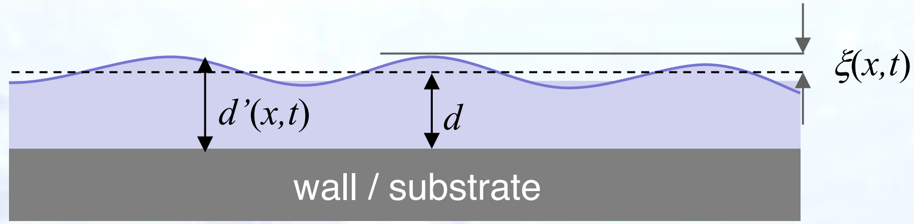
2.3 Properties of He-II described using the two-fluid model

(iii) Third sound

sound propagating in thin films



$$d'(x,t) = d + \xi(x,t)$$

mean film thickness

assumptions: thin films $\boldsymbol{v}_\mathrm{n} = 0$

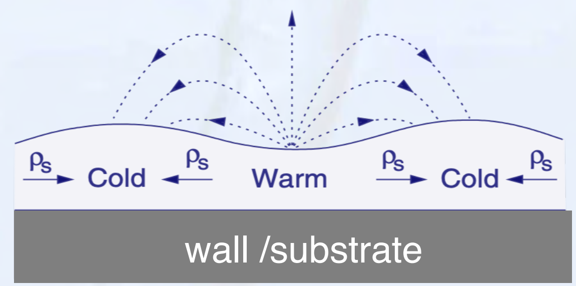
$\lambda \gg d \longrightarrow$ motion parallel to substrate in x-direction $(v_y = v_z = 0)$

$\mathrm{grad}\, T \approx 0$ (questionable ?)

$$\Rightarrow \quad \frac{\partial^2 \xi}{\partial t^2} = f d \frac{\varrho_\mathrm{s}}{\varrho} \frac{\partial^2 \xi}{\partial x^2} \quad \Rightarrow \quad \boxed{v_3^2 = \frac{\varrho_\mathrm{s}}{\varrho} 3 g z}$$

f: v. d. Waals force

z: height over liquid level



problem: evaporation and condensation

$\Rightarrow$ increases the amplitude and changes velocity

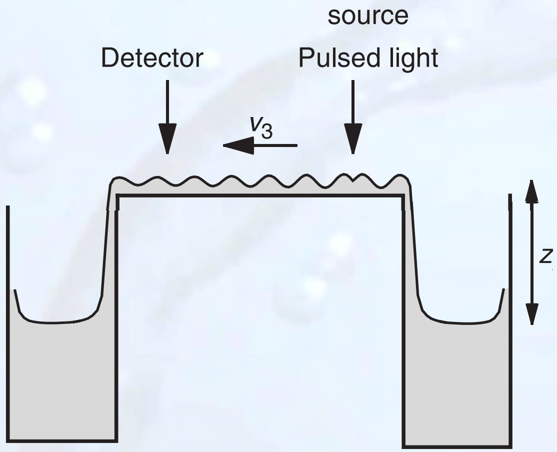
can be taken into account

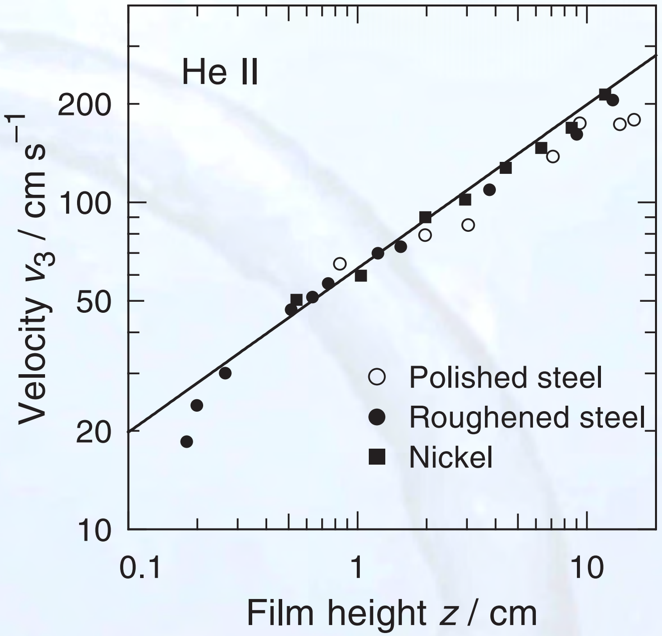
$$v_3^2 \approx \frac{\varrho_\mathrm{s}}{\varrho} 3 g z \left(1 + \frac{TS}{L}\right)$$

