2.6 Excitation Spectrum of He-II: Landau Model

Normalfluid component:

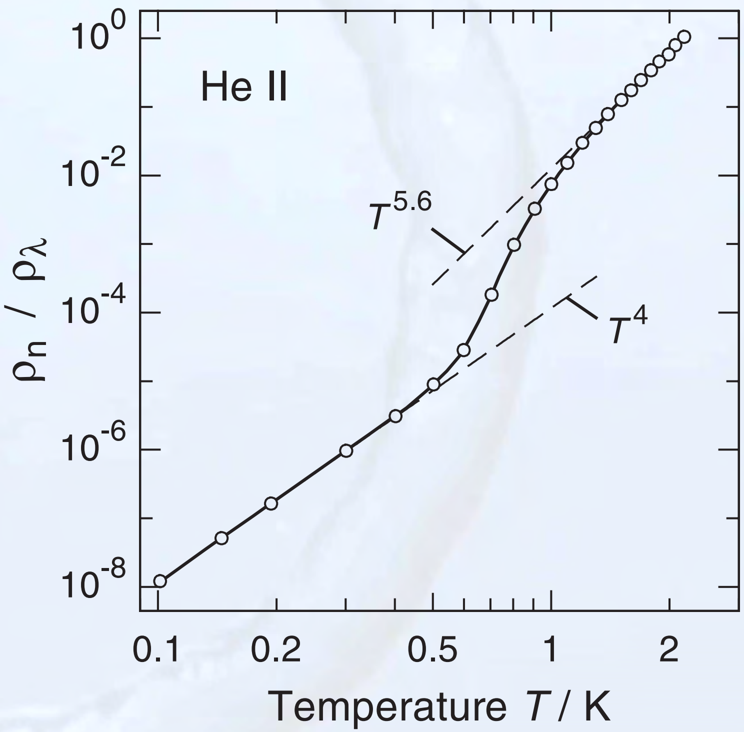
$$\varrho_{\mathrm{n}} = \varrho_{\mathrm{n,ph}} + \varrho_{\mathrm{n,r}}$$

$$\boxed{\varrho_{\mathrm{n,r}} = \frac{2\,p_0^4}{3\,\hbar^3}\sqrt{\frac{m^*}{(2\pi)^3 k_{\mathrm{B}} T}}\,\mathrm{e}^{-\Delta_{\mathrm{r}}/k_{\mathrm{B}}T}}$$

Rotons

$$\boxed{\varrho_{\mathrm{n,ph}} = \frac{2\pi^2 k_{\mathrm{B}}^4}{45\,\hbar^3\,v_1^5}\,T^4}$$

Phonons



- at low temperatures $\varrho_{\mathrm{n}} \propto T^4$ due to phonons
- rotons dominate between 0.5 K and 1.2 K
- above 1.2 K nature of excitations more complex

2.6 Excitation Spectrum of He-II: Landau Model

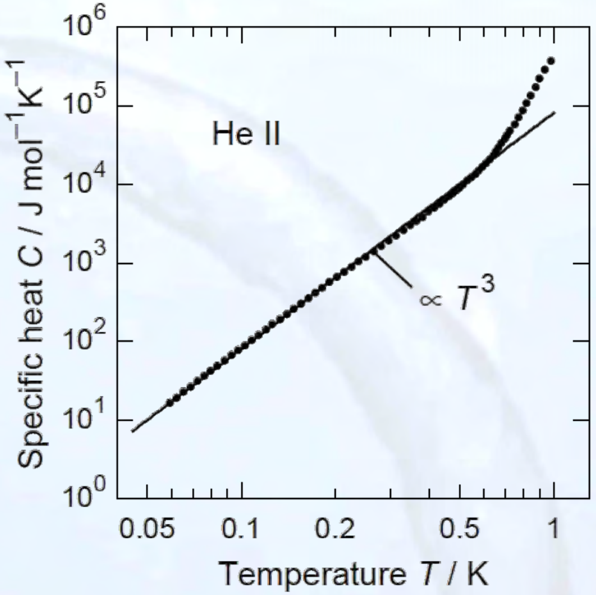
Specific heat:

a) low temperatures $T < 0.6\,\mathrm{K}$

only long wavelength phonons contribute

⟶ Debye model

$$\boxed{C_{\mathrm{ph}} = \frac{2\pi^2 k_{\mathrm{B}}^4}{15\varrho\hbar^3 v_1^3}\, T^3}$$

