3.2 The Landau Fermi-Liquid Theory

→ energy of one quasi particle is given by the energy of an isolated atom plus, the interaction with all other atoms

→ quasi particle states are not eigenstates

How does the distribution function look like? – does the Fermi distribution still hold?

Yes, as long as the energy levels (states) are well-defined!

but quasi particles aren't eigen states → transitions occur

→ broadening of levels $\delta E \approx \hbar/\tau$

collision time, lifetime

→ quasi particle states are well-defined as long as the uncertainty is small compared to the thermal broadening $\Delta E \approx k_\mathrm{B} T$

3.2 The Landau Fermi-Liquid Theory

this condition can always be fulfilled at sufficiently low temperatures, since

$$\tau \propto \frac{1}{T^2} \qquad \boxed{\delta E \propto T^2}$$

some numbers: $\tau \approx 5 \times 10^{-11} \frac{1}{T^2} \left[\frac{1}{\mathrm{s}}\right]$ Fermi gas

$\tau \approx 1 \times 10^{-12} \frac{1}{T^2} \left[\frac{1}{\mathrm{s}}\right]$ experimental result

$$\Delta E \approx k_\mathrm{B} T \longrightarrow T = 0.1\,\mathrm{K}$$

Fermi distribution holds $f(E,T) = \dfrac{1}{\mathrm{e}^{(E-\mu)/k_\mathrm{B}T} + 1}$

Landau theory is good for $T \ll 0.1\,\mathrm{K}$ in case of ^{3}He

3.2 The Landau Fermi-Liquid Theory

What is the dispersion relation ?

$T \to 0\,,$ states at E_F : $p_\mathrm{F} = \hbar\left(3\pi^2 n\right)^{1/3}$

general expression for states near Fermi level

$$E = E_\mathrm{F} + \left(\frac{\partial E}{\partial p}\right)_\mathrm{F} (p - p_\mathrm{F})$$

$$\left(\frac{\partial E}{\partial p}\right)_\mathrm{F} = \frac{p_\mathrm{F}}{m} = v_\mathrm{F}$$ Fermi gas

$$\left(\frac{\partial E}{\partial p}\right)_\mathrm{F} = \frac{p_\mathrm{F}}{m^*}$$ Fermi liquid

dispersion of quasi particles

$$E = E_\mathrm{F} + \frac{p_\mathrm{F}}{m^*}(p - p_\mathrm{F})$$

density of states at Fermi level

$$\boxed{D(E_\mathrm{F}) = \frac{m^* k_\mathrm{F}}{\pi^2\hbar^2} = \frac{m^*}{\pi\hbar^2}\sqrt[3]{\frac{3n}{\pi}}}$$

3.2 The Landau Fermi-Liquid Theory

Central problem: Interaction term

- energy of quasi particles depends on the configuration of all quasi particles
- $E(\boldsymbol{p},T)$ changes when the occupation of states with $\boldsymbol{p}'$ differ by $\delta f(\boldsymbol{p}')$ from the one at $T=0$

Phenomenological ansatz (without spin term)

cannot be derived

$$E(\boldsymbol{p},T) = E(\boldsymbol{p},0) + 2\varrho_k \int h(\boldsymbol{p},\boldsymbol{p}')\,\delta f'\,\mathrm{d}^3p'$$

$$h(\boldsymbol{p},\boldsymbol{p}') = \frac{\partial^2 U}{\partial f(\boldsymbol{p})\,\partial f'(\boldsymbol{p}')}$$ interaction term

- $D(E_\mathrm{F})\; h(\boldsymbol{p},\boldsymbol{p}')$ corresponds to the scattering amplitude
- like for a Fermi gas only states at the Fermi surface are important $p \approx p' \approx p_\mathrm{F}$

⟶ $h(\boldsymbol{p},\boldsymbol{p}')$ depends only on the angle Θ between $\boldsymbol{p}'$ and $\boldsymbol{p}$

