7. Conduction Electrons

Drude (1900) ⟶ Sommerfeld (1927) ⟶ Bloch (1940)

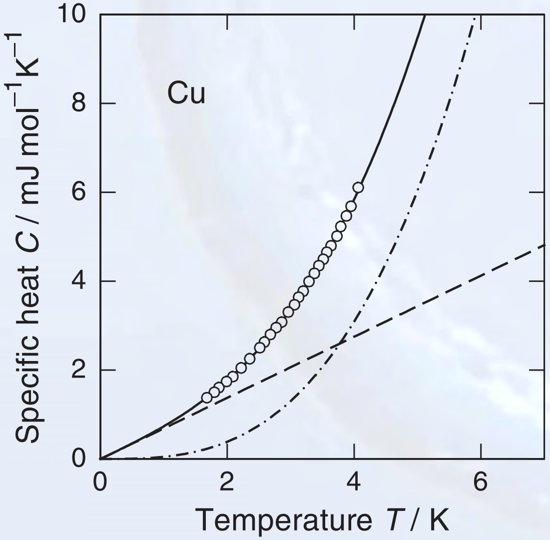
7.1 Specific heat

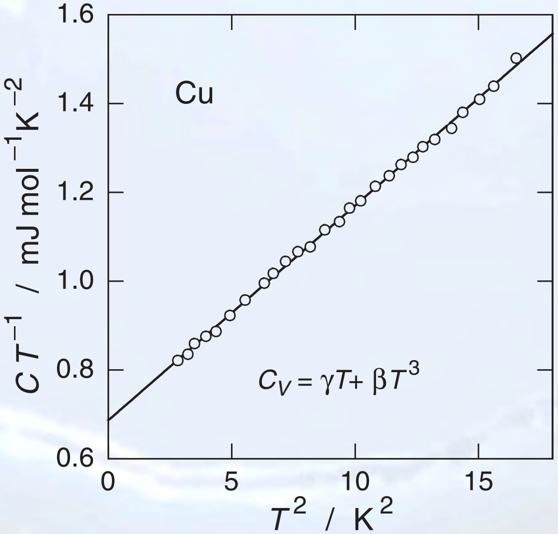
a) simple metals ⟶ free electrons gas

phonons

free electrons gas: $c_V = \gamma T + \beta T^3$

electrons: $\gamma = \dfrac{\pi^2 n k_\mathrm{B}^2}{2E_\mathrm{F}} \propto n^{1/3}\, m^*_\mathrm{th}$





- electrons dominate below ~ 4 K
- very good qualitative agreement

7.1 Specific Heat

Element	γ_{exp}	γ_{theo}	m^*_{th}/m	Element	γ_{exp}	γ_{theo}	m^*_{th}/m
Ag	0.64	0.64	1.00	Cu	0.69	0.50	1.37
Al	1.35	0.91	1.48	Ga	0.60	1.02	0.59
Au	0.69	0.64	1.08	In	1.66	1.26	1.31
Ba	2.70	1.95	1.38	K	2.08	1.75	1.19
Be	0.17	0.49	0.35	Li	1.65	0.75	2.19
Ca	2.73	1.52	1.80	Mg	1.26	1.00	1.26
Cd	0.69	0.95	0.73	Na	1.38	1.3	1.22
Cs	3.97	2.73	1.46	Pb	2.99	1.50	1.99

→ good qualitative agreement for simple metals

→ $\gamma_{\text{exp}}/\gamma_{\text{theo}} = m^*_{\text{th}}/m$ for quantitative agreement

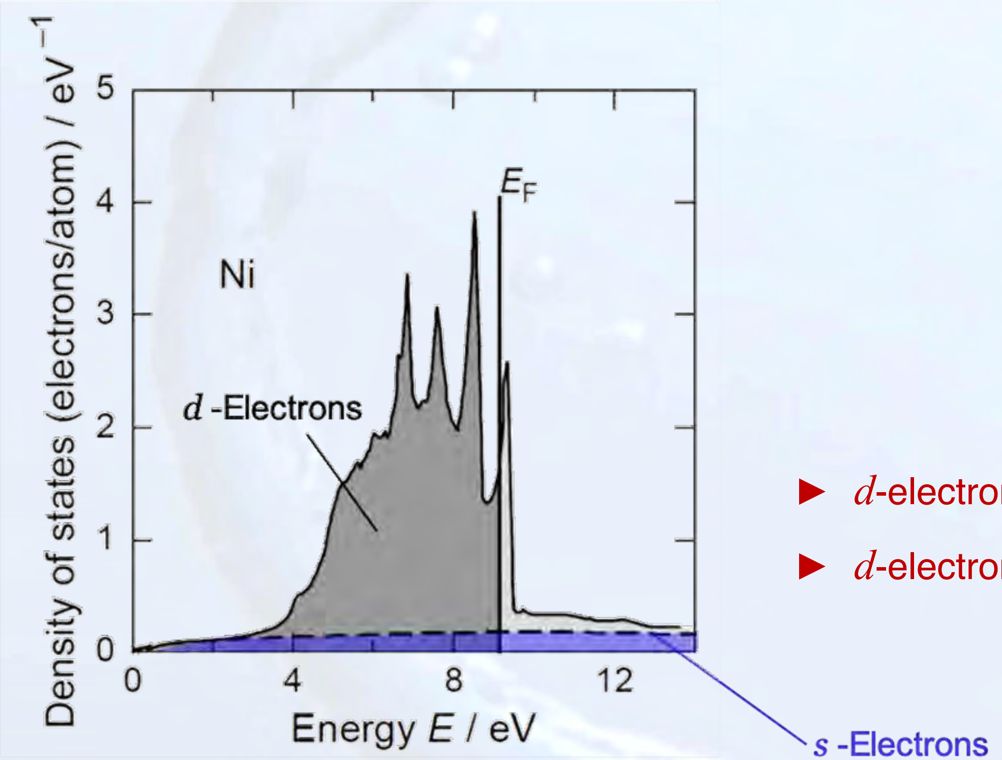
7.1 Specific Heat

but: transition series metals

example nickel: $m^*_{\mathrm{th}} \approx 15\,m$ $\longrightarrow$ reason is *d*-electrons contribute, which are not (completely) free

involved in covalent bond, highly oriented

$\longrightarrow$ no spherical Fermi surface

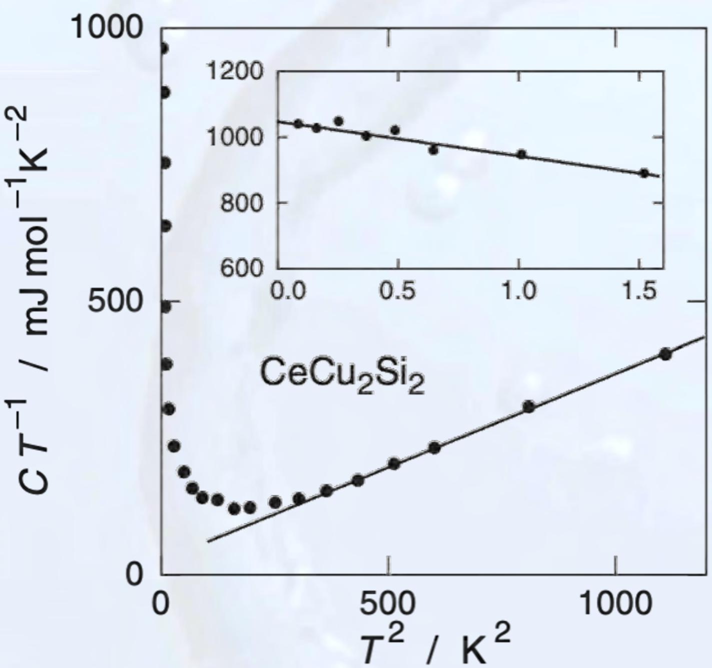


- d-electrons with large density of state dominate at E_F
- d-electrons are localized

