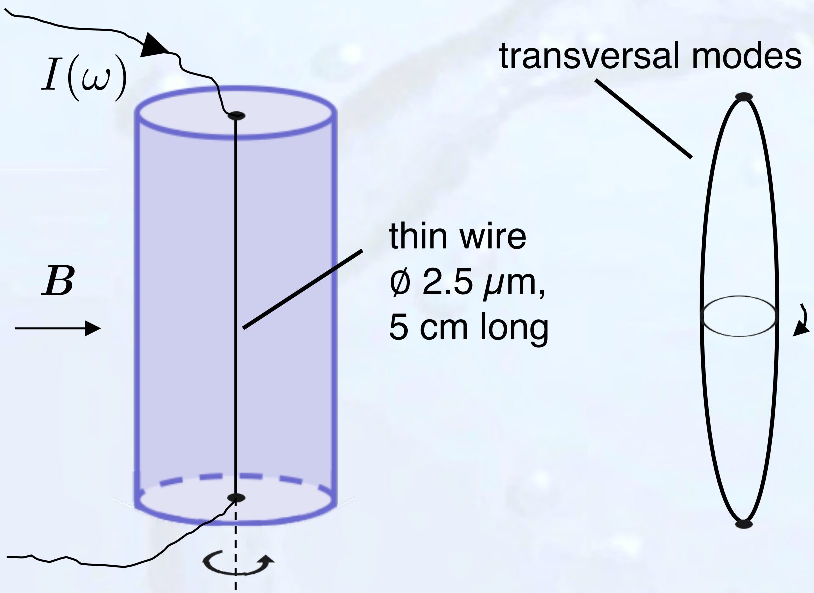
2.5 Macroscopic Quantum State

Experimental discovery of quantization of circulation

vibrating wire excited by current pules (Joe Vinen 1961)

transversal modes ≙ two circular polarized modes



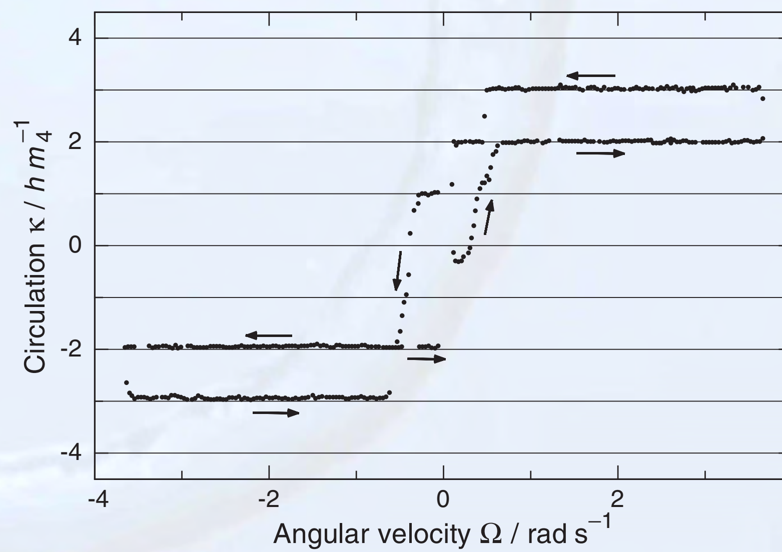
- without rotation: degenerate
- with rotation: lifting of degeneracy by Magnus force

frequency splitting: $$\Delta\nu = \frac{\varrho_s}{2\pi\,\mathcal{M}}\,\kappa$$

effective mass / length
(wire + ½ of expelled liquid)

experimental results

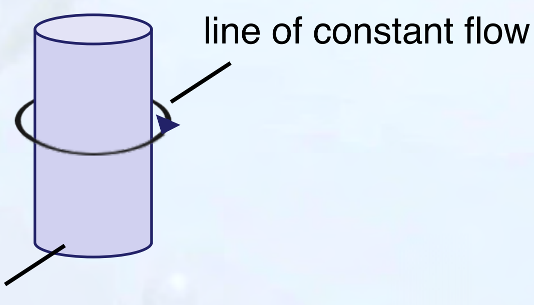
- quantization with expected value
- hysteresis effects are observed
- modern measurements up to $n = 4$

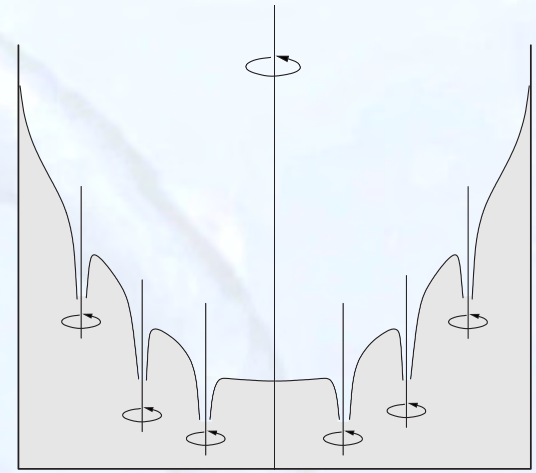


2.5 Macroscopic Quantum State

What has this to do with the rotation of bulk helium in a simply connected region?

