Scattering of Conduction Electrons on Localized Magnetic Moments

direct and exchange processes together

$$t^{(2)} = \sum_{\boldsymbol{k}'',\sigma} \frac{1}{E(\boldsymbol{k}) - E(\boldsymbol{k}'')} \Big[(1 - f_{\boldsymbol{k}''}) \langle \boldsymbol{k}' \uparrow | \mathcal{H}_{sd} | \boldsymbol{k}'' \sigma \rangle \langle \boldsymbol{k}'' \sigma | \mathcal{H}_{sd} | \boldsymbol{k} \uparrow \rangle + f_{\boldsymbol{k}''} \langle \boldsymbol{k}'' \sigma | \mathcal{H}_{sd} | \boldsymbol{k} \uparrow \rangle \langle \boldsymbol{k}' \uparrow | \mathcal{H}_{sd} | \boldsymbol{k}'' \sigma \rangle \Big]$$

- without spin flip → small temperature independent contribution (only terms with S_z contribute)
- with spin flip → Kondo effect

Kondo resistance

algebraic calculation leads to $t^{(2)} = J^2 S_z \sum_{\boldsymbol{k}''} \frac{2 f_{\boldsymbol{k}''} - 1}{E(\boldsymbol{k}) - E(\boldsymbol{k}'')}$

in addition: $D(E) \approx D(E_\mathrm{F})$ $\qquad f_{\boldsymbol{k}''} \longrightarrow$ step function ($T = 0$)

$\delta E < |E_\mathrm{F} \pm \mathcal{D}|$ $\qquad \sum \to \int$

$$t^{(1)} + t^{(2)} = -J S_z \left[1 - 2\, J\, D(E_\mathrm{F}) \ln \frac{\mathcal{D}}{|E_\mathrm{F} - E(\boldsymbol{k})|} \right]$$

Scattering of Conduction Electrons on Localized Magnetic Moments

scattering probability $w(\boldsymbol{k}\uparrow,\boldsymbol{k}'\uparrow) = t_{\mathrm{K}}^2$

$$\Rightarrow \quad w(\boldsymbol{k}\uparrow,\boldsymbol{k}'\uparrow) \propto J^2\, S_z^2 \left[1 - 4\,J\,D(E_{\mathrm{F}})\,\ln\frac{\mathcal{D}}{|E_{\mathrm{F}} - E(\boldsymbol{k})|} + \quad \ldots \quad\right]$$

terms of the order of $\mathrm{O}(J^4)$ are omitted, integration over all vectors and energies with $E(\boldsymbol{k}) \approx E_{\mathrm{F}} \pm k_{\mathrm{B}}T$

$$\Rightarrow \quad \varrho(T) \propto \varrho_0 \left[1 - 4J\,D(E_{\mathrm{F}})\,\ln\frac{\mathcal{D}}{k_{\mathrm{B}}T}\right]$$

$J < 0$ ⟶ increase with decreasing temperature

adding the lattice contribution $\varrho_{\mathrm{ph}} = aT^5$ ⟶ total resistance

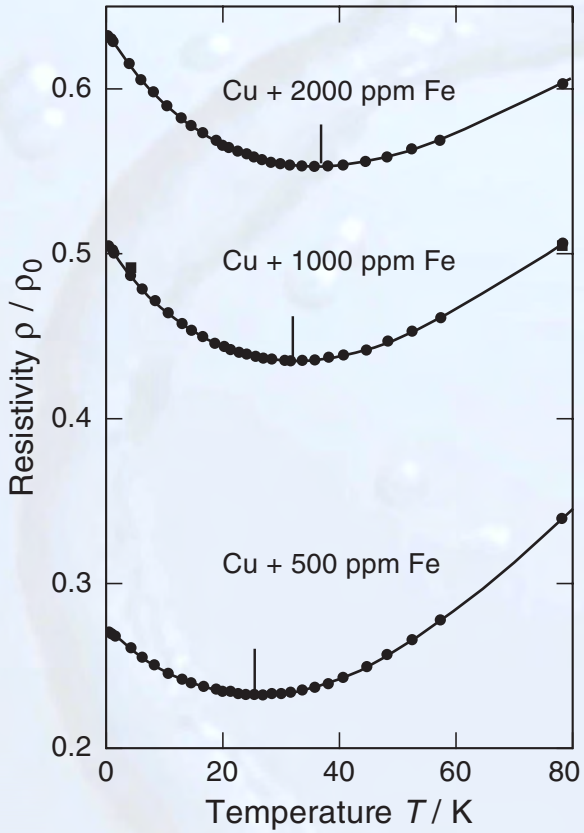
$$\Rightarrow \quad \varrho = aT^5 + c\varrho_0 + c\varrho_1\,\ln\frac{\mathcal{D}}{k_{\mathrm{B}}T}$$