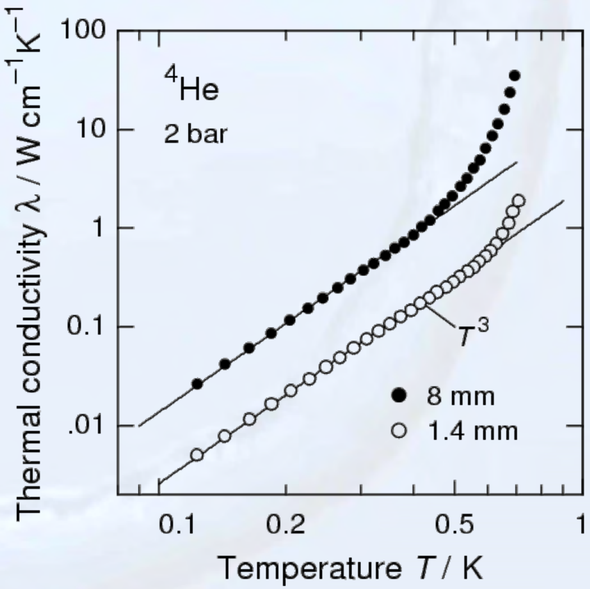


measurement of thermal conductivity

Casimir regime $\ell = d$

capillary cross section

$$\longrightarrow \quad \Lambda = \frac{1}{3} C_{\mathrm{ph}}\, v\, d \propto T^3$$



2.6 Excitation Spectrum of He-II: Landau Model

b) intermediate temperatures $\mathbf{0.6 < T < 1.2\,K}$

free energy $F_\mathrm{r} = -k_\mathrm{B} T n_\mathrm{r}$

$$n_\mathrm{r} = \frac{2p_0^2}{3\varrho\hbar^3}\sqrt{\frac{m^* k_\mathrm{B}T}{(2\pi)^3}}\,\mathrm{e}^{-\Delta_\mathrm{r}/k_\mathrm{B}T}$$

number density of rotons

$$S_\mathrm{r} = -\partial F_\mathrm{r}/\partial T$$

$$\downarrow$$

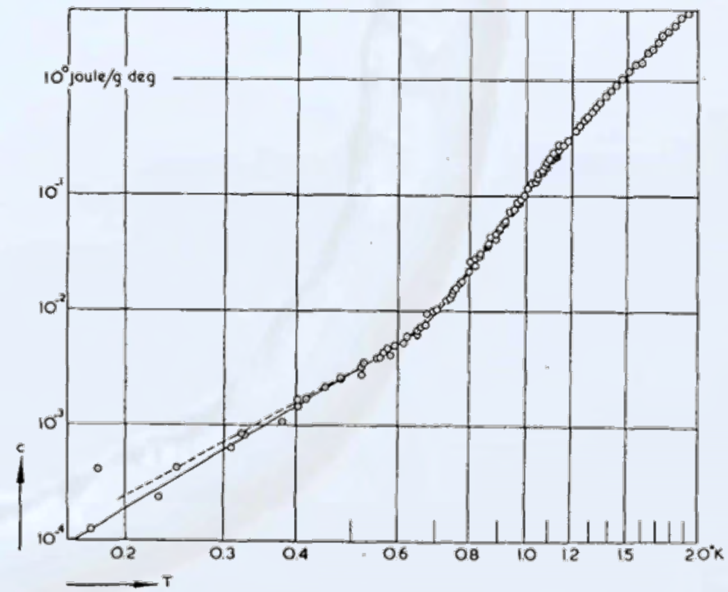
$$C_\mathrm{r} = T\partial S_\mathrm{r}/\partial T$$