7.1 Specific Heat

b) metal with heavy electrons

examples: $CeCu_2Si_2$ cer electronic configuration [Xe] $5d^1\ 4f^1\ 6s^2$



➡ γ not constant below 15 K

$\gamma = C_{el}/T$ → $\gamma \approx 30\,\mathrm{mJ\,(mol\,K^2)^{-1}}$ extrapolated from high T

$\gamma = C_{el}/T$ → $\gamma \approx 1050\,\mathrm{mJ\,(mol\,K^2)^{-1}}$ at low temperatures $T \rightarrow 0$

➡ $4f$ electrons are localized at high T and form a conduction band at low T

➡ effective mass: $m^* \approx 100\,m_e$

- $T > 15$ K, $D(E)$ and m^* are constant
- $T < 15$ K, C/T increase strongly with decreasing temperature

7.1 Specific Heat

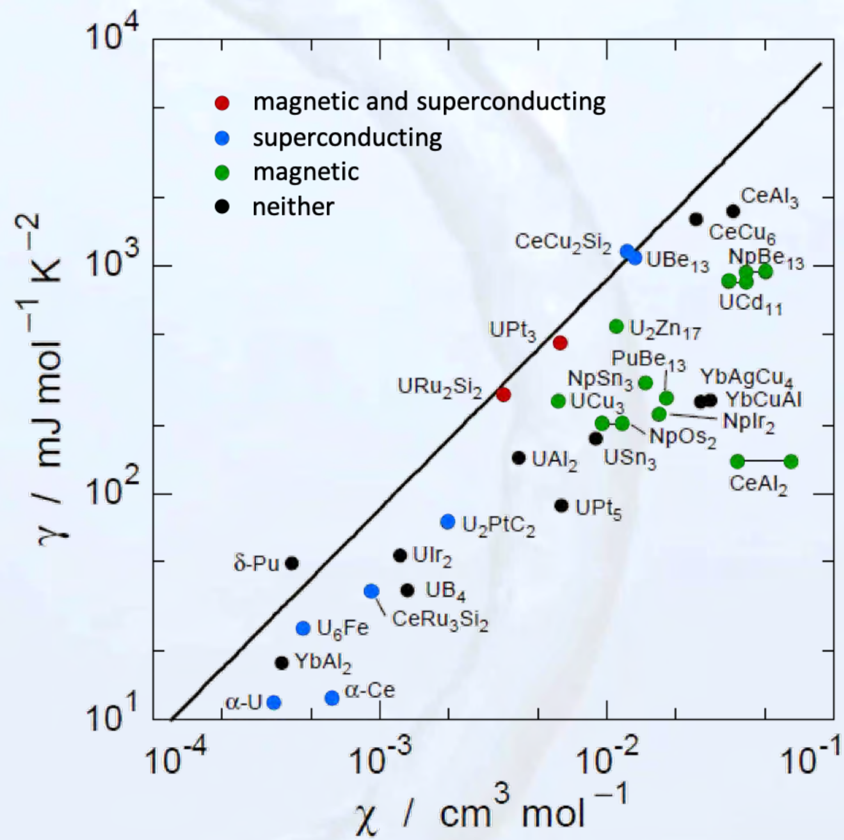
Heavy fermion systems

- interesting class of solids with strongly correlated electrons
- effective masses m^* up to 2000 m_e observed
- origin: interaction with localized spins

$$\chi = \mu_0 \mu_B^2 D(E_F) \propto n^{1/3} m^*$$

Wilson ratio: $$R = \frac{\chi}{\gamma} \frac{\pi^2 k_B^2}{\mu_0 \mu_{\mathrm{eff}}^2}$$

$$\gamma \propto n^{1/3} m^*_{\mathrm{th}}$$



important: Fermi liquid theory

$$C = \frac{m^*}{m} C_{FG} = \left(1 + \frac{1}{3} F_1\right) C_{FG}$$

analogy to ^{3}He reaches even further

⟶ some heavy fermion systems show unconventional superconductivity (S ≠ 0) : UPt_3, URu_2Si_2 …

7.2 Electrical Conductivity

metals, no superconductors, no semiconductors

Boltzmann equation ⟷ kinetic gas theory

- starting point: equilibrium distribution without external fields $f_0(\boldsymbol{k})$

Fermi-Dirac distribution

- with field: stationary non-equilibrium value of $f(\boldsymbol{k}, \boldsymbol{r}, t)$
- expand $f_0(\boldsymbol{k}) - f(\boldsymbol{k}, \boldsymbol{r}, t)$ in linear order + relaxation ansatz for collisions

→ linearized Boltzmann equation $f(\boldsymbol{k}) \approx f_0(\boldsymbol{k}) + \frac{e\tau(\boldsymbol{k})}{\hbar}\, \boldsymbol{\mathcal{E}} \cdot \frac{\partial f_0(\boldsymbol{k})}{\partial \boldsymbol{k}}$

scattering time

electric field

→ $$j_x = -e \int D(k)\, v_x(k)\, f(k)\, \mathrm{d}k = -\frac{e}{\pi^2} \int k^2\, v_x(k)\, f(k)\, \mathrm{d}k$$

→ $$\sigma = \frac{1}{3}\, e^2\, D(E_\mathrm{F})\, v_\mathrm{F}^2\, \tau(E_\mathrm{F}) \longrightarrow \boxed{\sigma = \frac{n\, e^2}{m}\, \tau(E_\mathrm{F})}$$

7.2 Electrical Conductivity

scattering time determined by:

- defect scattering
- phonon scattering
- magnon scattering (in ferromagnets)
- electron-electron scattering (can be neglected in most cases)

a) defect scattering

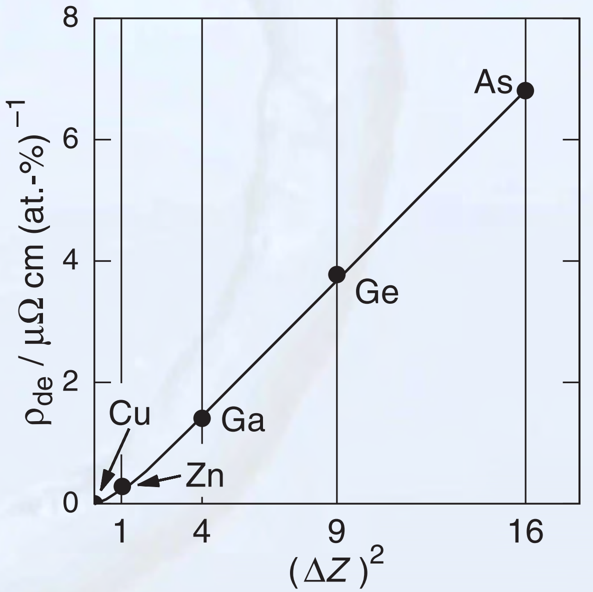
↗ local charge density variations

↘ local strain fields (less important)