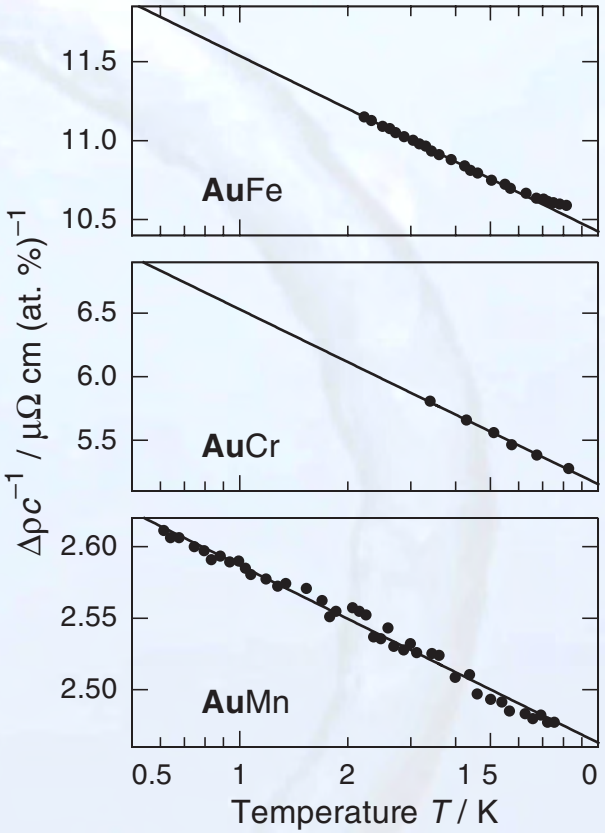
concentration of magnetic impurities

minimum expected at $T_{\min} = \left(\frac{c\varrho_1}{5a}\right)^{1/5}$

Scattering of Conduction Electrons on Localized Magnetic Moments

experimental observations:





► concentration-dependent minimum

► logarithmic temperature dependence

Scattering of Conduction Electrons on Localized Magnetic Moments

dependence for $T \to 0$? ⟶ logarithmic divergence is nonphysical

↗ $T > T_\mathrm{K}$ weak coupling regime $(-\ln T)$

↘ $T < T_\mathrm{K}$ strong coupling regime

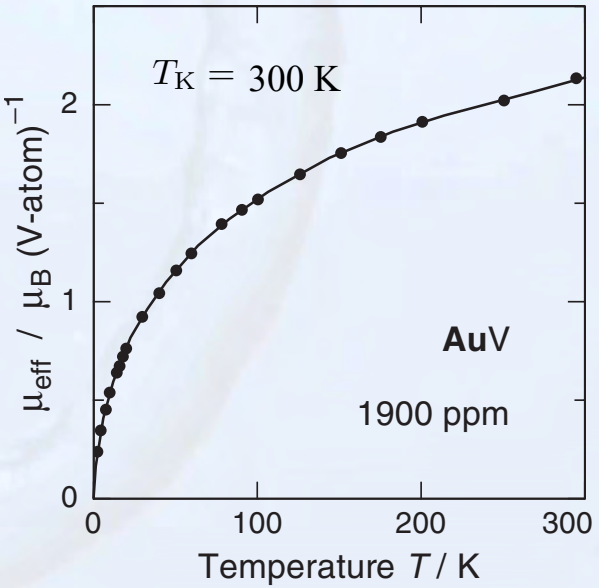
Kondo temperature: $T_\mathrm{K} \approx T_\mathrm{F}\, \mathrm{e}^{-1/JD(E_\mathrm{F})}$

strong coupling regime

- strong screening through surrounding conduction electrons
  - ⟶ spin-compensated singlet ground state
  - ⟶ coherent Kondo state
- transition from a magnetic to a non-magnetic system
- energy necessary to form the spin-compensated cloud: $k_\mathrm{B}T_\mathrm{K}$
- maximum in specific heat of Kondo systems at T_K
- magnetic moment disappears below T_K

