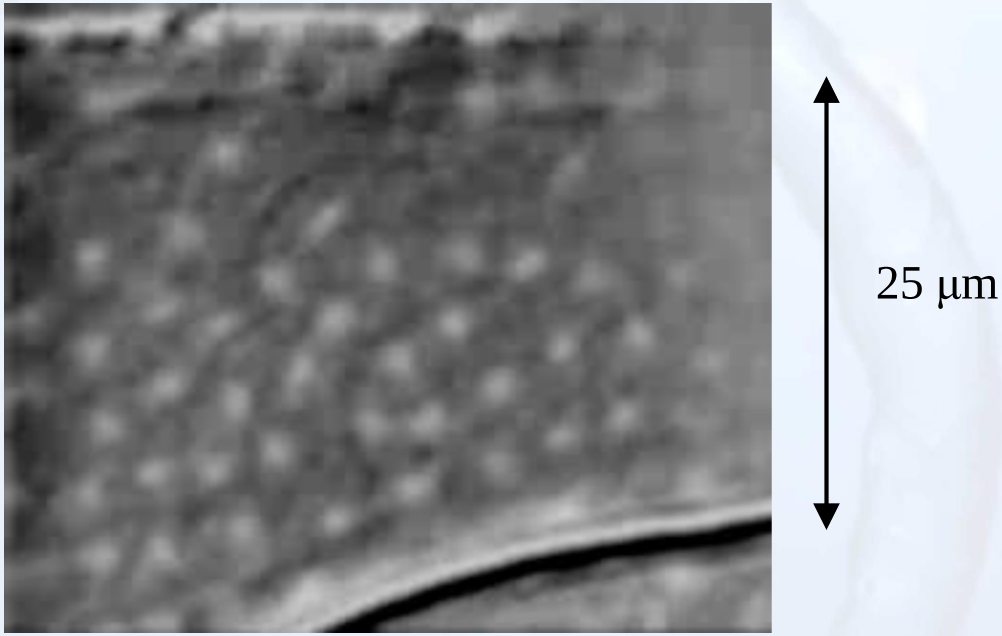




$NbSe_2$ T= 4.3 K, B = 0.3 mT

10. Superconductivity

penetration of magnetic flux into a superconductor



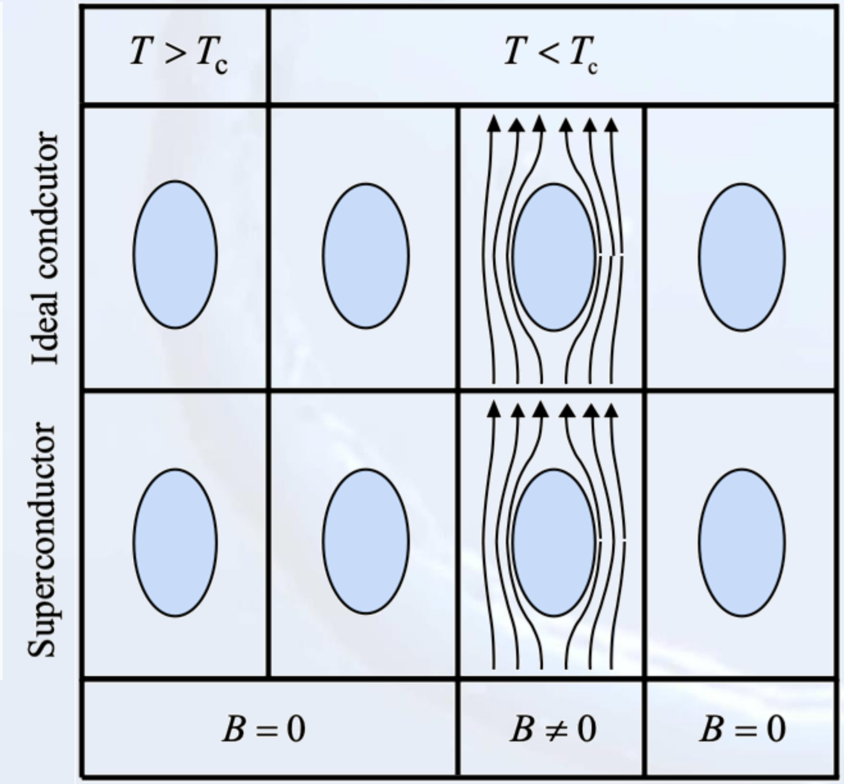
$NbSe_2$

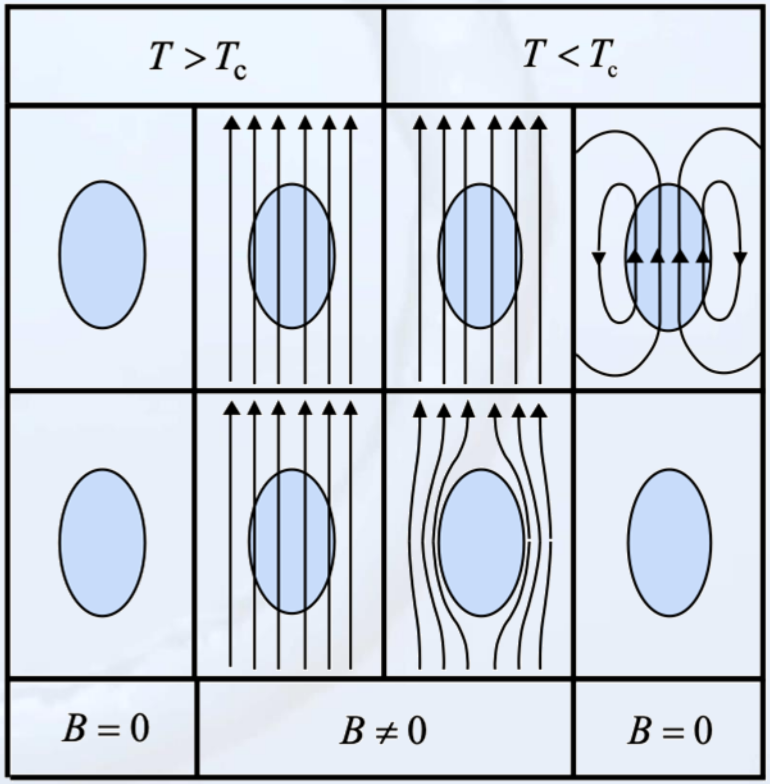
back to type I

Meißner-Ochsenfeld effect

comparison of ideal conductor and superconductor

Faraday law $\oint \boldsymbol{\mathcal{E}} \cdot \mathrm{d}\boldsymbol{s} = -\partial \Phi / \partial t$





10.1 Phenomenological Description

London theory, Fritz und Heinz London 1935

idea: Maxwell equations and the properties of ideal conductor and ideal diamagnet

electrical conductivity: $m\dot{\boldsymbol{v}} = -e\boldsymbol{\mathcal{E}} - \cancel{m\boldsymbol{v}/\tau}$

$\tau = \infty$ ideal conductor

with $\boldsymbol{j}_\mathrm{s} = -n_\mathrm{s} e_\mathrm{s} \boldsymbol{v}$

$\longrightarrow$ $$\boxed{\frac{\mathrm{d}\boldsymbol{j}_\mathrm{s}}{\mathrm{d}t} = \frac{n_\mathrm{s} e_\mathrm{s}^2}{m_\mathrm{s}} \boldsymbol{\mathcal{E}}}$$ 1st London equation

insert in Maxwell equation $\mathrm{rot}\,\boldsymbol{\mathcal{E}} = -\partial \boldsymbol{B}/\partial t$

