4.3 Quantum States of Superfluid ^{3}He

general spin wave function expressed using $\boldsymbol{d}$:

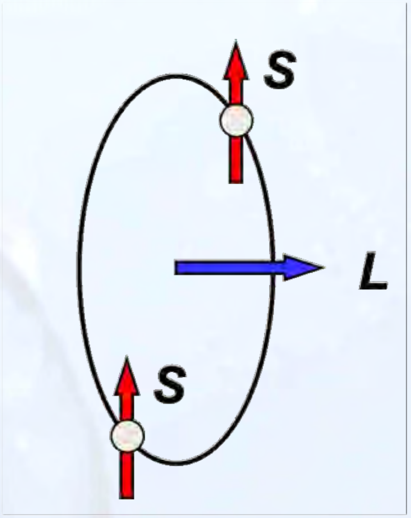
$$|\Psi\rangle = d_x\Big[\,|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle\Big] + \mathrm{i}d_y\Big[\,|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle\Big] + d_z\Big[\,|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big]$$

$$= -\,(d_x - \mathrm{i}d_y)\,|\uparrow\uparrow\rangle + (d_x + \mathrm{i}d_y)\,|\downarrow\downarrow\rangle + d_z\Big[\,|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big]$$

$|S_z = +1\rangle$ $\quad$ $|S_z = -1\rangle$ $\quad$ $|S_z = 0\rangle$

4.3 Quantum States of Superfluid ^{3}He

association with superfluid phases



^{3}He-A$_1$

(exist only in magnetic fields)

spins align parallel to applied magnetic field $(S_z = +1)$ ⟶ only pairs $|\uparrow\uparrow\rangle$

⟶ $d_x + \mathrm{i}d_y = 0 \qquad d_z = 0$

⟶ $|\Psi_{\mathrm{A}_1}\rangle = -2\, d_x\, |\uparrow\uparrow\rangle$

^{3}He-A

$S_z = \pm 1$ ⟶ $d_z = 0$

⟶ $|\Psi_{\mathrm{A}}\rangle = -(d_x - \mathrm{i}d_y)\,|\uparrow\uparrow\rangle + (d_x + \mathrm{i}d_y)\,|\downarrow\downarrow\rangle$ ABM state

Anderson, Brinkman, Morel 1961, 1963

^{3}He-B

⟶ general expression of wave function ⟶ quasi isotropic! total momentum: $J = L + S = 0$

⟶ $|\Psi_{\mathrm{B}}\rangle = d_x\Big[|\downarrow\downarrow\rangle - |\uparrow\uparrow\rangle\Big] + \mathrm{i}d_y\Big[|\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle\Big] + d_z\Big[|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\Big]$ BW state

Bailian, Werthammer 1963

4.3 Quantum States of Superfluid ^{3}He

at the phase transition 3 symmetries are (partially) broken at once

gauge (phase) ⟷ superfluidity

spin momentum ⟷ ferromagnets

orbital momentum ⟷ liquid crystals

group theory: symmetry of ^{3}He

$$G = SO(3)_L \times SO(3)_S \times U(1)_\varphi$$

$SO(3)_L \times SO(3)_S$: special orthogonal non-Ablian rotational group

$U(1)_\varphi$: unitary Ablian rotational group

example: ferromagnet $SO(3)_S$ magnetization

above T_c : isotropic (paramagnet)