⟶ $\boxed{h(\boldsymbol{p},\boldsymbol{p}') = h(\Theta)}$

3.2 The Landau Fermi-Liquid Theory

Treatment of interaction term

consider new function: $F(\Theta) = D(E_\mathrm{F})\, h(\Theta)$

expansion in terms of Legendre polynomials

$$F(\Theta) = \sum_i F_i\, P_i(\cos\Theta) = F_0 + F_1\, \cos\Theta + F_2\, \frac{3\cos^2\Theta - 1}{2} + \;\ldots$$

these coefficients can (only) be determined experimentally

general expression with spin term: $\mathcal{F}(\boldsymbol{p},\boldsymbol{s},\boldsymbol{p}',\boldsymbol{s}') = h(\boldsymbol{p},\boldsymbol{p}') + \underbrace{\xi(\boldsymbol{p},\boldsymbol{p}')\;\boldsymbol{s}\cdot\boldsymbol{s}'}_{\text{spin term}}$

consider new function for spin term: $G(\Theta) = D(E_\mathrm{F})\,\xi(\Theta)$

expansion in terms of Legendre polynomials

$$G(\Theta) = D(E_\mathrm{F})\,\xi(\Theta) = \sum_i G_i\, P_i(\cos\Theta) = G_0 + G_1\cos\Theta + \;\ldots$$

these coefficients can (only) be determined experimentally

3.2 The Landau Fermi-Liquid Theory

Application to liquid ^{3}He (not trivial)

(i) effective mass

$$\frac{1}{m} = \frac{1}{m^*} + \frac{p_F}{\hbar^3} \int F(\Theta) \cos\Theta \, d\Gamma$$

solid angle segment of Fermi surface

$$\boxed{\frac{m^*}{m} = \left(1 + \frac{1}{3} F_1\right)}$$

mean value of $F_1 \cos\Theta$

experimental results

pure ^{3}He: $\frac{m^*}{m} \approx 3$ normal pressure

$\frac{m^*}{m} \approx 6$ 30 bar

1% ^{3}He in ^{4}He: $\frac{m^*}{m} \approx 2.4$

Landau's Fermi liquid theory can be tested varying pressure and ^{3}He concentration

3.2 The Landau Fermi-Liquid Theory

(ii) specific heat

$$C = \frac{m^*}{m} C_{\mathrm{FG}} = \left(1 + \frac{1}{3} F_1\right) C_{\mathrm{FG}}$$

$C \propto T$ at $T \ll T_{\mathrm{F}}^*$ | $T_{\mathrm{F}}^* \approx 0.5\,\mathrm{K}$

(ii) sound velocity (first sound)

$$v_1^2 = \frac{p_{\mathrm{F}}^2}{3m^2} \frac{1+F_0}{1+\frac{1}{3}F_1} = \frac{1}{3} v_{\mathrm{F}}^2 \frac{1+F_0}{1+\frac{1}{3}F_1}$$

compare to:

$v = \frac{1}{3} v_{\mathrm{F}}$ Fermi gas

$v_1^2 = \left(\frac{\partial p}{\partial \rho}\right)_S = N \frac{\partial \mu}{\partial N}$ | $\operatorname{grad} \mu = \frac{1}{\rho} \operatorname{grad} p$

with $\mu = E_F = E(p_F)$

Since $p_F = \hbar \left(3\pi^2 \frac{N}{V}\right)^{1/3} \curvearrowright \frac{\partial \mu}{\partial N} \curvearrowright \frac{\partial \mu}{\partial p_F}$

$\curvearrowright$

$$v_1^2 = \frac{p_F}{3m} \frac{\partial \mu}{\partial p_F}$$

$$= \frac{p_F}{3m} \left[\frac{p_F}{m} + \frac{2 p_F^2}{\hbar^3} \int F(\theta)(1-\cos\theta)\, d\Gamma \right]$$

insert expansion

$$\curvearrowright v_1^2 = \frac{p_F^2}{3m^2} \left(\frac{1+F_0}{1+\frac{1}{3}F_1} \right)$$

(iii) magnetic susceptibility

$$\chi = \frac{m^*}{m} \left(\frac{1}{1+\frac{1}{4}G_0} \right) \chi_{\mathrm{FG}}$$

– 2.8

→ enhancement of susceptibility ↑↑ against Fermi statistics ↓↑

→ if exchanged interaction larger by a factor 2 ⟶ $1 + \frac{1}{4}G_0 < -1$ and ground state would be ferromagnetic

