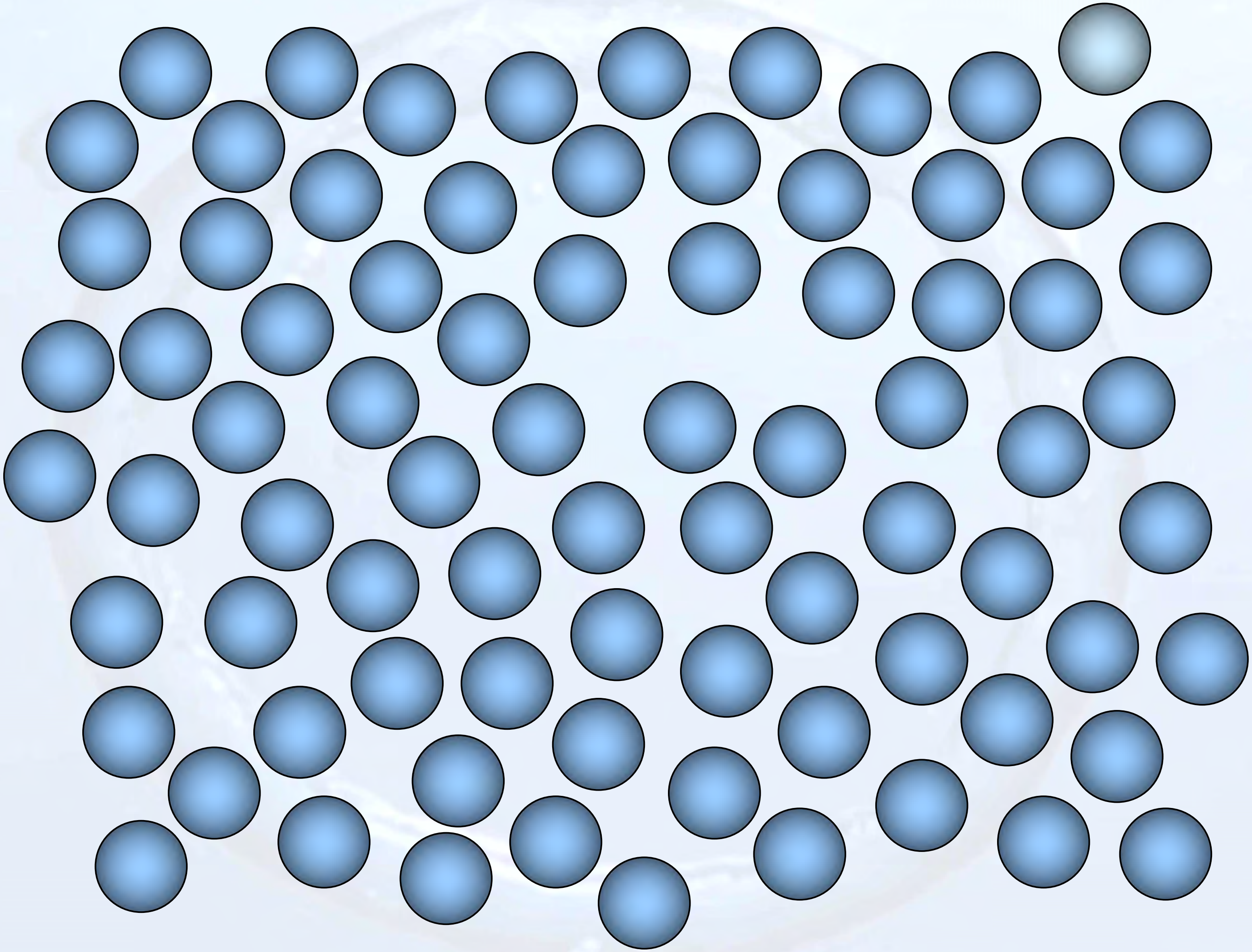
9.2 Tunneling Systems in Glasses



9.2 Tunneling Systems in Glasses

$\nearrow$ $\Delta \neq 0$ widely distributed

$\searrow$ Δ_0 also widely distributed

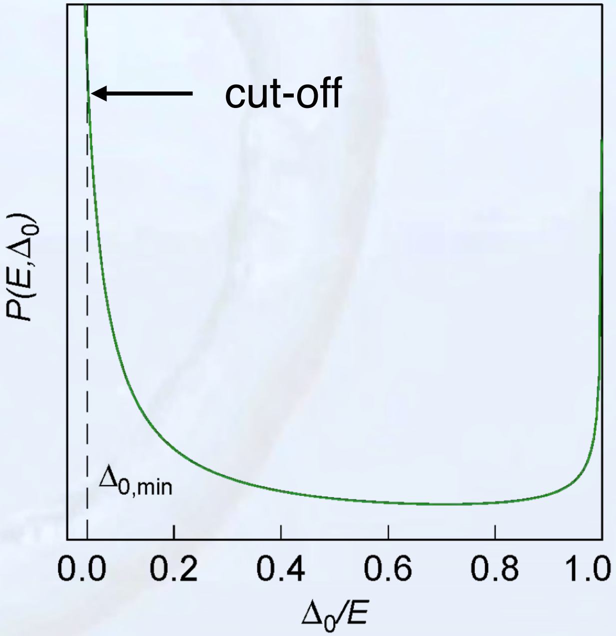
distribution function ⟶ standard tunneling model

$$P(\varDelta,\lambda)\,\mathrm{d}\varDelta\,\mathrm{d}\lambda = P_0\,\mathrm{d}\varDelta\,\mathrm{d}\lambda$$

$P(\varDelta,\lambda) \longrightarrow P(E,\varDelta_0)$ with $E^2 = \varDelta^2 + \varDelta_0^2$ and $\varDelta_0 = \hbar\varOmega \mathrm{e}^{-\lambda}$

$$P(E,\varDelta_0)\,\mathrm{d}\varDelta_0\,\mathrm{d}E = P(\varDelta,\lambda)\left|\frac{\partial\lambda}{\partial\varDelta_0}\right|\left|\frac{\partial\varDelta}{\partial E}\right|\mathrm{d}\varDelta_0\,\mathrm{d}E$$

$$= P_0\,\frac{E}{\varDelta_0\sqrt{E^2-\varDelta_0^2}}\,\mathrm{d}\varDelta_0\,\mathrm{d}E$$



density of states

$$D(E) = \int_{\varDelta_0^{\min}}^{E} P(\varDelta_0,E)\,\mathrm{d}\varDelta_0 = P_0\,\ln\frac{2E}{\varDelta_0^{\min}}$$