Local charge variations

- Rutherford scattering on ionic cores of impurity atoms
- scattering cross section : $\sigma_{\mathrm{cross}} \propto (\Delta Z)^2$
- resistivity $\varrho_{\mathrm{D}} \propto (\Delta Z)^2$

- residual resistance of copper with 1 at% impurities with different valence electrons configurations
- agrees well with: $\varrho_{\mathrm{D}} \propto (\Delta Z)^2$



7.2 Electrical Conductivity

Concentration dependence

example: binary mixture $\mathrm{A}_x\mathrm{B}_{1-x}$ $\longrightarrow$ mixing increases the resistance

average potential $U_0 = xU_\mathrm{A} + (1-x)U_\mathrm{B}$

deviations at atoms A and B: $(U_0 - U_\mathrm{A}) = (1-x)(U_\mathrm{B} - U_\mathrm{A})$

$$(U_0 - U_\mathrm{B}) = x(U_\mathrm{A} - U_\mathrm{B})$$

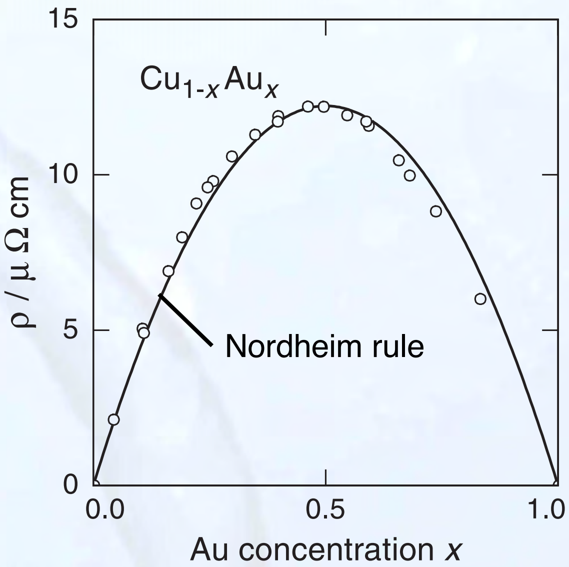
scattering probability:

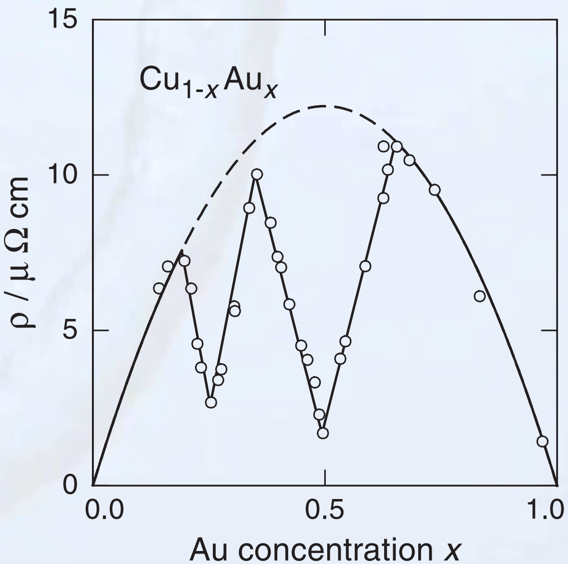
$$w_\mathrm{A} = (1-x)^2 \left| \int \psi^*(\boldsymbol{k})\,(U_\mathrm{B} - U_\mathrm{A})\psi(\boldsymbol{k}')\,\mathrm{d}^3k' \right|^2$$

$$w_\mathrm{B} = x^2 \left| \int \psi^*(\boldsymbol{k})\,(U_\mathrm{A} - U_\mathrm{B})\psi(\boldsymbol{k}')\,\mathrm{d}^3k' \right|^2 = \frac{x^2}{(1-x)^2}\,w_\mathrm{A}$$

resistivity: $\varrho_\mathrm{de} \propto x w_\mathrm{A} + (1-x) w_\mathrm{B}$

Nordheim rule: $\varrho_\mathrm{de} \propto x(1-x)$





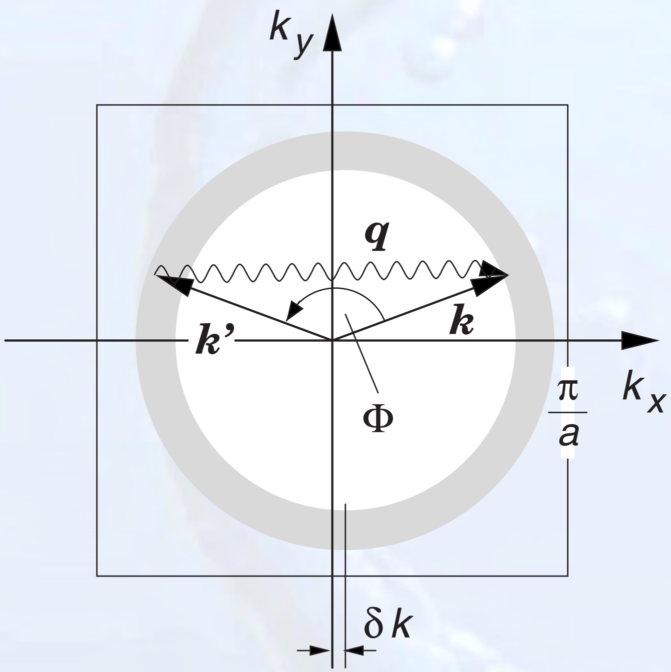
- $Cu_{1-x}Au_x$ data agree with Nordheim rule
- tempering of sample $\longrightarrow$ formation of ordered compounds

7.2 Electrical Conductivity

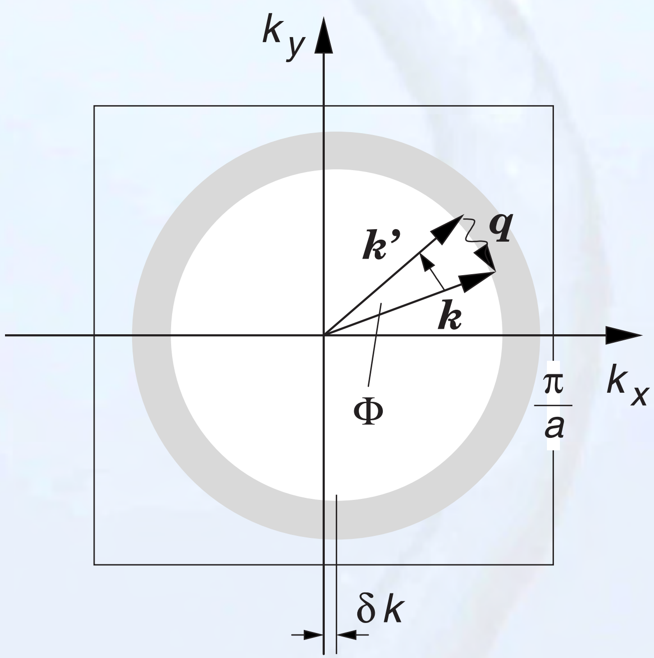
Electron-Phonon scattering

only electrons at the Fermi surface can participate $|\mathbf{k}| \approx |\mathbf{k}'|$.