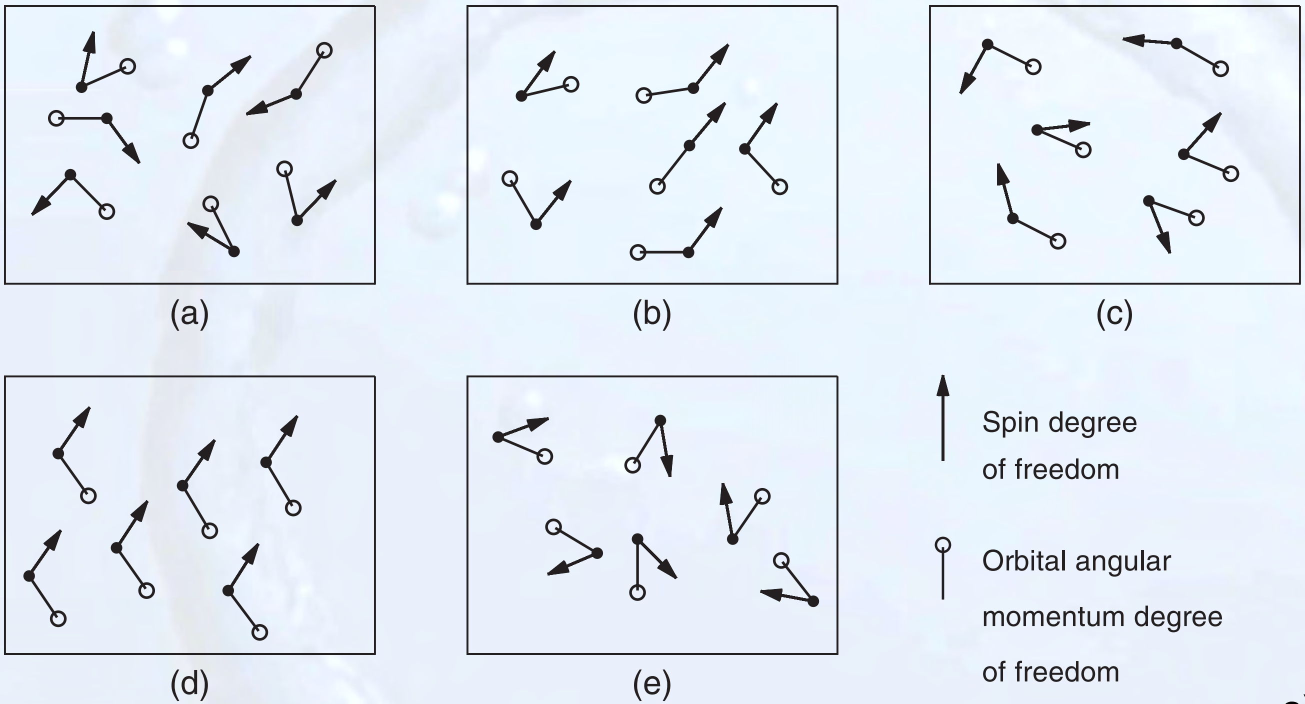
below T_c : one direction selected, but still rotational symmetry about axis of magnetization