9.2 Tunneling Systems in Glasses

internal energy

$D(E) \approx D_0 = \mathrm{const.}$

$$u = \int E\,D(E)\,f(E)\,\mathrm{d}E = D_0 (k_\mathrm{B}T)^2 \underbrace{\int_0^\infty \frac{x}{\mathrm{e}^x + 1}\,\mathrm{d}x}_{\pi^2/12}$$

Fermi-Dirac distribution

$$f(E) = \left[\exp\left(E/k_\mathrm{B}T\right) + 1\right]^{-1}$$

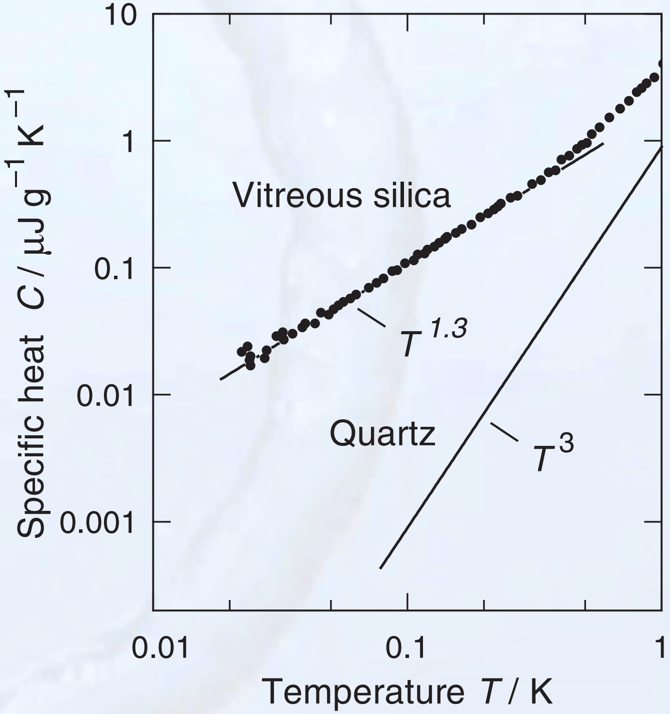
specific heat

$$C_V = \left(\frac{\partial u}{\partial T}\right)_V = \frac{\pi^2}{6} D_0 {k_\mathrm{B}}^2 T \;\propto\; T$$

total specific heat

$$C_V = AT + BT^3 + C_\mathrm{Debye}$$

- additional T^3 term ⟶ quasi-harmonic modes
- linear term $\sim T^{1.3}$ instead of $\sim T$
  - ⟶ good agreement but glass is non-equilibrium system
  - ⟶ not all TS can contribute in measuring time



9.2 Tunneling Systems in Glasses

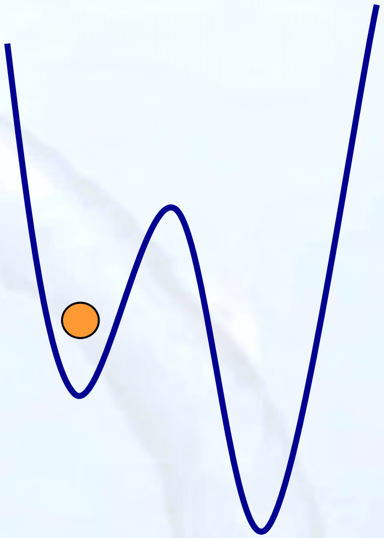
effective density of states

measuring time

$$D_{\rm eff}(E,t_0) = \int\limits_{\tau_{\rm min}}^{t_0} P(E,\tau)\,{\rm d}\tau = \frac{P_0}{2}\ln\frac{4t_0}{\tau_{\rm min}}$$

minimum relaxation time

$$P(E,\tau) = \frac{P_0}{2\tau\sqrt{1-\tau_{\rm min}/\tau}}$$

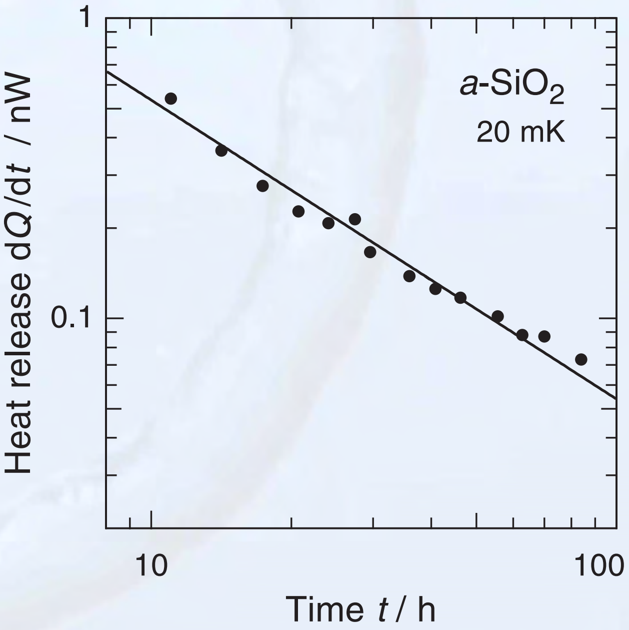


➡ $$C_V = \frac{\pi^2}{12} P_0 {k_{\rm B}}^2\, T \ln(4At_0T^3)$$

measuring time

➡ heat release of amorphous solids

$$\dot{Q} = \frac{\pi^2 k_{\rm B}^2}{24} P_0 (T_1^2 - T_0^2)\frac{1}{t}$$

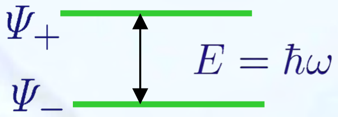


9.2 Tunneling Systems in Glasses

Echo experiments

coherent regime: $t \ll \tau_1, \tau_2 \;\to\; \infty$

two-level approximation:



applied rf field: $F = F_0[\exp(\mathrm{i}\omega t)+\exp(-\mathrm{i}\omega t)] = 2F_0\,\cos(\omega t)$