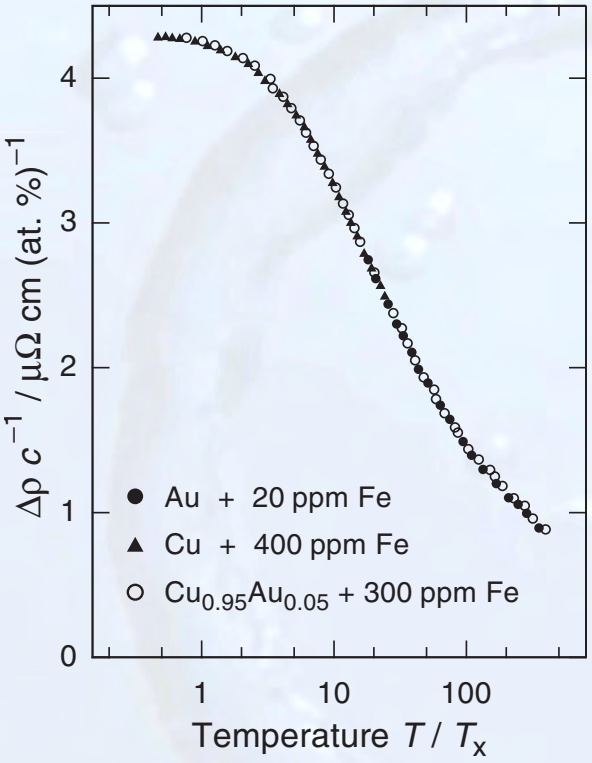
T_K ↗ **Cu**Mn 1 mK

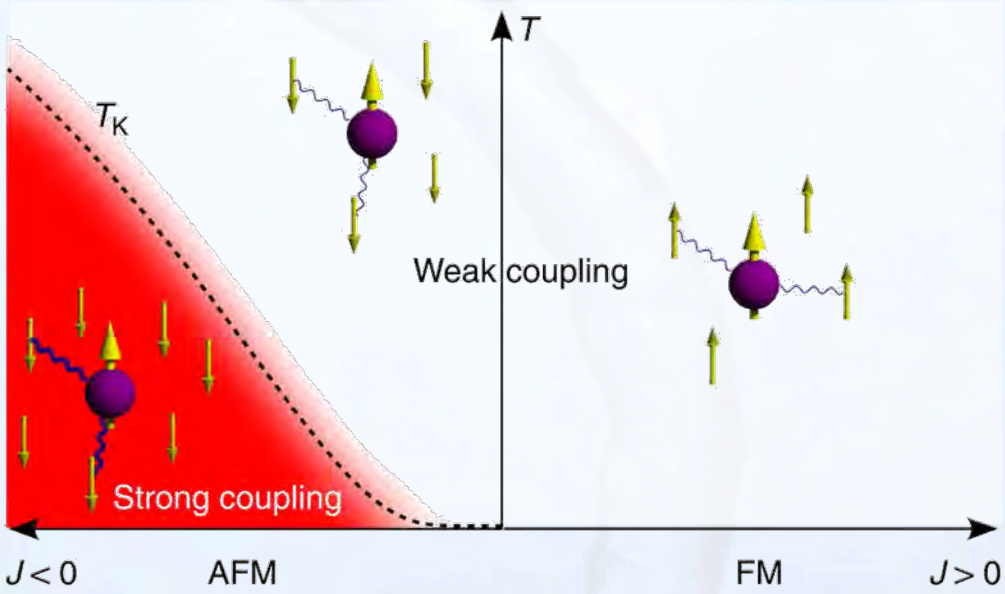
T_K ↘ **Al**Mn 1000 K



Scattering of Conduction Electrons on Localized Magnetic Moments

resistivity flattens towards low temperatures





- normalized temperature dependence
- Kondo resistance is scalable by $T/T_x \approx T/T_K$ and $\Delta\varrho/c$