- vortices may occur with normal fluid core
- resulting in a multiply connected region



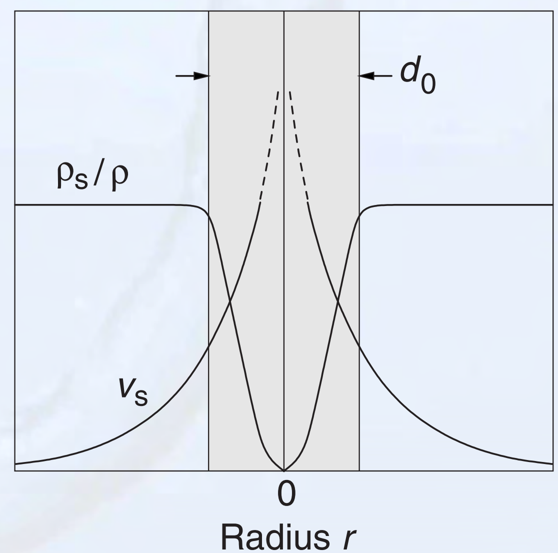


with $\kappa = \dfrac{h}{m_4}\, n$ and classical hydrodynamics one finds

$$v_\mathrm{s}(r) = \frac{\kappa}{2\pi\, r} = \frac{1.58 \times 10^{-8}}{r}\, n \qquad \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$$

- normal core: $v_\mathrm{s} \longrightarrow v_\mathrm{c}$

$d_0 \approx 2 - 3\,\text{Å}$ $\triangleq$ coherence length
healing length



2.5 Macroscopic Quantum State

Energy of a vortex

kinetic energy / volume

$$E_\mathrm{v} = \int_{a_0}^{b} \frac{\varrho_\mathrm{s} v_\mathrm{s}^2}{2} \, 2\pi r \, \mathrm{d}r$$

energy / length

a_0 : radius of vortex core

b : radius of vessel or ½ distance to next vortex

$$\kappa = v_\mathrm{s} \, 2\pi r \quad \longrightarrow \quad v_\mathrm{s}^2 = \frac{\kappa^2}{4\pi^2 r^2}$$

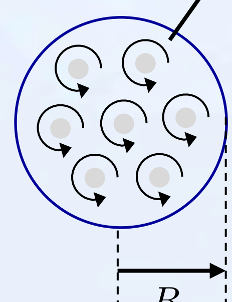
$$E_\mathrm{v} = \frac{\varrho_\mathrm{s} \kappa^2}{4\pi} \ln\left(\frac{b}{a_0}\right) \propto \kappa^2 \propto n^2$$

➡ vortex formation with $n = 1$ is preferred

Why is not a large vortex forming?

➡ splitting up in many small vortices
prohibits large kinetic energy in core
of vortex near the axis of rotation
(velocity at the edge of vessel is given)

N vortices



$$v_\mathrm{s}(R) = N \frac{h}{m_4} \frac{1}{2\pi R}$$

$N \propto R^2$ if evenly distributed

2.5 Macroscopic Quantum State

At what velocity vortices are formed ?

critical angular velocity

$$\omega_c = E_v / L_v$$

angular momentum

$$L_v = \int_0^R \varrho_s r \, v_s \, 2\pi r \, dr = \frac{1}{2} \varrho_s \kappa R^2$$

$$\Rightarrow \quad \omega_c = \frac{h}{2\pi m_4 R^2} \ln\left(\frac{R}{a_0}\right)$$

$$R = 1\,\mathrm{cm} \longrightarrow \omega_c \approx 10^{-3}\,\mathrm{s}^{-1}$$

comment:

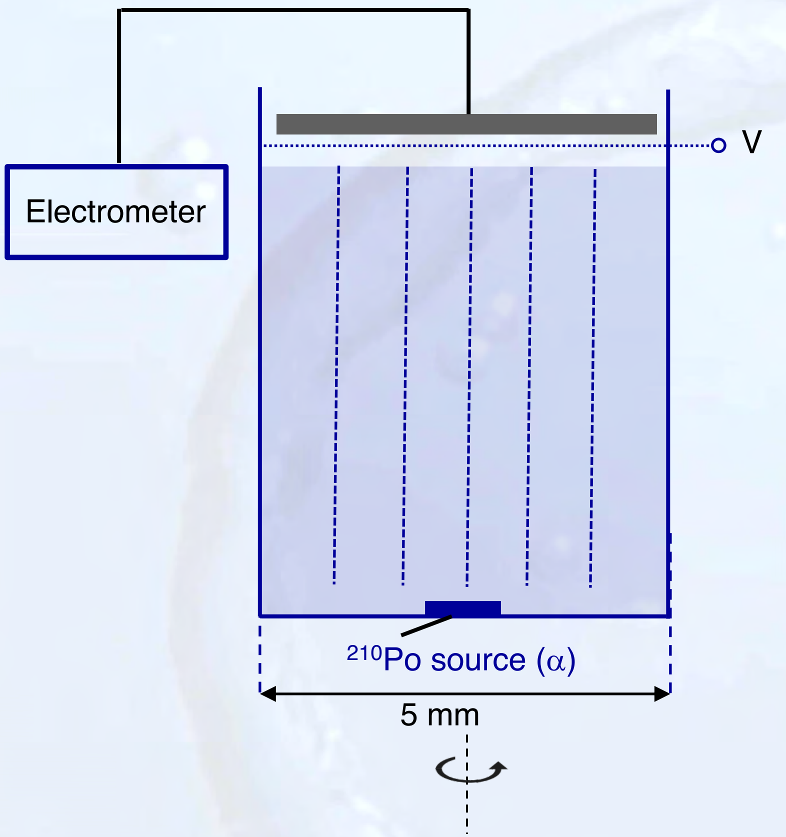
concept of critical velocity will be discussed in section 2.6