$SO(3)_S$ only partially broken ⟶ $R = U(1)$

R: residual symmetry

4.3 Quantum States of Superfluid ^{3}He

Two-dimensional model $G = U(1)_L \times U(1)_S$



a) isotope paramagnetic fluid

b) liquid ferromagnet

c) nematic liquid crystal

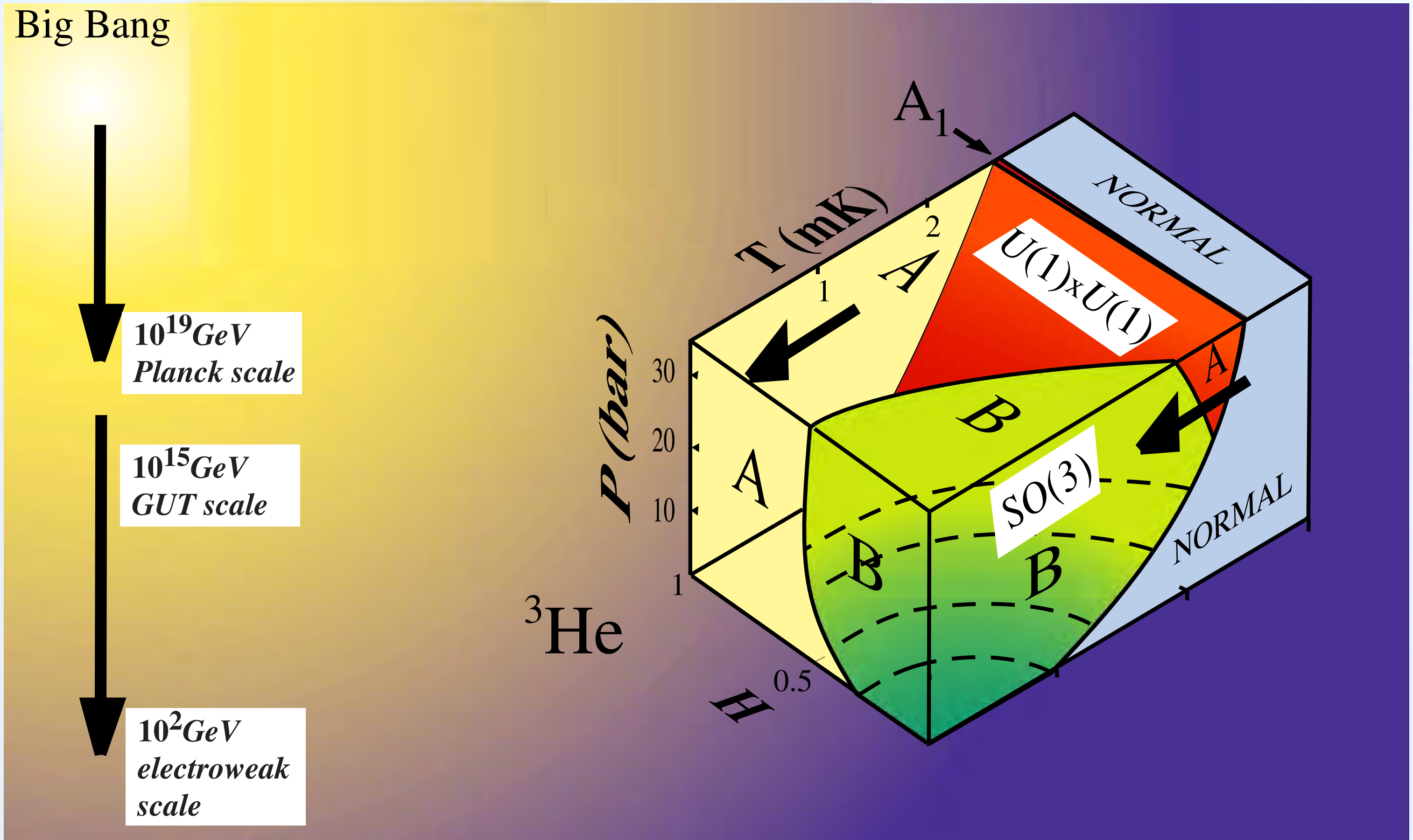
d) ^{3}He-A, ^{3}He-A$_1$

e) ^{3}He-B

4.3 Quantum States of Superfluid ^{3}He

Suprafluid ^{3}He – a Model System for „all“ Physics

Grigory E. Volovik

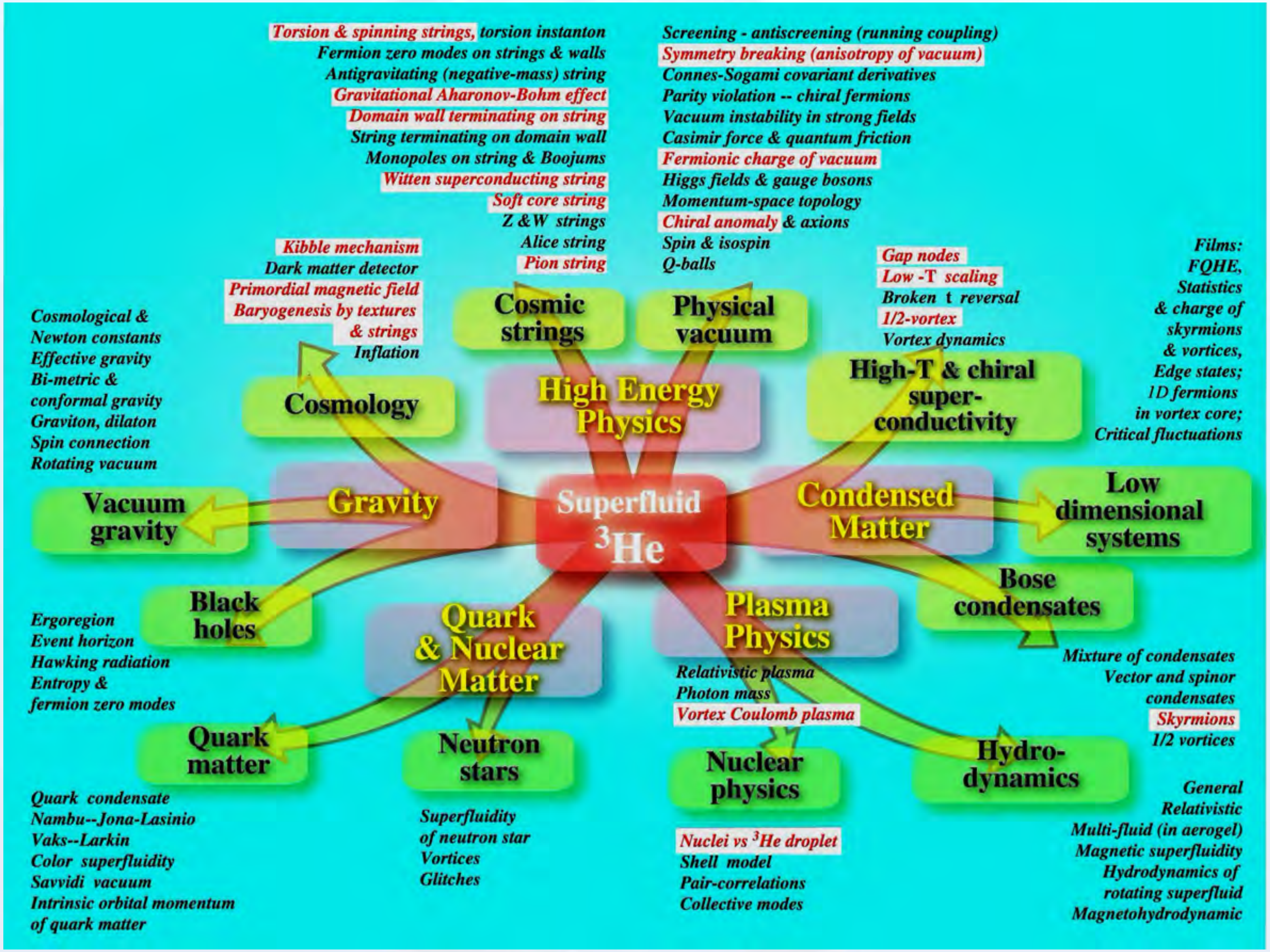


GUT in Standard Model *symmetry breaking phase transitions*	GUT in superfluid ^{3}He *symmetry breaking phase transitions*
$SO(10) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)$	$SO(3) \times SO(3) \times U(1) \rightarrow U(1) \times U(1) \rightarrow SO(3)$

4.3 Quantum States of Superfluid ^{3}He

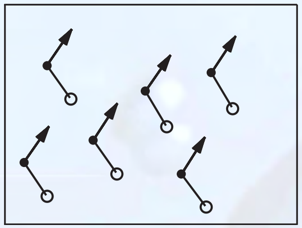
Suprafluides ^{3}He – a Model System for „all“ Physics

