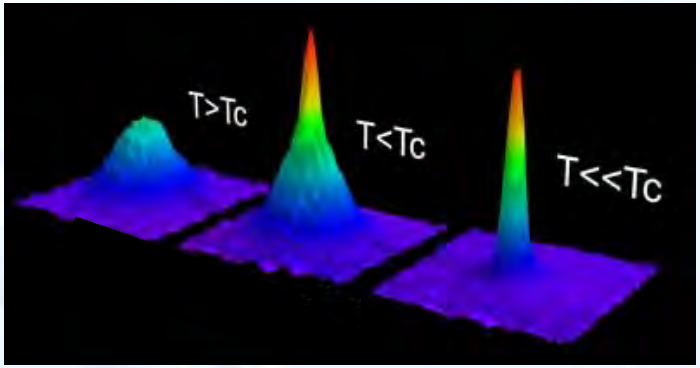
2.4 Bose-Einstein Condensation

What is the value of the condensation temperature?

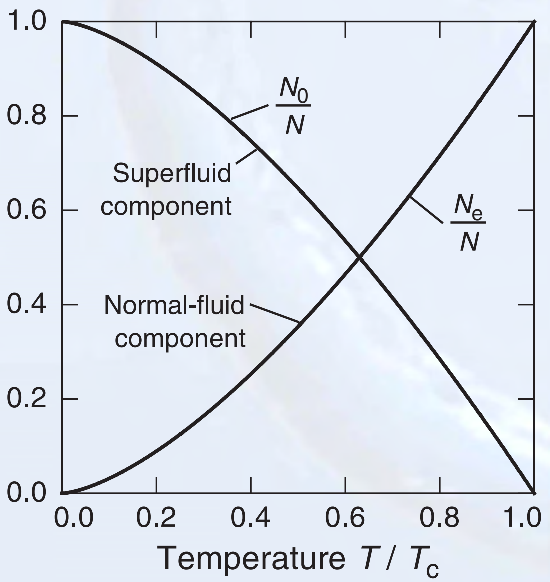
$$\left.\begin{aligned} N_0(T_c) &= 0 \\ N_e(T_c) &= N \end{aligned}\right\} \qquad T_c = \frac{2\pi\hbar^2}{k_B m}\left(\frac{N}{2.6V}\right)^{2/3}$$



Bose Einstein condensate of atomic gas

He → gas $T_c \approx 0.5\,\mathrm{K}$, but boiling point is at $4.2\,\mathrm{K}$

He → liquid $T_c = 3.1\,\mathrm{K}$, works well in comparison to $T_\lambda = 2.17\,\mathrm{K}$



$$\frac{N_e}{N} = \left(\frac{T}{T_c}\right)^{3/2}$$

the condensation of a normal gas in real space corresponds to the Bose-Einstein condensation in momentum space, which means all atoms have the same wave vector and are strongly correlated.

2.4 Bose-Einstein Condensation

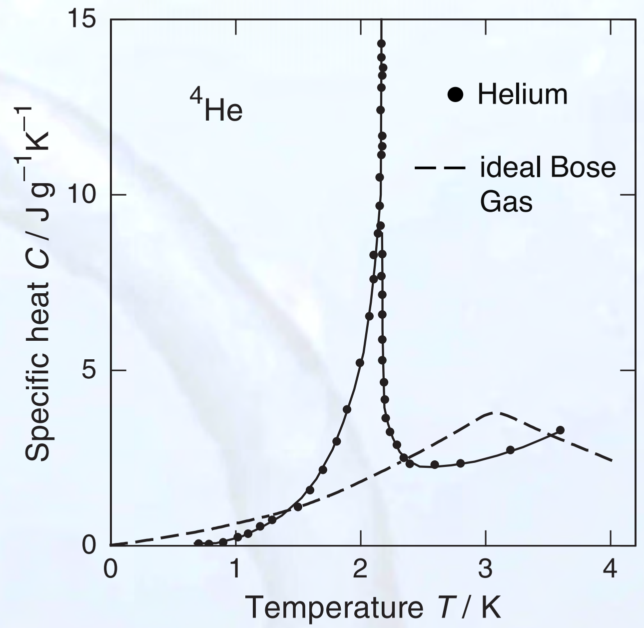
b) Interacting Bose "gas" (He)

specific heat

C_V → ideal Bose gas
C_V → experimental results

→ interactions are important

→ depletion of the ground state
→ collective excitation
(has first been proposed by Bogoliubov 1947)