high temperatures



low temperatures



Electron-Phonon scattering

a) high temperatures ($T > \theta_\mathrm{D}$) $\quad \ell \propto \tau v_\mathrm{F} \longrightarrow \ell^{-1} \propto n_\mathrm{ph} \propto T \longrightarrow \varrho \propto T$

b) intermediate temperatures ($T < \theta_\mathrm{D}$)

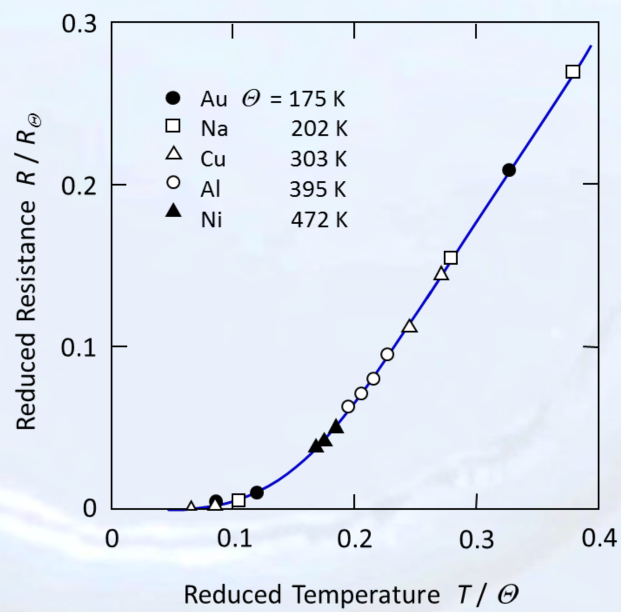
- cross-section depends on temperature $\quad e_0^2 \propto \omega/\Theta \propto T/\Theta$
- number of scattering centers (phonons) reduces $\quad n_\mathrm{ph} \propto T^2/\Theta^2$

$$\left.\begin{array}{l} e_0^2 \propto \omega/\Theta \propto T/\Theta \\ n_\mathrm{ph} \propto T^2/\Theta^2 \end{array}\right\} \tau^{-1} \propto n\Sigma \propto T^3/\Theta^3$$

- effectiveness of scattering process goes down $\quad (1-\cos\phi) \propto T^2/\Theta^2$

$$\longrightarrow \boxed{\varrho_\mathrm{ph} \propto \tau_\mathrm{eff}^{-1} \propto \left(\frac{T}{\Theta}\right)^5}$$

Bloch-Grüneisen law



- reduced plot → material independent
- defect scattering substracted
- good agreement with Bloch-Grüneisen

Small angles ϕ

$\rightsquigarrow (1-\cos\phi) \propto \phi^2$

with $q \simeq \left(\frac{T}{\Theta}\right) q_{max}$

and $k_F = \frac{\pi}{a}$

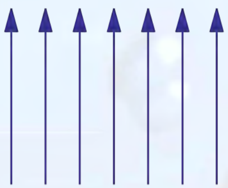
$\rightsquigarrow \phi = \frac{q}{k_F} \simeq \frac{T}{\Theta} \frac{\pi}{a} \frac{1}{k_F} = \frac{T}{\Theta}$

$\rightsquigarrow \phi^2 \propto \left(\frac{T}{\Theta}\right)^2$

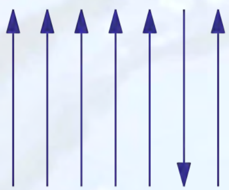
7.2 Electrical Conductivity

Electron magnon scattering

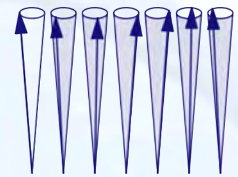
spin waves in ferromagnets



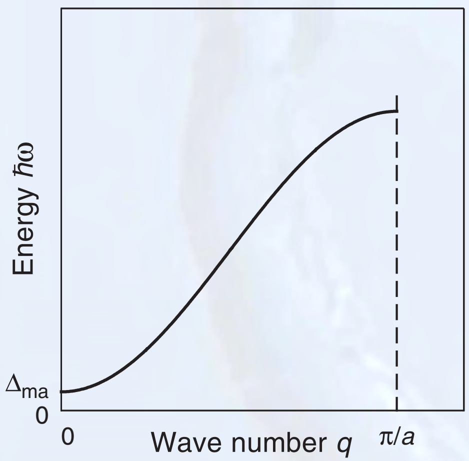
ground state



single spin excitation
(cost to much energy $E \approx k_\mathrm{B} T_\mathrm{C}$)



collective excitations
spin waves $\widehat{=}$ magnons



dispersion curve: (Terbium)

$$\hbar\omega = 2\mathcal{J}S\Big[1 - \cos(qa)\Big] + \Delta_\mathrm{ma}$$

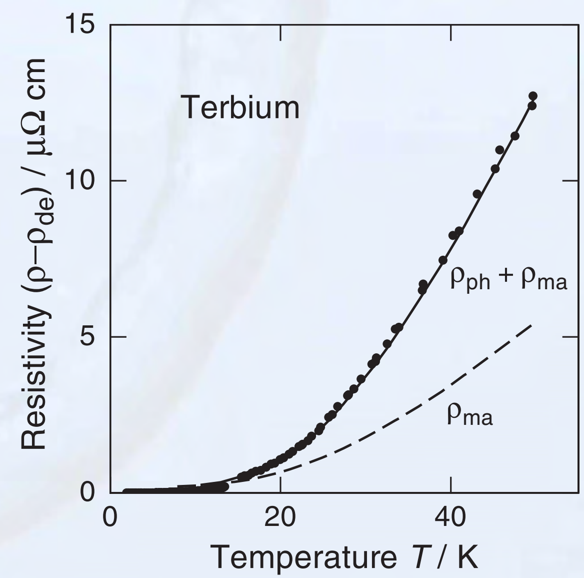
$\Delta_\mathrm{ma}/k_\mathrm{B} \approx 20\,\mathrm{K}$

anisotropic exchange interaction of spins

resistivity: (Terbium)

$$\tau^{-1} \propto n_\mathrm{ma} \propto \mathrm{e}^{-\Delta_\mathrm{ma}/k_\mathrm{B}T}$$

$$\longrightarrow \quad \varrho_\mathrm{ma} \propto \mathrm{e}^{-\Delta_\mathrm{ma}/k_\mathrm{B}T} \,.$$