3.2 The Landau Fermi-Liquid Theory

Landau Fermi liquid parameters for ^{3}He

p (bar)	V_m (cm^3)	F_0	F_1	G_0	m^*/m
0	36.84	9.30	5.39	−2.78	2.80
3	33.95	15.99	6.49	−2.89	3.16
6	32.03	22.49	7.45	−2.93	3.48
9	30.71	29.00	8.31	−2.97	3.77
12	29.71	35.42	9.09	−2.99	4.03
15	28.89	41.73	9.85	−3.01	4.28
18	28.18	48.46	10.60	−3.03	4.53
21	27.55	55.20	11.34	−3.02	4.78
24	27.01	62.16	12.07	−3.02	5.02
27	26.56	69.43	12.79	−3.02	5.26
30	26.17	77.02	13.50	−3.02	5.50
33	25.75	84.79	14.21	−3.02	5.74

3.3 Sound Propagation in ^{3}He: Zero Sound

normal (first) sound: quasi particles reach local equilibrium by collisions $\omega\tau \ll 1$

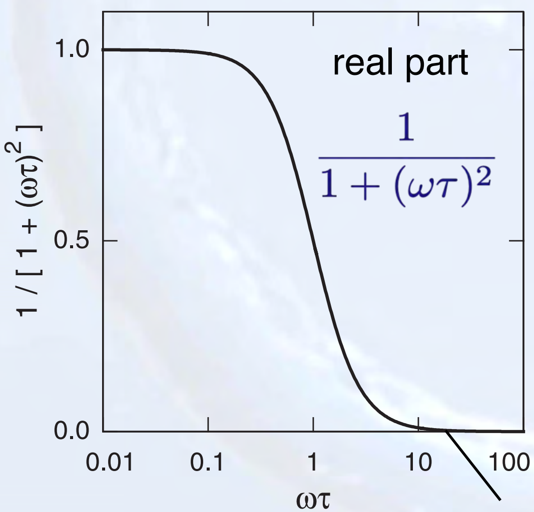
frequency of sound wave

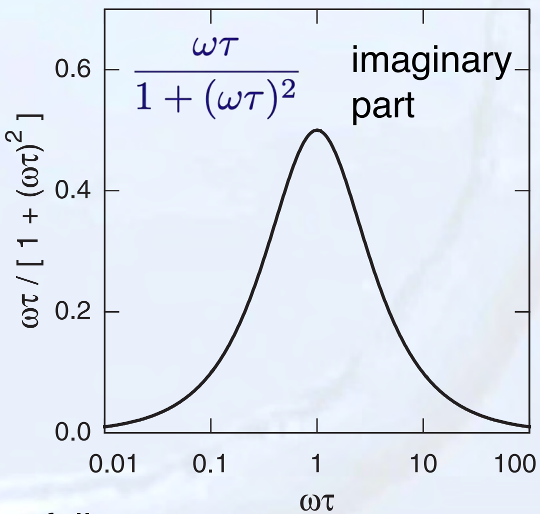
zero sound: collision-less propagation of sound $\omega\tau \gg 1$

^{3}He: particle density fluctuations in one region lead to density fluctuation in neighboring regions

→ propagation of sound-like modes → zero sound

Debye relaxation process (transition from hydrodynamic regime to collision-less regime)





systems cannot follow

3.3 Sound Propagation in ^{3}He: Zero Sound

- high T: $\omega\tau \ll 1$ ⟶ hydrodynamic regime: first sound
- low T: $\omega\tau \gg 1$ ⟶ collision-less regime: zero sound, longitudinal
  transversal
  collision-less spin waves

compare with classical gas

mean free path > wavelength ⟶ no sound propagation

but ^{3}He

- strongly interacting particles
- force on quasiparticle does not stem from direct neighbors, but from all atoms
- density fluctuations can propagate without collisions
- transversal modes are also possible

General theoretical description of zero sound is rather complicated ⟶ here only results

⟶ collective modes with $\omega\tau \gg 1$ ⟶ zero sound

⟶ 2 different sound modes (similar to first sound) and collision-less spin waves

3.3 Sound Propagation in ^{3}He: Zero Sound

longitudinal sound:

$$v_1 = \frac{v_\mathrm{F}}{3}\sqrt{\frac{(1+F_0)}{(1+\frac{1}{3}F_1)}} \qquad \omega\tau \ll 1$$

$$v_0 = v_1\left[1+\frac{2}{5}\left(\frac{1+\frac{1}{5}F_2}{1+F_0}\right)\right] \qquad \omega\tau \gg 1$$

difference of zero and first sound: $\dfrac{v_0 - v_1}{v_1} = \dfrac{2}{5}\dfrac{(1+\frac{1}{5}F_2)}{(1+F_0)}$