Schrödinger equation:

$$\mathrm{i}\hbar\frac{\partial \Psi}{\partial t} = [H_0 + H_\mathrm{S}]\,\Psi = \left[H_0 + p\frac{\Delta_0}{E}F_0\left(\mathrm{e}^{\mathrm{i}\omega t} + \mathrm{e}^{-\mathrm{i}\omega t}\right)\right]\Psi$$

ansatz:

$$\Psi(t) = a_1(t)\,\Psi_-\,\mathrm{e}^{-\mathrm{i}\omega_1 t} + a_2(t)\,\Psi_+\,\mathrm{e}^{-\mathrm{i}\omega_2 t} \longrightarrow \begin{cases} a_1(t) = \cos\left(\Omega_\mathrm{R} t\right) \\ a_2(t) = -\mathrm{i}\sin\left(\Omega_\mathrm{R} t\right) \end{cases}$$

$$\Omega_\mathrm{R} = \frac{1}{\hbar}\frac{\Delta_0}{E}\,\boldsymbol{p}\cdot\boldsymbol{F}_0$$

Rabi frequency

occupation number difference varies with Rabi frequency

9.2 Tunneling Systems in Glasses

polarization vector:

$$\boldsymbol{P} = \begin{pmatrix} ab^* + ba^* \\ \mathrm{i}(ab^* - ba^*) \\ aa^* - bb^* \end{pmatrix} = \begin{pmatrix} -\sin(\Omega_\mathrm{R} t)\sin(\omega t) \\ \sin(\Omega_\mathrm{R} t)\cos(\omega t) \\ \cos(\Omega_\mathrm{R} t) \end{pmatrix} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix}$$

Bloch equations:

$$\frac{\mathrm{d}\langle S_x\rangle}{\mathrm{d}t} = -\frac{2}{\hbar}\left(\frac{E}{2} + \frac{\Delta}{E}\boldsymbol{p}\cdot\boldsymbol{F}\right)\langle S_y\rangle - \frac{\langle S_x\rangle}{\tau_2}$$

$$\frac{\mathrm{d}\langle S_y\rangle}{\mathrm{d}t} = \frac{2}{\hbar}\left(\frac{E}{2} + \frac{\Delta}{E}\boldsymbol{p}\cdot\boldsymbol{F}\right)\langle S_x\rangle - \frac{2}{\hbar}\left(\frac{\Delta_0}{E}\boldsymbol{p}\cdot\boldsymbol{F}\right)\langle S_z\rangle - \frac{\langle S_x\rangle}{\tau_2}$$

$$\frac{\mathrm{d}\langle S_z\rangle}{\mathrm{d}t} = \frac{2}{\hbar}\left(\frac{\Delta_0}{E}\boldsymbol{p}\cdot\boldsymbol{F}\right)\langle S_y\rangle - \frac{[\langle S_z\rangle - S_z^0(\boldsymbol{F})]}{\tau_1}$$

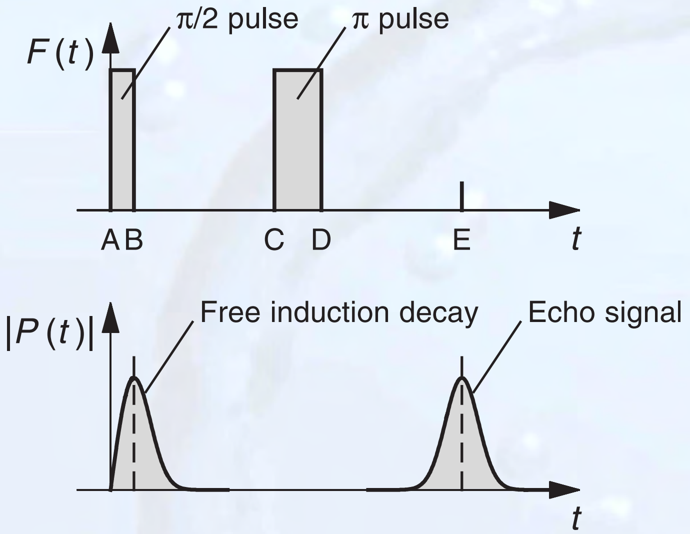
energy relaxation τ_1: $T < 1$ K ⟶ one phonon process

phase coherence time τ_2:

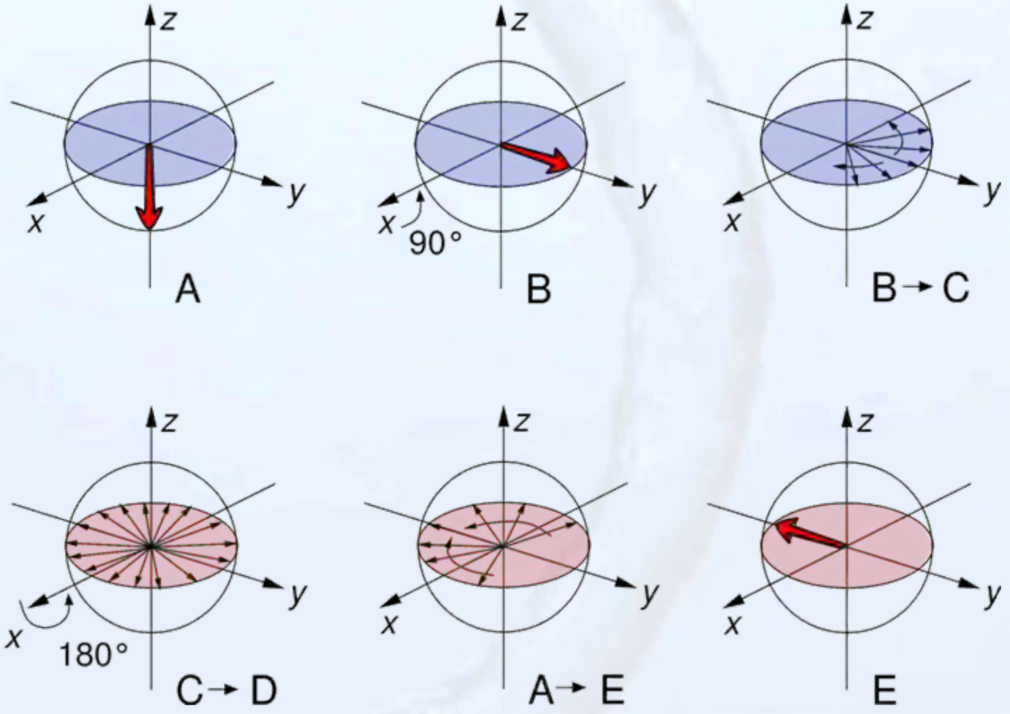
τ_1 processes
spectral diffusion ⬅
spin diffusion