Comparison of Electrical and Thermal Conductance

Thermal conductivity of a free Fermi gas

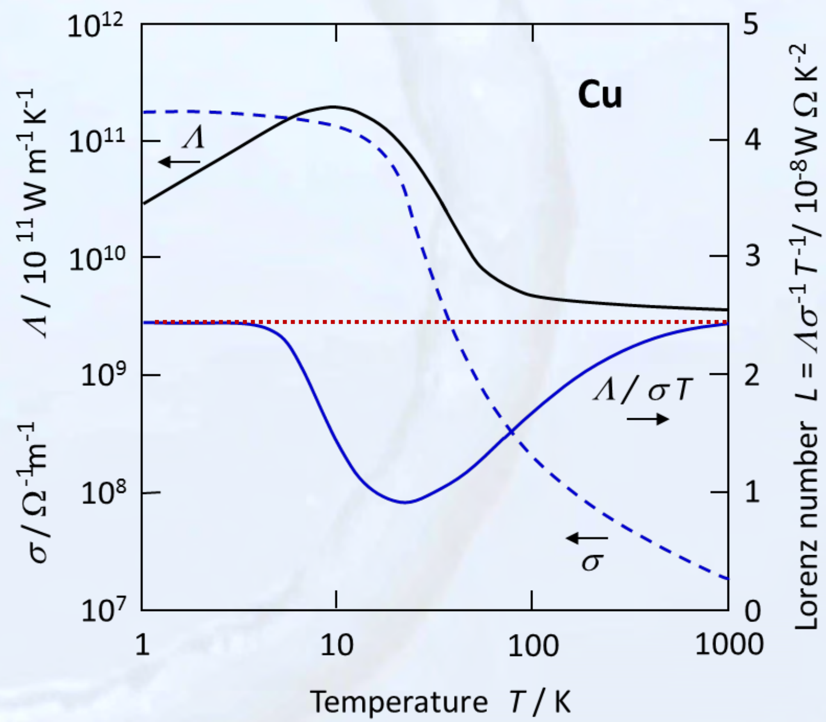
$$\Lambda_{\mathrm{el}} = \frac{1}{3} c_V^{\mathrm{el}} \, v \, \ell = \frac{1}{3} \frac{\pi^2 n k_{\mathrm{B}}^2 T}{m v_{\mathrm{F}}^2} \, v_{\mathrm{F}} \, \ell$$

if scattering processes are identical for electrical and thermal transport

➡ Widemann-Franz law

$$\boxed{\frac{\Lambda_{\mathrm{el}}}{\sigma} = \frac{\pi^2}{3} \left(\frac{k_{\mathrm{B}}}{e} \right)^2 T = \mathcal{L} T}$$

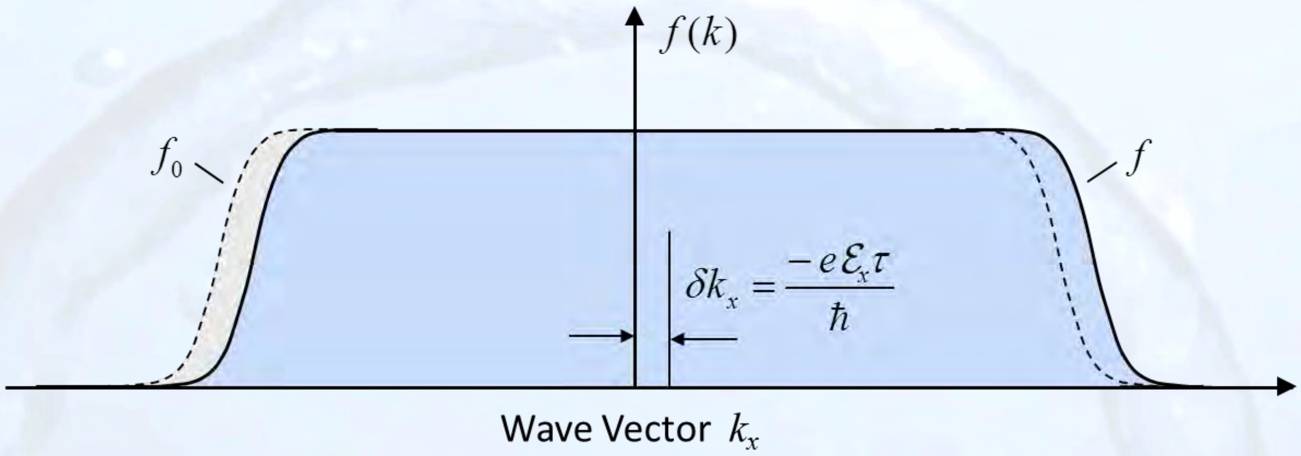
$$\mathcal{L} = 2.45 \times 10^{-8} \, \mathrm{V^2 \, K^{-2}}$$



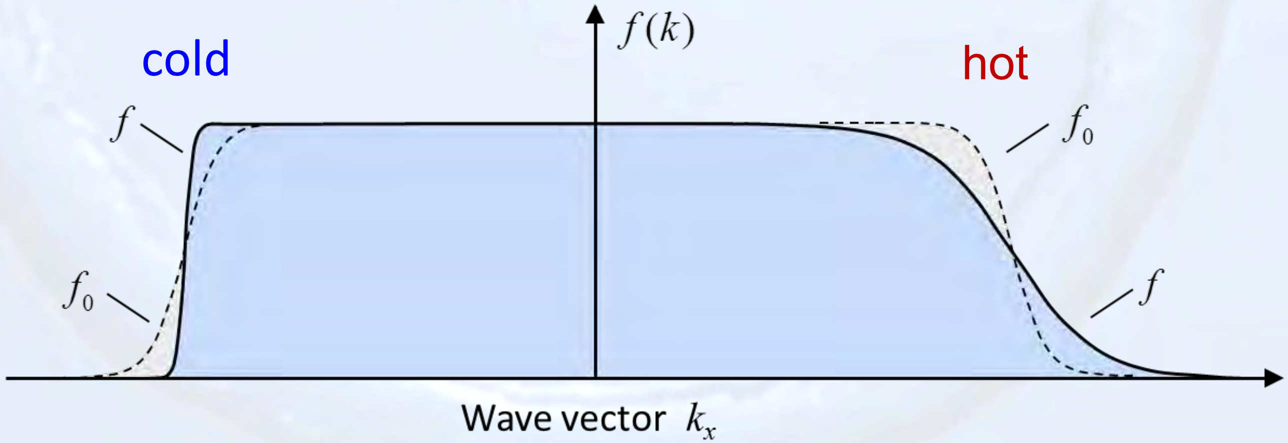
- Lorenz number depends on temperature
- works well at very low and very high temperatures