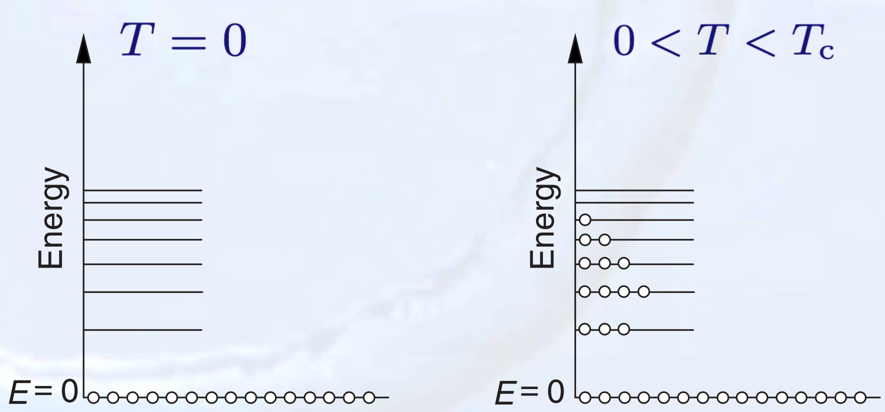


ideal Bose gas

$T = 0\,,\ N_0 = N$

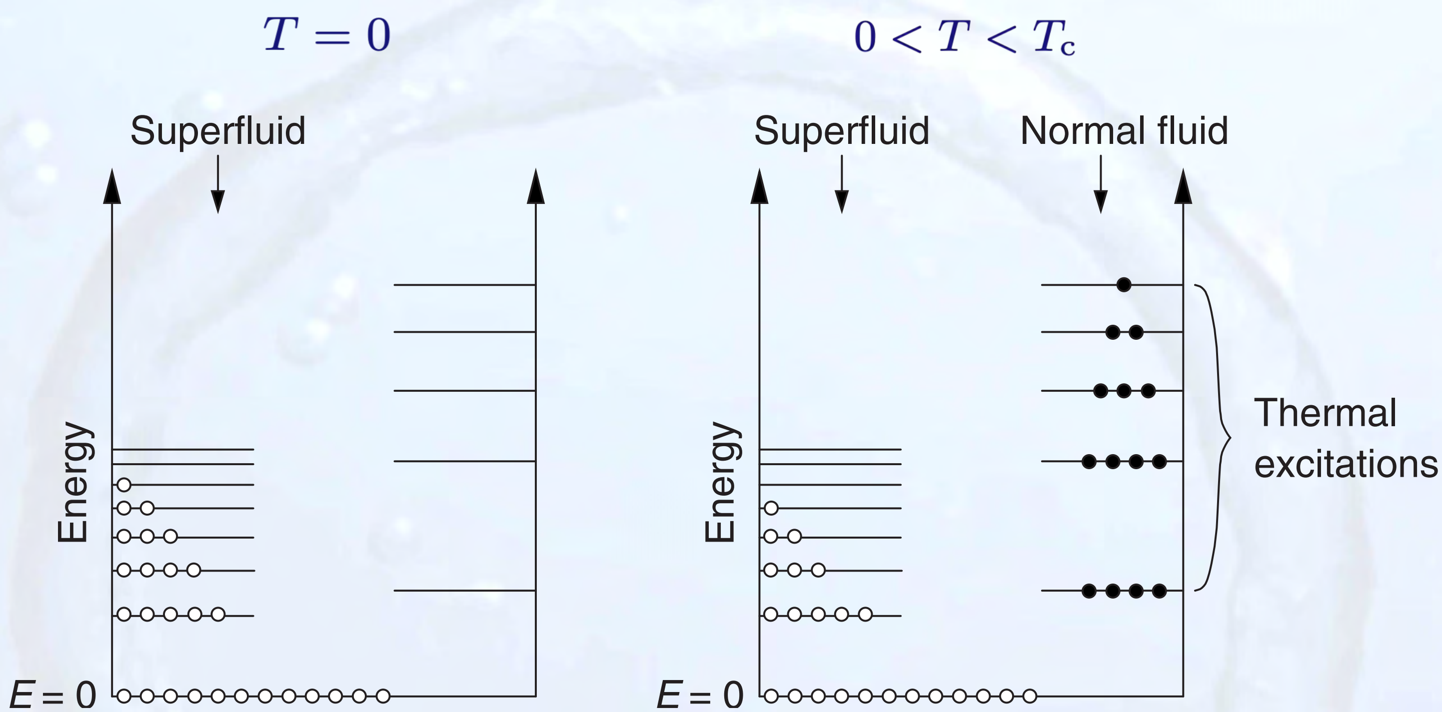
$T \neq 0\,,\ N_0 \leq N$

excited atoms



2.4 Bose-Einstein Condensation

interacting Bose gas

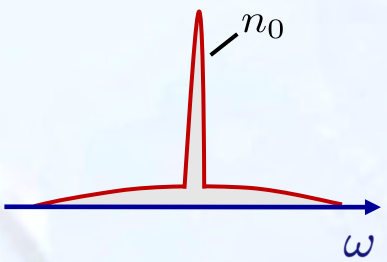


$T = 0\,,\; N_0 < N:$ significant number of atoms are not in the ground state

$T \neq 0\,,\; N_0 < N:$ in addition, collective excitations, nature of excitations changes

Experimental determination of the condensate

there is no direct way to measure the condensate fraction: $N_0/N = n_0$



a) neutron scattering: measuring the dynamic structure factor $S(Q,\omega) \mathrel{\widehat{=}} n(p)$ via inelastic neutron scattering

momentum distribution

b) X-ray scattering: pair correlation function $g(r)$ at transition to superfluid state becomes broader because of the condensation in momentum space