$\left.\begin{array}{l} TS/L = 0.01 \text{ at } 1\text{ K} \\ TS/L = 0.15 \text{ at } T_\lambda \end{array}\right\}$ not really a problem

2.3 Properties of He-II described using the two-fluid model

3rd sound experiment





Procedure

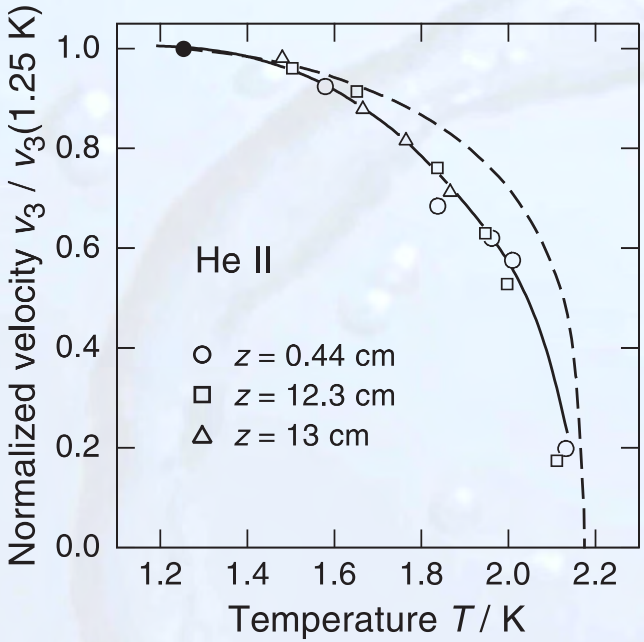
- periodic local heating
- ϱ_s flows to warm location ➡ thickness changes
- surface wave ≙ 3rd sound
- optical detection of thickness

Measurement and results

- 3rd sound velocity vs. z (log/log plot)
- different surfaces: v_3 almost independent
- line ≙ theory $v_3 \propto \sqrt{z}$
- good agreement except for very thick films

2.3 Properties of He-II described using the two-fluid model

3rd sound experiment: temperature dependence



Measurement and results

- 3rd sound velocity vs T
- points at T = 1.25 K normalized to (●)
- v_3 is rising with decreasing T
- $T \to 0$: v_3 = 1.5 m/s (very slow)
- dashed line ≙ theory $v_3 \propto \sqrt{\varrho_s}$
- systematic deviations: origin unknow, but likely due to generation process

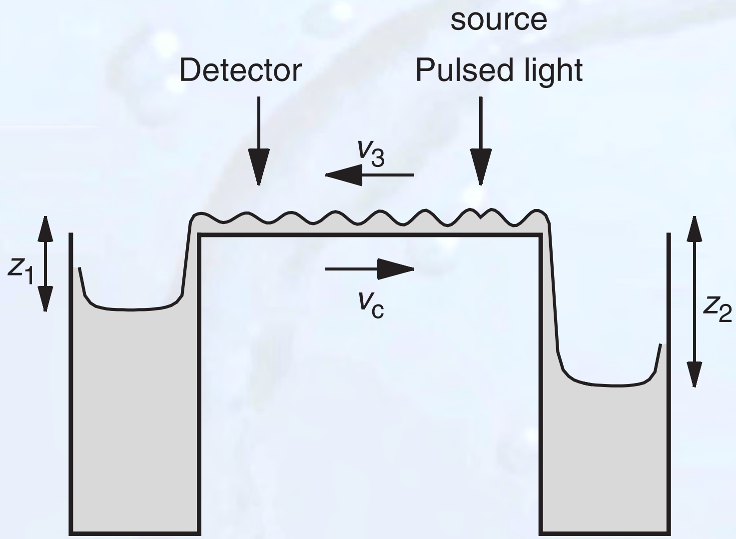
3rd sound in very thin films:

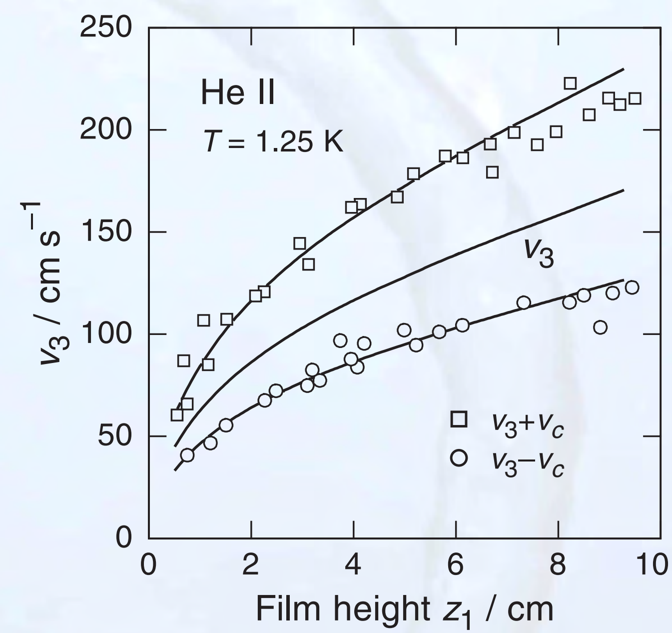
3rd sound propagation can be observed down to 2.1 monolayers

➡ onset of superfluidity

2.3 Properties of He-II described using the two-fluid model

3rd sound in moving films:





3rd sound propagation in moving films ➡ Doppler effect $v_3 \pm v_f$

v_c critical velocity

2.3 Properties of He-II described using the two-fluid model

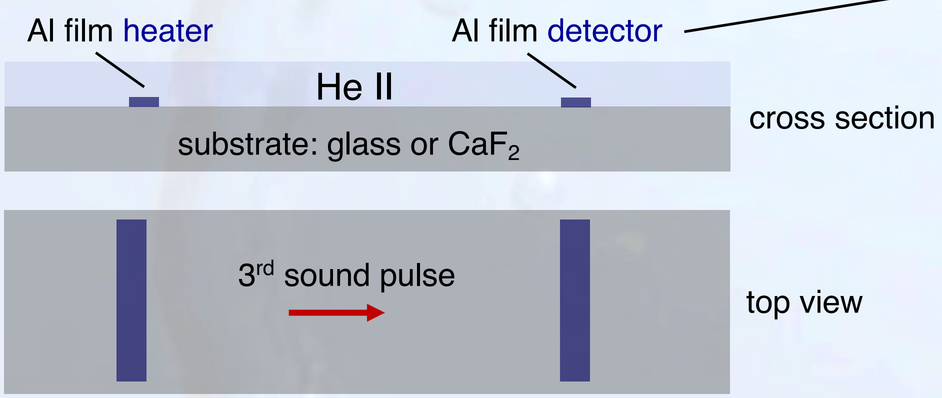
Detection of 3rd sound experiment in ultralow films:

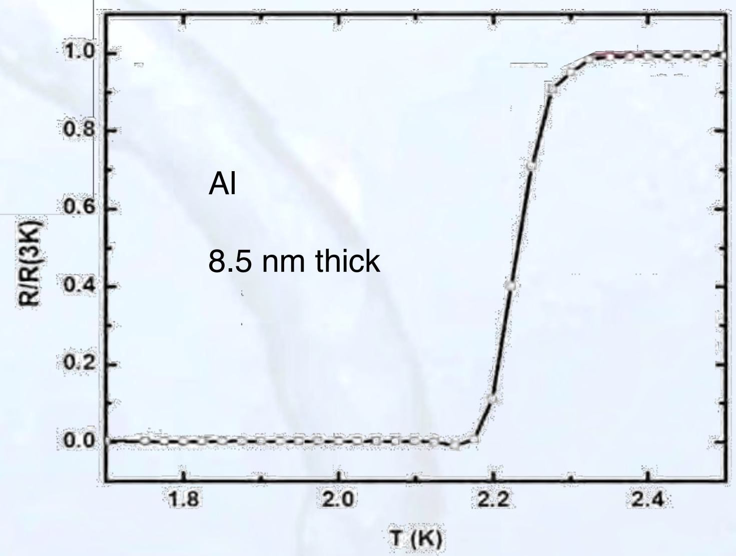
Third Sound and the Healing Length of He II in Films as Thin as 2.1 Atomic Layers*