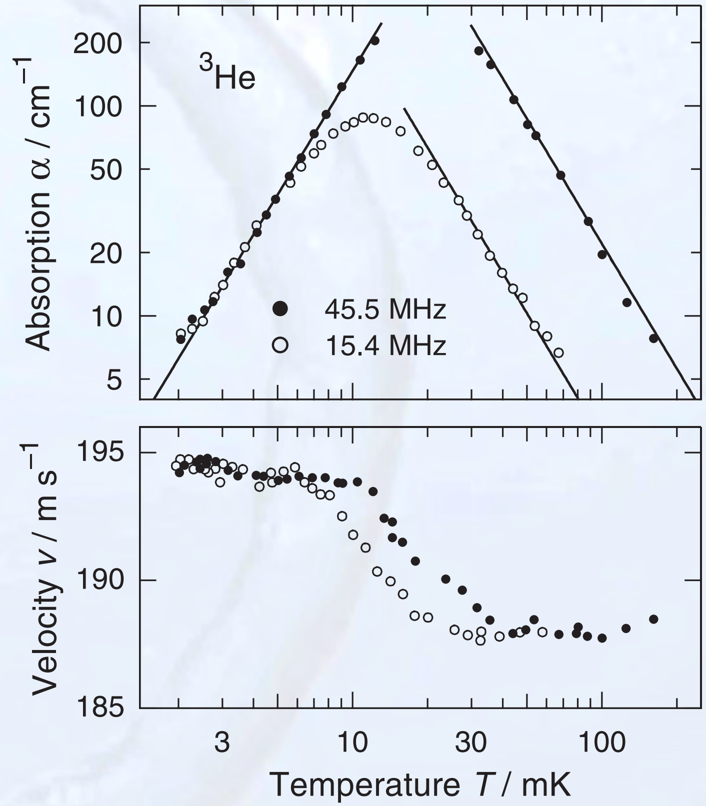
$$\longrightarrow \boxed{(v_0 - v_1) \approx 6\,\mathrm{m\,s^{-1}}}$$

intermediate temperatures: $\dfrac{v}{v_1} = 1 + \dfrac{v_0 - v_1}{v_1}\dfrac{\omega^2\tau_\mathrm{s}^2}{1+\omega^2\tau_\mathrm{s}^2}$

sound attenuation: $\dfrac{\alpha v}{\omega} = -2\,\dfrac{v_0 - v_1}{v_1}\dfrac{\omega\tau_\mathrm{s}}{1+\omega^2\tau_\mathrm{s}^2}$

limiting cases: $\alpha_1 = A_1\omega^2\tau \boxed{\propto\ \omega^2 T^{-2}}$

$$\alpha_0 = A_0\tau^{-1} \boxed{\propto\ T^2}$$



→ excellent agreement with Landau theory

3.3 Sound Propagation in ^{3}He: Zero Sound

transversal sound:

ordinary liquids $\longrightarrow$ no transversal sound mode

^{3}He

$\omega\tau \ll 1$ hydrodynamical regime $\longrightarrow$ diffuse shear mode

$\omega\tau \gg 1$ real solution for $F_1 > 6$

impossible at normal pressure: $F_1 = 5.2$

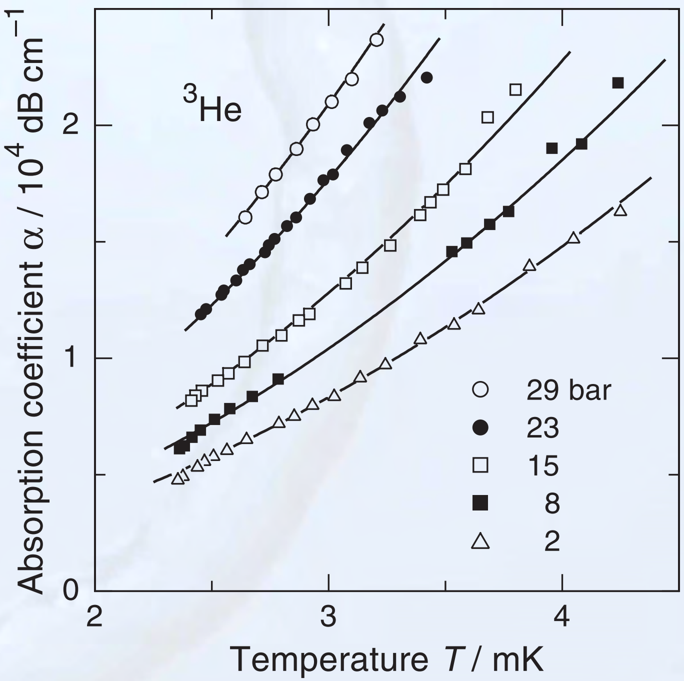
but F_1 depends on pressure

$F_1 = 5.2 \ldots 15$ melting pressure

attenuation: $\alpha_0 \propto T^2$

experimental results

- narrow T range, very high damping
- sound transducers spaced by 25 μm
- damping depends on pressure



3.3 Sound Propagation in ^{3}He: Zero Sound

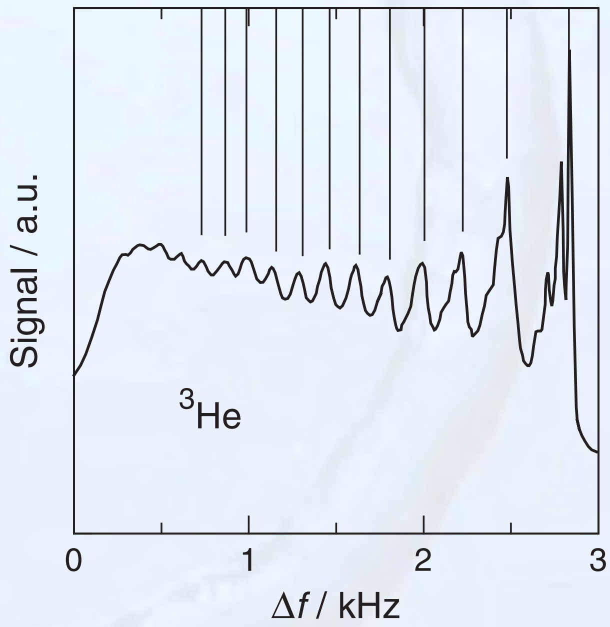
collision-less spin waves: (predicted by Silin 1957)

spin transport $D_\mathrm{s} = \frac{1}{3}\tau_\mathrm{D}\, v_\mathrm{F}^2 \left(1 + \frac{1}{4G_0}\right)$

spin transport
- $\omega\tau \ll 1 \longrightarrow$ normal spin diffusion
- $\omega\tau \gg 1 \longrightarrow$ collision-less spin waves

experimental results

- standing spin waves
- linear magnetic field gradient $44\,\mathrm{mT\,m^{-1}}$
- rectangular absorption "line"
- maxima of spin wave resonance on top



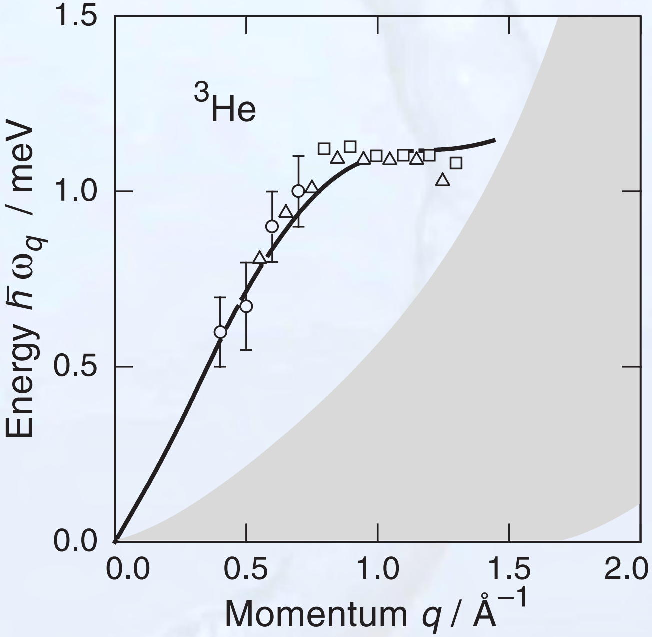
3.3 Sound Propagation in ^{3}He: Zero Sound

Dispersion of zero sound modes:

experimental determination very difficult

capture cross section very high

ultralow temperatures $T < 20$ mK



3.3 Sound Propagation in ^{3}He: Zero Sound