Quantized Electrical Conductance – 1D Conductors

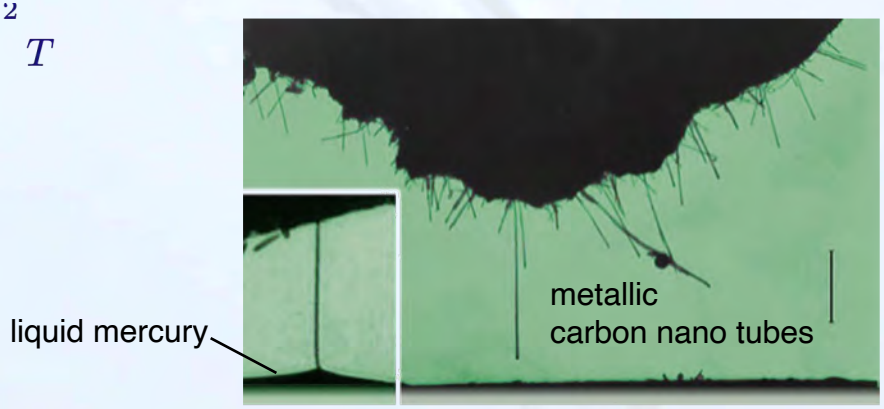
Starting point

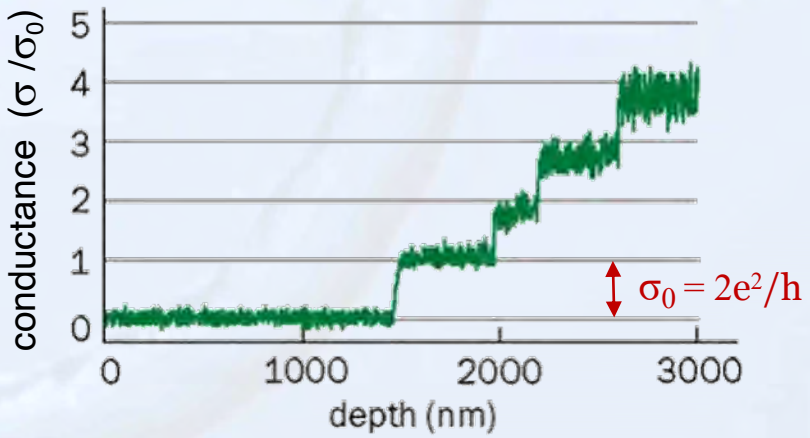
quantized thermal conductivity: $\Lambda_0 = \frac{\pi^2}{3}\frac{k_B^2 T}{h}$

Wiedemann-Franz law: $\frac{\Lambda_{el}}{\sigma} = \frac{\pi^2}{3}\left(\frac{k_B}{e}\right)^2 T$

$$\rightarrow \quad \widetilde{\sigma_0} = \frac{\Lambda_0}{(\pi^2/3)(k_B^2/e^2)T} = \frac{e^2}{h}$$

2 spin directions: $\boxed{\sigma_0 = 2\widetilde{\sigma_0} = \frac{2e^2}{h}}$





magnetic order of nuclear spins

first observation 1969 in CaF_2 (Chapellier, Goldman, Chan, Abragam)

^{19}F

isolators: dipole-dipole interaction $E \propto \frac{\mu_n^2}{r^3}$

metals: dipole-dipole and indirect exchange interaction $H = H_{dip} + H_{RK}$

Ruderman Kittel

- weak electron-nuclei coupling $H_{RK} \approx H_{dip}$
- strong electron-nuclei coupling $H_{RK} > H_{dip}$

metals:

strength of coupling important for relaxation time τ

→ Korringa relation: $\tau = \frac{\kappa}{T_e}$

8. Magnetic Moments – Spins

a) strong electron-nuclei coupling

nuclear ordering in thermal equilibrium ($T_e = T_n$): there are only a few examples

- solid ^{3}He antiferromagnetic ordering at 0.9 mK
- $PrCu_6$, $PrNi_5$, … ferromagnetic ordering at 2.5 mK, 0.4 mK, ….

  Van Vleck – paramagnets ⟶ magnetic field at the nuclei are enhanced by hyperfine interaction through polarization of the electrons

- $AuIn_2$ is the only non-hyperfine-enhanced compound showing nuclear ordering in thermal equilibrium

  I = 9/2, μ = 5.5 μ_n, small Korringa constant κ = 0.09 Ks (pure In)