Grigory E. Volovik

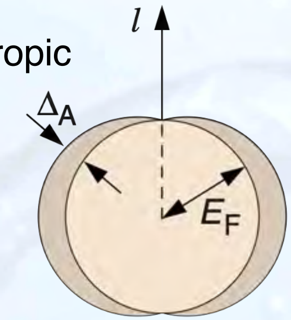


4.3 Quantum States of Superfluid ^{3}He

Energy gap

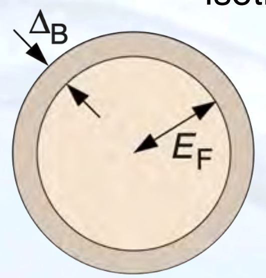


non-isotropic

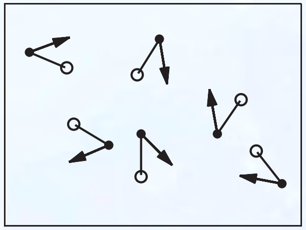


^{3}He–A

isotropic



^{3}He–B



BCS theory

$$T \to 0 \quad \Delta_\mathrm{B} \approx 1.76\, k_\mathrm{B}\, T_\mathrm{c}$$

$$\boldsymbol{d}(\boldsymbol{k}) = \sqrt{3/2}\, \Delta_\mathrm{m}(T)\, \sin(\widehat{\boldsymbol{k}},\widehat{\boldsymbol{l}})\, \widehat{\boldsymbol{d}} = \Delta_\mathrm{A}(\widehat{\boldsymbol{k}},T)\, \widehat{\boldsymbol{d}}$$

maximal gap — orbital momentum

$$\Delta_\mathrm{m}(0) = 2.029\, k_\mathrm{B} T_\mathrm{c}$$

$$\boldsymbol{d}(\boldsymbol{k}) = \Delta_\mathrm{B}(T)\, \widehat{\boldsymbol{d}}$$

$$\widehat{\boldsymbol{d}} = \overleftrightarrow{\mathbf{R}}\, (\widehat{\boldsymbol{n}}, \Theta)\widehat{\boldsymbol{k}}$$