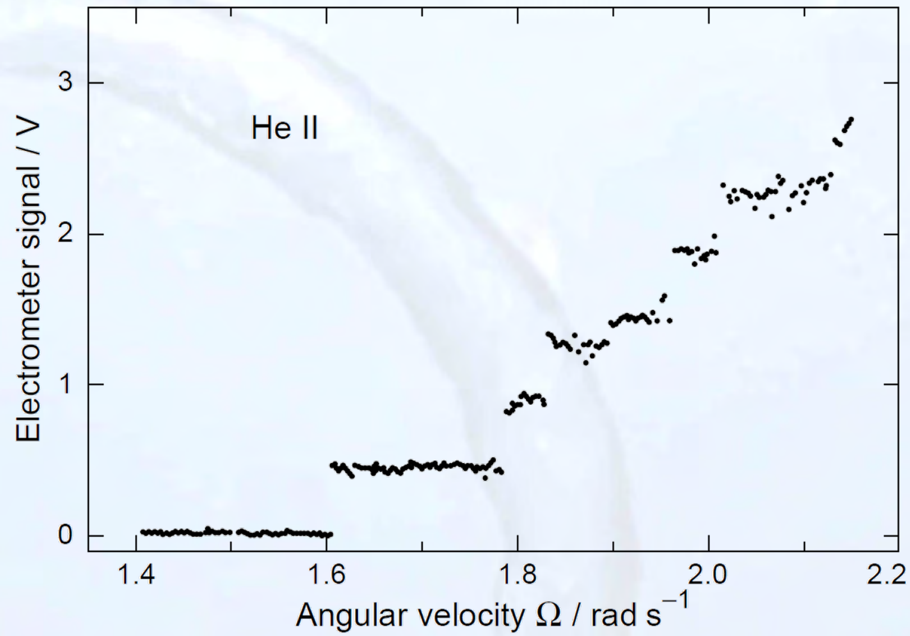
Experimental observation of vortices

- meniscus is rotating vessels
- damping of second sound
- electrometer experiments
- exploding electron bubbles
- decorating with hydrogen ice particles

2.5 Macroscopic Quantum State

Experimental observation of vortices

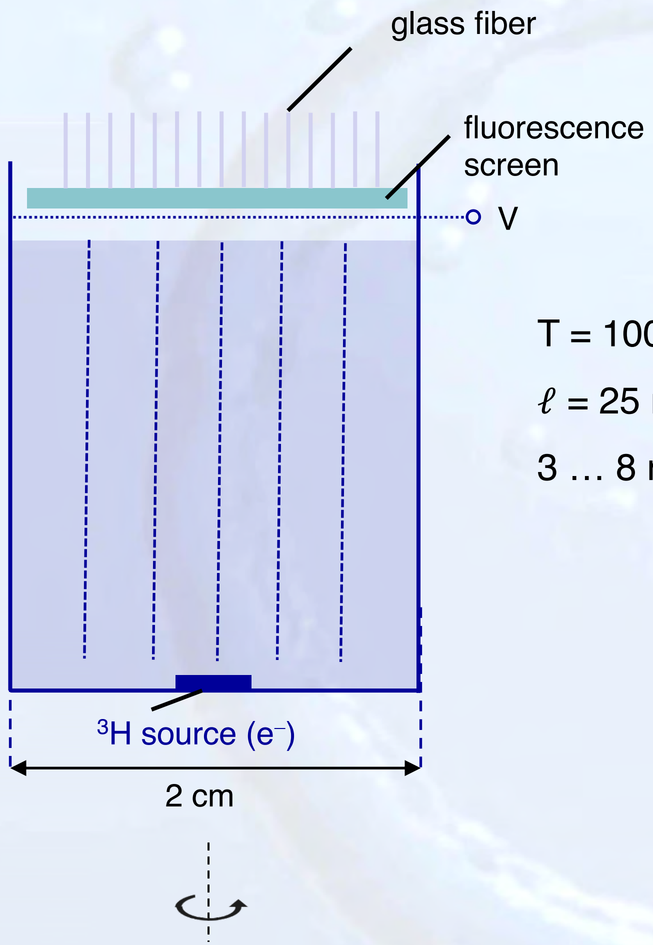




- α source ⟶ helium ionized ⟶ electrons form bubbles
- bubbles are captured by vortex lines via Magnus force
- *E* field is pulling bubbles alongside of vortex line to surface
- measurement of charge ⟶ is proportional to number vortex lines
- uniform acceleration over 10 h to 10 rot/min

