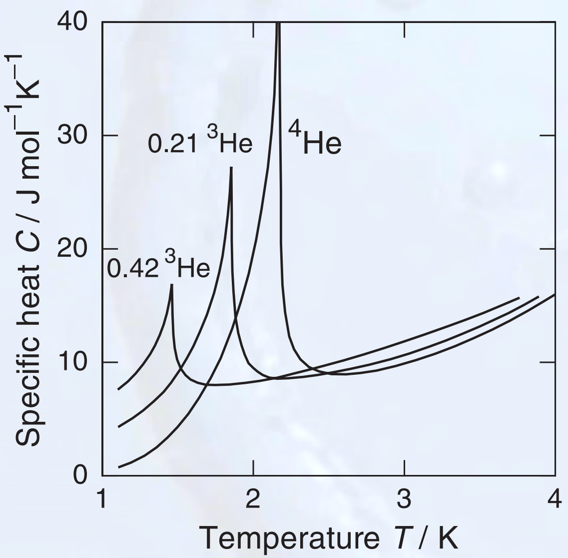
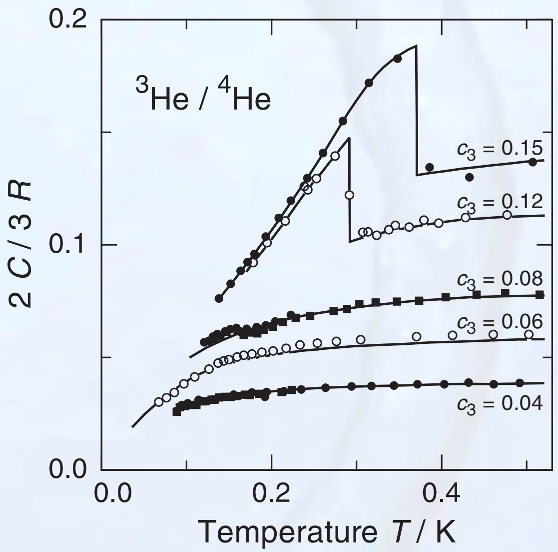
5) ^{3}He/^{4}He Mixtures

- interesting for technical reasons: dilution cryostats
- test for different theories: Fermi liquids, RGT, …

first experiments 1947
observation of second sound 1950

5.1 Specific heat and phase diagram





► lambda transition shifts to lower temperatures with $c_3 = N_3/(N_3 + N_4)$

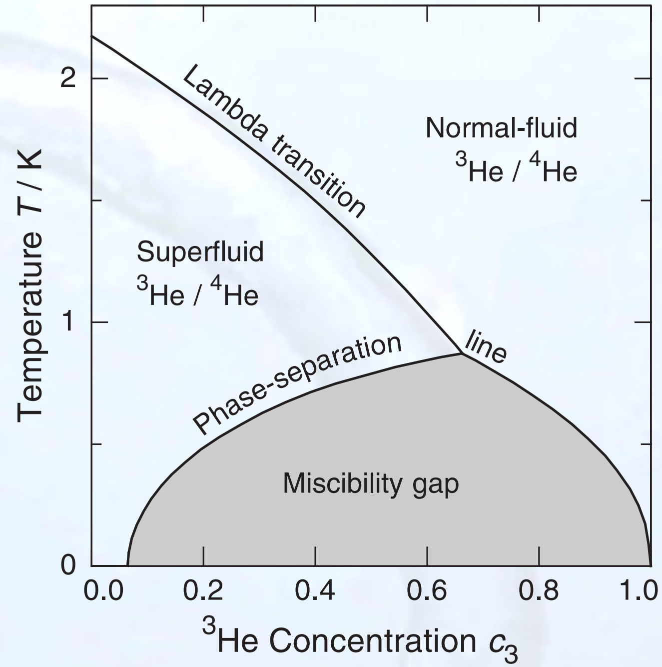
► low temperatures: jump in specific heat