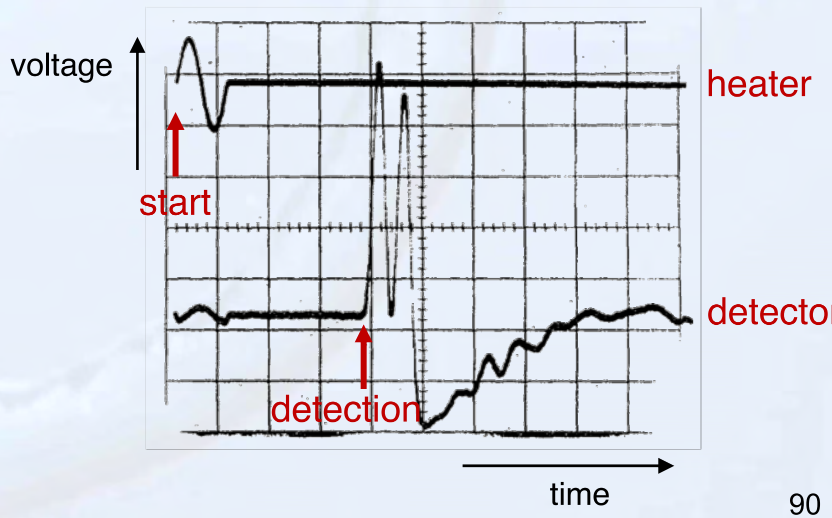
J. H. Scholtz, E. O. McLean,† and I. Rudnick
University of California, Los Angeles, California 90024
(Received 23 August 1973)

Measurements of the velocity of third sound on films as thin as 2.1 atomic layers yield the healing length of superfluid He II down to temperatures of 0.1 K. It is argued that these films are two-dimensional superfluids.

PRL **32** 147 (1974)