rotation of spin coordinates relative to $\boldsymbol{k}$ vector of pair

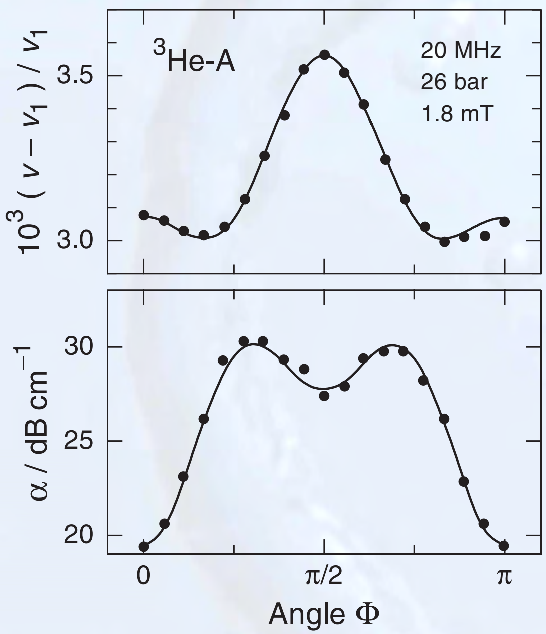
- in A phase pairs can be broken at arbitrarily small energy …. still it is a superfluid!
  ⟶ metastable persistent flow
- but: massive objects cannot be moved without friction in ^{3}He-A

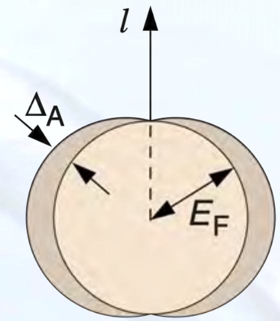
- like He-II ⟶ stable persistent flow
- massive objects can be moved without friction in ^{3}He-B for $v < v_\mathrm{c}$

4.3 Quantum States of Superfluid ^{3}He

experimental determination of anisotropy of gap of ^{3}He-A

propagation of longitudinal zero sound





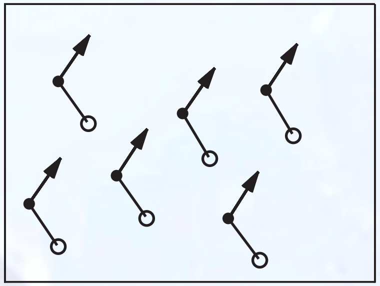
^{3}He–A

- in this experiment $\boldsymbol{l}$ is oriented by a small magnetic field 1.8 mT
- ϕ is the angle between $\boldsymbol{B}$ and $\boldsymbol{q}$

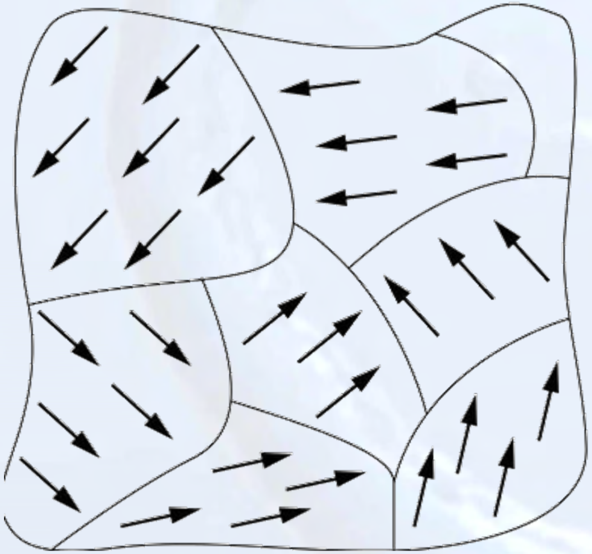
  wave vector of sound wave
- expected anisotropy is clearly observed

4.4 Orderparameter Orientation — Textures

Textures:



- this term was introduced by de Gennes (similar to liquid crystals)
- denotes orientational effects of $\boldsymbol{l}$ and $\boldsymbol{d}$
- texture depends on many things: dipole-dipole interaction, magnetic and electric fields, geometry, …
- often no uniform texture ⟶ texture domains