  pure In: tetragonal ⟶ nuclear quadruple interaction ⟶ suppresses nuclear order

  and B_c = 28 mT (superconductivity) ⟶ $B_{ext} > B_c$ to stay normal conducting

  ⟶ demagnetization has to stop above B_c

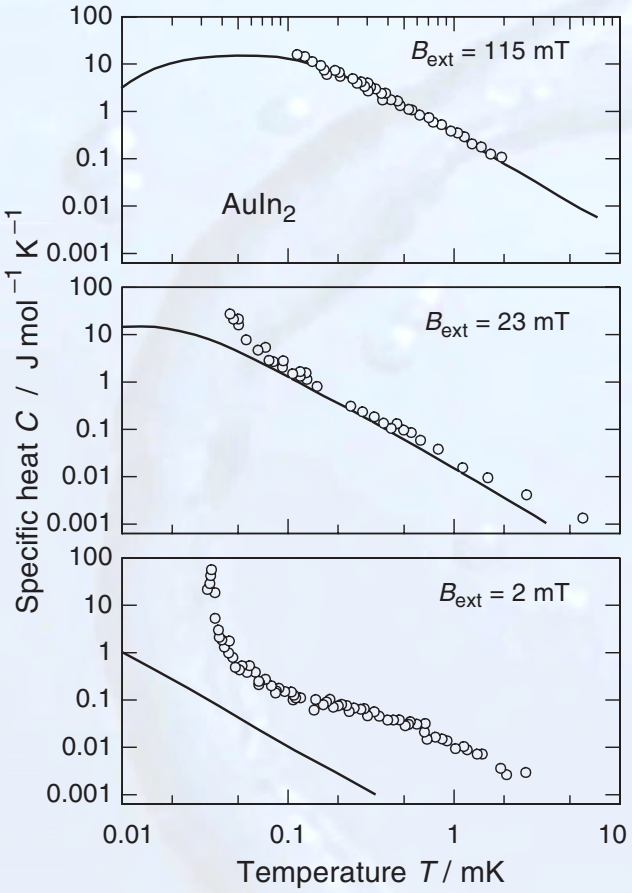
  ⟶ cannot demagnetized as deep

  $AuIn_2$: fcc lattice, B_c = 1.45 mT , κ = 0.11 Ks (similar to pure In)

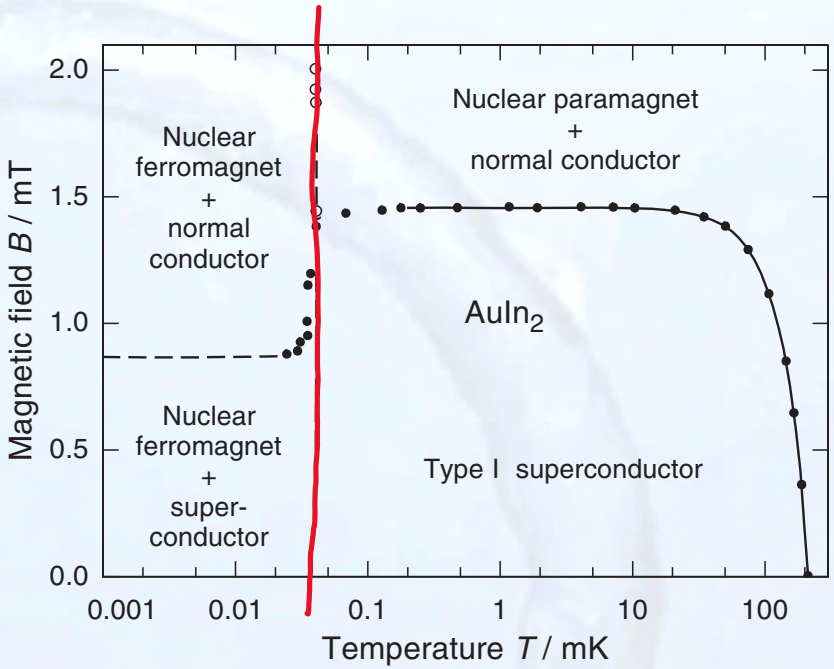
  no quadruple interaction

  100 μK ⟶ ~ 1000 s (doable)

8. Magnetic Moments – Spins



Phase diagram AuIn$_2$



- B_{ext} = 2 mT ⟶ phase transition at 35 µK
- heat capacity is very large in absolute terms
- this phase transition is suppressed at higher fields

- the phase transition at 35 µK is related to nuclear ferromagnetic ordering
- as a consequence, the critical field for superconductivity is partially suppressed