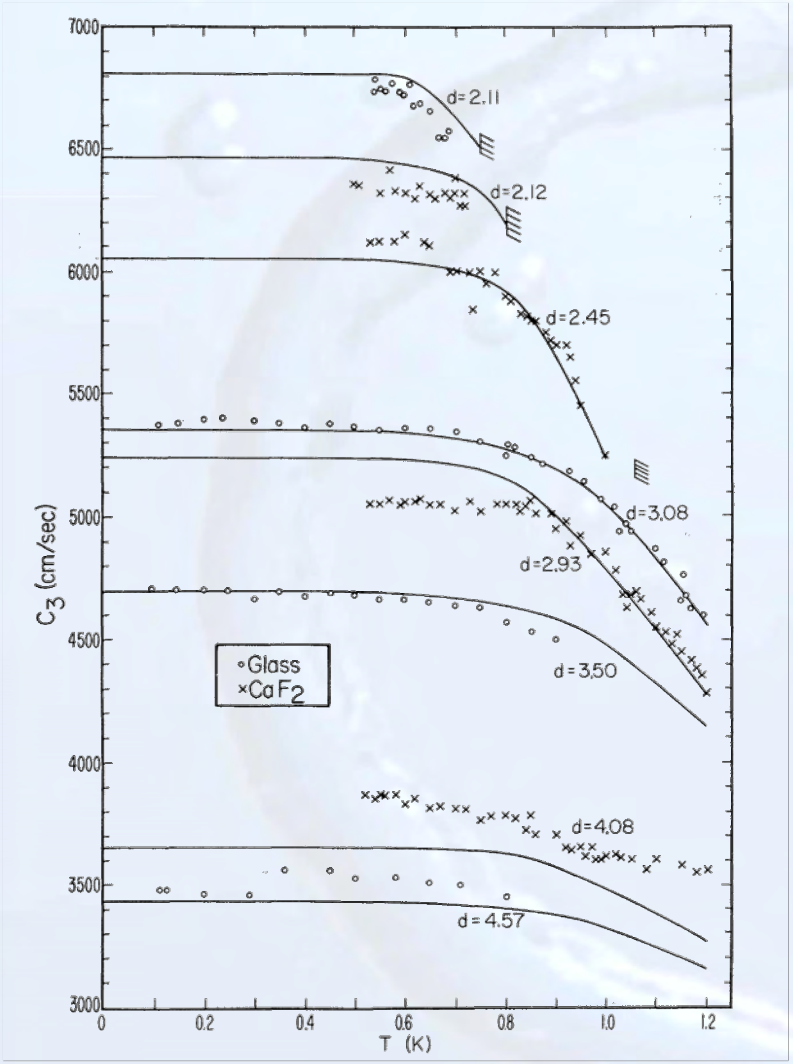




- ► time of flight detection of 3rd sound pulse
- ► thin film Al heater and detector
- ► detector operated at the transition temperature
- ► detection of heat pulse associated with 3rd sound pulse
- ► very sensitive because of steep transition curve

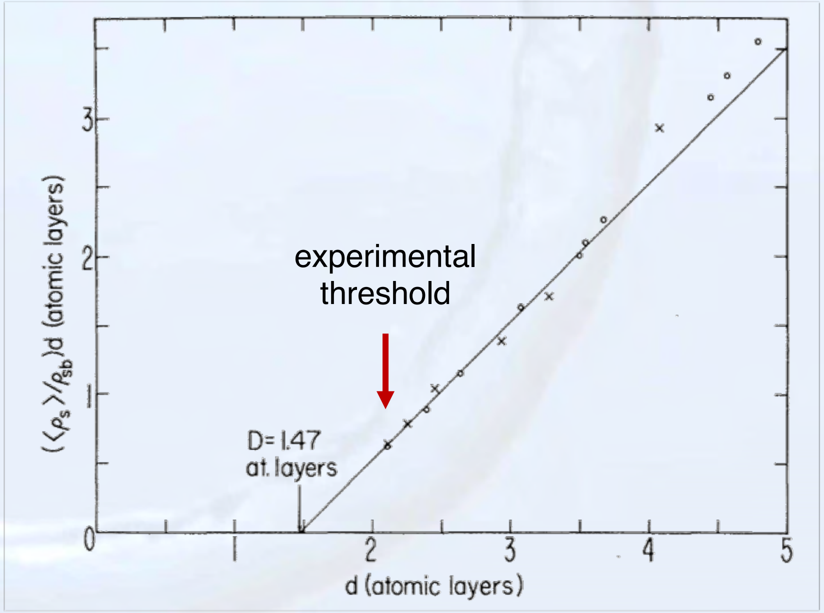
2.3 Properties of He-II described using the two-fluid model

Experimental results:



for ultrathin films: $v_3^2 = \frac{\overline{\rho_s}}{\rho_{s,\mathrm{bulk}}} \frac{3RT}{m} \ln \frac{p_0}{p}$

- experimental threshold of 2.1 monolayers independent of substrate
- film thickness determine by amount of helium and surface area
- extrapolation suggests that 1.57 monolayers might be the onset threshold



2.3 Properties of He-II described using the two-fluid model

(iv) Fourth sound

sound propagation in fine powders / slits $\boldsymbol{v}_\mathrm{n} \approx 0$

➡ oscillations in total density, in ratio of superfluid to normalfluid density, in pressure, in temperature, in entropy

$$v_4^2 = \frac{\varrho_\mathrm{s}}{\varrho} v_1^2 \left[1 + \underbrace{\frac{2ST}{\varrho C_p} \left(\frac{\partial \varrho}{\partial T} \right)_p}_{\ll 1} \right] + \frac{\varrho_\mathrm{n}}{\varrho} v_2^2$$

$$\boxed{v_4 \approx \sqrt{\frac{\varrho_\mathrm{s}}{\varrho} v_1^2 + \frac{\varrho_\mathrm{n}}{\varrho} v_2^2}} \approx \sqrt{\frac{\varrho_\mathrm{s}}{\varrho} v_1^2}$$