→ 1st order phase transition

→ de-mixing of ^{3}He and ^{4}He

5.1 Specific heat and phase diagram

Phase diagram



- tri-critical point $T = 0.87$ K, $c_3 = 0.67$
- miscibility gap is observed

light phase (^{3}He rich): $c_4 = (1 - c_3) = a\sqrt{T^3}\,\mathrm{e}^{-b/T}$

$a = 0.85\,\mathrm{K}^{-3/2}$, $b = 0.56\,\mathrm{K}$

heavy phase (^{4}He rich): $c_3 = c_{3,0}\,(1 + \tilde{a}\,T^2 + \tilde{b}\,T^3)$

$\tilde{b} = 9.4\,\mathrm{K}^{-3}$, $\tilde{a} = 8.4\,\mathrm{K}^{-2}$, $c_{3,0} = 0.0648$

5.1 Specific heat and phase diagram

dilute solutions of ^{3}He in He-II ($c_3 < 0.15$, $T < 0.5$ K)

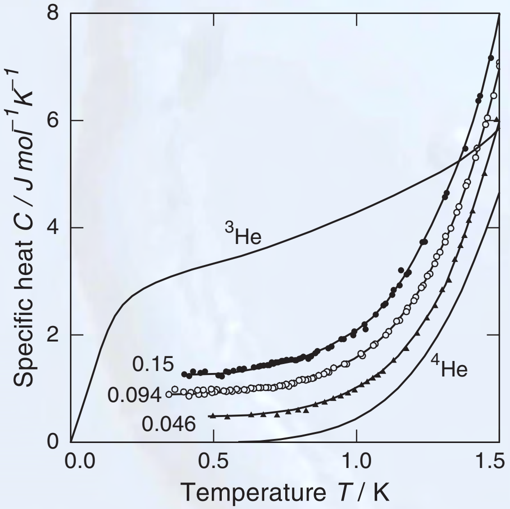
→ ^{4}He: passive background fluid

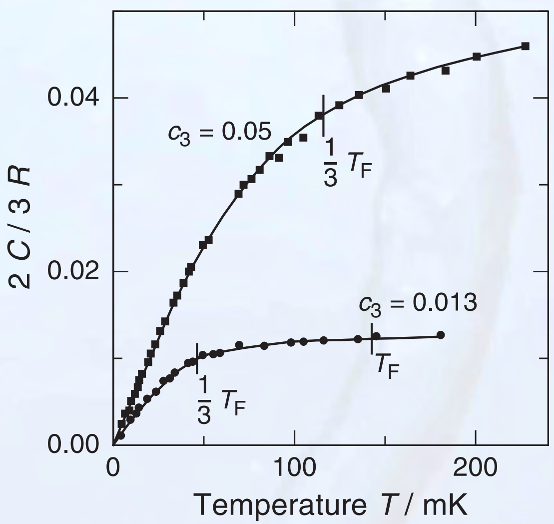
→ ^{3}He: "free" atoms in a quasi vacuum and effective mass $m_3^* = 2.4\, m_3$

Fermi gas $$T_\mathrm{F} = \frac{\hbar^2}{2m_3^* k_\mathrm{B}} \left(3\pi^2 n_3\right)^{2/3} \propto c_3^{2/3}$$

C → $T > T_\mathrm{F}, \quad C \propto c_3 T^0 \quad (\widehat{=} \frac{3}{2}R)$

C → $T < \frac{1}{3}T_\mathrm{F}, \quad C \propto T$





- T_λ depends on c_3
- pure ^{3}He: transition Fermi gas → Fermi liquid
- high T, dilute solution: classical gas with m*

- low T: transition classical gas → Fermi gas
- lines correspond to theory

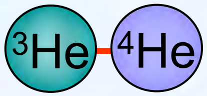
5.1 Specific heat and phase diagram

Finite solubility of ^{3}He in liquid ^{4}He at $T = 0$

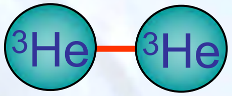
reason: difference in zero-point motion of ^{3}He and ^{4}He

v. Waals interaction identical for ^{3}He-^{3}He and ^{3}He-^{4}He
but: larger zero-point motion of ^{3}He weakens the bonding