8. Magnetic Moments – Spins

b) weak electron-nuclei coupling $H_{\mathrm{RK}} \approx H_{\mathrm{dip}}$

⟶ nuclear order expected in the nK range

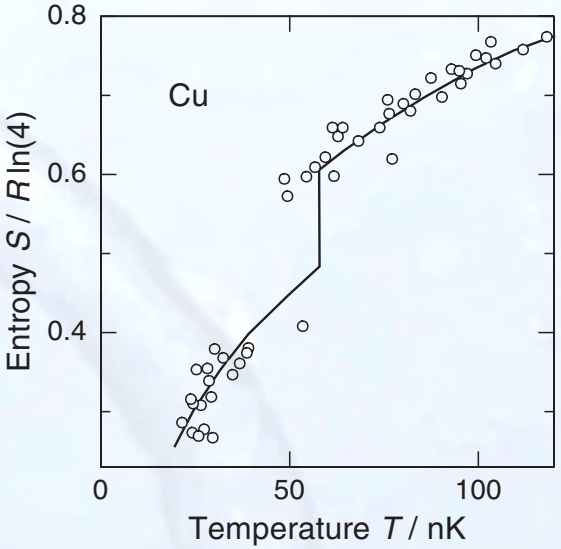
not yet possible in thermal equilibrium

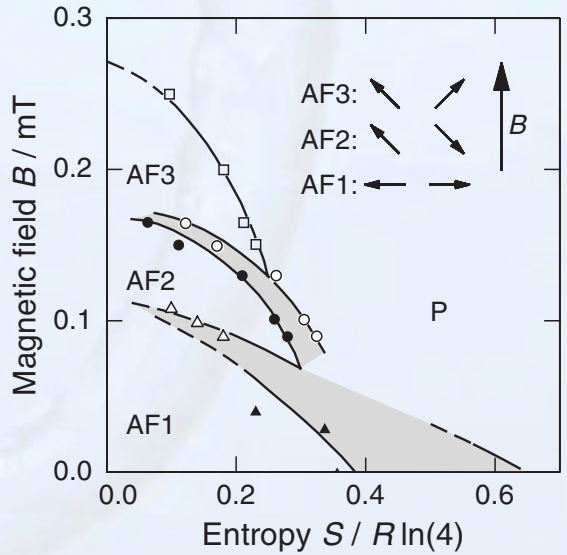
Example: Copper, I = 3/2, 69.1% ^{63}Cu, 30.9% ^{65}Cu

Korringa constant κ = 1.2 Ks

⟶ 100 μK ⟶ ~ 3 hours

fast cooling: $T_{\mathrm{n}} \neq T_{\mathrm{e}}$





- phase transition at $T_{\mathrm{n}} = 58$ nK
- first order phase transition ⟶ reduction of entropy
  ⟶ antiferromagnetic phase
- in magnetic fields 3 different antiferromagnetic phases

Negative temperatures

1938 Casimir, DuPré : concept

1951 Purcell, Pound : first realization LiH

$\tau_1 = 5$ min, $T = -1$ K

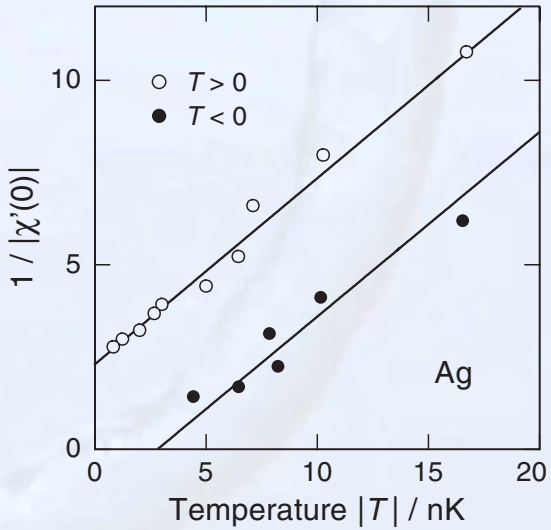
prerequisites:

- system with finite number of levels
- thermal equilibrium within the system $\tau_1 \gg \tau_2$

example: silver: $T_e = 200\ \mu K$ $\quad \tau_1 = 5 \times 10^4\,\mathrm{s}$

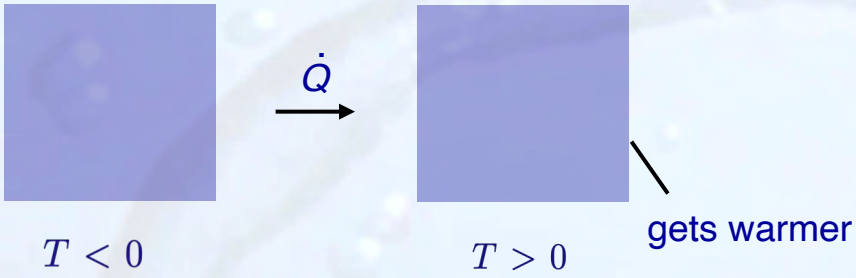
$\tau_2 = 10\,\mathrm{ms}$

susceptibility at positive and negative temperature



Thermodynamic at negative temperatures

negative temperatures are warmer than positive temperatures

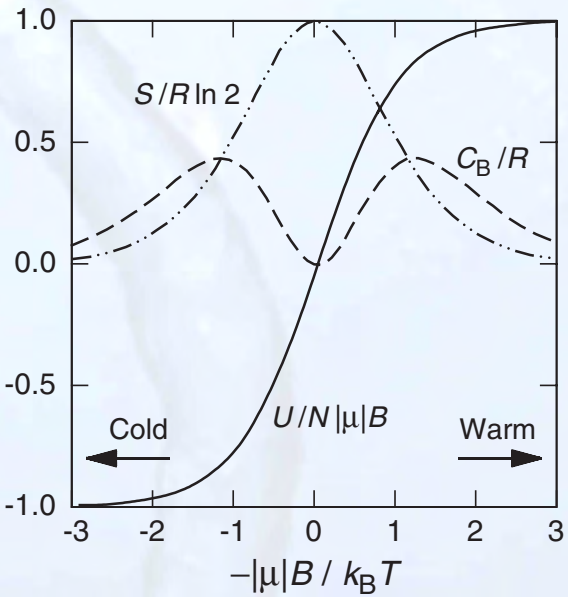


$$+0\,\mathrm{K} \rightarrow 300\,\mathrm{K} \rightarrow \pm\infty\,\mathrm{K} \rightarrow -300\,\mathrm{K} \rightarrow -0\,\mathrm{K}$$

system cannot be cooled further, it cannot release any energy

system cannot be warmed further, it cannot take up any energy

internal energy, entropy, specific heat of two-level systems at positive and negative temperatures