$$\boxed{C_\mathrm{r} = \frac{2k_\mathrm{B}p_0^2}{3\varrho\hbar^3}\sqrt{\frac{m^* k_\mathrm{B}T}{(2\pi)^3}}\left\{\frac{3}{4} + \frac{\Delta_\mathrm{r}}{k_\mathrm{B}T} + \left(\frac{\Delta_\mathrm{r}}{k_\mathrm{B}T}\right)^2\right\}\mathrm{e}^{-\Delta_\mathrm{r}/k_\mathrm{B}T}}$$

c) high temperatures $\mathbf{1.2\,K < T < T_\lambda}$

additional excitations contribute: maxons
lifetime of rotons becomes very short

$\longrightarrow$ excitations are not well-defined



2.6 Excitation Spectrum of He-II: Landau Model

Landau's concept of critical velocity

superconductors ⟶ energy gap

superfluid He-II ⟶ no energy gap, but velocity gap!

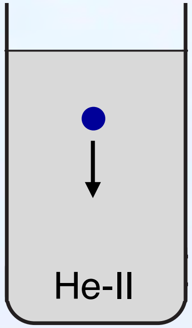
comment:

phonons can be excited at arbitrary small energies

Landau's Gedankenexperiment: dropping a massive sphere in He-II at $T = 0$

let's assume that sphere generates one excitation with energy $\mathcal{E}$ and momentum $\boldsymbol{p}$

How fast must this sphere fall in He-II to generate dissipation ?



energy conservation

$$\frac{1}{2}\mathcal{M}v^2 = \frac{1}{2}\mathcal{M}v'^2 + \mathcal{E} \qquad (1)$$

momentum conservation

$$\mathcal{M}\boldsymbol{v} - \boldsymbol{p} = \mathcal{M}\boldsymbol{v}' \qquad (2)$$

not all combinations of $\mathcal{E}$ and $\boldsymbol{p}$ fulfill both conservation law's at the same time, even if the direction of the excitation is not fixed

2.6 Excitation Spectrum of He-II: Landau Model

let's test this: square (2) and divide by $2\mathcal{M}$ $\longrightarrow$ $\frac{1}{2}\mathcal{M}v^2 - \boldsymbol{v}\cdot\boldsymbol{p} + \frac{1}{2\mathcal{M}}p^2 = \frac{1}{2}\mathcal{M}v'^2$

comparison with (1) results in: $\boldsymbol{v}\cdot\boldsymbol{p} - \cancel{\frac{1}{2\mathcal{M}}p^2} = \mathcal{E}$

with $\boldsymbol{v} \parallel \boldsymbol{p}$

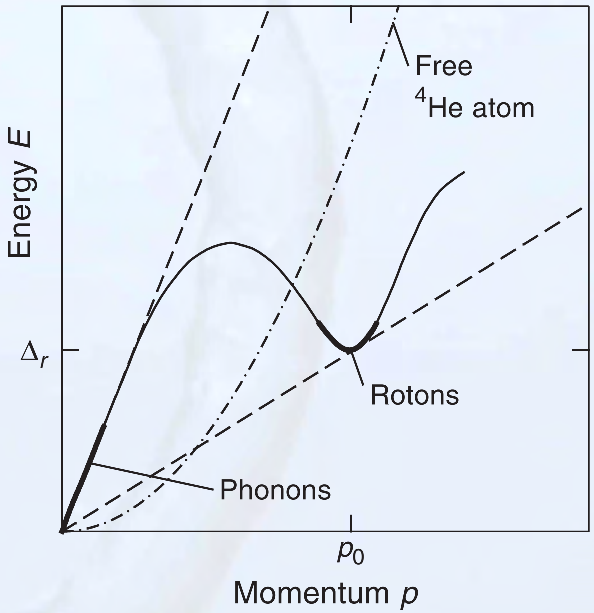
mass of sphere is large

$\Rightarrow$ $\boxed{v_\mathrm{c} = \frac{\mathcal{E}}{p}}$ independent of nature of excitation