5th sound

4th sound generation like for 1st sound, but $\boldsymbol{v}_\mathrm{n} \approx 0$

2.3 Properties of He-II described using the two-fluid model

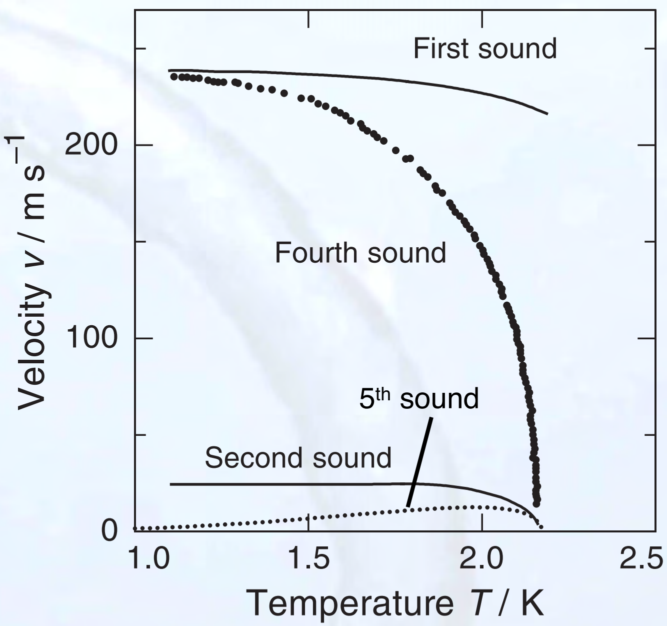
4th sound experiments

4th sound generation like for 1st sound, but $\boldsymbol{v}_\mathrm{n} \approx 0$

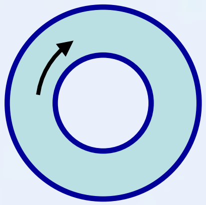
$T \to 0 \quad v_4 = v_1 \approx 238\,\mathrm{m/s}$, since $\varrho_\mathrm{s} = \varrho$

$T = T_\lambda \quad v_4 = 0$

$$v_4 \approx \sqrt{\frac{\varrho_\mathrm{s}}{\varrho}\, v_1^2 + \frac{\varrho_\mathrm{n}}{\varrho}\, v_2^2}$$



Persistent flow and 4th sound



persistent flow velocity

$$v_4 \approx v_{4,0} \pm \frac{\varrho_\mathrm{s}}{\varrho}\, v_\mathrm{D}$$

coupling of a compression wave to second sound

2.4 Bose-Einstein Condensation

Einstein 1924
Bose 1925
London 1938

Basic idea of Fritz London:

dissipation-less motion ⟷ macroscopic wave function

a) Ideal Bose gas

non-interacting Bose gas (rough approximation for liquid He)

let's consider: 1 cm^3 cube of liquid ^{4}He ≙ 10^{22} atoms with mass *m*

eigenstates for free particles in a cube:

$$E_n = \frac{\hbar^2}{2m}\left(\frac{\pi}{L}\right)^2 n^2 \qquad \text{with} \qquad n^2 = n_x^2 + n_y^2 + n_z^2$$

$T = 0 \longrightarrow$ all atoms are in the ground state E_{111} trivial !

But at finite temperatures?

2.4 Bose-Einstein Condensation

consider energy difference between ground state and first excited state

$$\Delta E/k_{\mathrm{B}} = (E_{211} - E_{111})\,/k_{\mathrm{B}} \approx 2 \times 10^{-14}\,\mathrm{K}$$

if Boltzmann statistics would hold ➡ no condensate at 1 K!!!

however, Bose-Einstein distribution is relevant here

$$f(E,T) = \frac{1}{\mathrm{e}^{(E-\mu)/k_{\mathrm{B}}T} - 1} \,.$$

chemical potential $\mu = \dfrac{\partial F}{\partial N}$

what we know: $\mu < E_{111}$ $\longrightarrow$ otherwise, negative occupation

$\mu \neq 0$ $\longrightarrow$ since particle number conserved

2.4 Bose-Einstein Condensation

Occupation of ground state $E_{111} = 0$

$$f(0,T) = \frac{1}{\mathrm{e}^{-\mu/k_\mathrm{B}T} - 1}$$ ➡ occupation depends critically on μ