9.2 Tunneling Systems in Glasses

Origin of echo

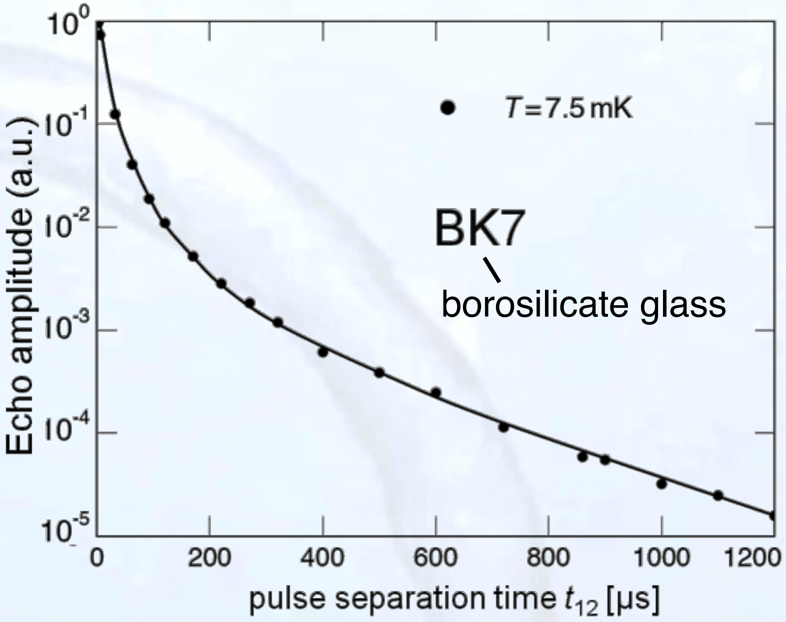


echo generation explained in Bloch sphere



9.2 Tunneling Systems in Glasses

two-pulse echo decay



- sensitivity five orders of magnitude
- non-exponential decay

what determines the decay: spectral diffusion

interaction between resonant TS and thermally fluctuating TS

energy splitting of single TS fluctuating with time

9.2 Tunneling Systems in Glasses

spectral diffusion: decay regimes

- short-time limit (no-flip limit): $t_{12} \ll \tau_{\min}$ — fastest thermal systems

$$\longrightarrow \quad A(2t_{12}) = A(0)\,\mathrm{e}^{-(2t_{12}/\tau_2)^2}$$ Gaussian decay

- long-time limit (multiple-flip limit): $t_{12} \gg \tau_{\min}$

$$\longrightarrow \quad A(2t_{12}) = A(0)\,\mathrm{e}^{-2t_{12}/\tau_2}$$ exponential decay

temperature dependence

- short-time limit (no-flip limit):

$$A(2t_{12},T) \;\propto\; \tanh\left(\frac{E}{2k_{\mathrm{B}}T}\right)\mathrm{e}^{-m_0T^4(\Delta/E)t_{12}^2} \;=\; A_0(T)\,\mathrm{e}^{-m(T)(\Delta/E)t_{12}^2}$$

$$\longrightarrow \quad \tau_2 \;\propto 1/\sqrt{m(T)} \;\propto\; T^{-2}$$

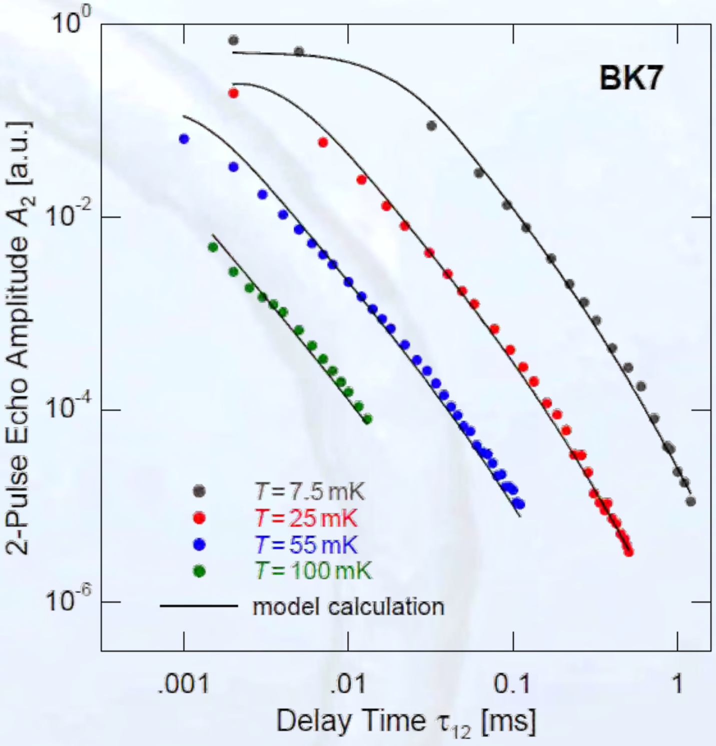
- long-time limit (multiple-flip limit):