Comparison of Electrical and Thermal Conductance

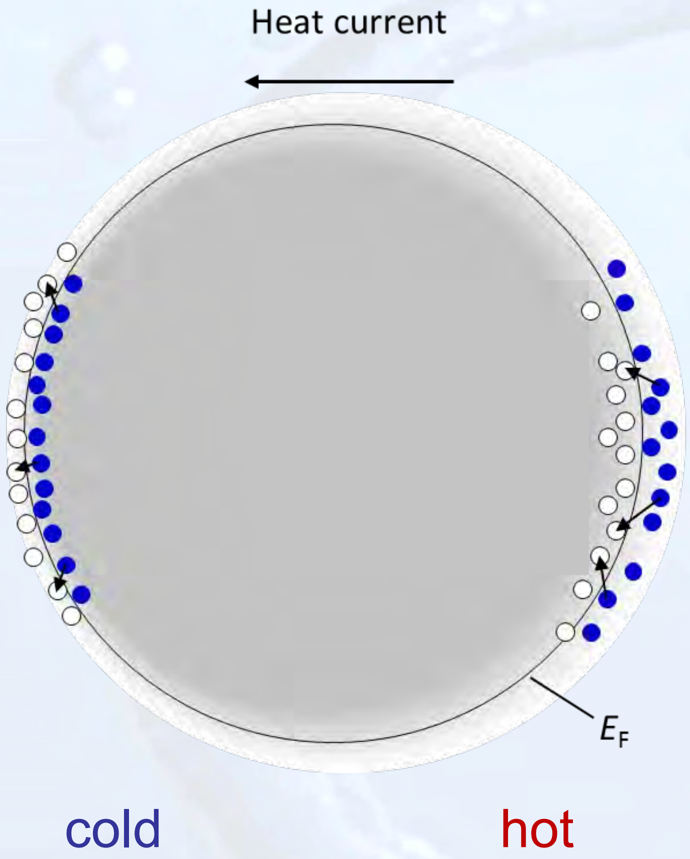
Electric Field

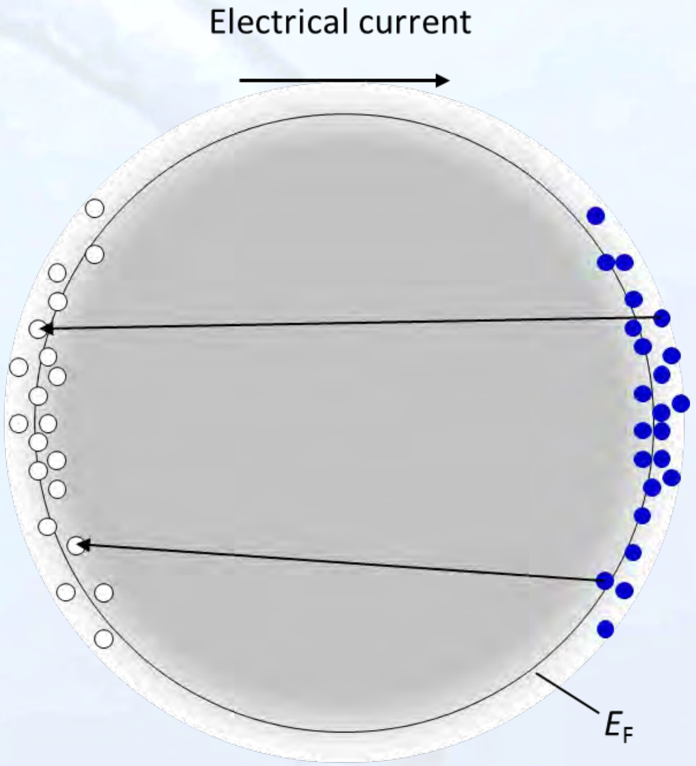


Thermal Gradient



Comparison of Electrical and Thermal Conductance

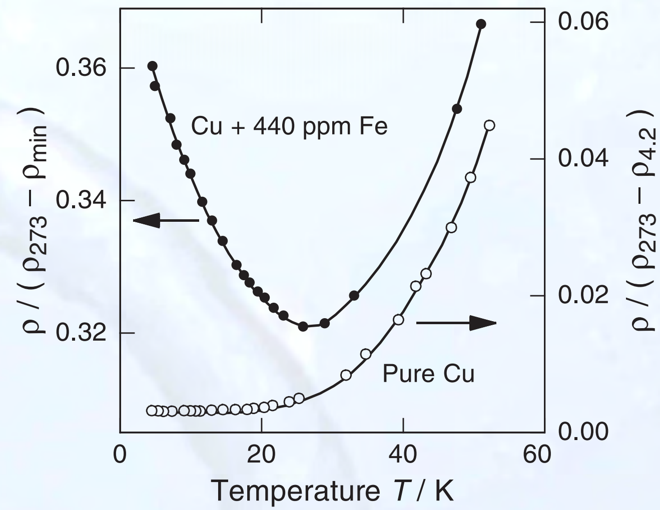




Interaction of Conduction Electrons and Localized Magnetic Moments

1930 Meissner and Voigt observe a resistance minimum for Au and Cu with magnetic impurities

- example: Cu + 440 ppm Fe
- resistance minimum at 27 K



1964 explanation by Kondo by spin dependent scattering of electrons on magnetic impurities

a) Influence of conduction electrons on localized magnetic moments

example: d-levels of transition metals in simple metals

- d-d interaction $\longrightarrow$ splitting and polarization of d-levels, because of crystal field
- interaction of d-electrons with conduction electrons (s) $\longrightarrow$ hybrid states

$\downarrow$

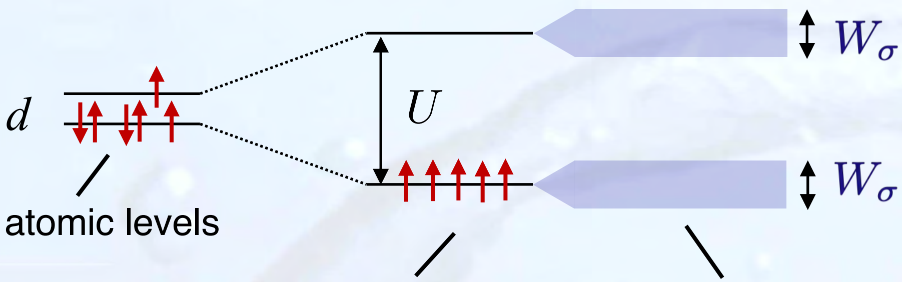
width determined by $s \leftrightarrow d$ transition rate

matrix element

$\longrightarrow$ golden rule: $\dfrac{W_\sigma}{\hbar} = \dfrac{\pi}{\hbar}\,\mathcal{V}^2\, D_s(E_{d\sigma})$

Density of states of s-electrons at $E_{d\sigma}$

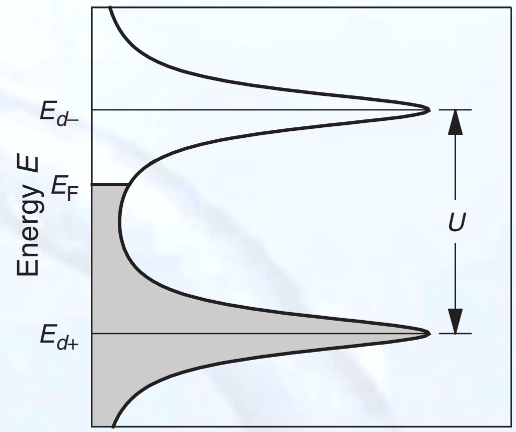
Interaction of Conduction Electrons and Localized Magnetic Moments



atomic levels

crystal field splitting U
(magnetic atoms in a solid)

s-d hybrid states



Density of states $D_d(E)$

AgMn:

$W_\sigma = 0.5\,\mathrm{eV}$

$U = 4.8\,\mathrm{eV}$

localized moments remain if interaction (W_σ) is not too strong $W_\sigma < U$

spin-dependent interaction of s electrons with d-electrons

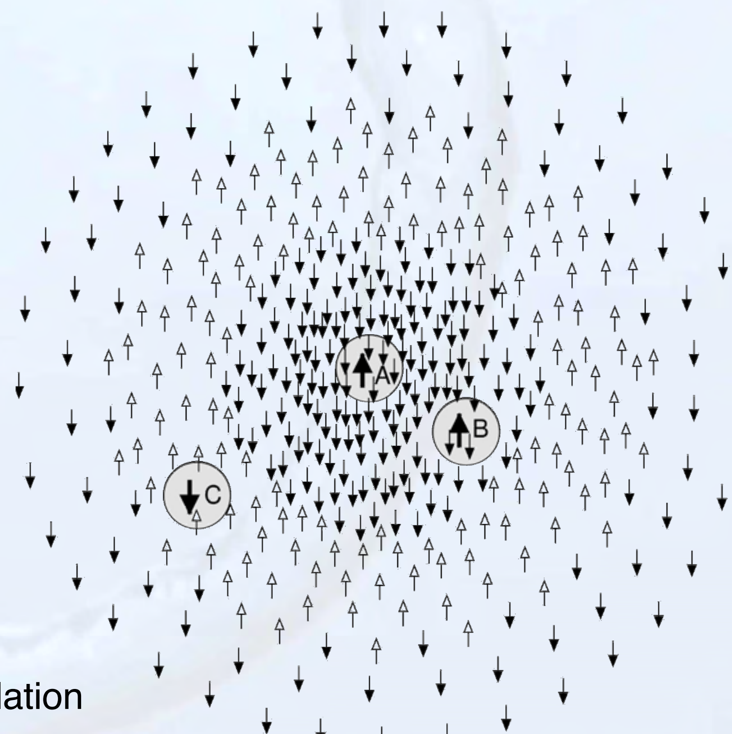
$$J \approx -\mathcal{V}^2/U$$

position of localized spin

$$\mathcal{H}_{sd} = -J\,\boldsymbol{S}\cdot\boldsymbol{s}\;\delta(\boldsymbol{r}-\boldsymbol{R}) \qquad (\star)$$

localized spins

conduction electron



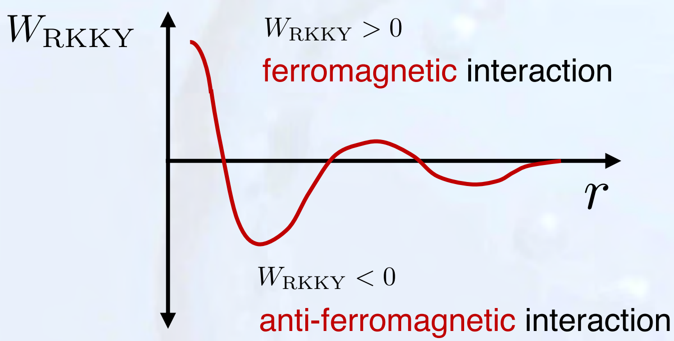
Friedel oscillation

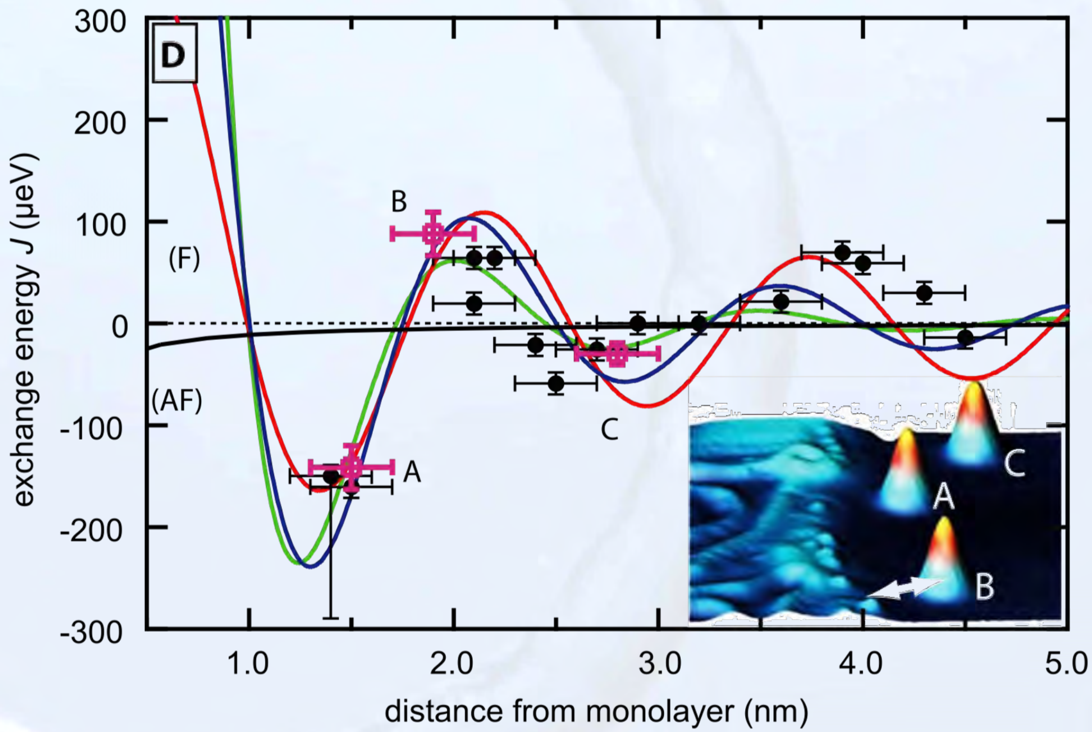
Interaction of Conduction Electrons and Localized Magnetic Moments