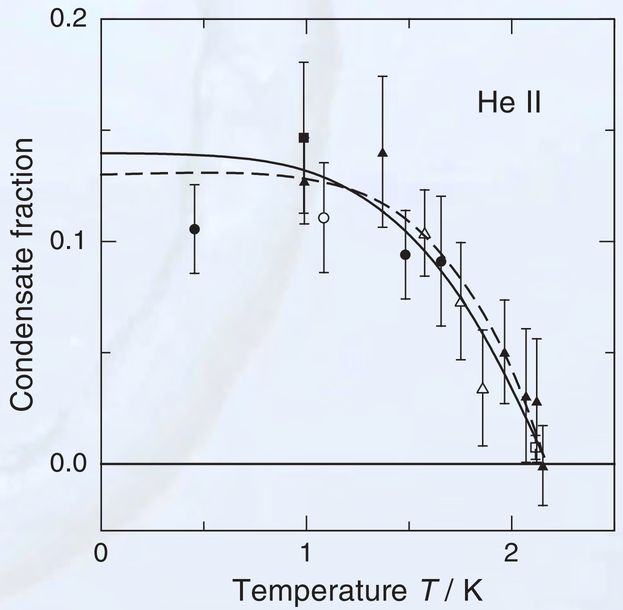
$$g(r) - 1 = (1 - n_0)^2 \left[g^*(r) - 1\right]$$

$g(r)$ above

c) surface tension: complicated but possible

➡ condensate fraction for $T \to 0$ just 13 %

ϱ_s is not equal with condensate fraction



2.5 Macroscopic Quantum State

→ quantization of circulation

→ Josephson effects

wave function of superfluid component

$$\psi(\boldsymbol{r}) = \psi_0\, \mathrm{e}^{\mathrm{i}\varphi(\boldsymbol{r})} \qquad (*) \qquad \text{with} \quad \psi^\star\psi = |\psi_0|^2 = \frac{\varrho_\mathrm{s}}{m_4}$$

m_4: mass of a ^{4}He atom

Schrödinger equation

$$-\mathrm{i}\hbar\nabla\psi = \boldsymbol{p}\,\psi$$

with $(*)$ ⟶ $\boldsymbol{p} = \hbar\nabla\varphi(\boldsymbol{r}) = m_4\boldsymbol{v}_\mathrm{s}$ ⟶

$$\boldsymbol{v}_\mathrm{s} = \frac{\hbar}{m_4}\nabla\varphi(\boldsymbol{r})$$

comment:

only valid at sufficiently low velocity were ϱ_s is constant

➡ $\boldsymbol{v}_\mathrm{s}$ determines the phase shift of wave function

→ $\boldsymbol{v}_\mathrm{s} = 0$ ⟶ phase is constant

→ $\boldsymbol{v}_\mathrm{s} =$ const. ⟶ phase is changes uniformly

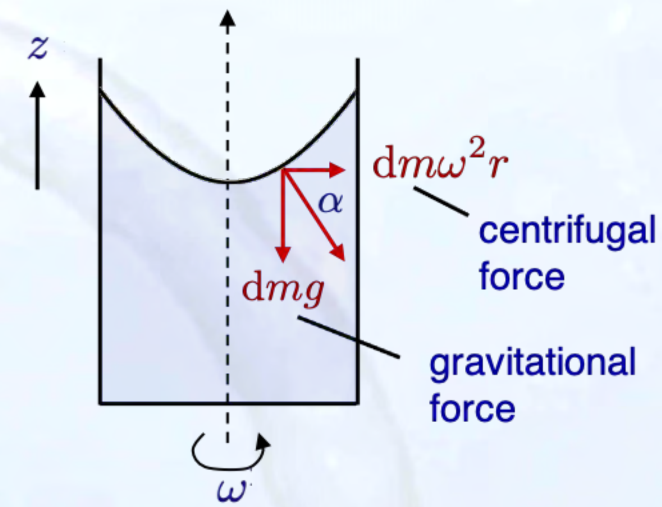
Interpretation

- phase is well-defined in entire liquid
- macroscopic wave function
- “rigid” coupling in momentum space

2.5 Macroscopic Quantum State

Proof of the concept: He-II under rotations

measurement of liquid meniscus



classical fluid $\triangleq$ normalfluid component ϱ_n

➡ solid body rotation $v_\mathrm{n} = \omega r$ — distance from axis of rotation

➡ profile of liquid surface ⟶ parabola

$$\tan\alpha = \frac{\mathrm{d}z}{\mathrm{d}r} = \frac{\omega^2 r}{g} \longrightarrow z = \frac{\omega^2}{2g} r^2$$

what about the superfluid component ?

two-fluid model $\operatorname{curl} \boldsymbol{v}_\mathrm{s} = 0$!

for a simply-connected region this means every loop can be contracted to a point

$$\int_A \underbrace{\operatorname{curl} \boldsymbol{v}_\mathrm{s}}_{=0} \cdot \mathrm{d}\boldsymbol{f} = \oint_L \boldsymbol{v}_\mathrm{s} \cdot \mathrm{d}\boldsymbol{l} = 0$$

area enclosed by contour L

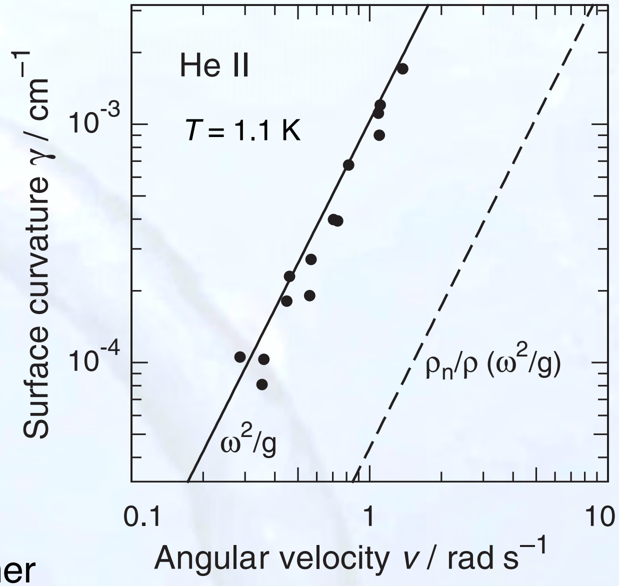
Stokes

- ϱ_s should not rotate (should be at rest)
- if so, centrifugal force is reduced

➡ $$z = \frac{\varrho_\mathrm{n}}{\varrho} \frac{\omega^2}{2g} r$$

2.5 Macroscopic Quantum State

Experimental results



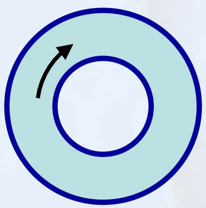
surface curvature: $\gamma = \omega^2/g$ all liquid

$\gamma = (\varrho_\mathrm{n}/\varrho)\omega^2/g$ only normalfluid

➡ curvature for all liquid is observed in Osborn experiment

Why is this the case?

⟶ let's do a thought experiment with an annular-shaped container — multiply-connected region



circulation:

$$\kappa = \oint_L \boldsymbol{v}_\mathrm{s} \cdot \mathrm{d}\boldsymbol{l} \qquad \boldsymbol{v}_\mathrm{s} = \frac{\hbar}{m_4}\nabla\varphi(\boldsymbol{r})$$

➡ $$\kappa = \frac{\hbar}{m_4}\Delta\varphi_L$$

- since $\psi(\boldsymbol{r})$ is a uniquely-defined function
  ⟶ phase can only be changed by $2\pi\,n$ for full cycle
- $\Delta\varphi = 2\pi\,n \qquad n = 0,\ 1,\ 2,\ 3,\ \ldots\ .$

circulation is quantized !

$$\kappa = \frac{h}{m_4}n$$