4.4 Orderparameter Orientation — Textures

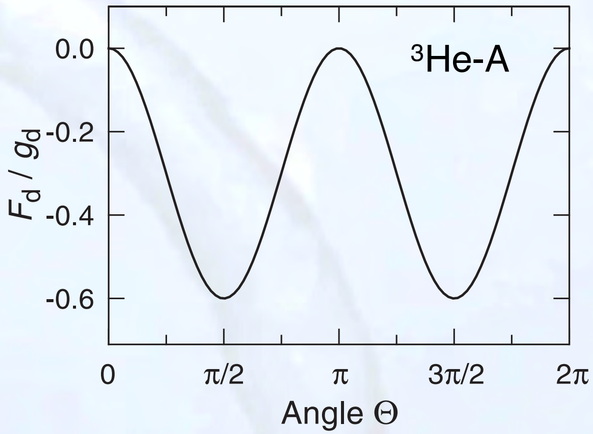
a) orientation of $\boldsymbol{l}, \boldsymbol{d}$ without external field

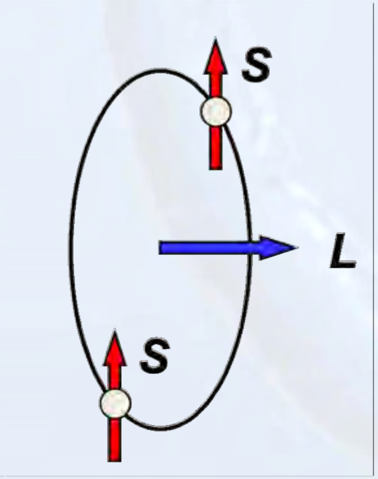
^{3}He-A: macroscopic orientation
dipole-dipole energy is minimal, if $\boldsymbol{l} \parallel \boldsymbol{d} \;\widehat{=}\; \boldsymbol{l} \perp \boldsymbol{S}$

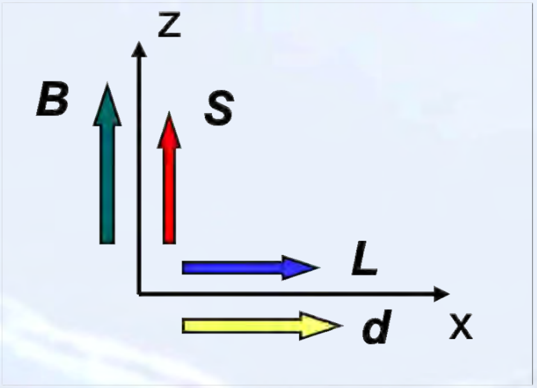
free energy: dipole-dipole interaction

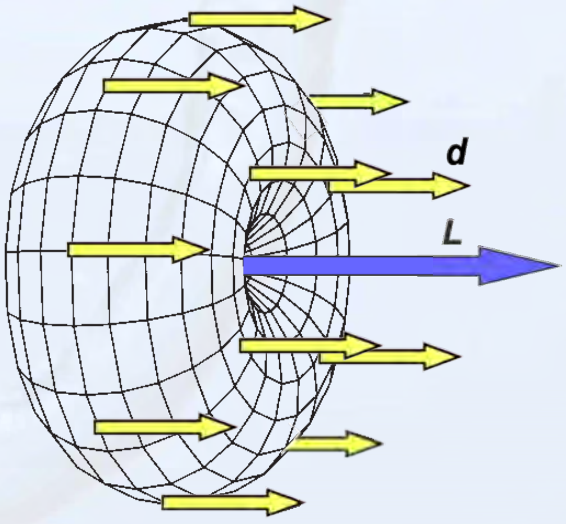
$$F_\mathrm{d} = -\frac{3}{5}\, g_\mathrm{d}(T)\left[1 - (\hat{\boldsymbol{d}} \cdot \hat{\boldsymbol{l}})^2\right] = -\frac{3}{5}\, g_\mathrm{d}(T)\, \sin^2 \Theta$$

$$g_\mathrm{d} \approx 10^{-10}(1 - T/T_\mathrm{c})\,\mathrm{J\,cm^{-3}} \propto \varrho_\mathrm{s}$$