} stronger effective binding for

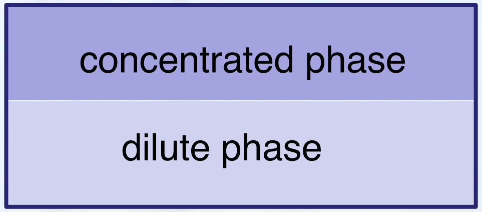


compared to



in equilibrium on finds $\mu_{3,\mathrm{d}}(T, c_{3,\mathrm{d}}) = \mu_{3,\mathrm{c}}(T, c_{3,\mathrm{c}})$

dilute — concentrated



- $T = 0 \longrightarrow c_3 = 1$ for concentrated phase (pure ^{3}He)
- necessary energy to bring one ^{3}He atom into "vacuum" $L_3(T = 0)$ — latent heat

$$\longrightarrow \mu_{3,\mathrm{c}}(0,1) = \mu_3(0) = -L_3(0) = -2.473\,\mathrm{K}$$

- dilute phase: $E_3 = -\mu_{3,\mathrm{d}}(0,0) \longrightarrow$ binding energy

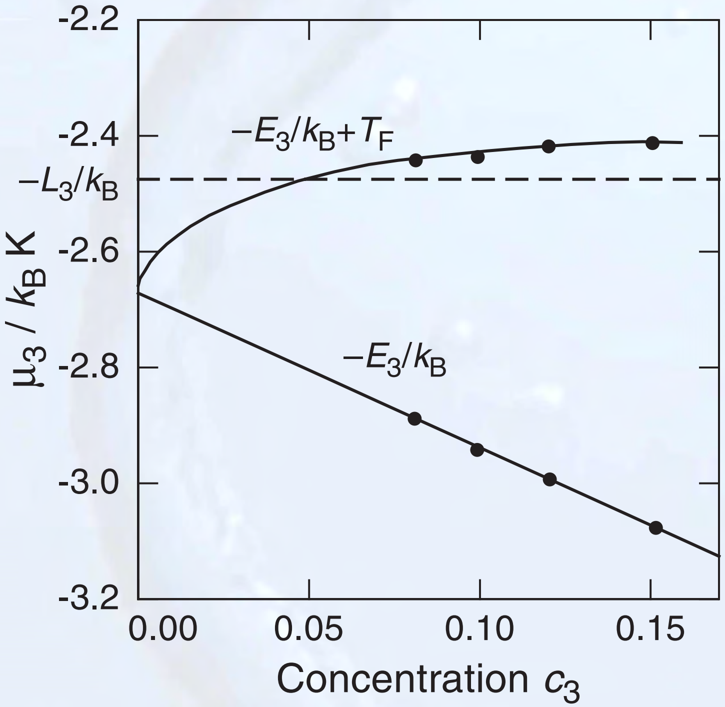
$$c_{3,\mathrm{d}} \to 0$$

- with increasing concentration the effective binding energy for ^{3}He is reduced because of the Pauli principle $\longrightarrow$ Fermi gas: $E_\mathrm{F} = k_\mathrm{B} T_\mathrm{F}(c_3)$