$$A(2t_{12},T) \;\propto A_0(T)\,\mathrm{e}^{-2t_{12}^2/\tau_2}$$

$$\longrightarrow \quad \tau_2 \;\propto\; T^{-1}$$

9.2 Tunneling Systems in Glasses

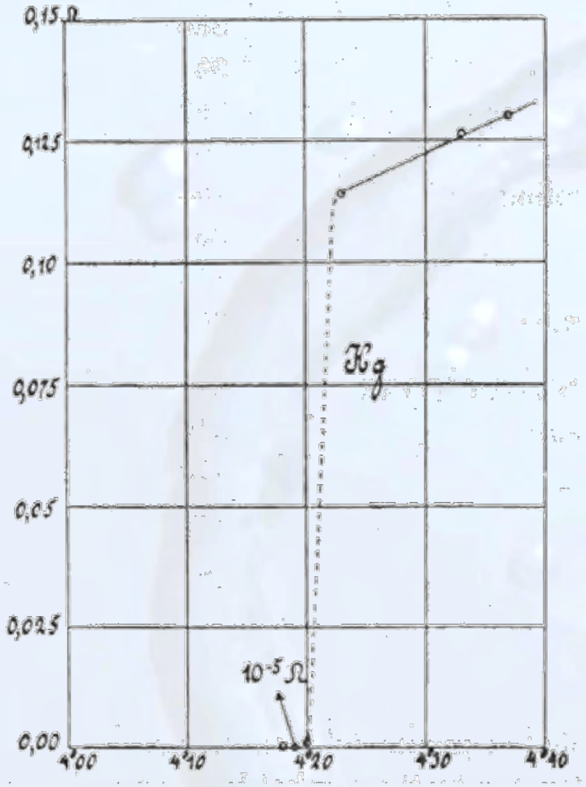
experimental observation

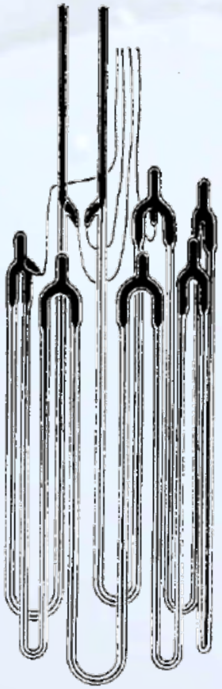


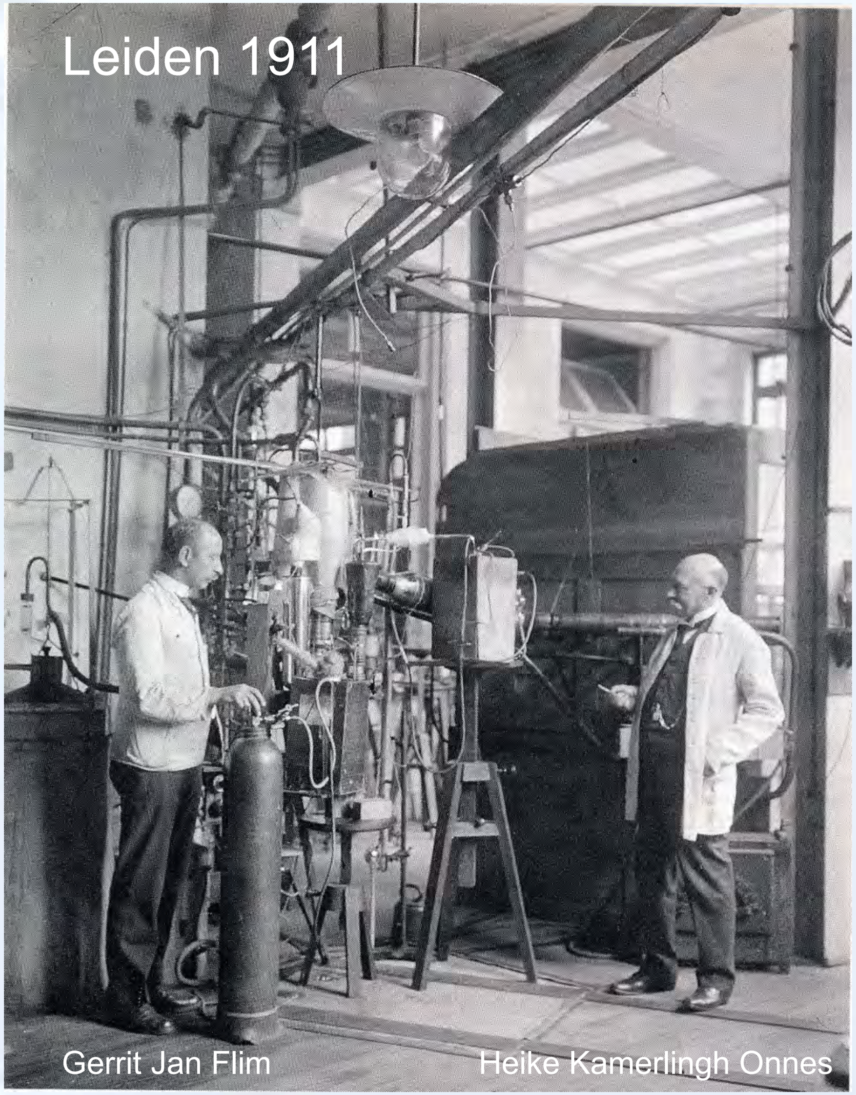
- ► transition between regimes is visible at short times and low temperatures.
- ► with increasing temperature, the Gaussian regime shifts to shorter times