2.5 Macroscopic Quantum State

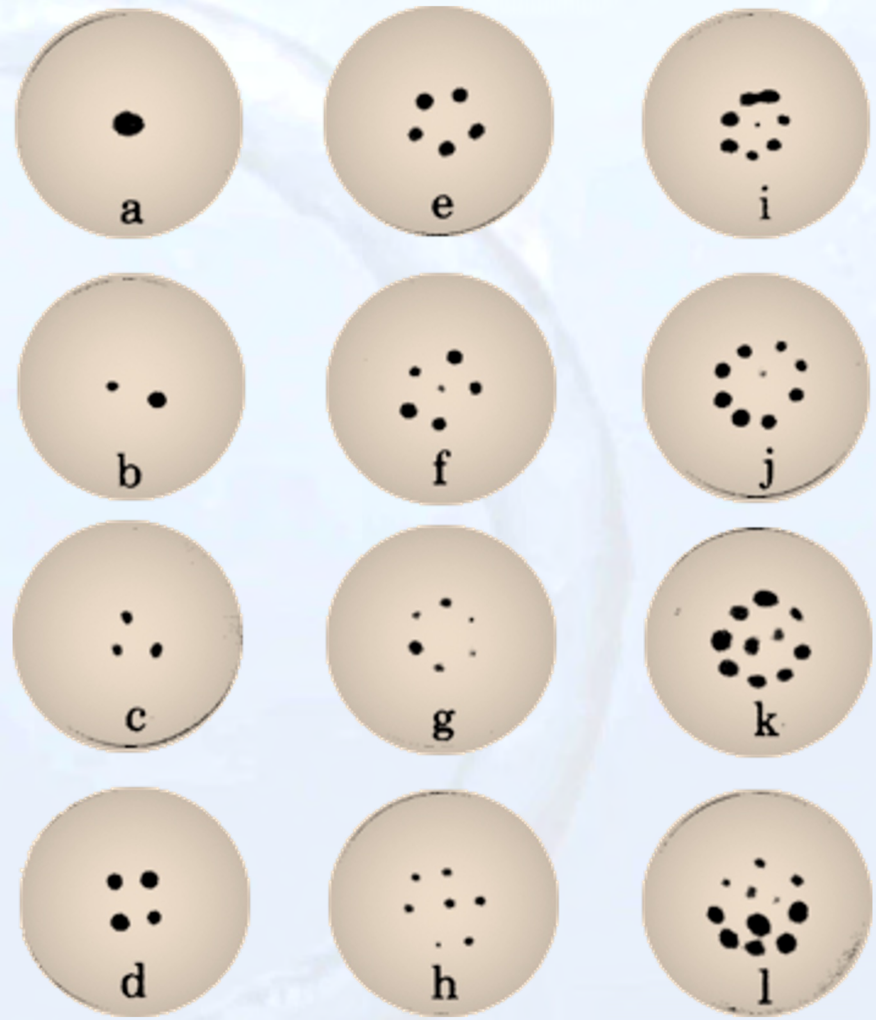
Experimental observation of vortices



T = 100 mK

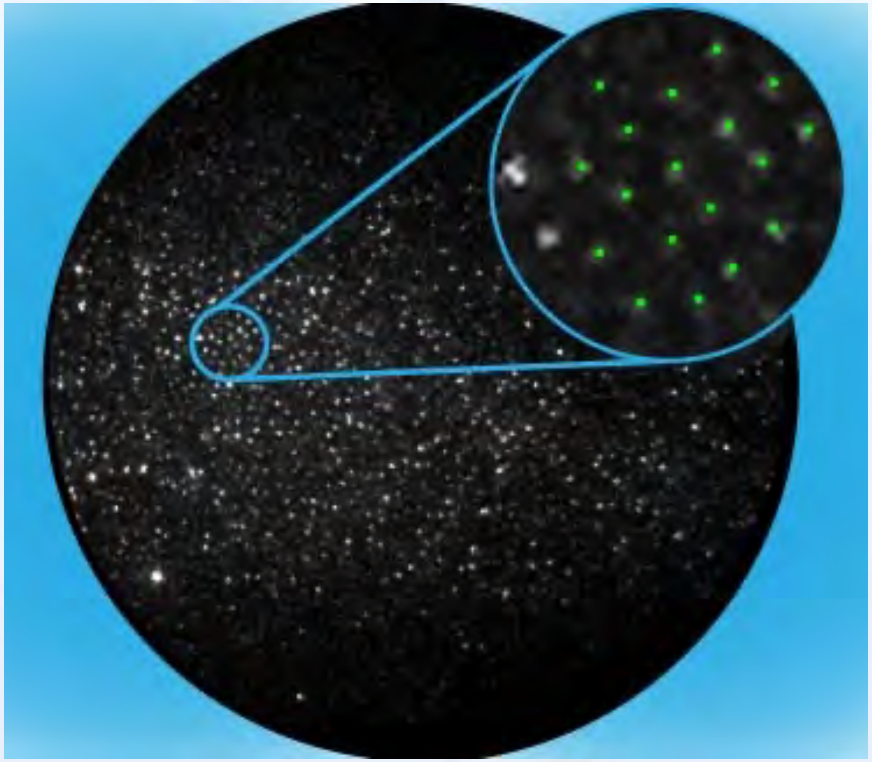
ℓ = 25 mm

3 … 8 rot/min



2.5 Macroscopic Quantum State

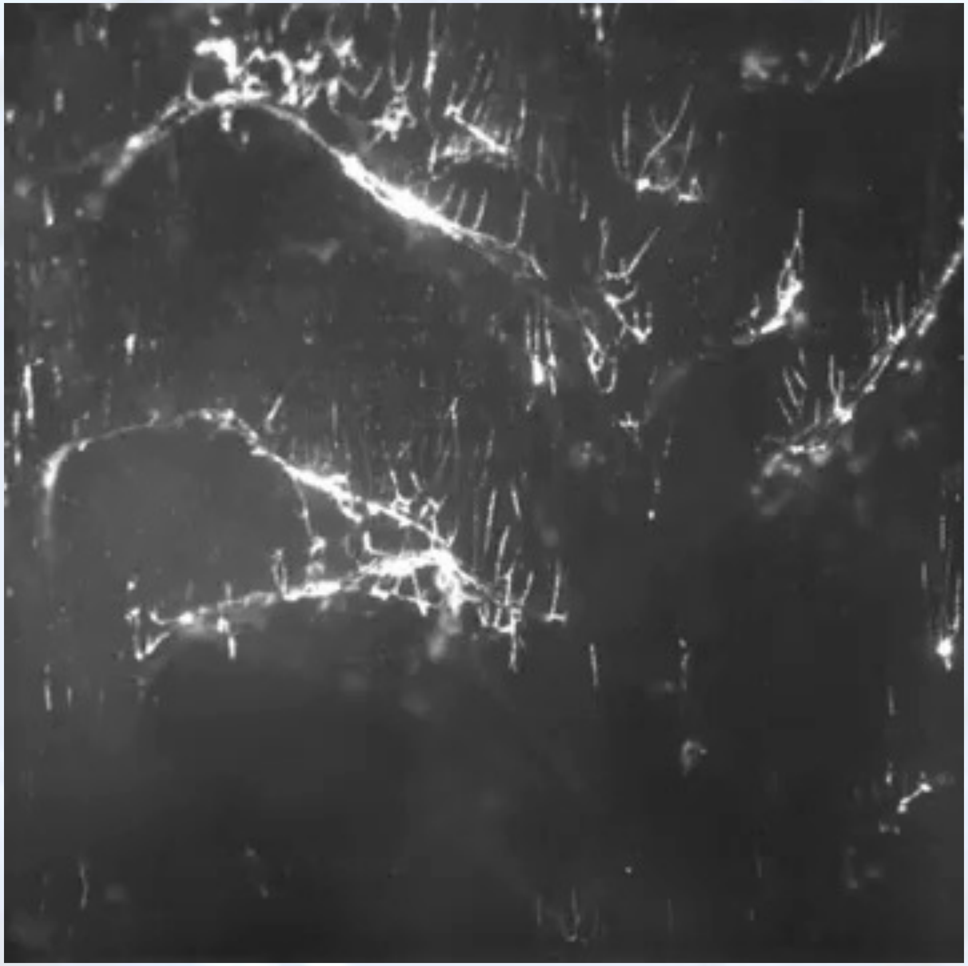
Experimental observation of vortices



Abrikosov lattice ⟶ Type 2 superconductor

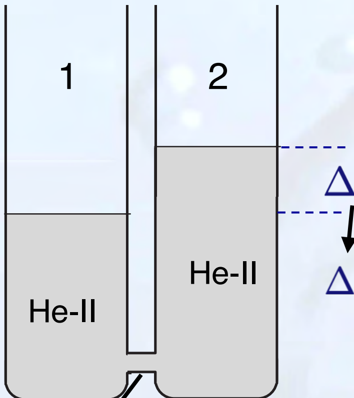
2.5 Macroscopic Quantum State

Experimental observation of vortices



2.5 Macroscopic Quantum State

Josephson Effects



$$\Delta\mu = \mu_2 - \mu_1 = m_4 g \Delta z$$

weak link

healing length

$$d \approx \xi = 1 \ldots 2\,\text{Å}$$

$$\xi_4 = \frac{0.3\,\text{nm}}{(1 - T/T_\lambda)^{2/3}}$$

diverges for $T \to T_\lambda$

Schrödinger Eq.