phonons: $v_\mathrm{c} = 238\,\mathrm{m\,s^{-1}}$

rotons: $v_\mathrm{c} \approx 60\,\mathrm{m\,s^{-1}}$

free ^{4}He atom: $v_\mathrm{c} = 0$ $\longleftarrow$ do not exist in He-II



- no excitation for $v < v_\mathrm{c}$
- no dissipation $\longrightarrow$ superfluidity
- for $v \geq v_\mathrm{c}$ sudden onset of dissipation, laminar $\longrightarrow$ turbulent flow

2.7 Motion of Ions in He-II

Testbed for the generation of excitations and the critical velocity

type of ions:

- electrons (–) : zero-point motion ⟶ bubbles $r = 19$ Å
- $^4He^+$, H_2^+ (+) : attract He atoms ⟶ snowballs $r \approx 7$ Å
- other ions (–, +) : properties depend on wave function

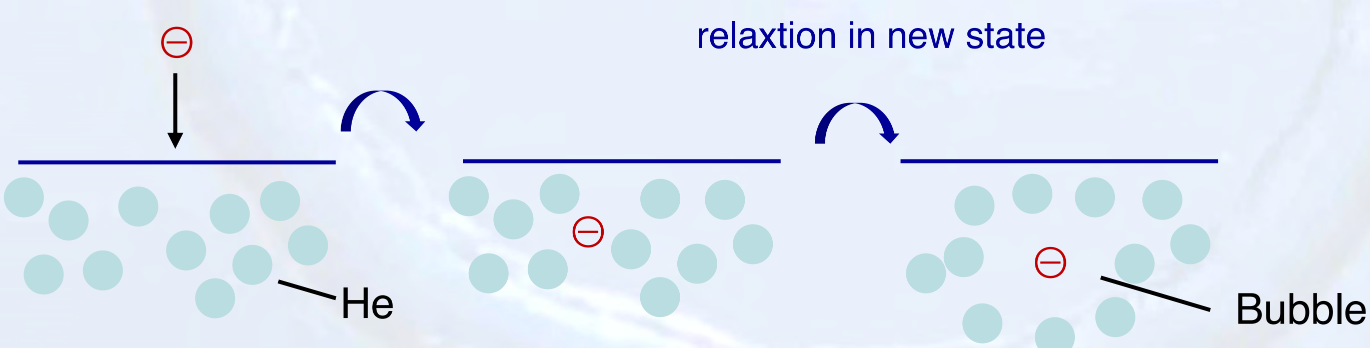
Electrons in liquid He

electrons need energy to be emerged in helium ~ 1 eV, which means they need more that 1 eV of kinetic energy to enter liquid He.

comment:

similar to work function of electrons in metals

bubble formation



2.7 Motion of Ions in He-II

Energy of bubble

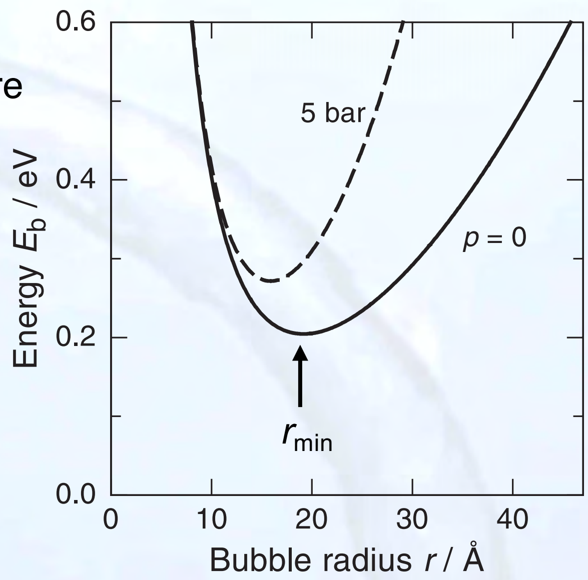
$$E_{\mathrm{b}} = \frac{h^2}{8mr^2} + 4\pi r^2 \alpha + \frac{4}{3}\pi r^3 p$$