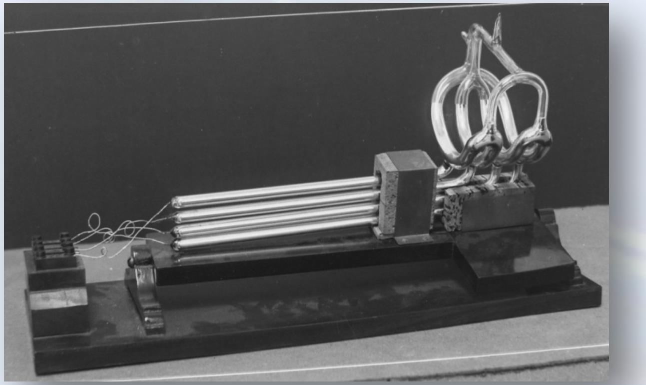
10. Superconductivity

23 May 1911









10. Superconductivity

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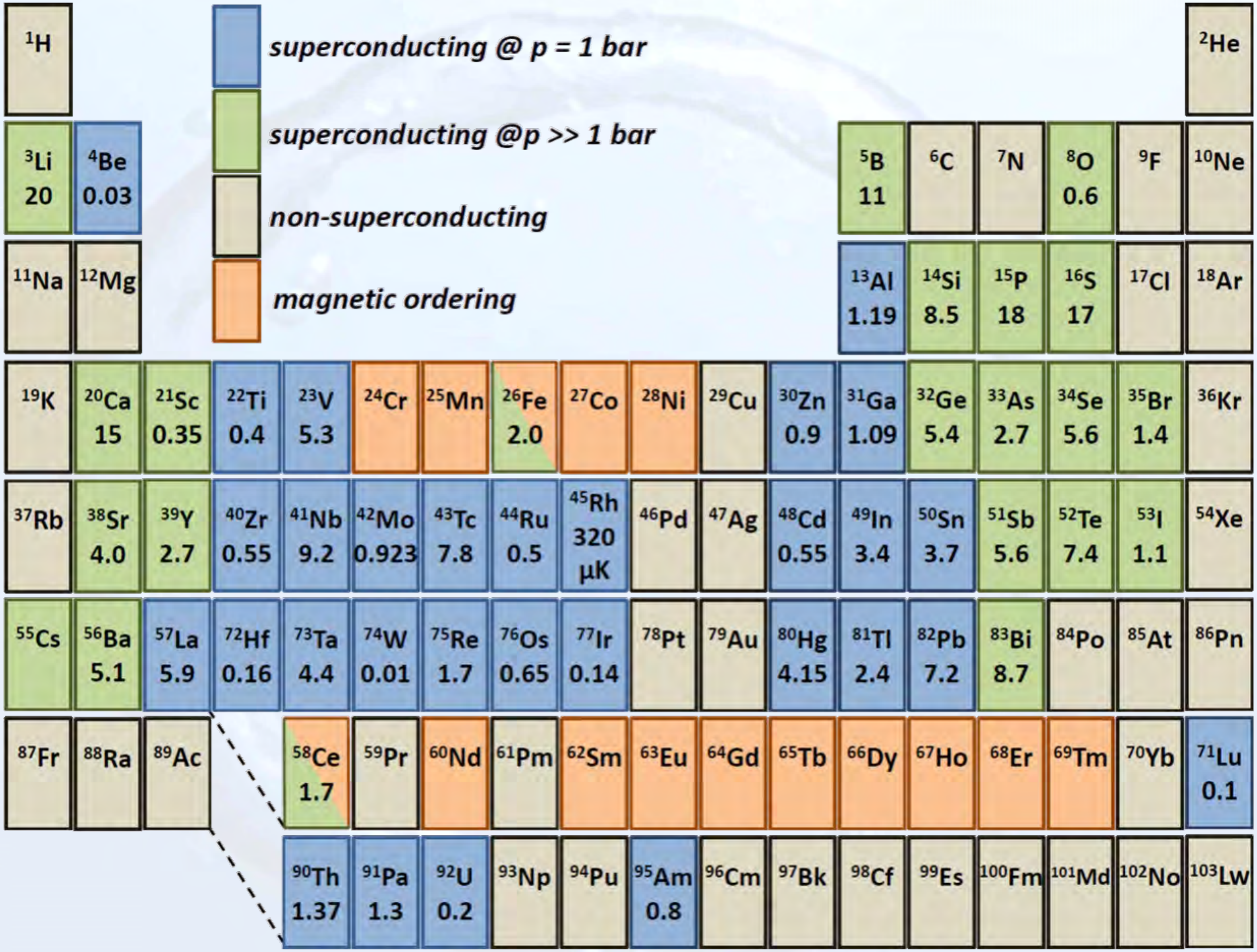