Indirect exchange interaction (d-d) – RKKY interaction

Ruderman
Kittel
Kusuya
Yosida

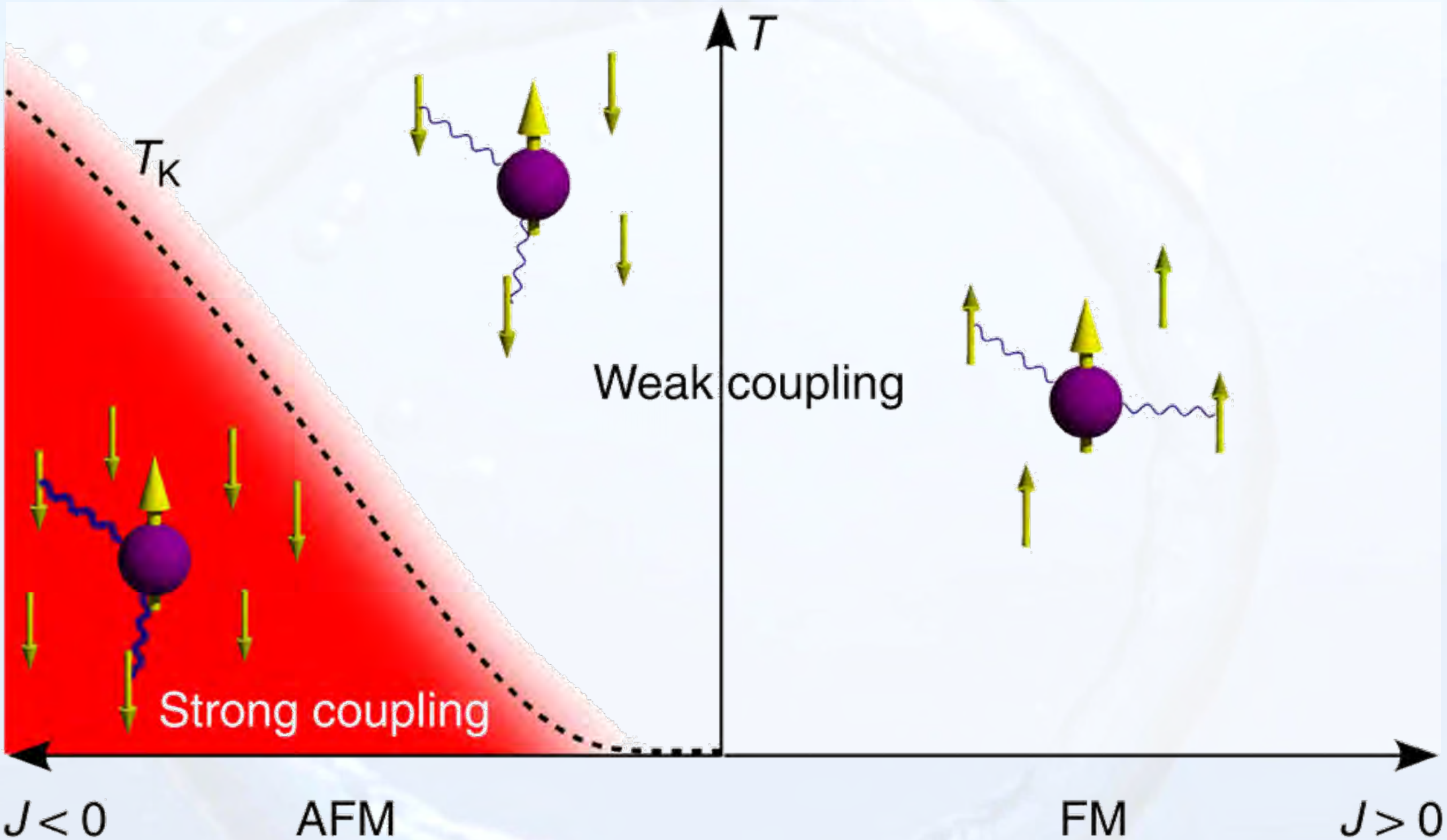
$$W_{\mathrm{RKKY}} \propto J_{\mathrm{RKKY}}^2 D(E_{\mathrm{F}}) \frac{\cos(2k_{\mathrm{F}}r)}{r^3}$$





MFM measurement using magnetic moments on a noble metal surface

Interaction of Conduction Electrons and Localized Magnetic Moments



Scattering of Conduction Electrons on Localized Magnetic Moments

Jun Kondo

electrical resistance → inelastic scattering

electrical resistance → elastic scattering ← Kondo effect

consider (★) in "*N* representation" | $\mathcal{H}_{sd} = -J\,\boldsymbol{S}\cdot\boldsymbol{s}\;\delta(\boldsymbol{r}-\boldsymbol{R})$

$$\mathcal{H}_{sd} = -J \sum_{\boldsymbol{k}\boldsymbol{k}'} S_z(c^+_{\boldsymbol{k}'\uparrow}\,c_{\boldsymbol{k}\uparrow} - c^+_{\boldsymbol{k}'\downarrow}\,c_{\boldsymbol{k}\downarrow}) + S_+ c^+_{\boldsymbol{k}'\downarrow}\,c_{\boldsymbol{k}\uparrow} + S_- c^+_{\boldsymbol{k}'\uparrow}\,c_{\boldsymbol{k}\downarrow}$$

const. < 0!

$c^+_{\boldsymbol{k}}$ creation operator

$c_{\boldsymbol{k}}$ annihilation operator

} act on conduction electrons

$S_+ = S_x + \mathrm{i}S_y$

$S_- = S_x - \mathrm{i}S_y$

} spin states

$\boldsymbol{k}$ wave vector of conduction electrons

Harmonic Oscillator

$H = \hbar\omega\left(a^+a + \frac{1}{2}\right) \quad \Big| \quad H = \frac{P^2}{2m} + \frac{m\omega^2 Q^2}{2}$

$Q = \sqrt{\frac{\hbar}{2m\omega}}\,(a + a^+)$

$P = \frac{1}{i}\sqrt{\frac{m\omega\hbar}{2}}\,(a - a^+)$

$a^+ = \sqrt{\frac{m\omega}{2\hbar}}\left(Q - \frac{i}{m\omega}P\right)$

$a = \sqrt{\frac{m\omega}{2\hbar}}\left(Q + \frac{i}{m\omega}P\right)$

$a^+a = N$

$a^+|\psi_n\rangle = \sqrt{n+1}\,|\psi_{n+1}\rangle$

Scattering of Conduction Electrons on Localized Magnetic Moments

(i) first-order perturbation theory $|\boldsymbol{k}\uparrow\rangle \longrightarrow |\boldsymbol{k}'\uparrow\rangle$

$$t^{(1)} = \langle \boldsymbol{k}'\uparrow|\mathcal{H}_{sd}|\boldsymbol{k}\uparrow\rangle = -J\,S_z$$

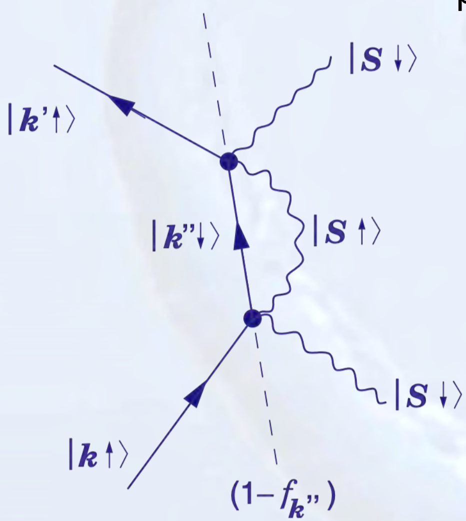
scattering amplitude

temperature independent constant

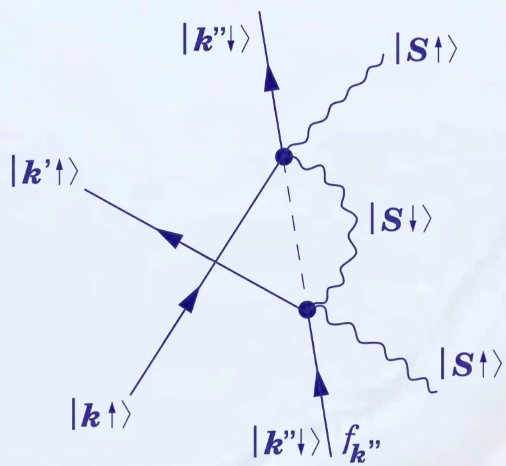
(ii) second-order perturbation theory: two possible process

- direct process (normal order)
- exchange process (reverse order)

processes without spin flip



direct process



exchange process

Scattering of Conduction Electrons on Localized Magnetic Moments

all first and second order processes