applied pressure

volume

zero-point energy

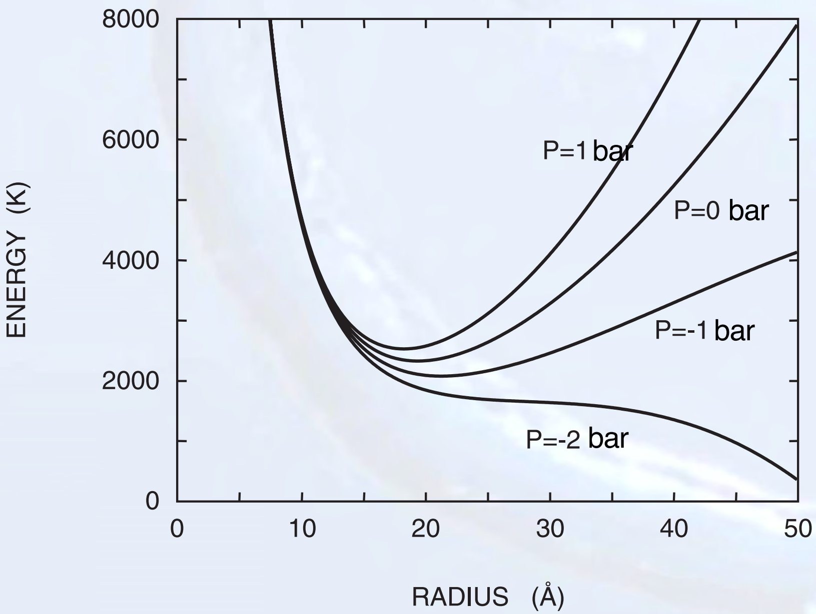
surface tension

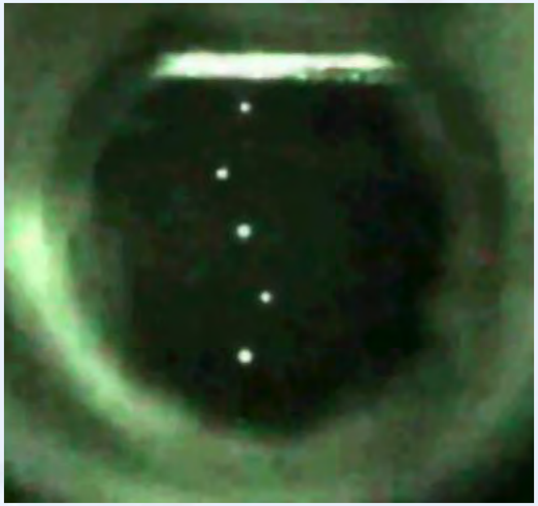


bubble size:

$$\frac{\partial E}{\partial r} = 0 \longrightarrow r_{\min}(p=0) = 19\ \text{Å}$$

size depends on pressure





exploding bubbles at negative pressure

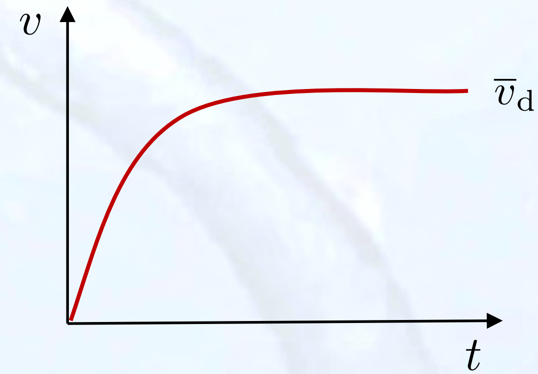
2.7 Motion of Ions in He-II

Acceleration of ions in constant field

→ constant drift velocity is reached $\overline{v}_\mathrm{d} = \dfrac{q\mathcal{E}}{6\pi\eta r}$