NMR: absorption → stimulated emission (no laser)

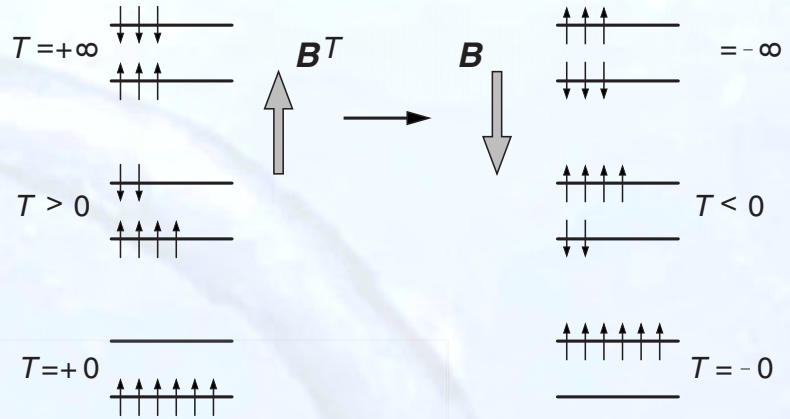
TS term in free energy $F = U - TS$ changes sign → spin system maximizes free energy!

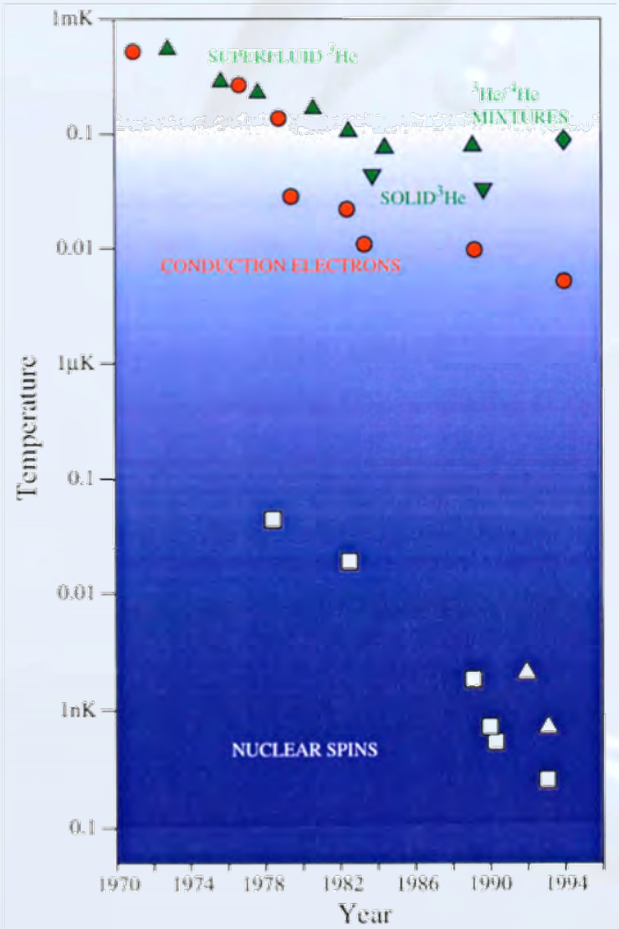
systems that orders antiferromagnetically at positive temperatures