$\longrightarrow$ $$\frac{\partial}{\partial t} \underbrace{\left(\mathrm{rot}\,\boldsymbol{j}_\mathrm{s} + \frac{n_\mathrm{s} e_\mathrm{s}^2}{m_\mathrm{s}} \boldsymbol{B} \right)}_{=0} = 0$$

magnetic flux is constant for ideal conductors, but Meißner effect demands const. = 0, therefore not $\mathrm{rot}\,\partial \boldsymbol{B}/\partial t = 0$ but $B = 0$

$\longrightarrow$ $$\boxed{\mathrm{rot}\,\boldsymbol{j}_\mathrm{s} = -\frac{n_\mathrm{s} e_\mathrm{s}^2}{m_\mathrm{s}} \boldsymbol{B}}$$ 2nd London equation

10.1 Phenomenological Description

application of London theory: penetration depth

fields enter a „little bit“, otherwise $j = \infty$

screening current

Maxwell equation $\mathrm{rot}\,\boldsymbol{B} = \mu_0 \boldsymbol{j}$

insert in 2nd London equation

$$\longrightarrow \quad \mathrm{rot}\,\mathrm{rot}\,\boldsymbol{B} = \mu_0\,\mathrm{rot}\,\boldsymbol{j} = -\mu_0 \frac{n_\mathrm{s} e_\mathrm{s}^2}{m_\mathrm{s}} \boldsymbol{B}$$

special geometry:

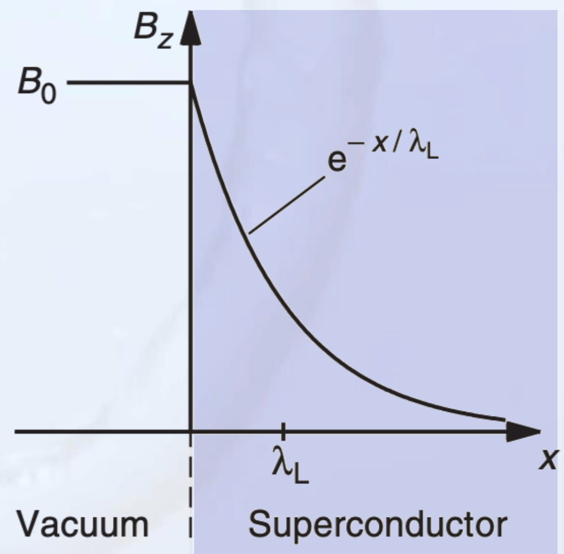
- superconductor fills half space $x > 0$
- magnetic field in z-direction

$$\frac{\mathrm{d}^2 B_z(x)}{\mathrm{d}x^2} - \frac{\mu_0 n_\mathrm{s} e_\mathrm{s}^2}{m_\mathrm{s}} B_z(x) = 0$$

ansatz $B_z(x) = B_0\,\mathrm{e}^{-x/\lambda_\mathrm{L}}$

$$\longrightarrow \quad \boxed{\lambda_\mathrm{L} = \sqrt{\frac{m_\mathrm{s}}{\mu_0 n_\mathrm{s} e_\mathrm{s}^2}}}$$

London penetration depth



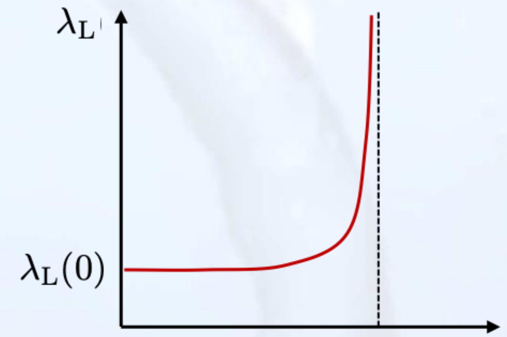
10.1 Phenomenological Description

insert in Maxwell equation $\mathrm{rot}\,\boldsymbol{B} = \mu_0 \boldsymbol{j}$ $\longrightarrow$ $j_{s,y}(x) = j_0\,\mathrm{e}^{-x/\lambda_\mathrm{L}}$ screening current