Stokes law of viscos friction

constant electrical field $\mathcal{E}$



mobility:

$$\mu = \frac{\overline{v}_\mathrm{d}}{\mathcal{E}} = \frac{q}{6\pi\eta r}$$

snowball (electrons 4π)

collision partners: phonons, rotons, ³He, …

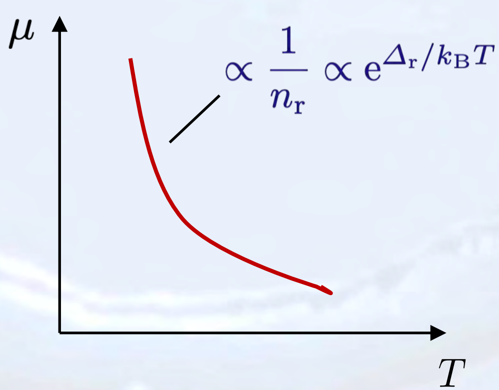
impurities, which at some level are always present

0.7 K < *T* < 1.8 K: rotons should dominate however, difficult to observe because of other excitations / impurities

mobility for roton scattering

$$\mu \propto \frac{1}{\eta} \propto \frac{1}{\tau} \propto \frac{1}{n_\mathrm{r}}$$

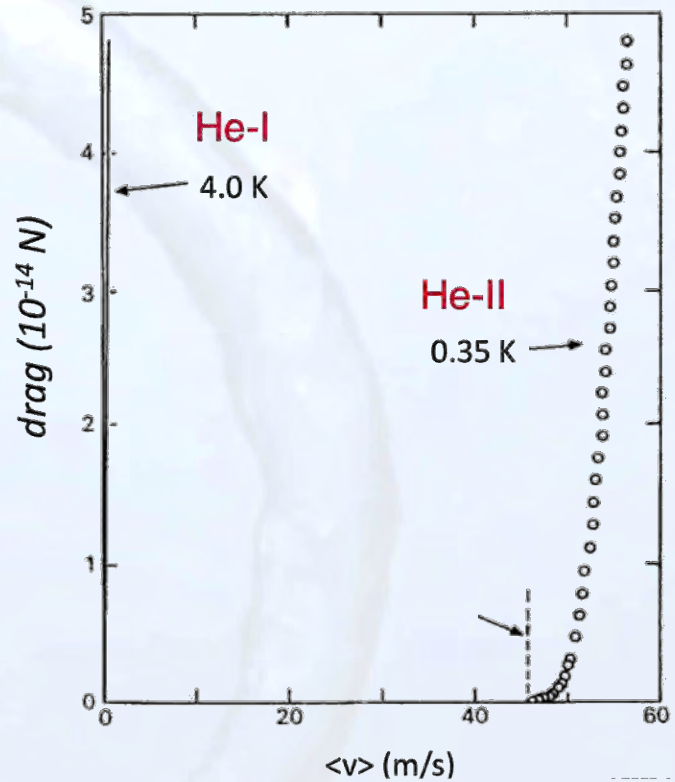
$$\eta = \frac{1}{3}\varrho v^2 \tau = \frac{1}{3}\varrho v \ell$$