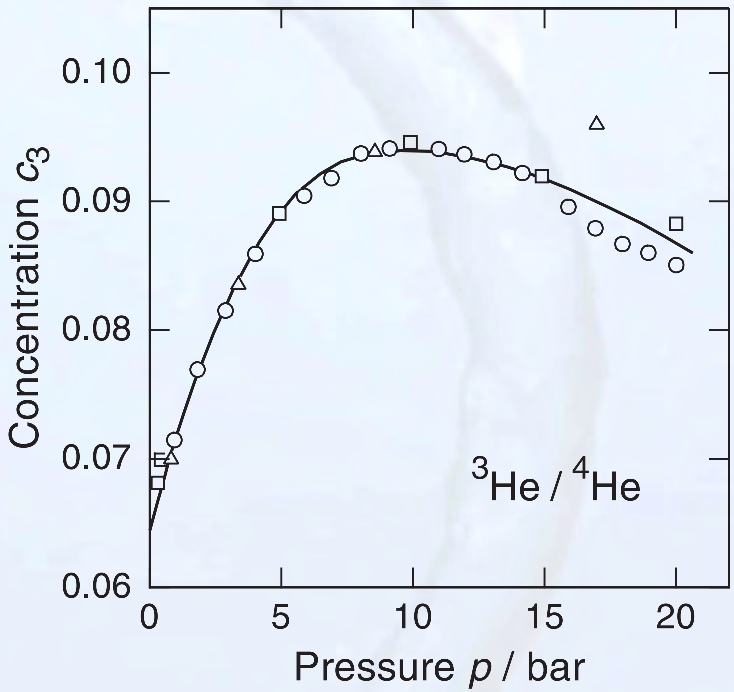
5.1 Specific heat and phase diagram

equilibrium concentration at $T = 0$

$$-L_3(0) = -E_3(0, c_3) + k_B T_F(c_3)$$



pressure dependence



- calculation of $E_3(0,c_3)$ is not trivial

  ⟶ Bardeen, Baym, Pines model

- maximum at 8.7 bar
- concentration $c_3 = 0.096$

5.2 Normalfluid Component

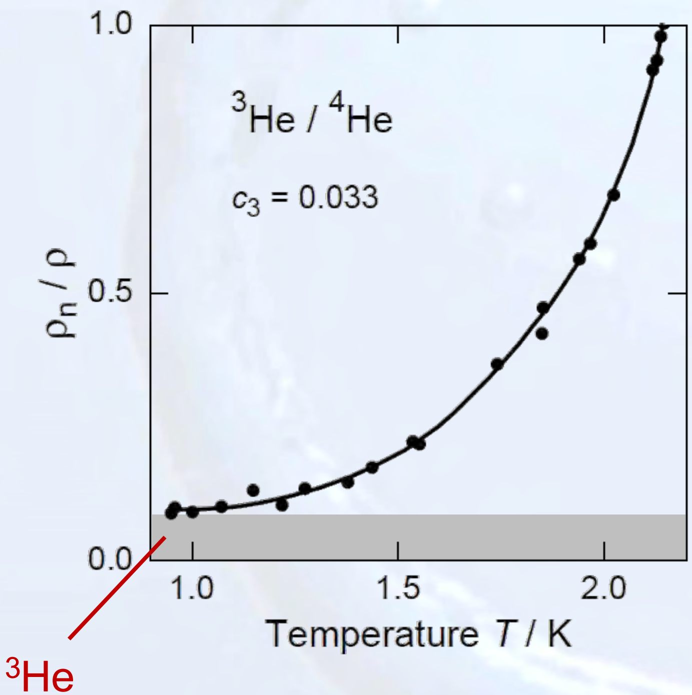
determination of ϱ_n ⟶ Andronikasvili-type experiment

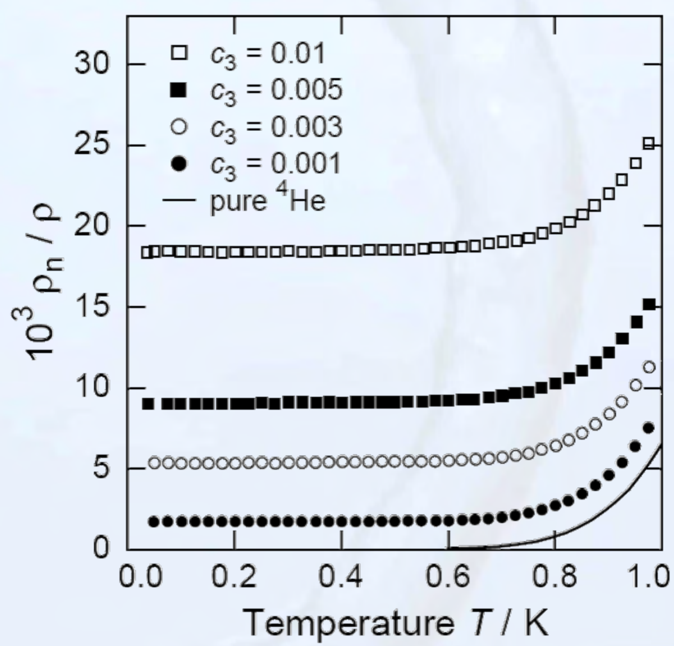
15 mica sheets
4 cm diameter
190 μm spacing