$$\mathrm{i}\hbar\dot{\Psi}_1 = \mu_1\Psi_1 + \mathcal{K}\Psi_2$$

$$\mathrm{i}\hbar\dot{\Psi}_2 = \mu_2\Psi_2 + \mathcal{K}\Psi_1$$

with $\Psi_1 = \sqrt{\varrho_\mathrm{s}}\,\mathrm{e}^{\mathrm{i}\varphi_1}$ and $\Psi_2 = \sqrt{\varrho_\mathrm{s}}\,\mathrm{e}^{\mathrm{i}\varphi_2}$

$$\frac{\partial \varrho_\mathrm{s}}{\partial t} = \frac{2\mathcal{K}}{\hbar}\,\varrho_\mathrm{s}\sin(\varphi_2 - \varphi_1)$$

$T = 0$

$$\frac{\partial}{\partial t}(\varphi_2 - \varphi_1) = -\frac{1}{\hbar}(\mu_2 - \mu_1) = -\frac{1}{\hbar} m_4 g \Delta z$$

$\Delta\mu = 0$ phase difference constant → Josephson dc effect

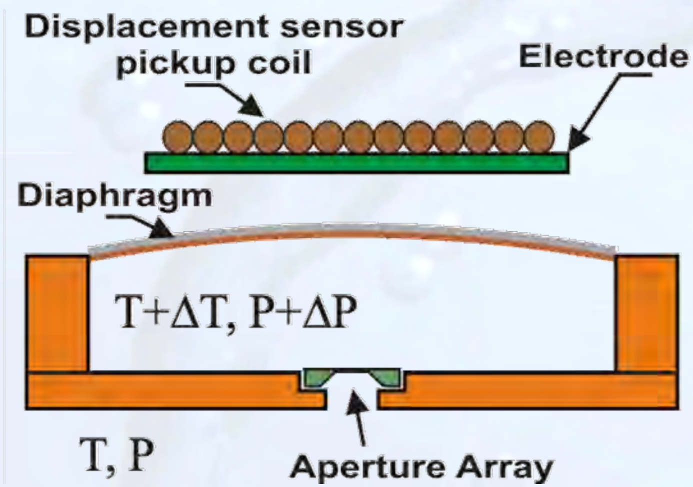
$\Delta\mu \neq 0$ phase difference changes → Josephson ac effect

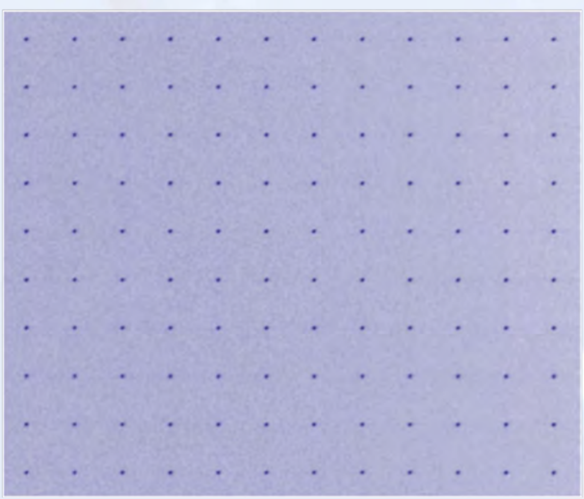
$$T = 0 \longrightarrow \omega_\mathrm{J} = \frac{\Delta\mu}{\hbar} = \frac{m_4 \Delta p}{\hbar\varrho} \quad \text{with } \Delta p = \varrho g \Delta z$$

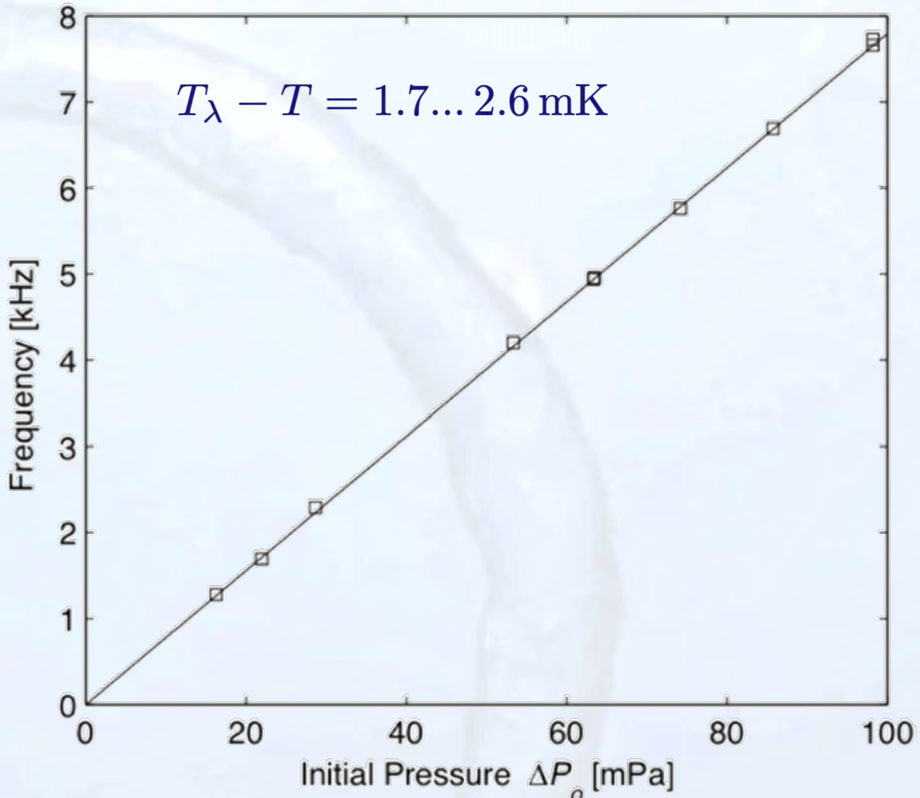
$$T \neq 0 \longrightarrow \omega_\mathrm{J} = \frac{\Delta\mu}{\hbar} = \frac{m_4 \Delta p}{\hbar\varrho} - m_4 \frac{S \Delta T}{\hbar}$$