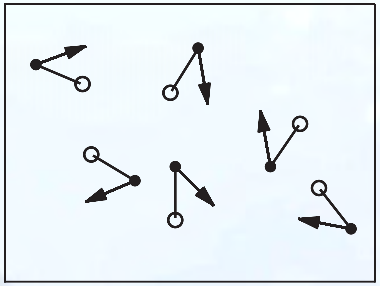








4.4 Orderparameter Orientation — Textures



^{3}He-B: isotropic regarding spin and orbital momentum

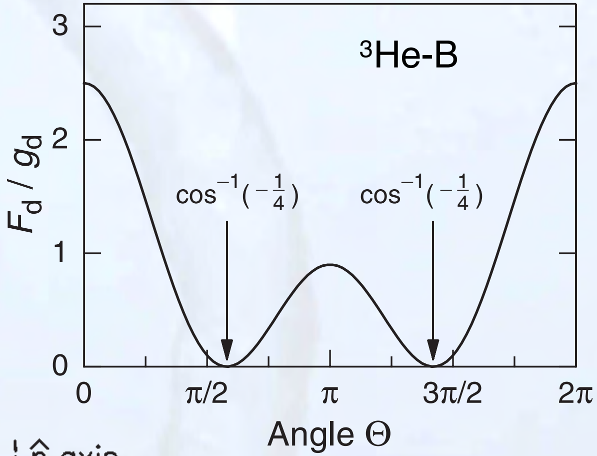
⟶ no macroscopic orientation

but: dipole-dipole interaction leads to a relative

orientation of $\boldsymbol{l}$, $\boldsymbol{d}$ locally for each point on the Fermi surface

described by a rotation about $\hat{\boldsymbol{n}}$ described by $\hat{\boldsymbol{d}} = \overleftrightarrow{\mathbf{R}}\,(\hat{\boldsymbol{n}},\Theta)\hat{\boldsymbol{k}}$

⟶ leads to weak texture effects

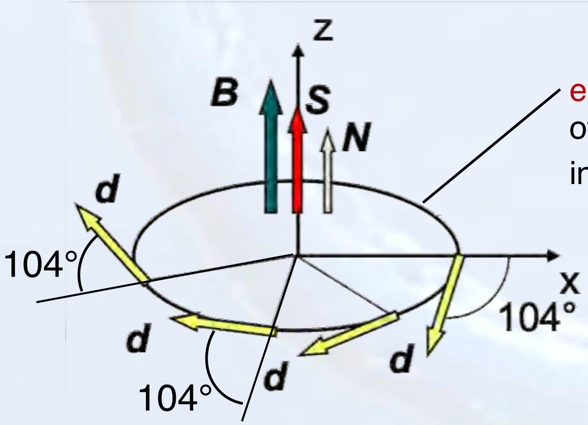


free energy: dipole-dipole interaction

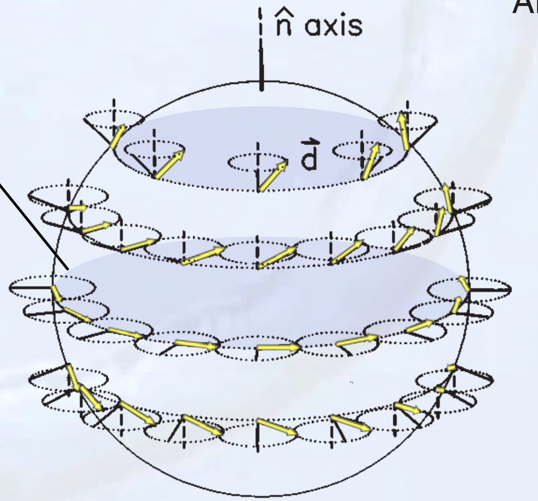
$$F_\mathrm{d} = \frac{8}{5}\, g_\mathrm{d}(T) \left(\cos\Theta + \frac{1}{4}\right)^2$$