$$j_0 = B_0 / \mu_0\,\lambda_\mathrm{L}$$

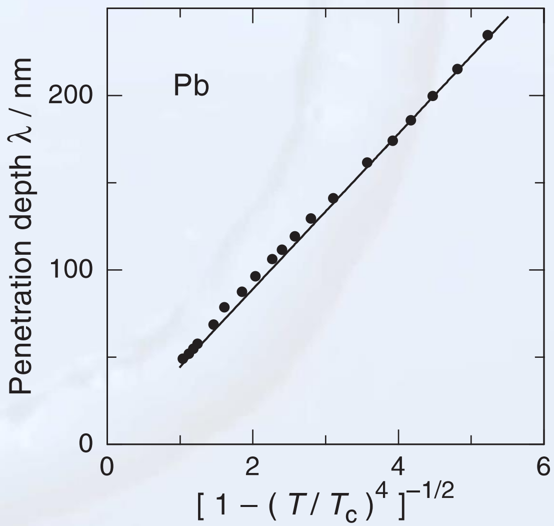
some numbers: $n_s \cong 10^{23}\ \mathrm{cm}^{-3}$ $\longrightarrow$ $\lambda_\mathrm{L} = 30$ nm

experimental observation: susceptibility of thin lead cylinders



temperature dependence

$$n_s \propto 1 - (T/T_c)^4 \longrightarrow \lambda_\mathrm{L}(T) = \frac{\lambda_\mathrm{L}(0)}{\sqrt{1-(T/T_c)^4}}$$



- penetration depth of lead in the 100 nm range
- solid line: two-fluid model for superconductors

10.2 Microscopic Theory

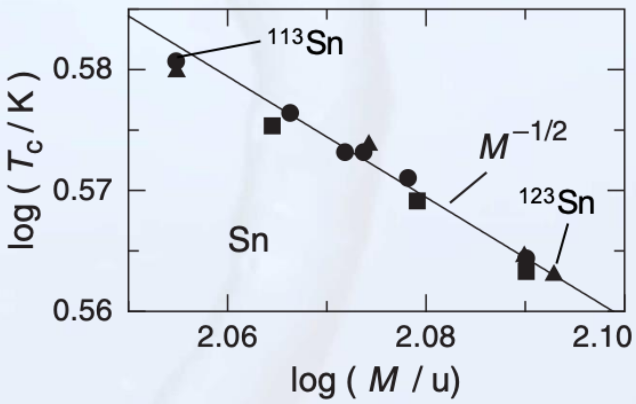
superconductivity occurs in many different materials

low transition temperatures ⟶ small energy differences matter ⟷ electrons have Fermi energy!

1950 Fröhlich ⟶ interaction between electrons and lattice can mediate attraction between electrons
(Bardeen)

Isotope effect, discovered 1950

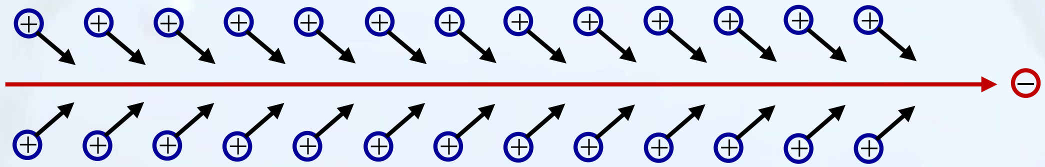
- T_c depends on atomic mass $T_c \propto 1/\sqrt{M}$
- for m = 113 u … 123 u T_c = 3.8 K … 3.66 K
- lattice properties are important for superconductivity



10.2 Microscopic Theory

schematic picture

- electron passes through lattice and attracts positive ions
- positive charge density maximum occurs long after electron has passed
- a second electron is attracted, but Coulomb repulsion is small since it is far away from first electron



estimated distance between electron and positive charge density maximum

$$s = v_{\mathrm{F}} t \approx 10^8 \times 10^{-13}\,\mathrm{cm} = 1000\,\mathrm{\AA}$$

time for ions to react $1/\omega_{\mathrm{D}}$