2.5 Macroscopic Quantum State

Josephson Effects







measurement directly after pressure applied

$\longrightarrow \Delta T \approx 0$

65×65 array

$70\,\mathrm{nm}$ apertures spaced $3\,\mu\mathrm{m}$

50 nm-thick silicon nitride membrane

2.6 Excitation Spectrum of He-II: Landau Model

Is the occurrence of the condensate equivalent to superfluidity ?

comment:

superconductivity in metals is related to an energy gap

nature of excitations is important

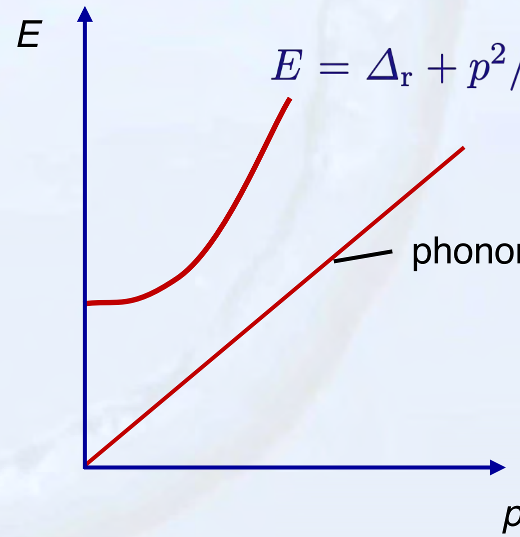
ideal Bose gas:

$$E = \frac{p^2}{2m} \longrightarrow$$ arbitrary energy transfer possible

➡ no superfluidity !

Idea of Landau 1941

- at low temperatures: only longitudinal phonons with linear dispersion
- at “high” temperatures: more and other kinds of excitations contribute, but with energy gap



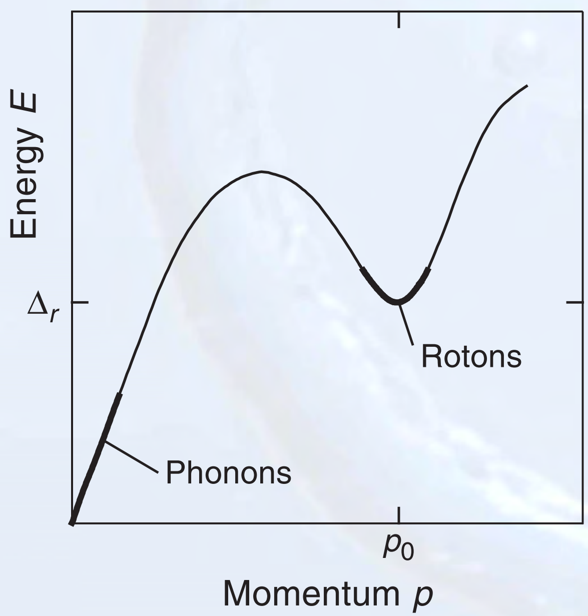
2.6 Excitation Spectrum of He-II: Landau Model