Leggett angle

dipole-dipole energy is minimal, if $\Theta = \arccos(-1/4) \approx 104°$



equatorial plane of Fermi sphere in k-space



4.4 Orderparameter Orientation — Textures

b) external influences on the orientation of $\boldsymbol{l}\,,\boldsymbol{d}$

⟶ changes of the texture

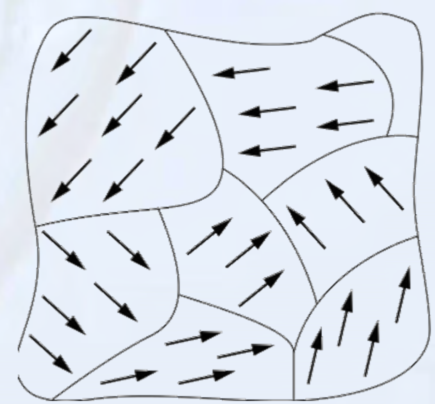
textures in ^{3}He-A

preferred alignment and relative strength of different influences

	Preferred Alignment	$\Delta E/(1-T/T_\mathrm{c})$ $(\mathrm{J\,m^{-3}})$
magnetic dipole interaction	$\boldsymbol{d} \parallel \boldsymbol{l}$	$-6\times10^{-5}\,(\widehat{\boldsymbol{d}}\cdot\widehat{\boldsymbol{l}})^2$
electric field	$\boldsymbol{l} \perp \boldsymbol{\mathcal{E}}$	$2\times10^{-7}\,(\widehat{\boldsymbol{l}}\cdot\boldsymbol{\mathcal{E}})^2$
magnetic field	$\boldsymbol{d} \perp \boldsymbol{B}$	$5\,(\widehat{\boldsymbol{d}}\cdot\boldsymbol{B})^2$
mass flow	$\boldsymbol{l} \parallel \boldsymbol{v}_\mathrm{s}$	$-10\,(\widehat{\boldsymbol{l}}\cdot\boldsymbol{v}_\mathrm{s})^2$
wall alignment	$\boldsymbol{l} \parallel \boldsymbol{N}$	$-30\,(\widehat{\boldsymbol{l}}\cdot\widehat{\boldsymbol{N}})^2$

- most important are walls $\boldsymbol{l} \parallel \boldsymbol{N}$ and mass flow $\boldsymbol{l} \parallel \boldsymbol{v}_\mathrm{s}$
- strength compared to intrinsic alignment:
  $\mathcal{E} = 17\,\mathrm{V\,m^{-1}}$, $B = 3.3\,\mathrm{mT}$ and $v_\mathrm{s} = 2.4\,\mathrm{mm\,s^{-1}}$
- for in homogenies textures ⟶ gradient energy must be considered

costs energy



4.4 Orderparameter Orientation — Textures

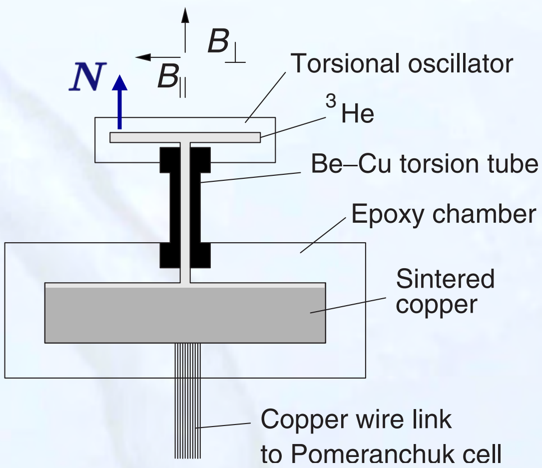
Example for influence of wall and magnetic field

Determination of $\varrho_\mathrm{s}/\varrho$ with with a disc like resonator

- ϱ_n is dragged with resonator because of η_n
- mass of ϱ_n adds to moment of inertia
- resonance frequency depends on $\varrho_\mathrm{n}/\varrho$

$\longrightarrow \varrho_\mathrm{s}/\varrho$

Andronikasvili-like experiment



(i) $\boldsymbol{B}$ parallel to wall $\boldsymbol{B}_{||} \perp \boldsymbol{N}$

$$\left.\begin{array}{l} \boldsymbol{l} \parallel \boldsymbol{N} \\ \boldsymbol{S} \parallel \boldsymbol{B}_{||} \curvearrowright \boldsymbol{d} \perp \boldsymbol{B}_{||} \end{array}\right\} \boldsymbol{d} \parallel \boldsymbol{l}$$

optimal even without external field

(ii) $\boldsymbol{B}$ perpendicular to wall $\boldsymbol{B}_\perp \parallel \boldsymbol{N}$

$$\left.\begin{array}{l} \boldsymbol{l} \parallel \boldsymbol{N} \\ \boldsymbol{S} \parallel \boldsymbol{B}_\perp \curvearrowright \boldsymbol{d} \perp \boldsymbol{B}_\perp \end{array}\right\} \boldsymbol{d} \perp \boldsymbol{l}$$

not optimal for dipole dipole interaction

$\longrightarrow \varrho_{\mathrm{s}\perp} < \varrho_{\mathrm{s}||}$