$$\varrho_n = \varrho_{n,4} + \varrho \frac{m_3^*}{m_4} c_3$$

$\varrho_{n,4}$: pure He-II

$\frac{m_3^*}{m_4}$: const

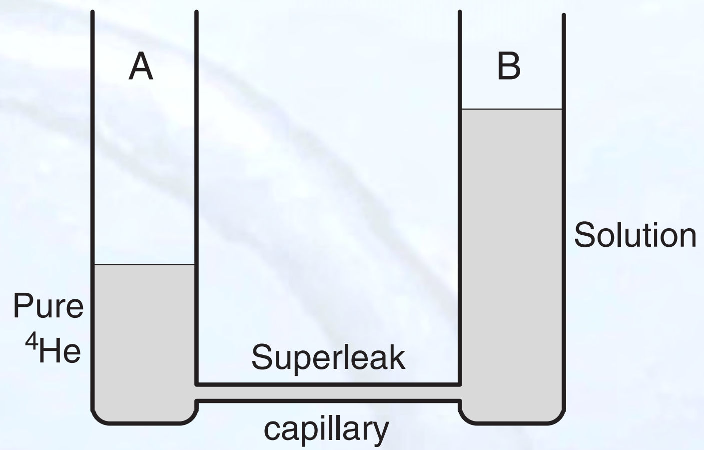




$$\Rightarrow \varrho_n(T \to 0) = \text{const} \propto c_3$$

5.2 Normalfluid Component

Osmotic pressure



- ^{4}He flows to solution to thin the ^{3}He concentration
- ^{3}He is blocked

→ osmotic pressure

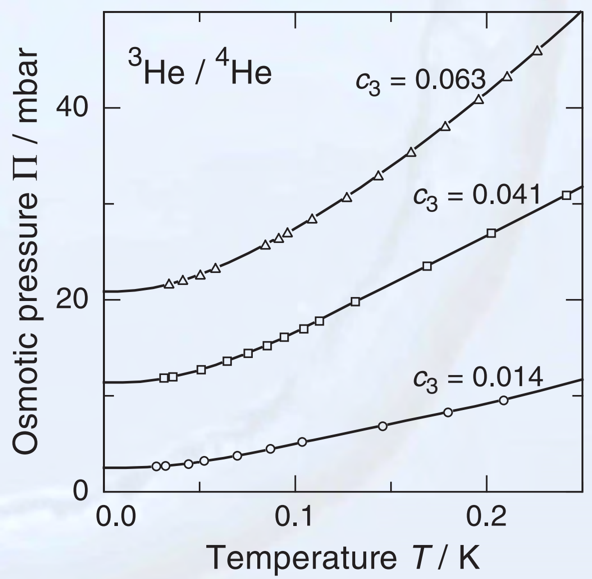
van't Hofft law ($T \gg T_F$, classical regime)