🟦 *superconducting @ p = 1 bar*

🟩 *superconducting @p >> 1 bar*

⬜ *non-superconducting*

🟧 *magnetic ordering*

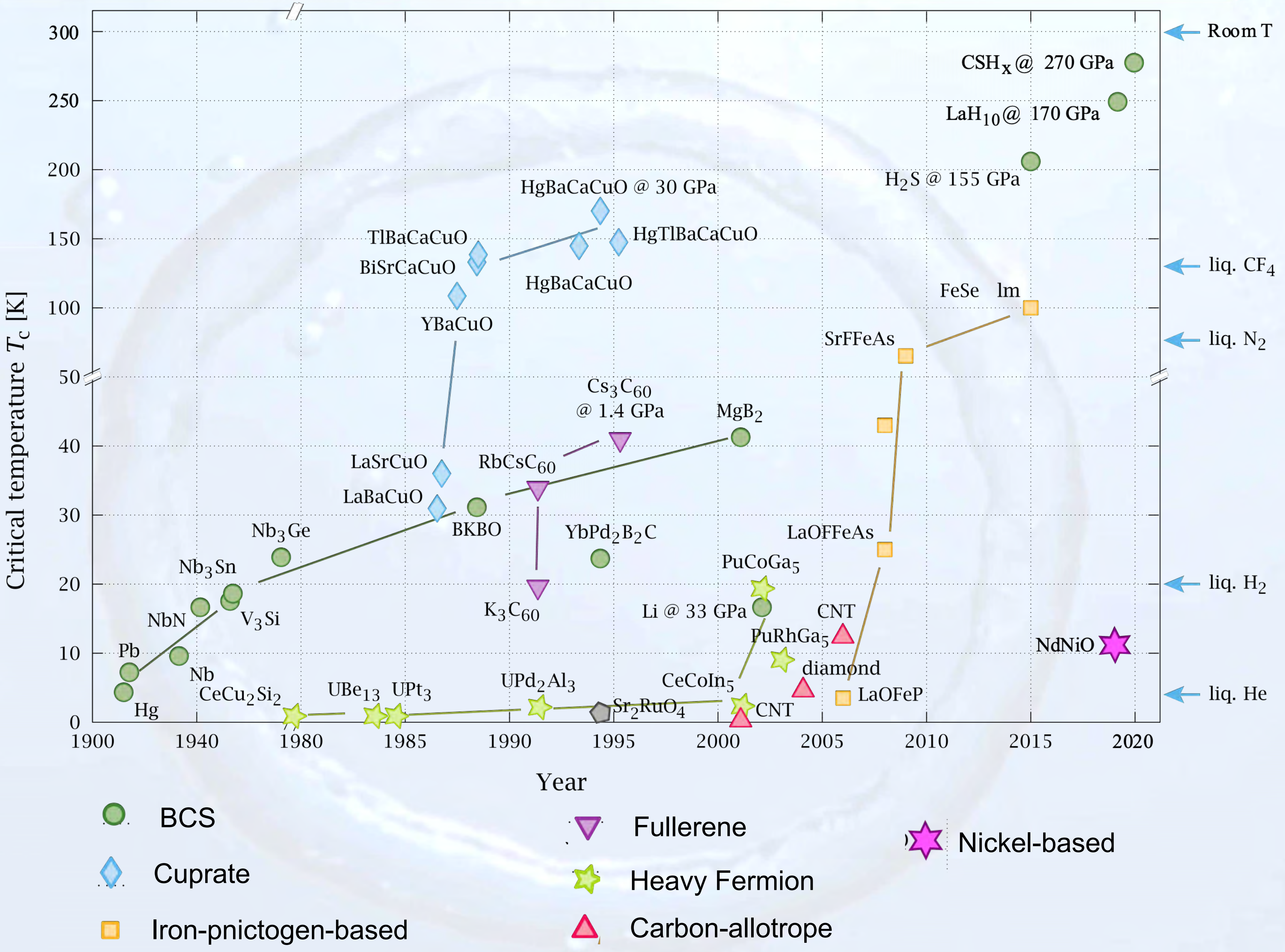
⬜ 1H																	⬜ 2He
🟩 3Li 20	🟦 4Be 0.03											🟩 5B 11	⬜ 6C	⬜ 7N	🟩 8O 0.6	⬜ 9F	⬜ 10Ne
⬜ 11Na	⬜ 12Mg											🟦 13Al 1.19	🟩 14Si 8.5	🟩 15P 18	🟩 16S 17	⬜ 17Cl	⬜ 18Ar
⬜ 19K	🟩 20Ca 15	🟩 21Sc 0.35	🟦 22Ti 0.4	🟦 23V 5.3	🟧 24Cr	🟧 25Mn	🟧🟩 26Fe 2.0	🟧 27Co	🟧 28Ni	⬜ 29Cu	🟦 30Zn 0.9	🟦 31Ga 1.09	🟩 32Ge 5.4	🟩 33As 2.7	🟩 34Se 5.6	🟩 35Br 1.4	⬜ 36Kr
⬜ 37Rb	🟩 38Sr 4.0	🟩 39Y 2.7	🟦 40Zr 0.55	🟦 41Nb 9.2	🟦 42Mo 0.923	🟦 43Tc 7.8	🟦 44Ru 0.5	🟦 45Rh 320 μK	⬜ 46Pd	⬜ 47Ag	🟦 48Cd 0.55	🟦 49In 3.4	🟦 50Sn 3.7	🟩 51Sb 5.6	🟩 52Te 7.4	🟩 53I 1.1	⬜ 54Xe
🟩 55Cs	🟩 56Ba 5.1	🟦 57La 5.9	🟦 72Hf 0.16	🟦 73Ta 4.4	🟦 74W 0.01	🟦 75Re 1.7	🟦 76Os 0.65	🟦 77Ir 0.14	⬜ 78Pt	⬜ 79Au	🟦 80Hg 4.15	🟦 81Tl 2.4	🟦 82Pb 7.2	🟩 83Bi 8.7	⬜ 84Po	⬜ 85At	⬜ 86Pn
⬜ 87Fr	⬜ 88Ra	⬜ 89Ac		🟧🟩 58Ce 1.7	⬜ 59Pr	🟧 60Nd	⬜ 61Pm	🟧 62Sm	🟧 63Eu	🟧 64Gd	🟧 65Tb	🟧 66Dy	🟧 67Ho	🟧 68Er	🟧 69Tm	⬜ 70Yb	🟦 71Lu 0.1
				🟦 90Th 1.37	🟦 91Pa 1.3	🟦 92U 0.2	⬜ 93Np	⬜ 94Pu	🟦 95Am 0.8	⬜ 96Cm	⬜ 97Bk	⬜ 98Cf	⬜ 99Es	⬜ 100Fm	⬜ 101Md	⬜ 102No	⬜ 103Lw

10. Superconductivity

Observations regarding superconductivity

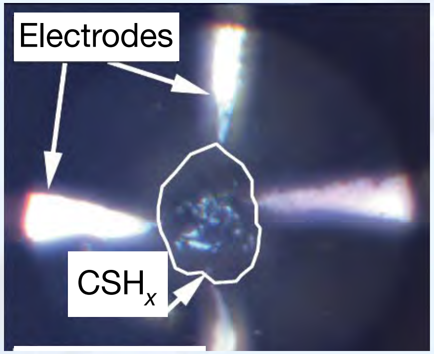
- ► small atomic volume appears to favor superconductivity
- ► metals, semi-metals, semi-conductors (highly doped)
- ► not superconducting: good conductors Ag, Au, Cu, K, …. and magnetic systems Fe, Ni, Co, …
- ► impurities are unimportant, except magnetic impurities
- ► structural order is unimportant: single crystals, poly crystals, alloys, amorphous solids
- ► transition temperatures are material dependent and spread over a wide range
- ► sufficiently large magnetic fields destroy superconductivity