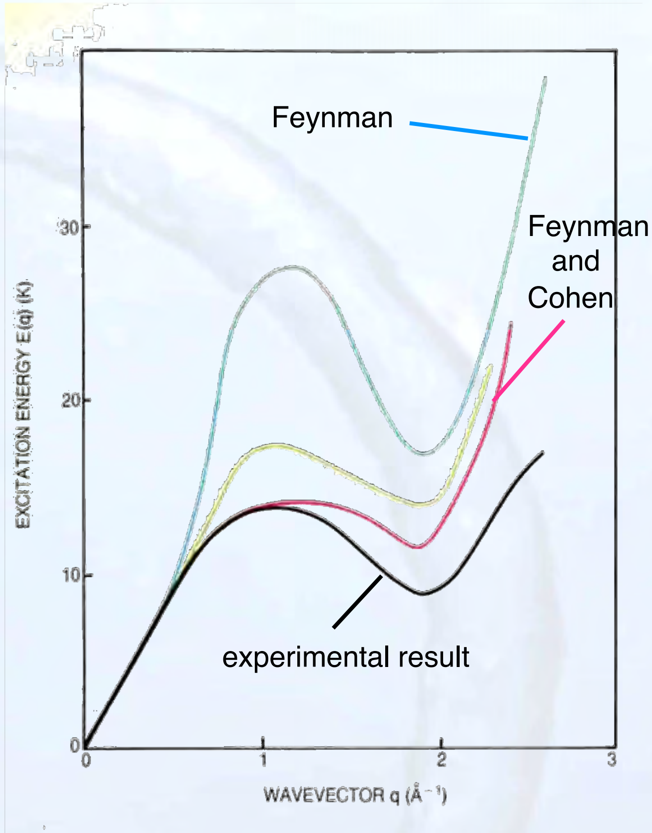
Landau's modification in 1947:

→ common dispersion curve

roton dispersion:

$$E = \Delta_\mathrm{r} + \frac{(p - p_0)^2}{2m^*}$$





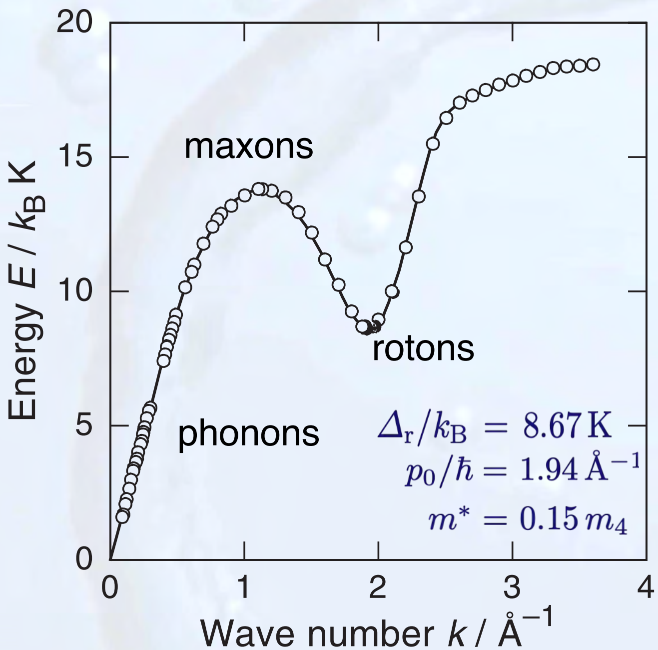
Feynman 1954:

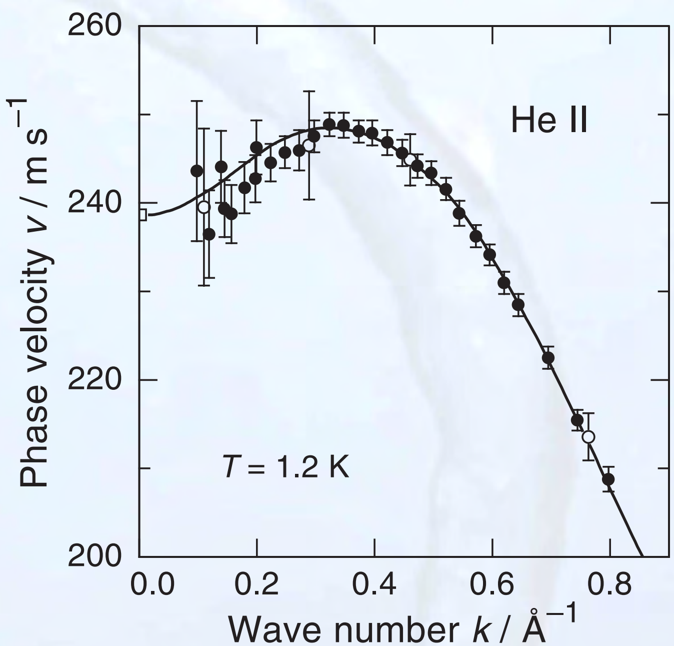
- QM calculation of dispersion curve from symmetry considerations
- improved by Feynman and Cohen in 1955

2.6 Excitation Spectrum of He-II: Landau Model

Experimental determination of the dispersion

Feynman's idea: inelastic neutron scattering





- good agreement with q_{min}, q_{max}
- linear dispersion with v = 238 m/s
- sharp excitations even at high q vectors
- single particle excitations are suppressed

- dispersion not perfectly linear
- anomaly at low wave vectors
  - → causes damping by three phonon scattering
  - → anomaly disappears at $p > 20$ bar

2.6 Excitation Spectrum of He-II: Landau Model

Experimental Determination of the Dispersion

new high-precision measurement