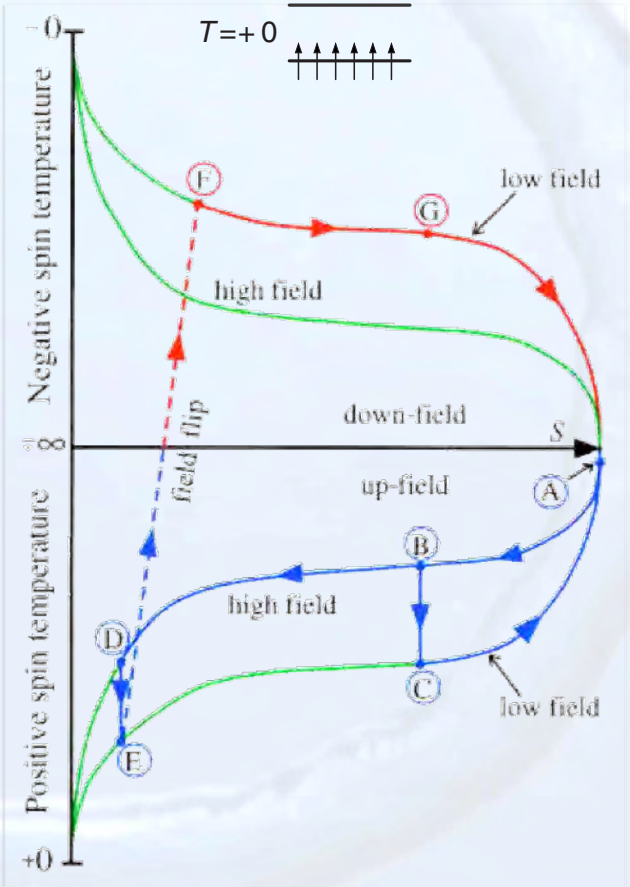
order ferromagnetically at negative temperatures and vice versa

Production of negative temperatures

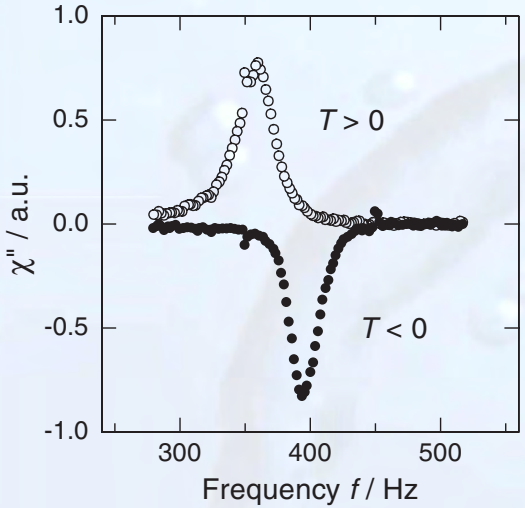
- precool system in magnetic field
- isolate system sufficiently
- adiabatic reversal of magnetic field







NMR experiments

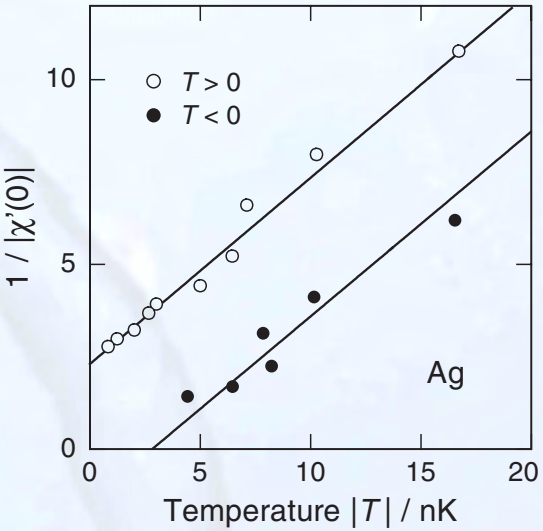


- normal NMR absorption $T > 0$
- stimulated emission $T < 0$

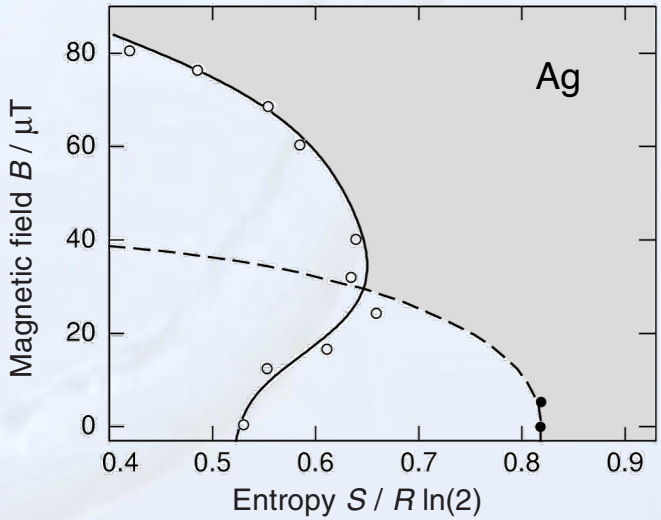
$T > 0$ anti-ferromagnetic order
$T_\mathrm{N} = 560 \pm 60\,\mathrm{pK}$

$T < 0$ ferromagnetic order
$T_\mathrm{c} = -1.9 \pm 0.4\,\mathrm{nK}$

Nuclear Suceptibility



Phase diagram of silver at positive and negative temperature



antiferromagnetic phase region $T > 0$

ferromagnetic phase region $T < 0$

paramagnetic phase region