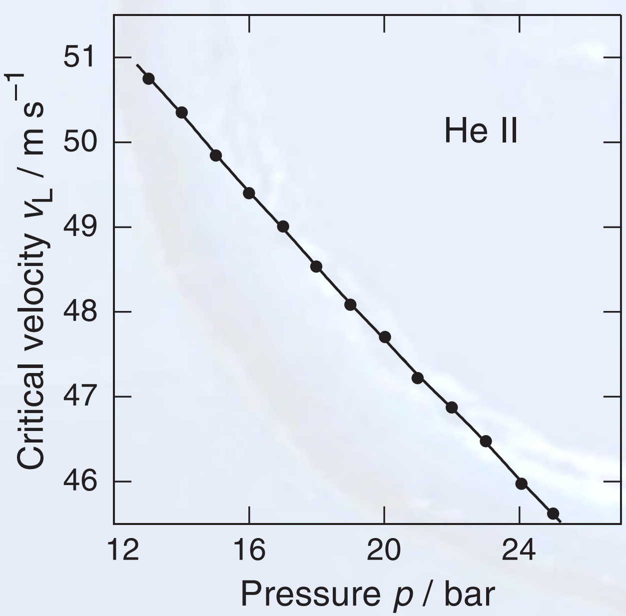
2.7 Motion of Ions in He-II

in ultra-pure He-II under pressure ions can be accelerated up to Landau velocity

- negative ions accelerated in electric field under high pressure
- drag is measured by time-of-flight method
- in He-I: drag proportional to velocity
- in He-II: drag is not observable until critical velocity is reached



pressure dependence of v_c



- $v_L \mathrel{\widehat{=}} v_c$ Landau velocity
- roton pair production
- $p \uparrow \longrightarrow v_L \downarrow$ since $\Delta_r(p)$ decreases with pressure

2.7 Motion of Ions in He-II

***T* < 0.3 K**

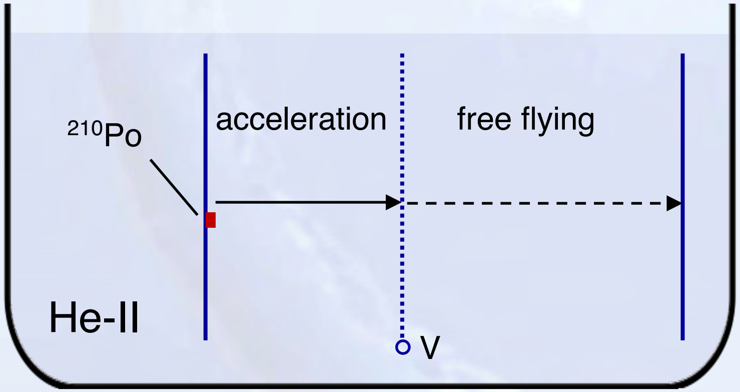
no thermal rotons are excited

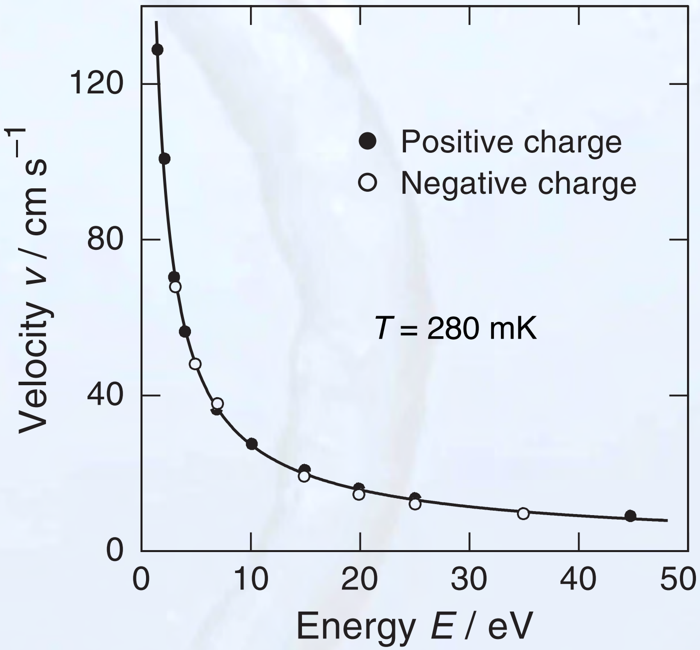
phonons mean free path becomes very large ⟶ several cm! $v_c \to 238\ \frac{\mathrm{m}}{\mathrm{s}}$?

experimental answer: no! $\overline{v}_\mathrm{d}$ = 10 … 100 cm/s

in addition: $\overline{v}_\mathrm{d}$ decreases with energy of ions, which means it decreases with accelerating field

Experiment by Rayfield and Reif 1964





➡ measurement of ion velocity by time of flight

explanation:

- creation of vortex rings and trapping of ions
- experiment observes motion of vortex rings