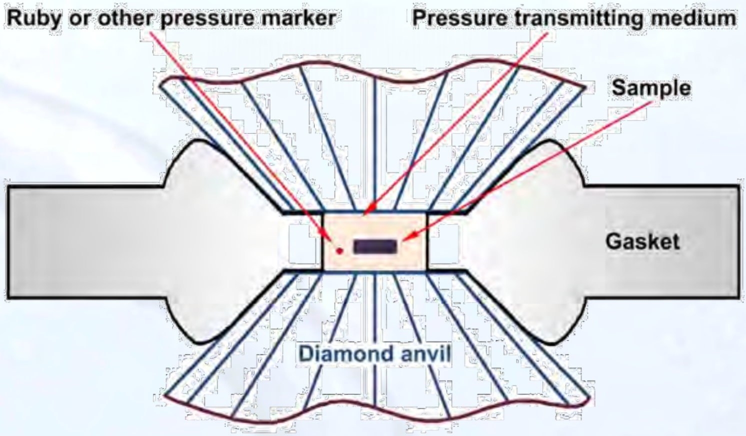
10. Superconductivity

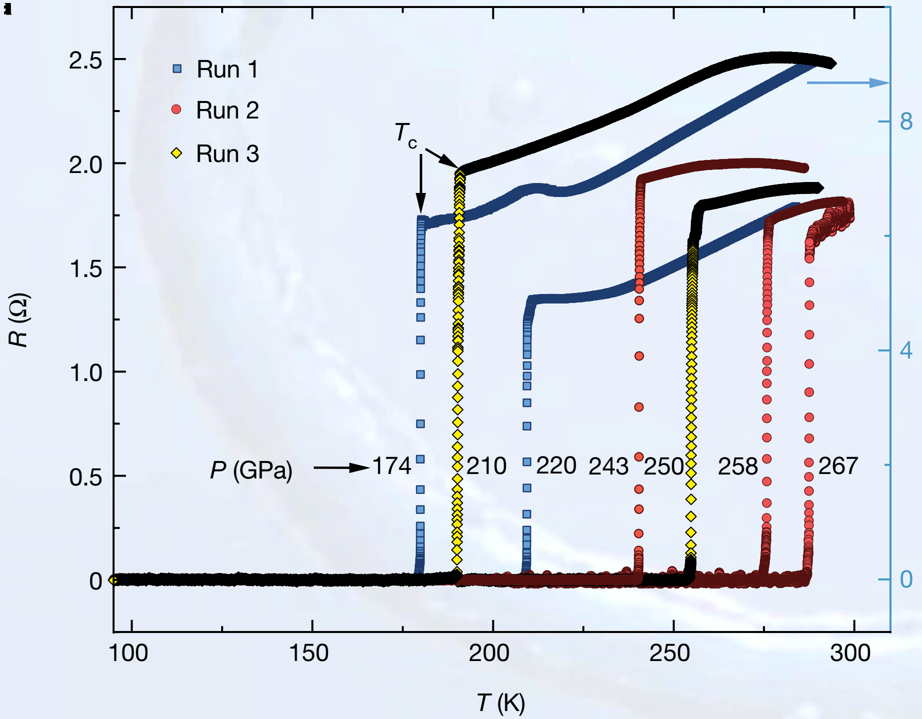


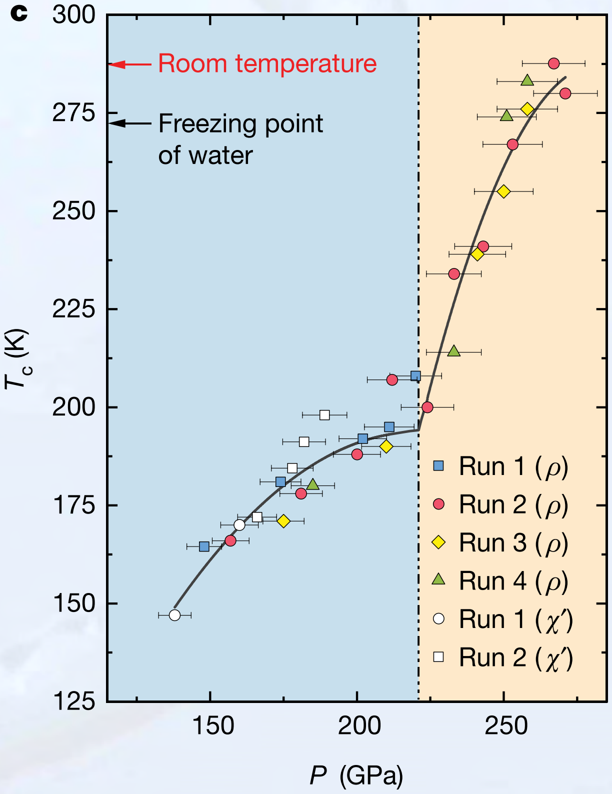
10. Superconductivity

Superconducting Transition for CSH_x @ 174 - 267 GPa



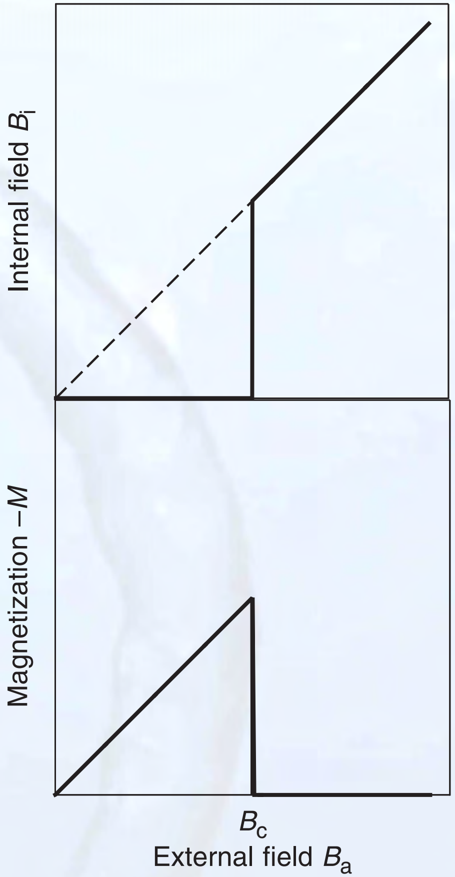






10. Superconductivity

Superconductors in magnetic fields



Type-I superconductors (pure metals like Pb, Hg, In, Al, …

$B < B_c$

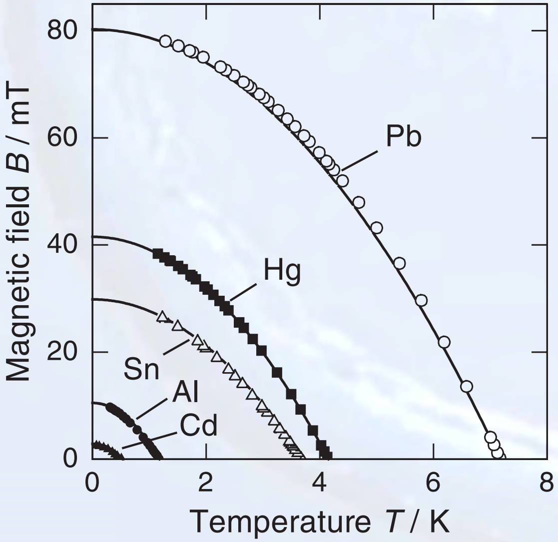
field expelled: $B_i = B_a + \mu_0 M = 0$ Meißner phase

$\longrightarrow$ $M = -B_a/\mu_0$

$\longrightarrow$ $\chi = \frac{\mu_0 M}{B_a} = -1$

↖ ideal diamagnet

$B_c(T)$



- ► critical fields are low (mT)
- ► empirical relation for $B_c(T)$

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$