$$\Pi = n_3 k_B T \;\propto\; c_3 T$$

$T \ll T_F$, degenerate Fermi gas

$$\Pi = \frac{2}{5} n_3 k_B T_F \propto c_3^{5/3} = \;\text{const}$$

depends on c_3

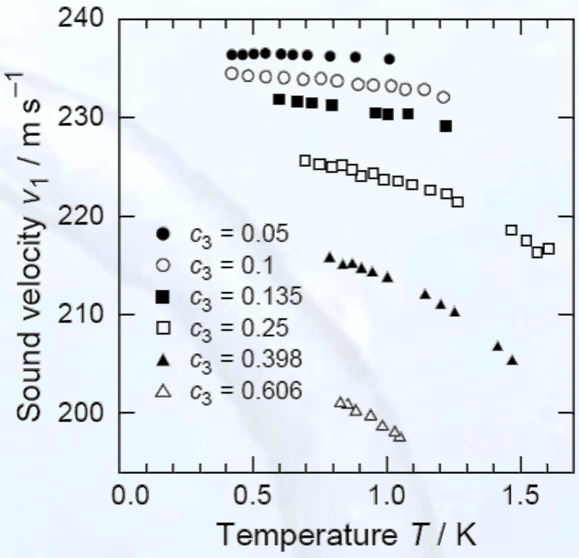


→ transition from FG to classical gas

5.3 Sound Propagation

First sound

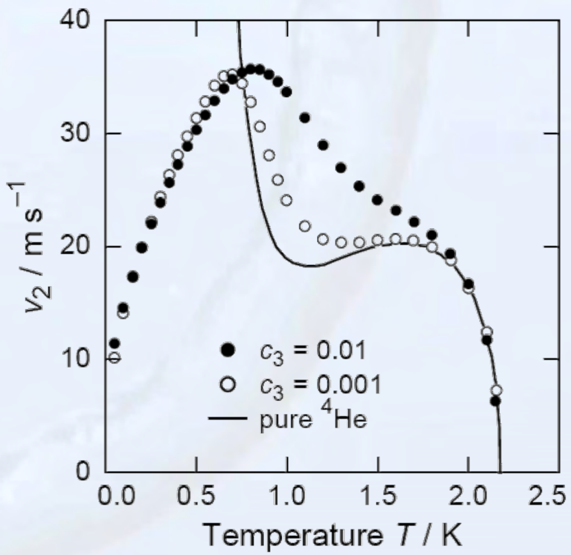
$$v_1^2 = \left(\frac{\partial p}{\partial \varrho}\right)_{S,c_3} \left[1 + \frac{\varrho_s}{\varrho_n}\left(\frac{\partial \varrho}{\partial c_3}\frac{c_3}{\varrho}\right)^2\right]$$



Second sound

$$v_2^2 = \frac{\varrho_s}{\varrho_n}\left[\overline{S}\left(\frac{\partial T}{\partial S}\right)_{\varrho,c_3} + c_3^2\frac{\partial(\mu_3-\mu_4)}{\partial c_3}\right]\left[1 + \frac{\varrho_s}{\varrho_n}\left(\frac{\partial \varrho}{\partial c_3}\frac{c_3}{\varrho}\right)^2\right]^{-1}$$

$$\overline{S} = S_{4,0} - \frac{k_B}{m_4}\left[c_3 + \ln(1-c_3)\right] + \frac{k_B}{m_3}c_3$$



5.4 Transport Properties

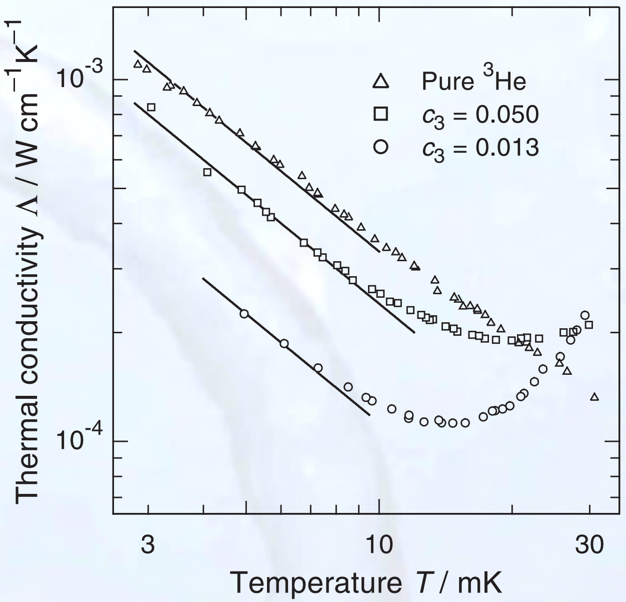
Thermal transport (rather complex)

high Temp.: ϱ_{n} flow leads to ^{3}He concentration gradient

→ ^{3}He atoms diffuse back

→ ^{3}He form scattering centers for ϱ_{n}

→ reduction of heat transport



low Temp.: ^{3}He atoms from FG

$$\Lambda = \frac{1}{3} C\, v_{\mathrm{F}}\, \ell \propto \frac{c_3}{T}$$

$$\ell = v_{\mathrm{F}} \tau \propto (T_{\mathrm{F}}/T)^2$$

$$v_{\mathrm{F}} = (\hbar/m_3^*)(3\pi^2 n_3)^{1/3}$$

$$C \propto T/T_{\mathrm{F}}$$

$$m_3^* = (1 + F_1/3) m_3$$

5.5 Search For a Superfluid Phase of ^{3}He in Mixture

very interesting: 3 superfluid phases in the same container ⟶ ^{4}He, ^{3}He, and dilute ^{3}He

Problem: ^{3}He/^{4}He mixtures are hard to cool because of Kapitza resistance

↓

acoustic mismatch hinders cooling

new initative:

⟶ cooling by melting of ^{4}He crystal

⟶ lowest temperature so far 90 μK