$f(0, T \to 0) \to \infty$ if $\mu \to 0$ faster than $T \to 0$ $\left(\frac{1}{e^0 - 1} \to \infty \right)$

What is the temperature dependence of $\mu(T)$?

for this let us consider a real, but non-interacting gas

$$\mu = -k_\mathrm{B} T \ln \left(\frac{V_\mathrm{A}}{V_\mathrm{Q}} \right)$$

quantum volume $V_\mathrm{Q} = \left(\frac{h}{\sqrt{2\pi m k_\mathrm{B} T}} \right)^3 = \lambda_\mathrm{B}^3$

thermal de Broglie wavelength

For ^{4}He ⟶ $\lambda_\mathrm{B}^3 =$ (8.7 Å)^3 at 1 K

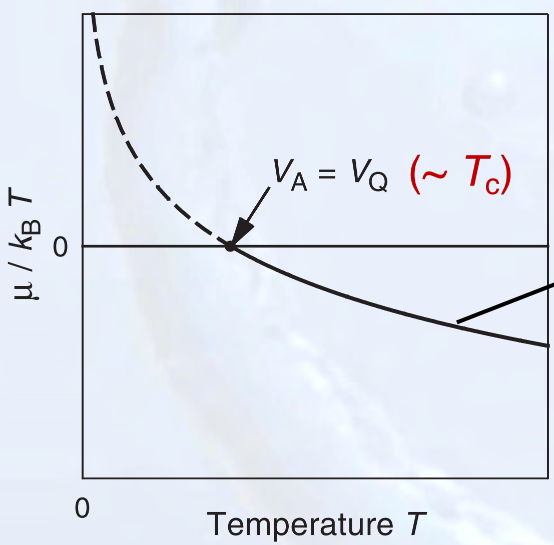
$V_\mathrm{A} = V/N =$ (3.8 Å)^3 in comparison

2.4 Bose-Einstein Condensation

for $T \to 0$ ➡ $V_\mathrm{Q} \to \infty$

at sufficiently low temperatures $V_\mathrm{A} = V_\mathrm{Q}$ **!**

$$\ln\left(\frac{V_\mathrm{A}}{V_\mathrm{Q}}\right) \to 0 \quad ➡ \quad \mu \to 0$$

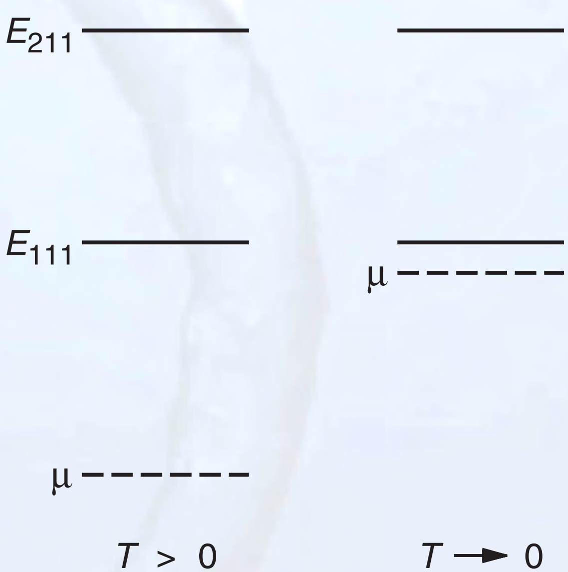


classical regime

μ is negative

He gas at 300 K:

$\mu/k_\mathrm{B} = -3800\,\mathrm{K}$



➡ this means $|\mu|$ becomes smaller than $\Delta E/k_\mathrm{B} = (E_{211} - E_{111})$ at finite T **!**

2.4 Bose-Einstein Condensation

Calculation of μ: how large is μ at 1K? (revers argument)

for $T \to 0$ ➡ $f_{111} \to N$

$$\lim_{T\to 0} f(0,T) = N_0(T) = \lim_{T\to 0}\left(\frac{1}{e^{-\mu/k_B T} - 1}\right)$$

$E_{111} = 0$, ground state

$$\approx \lim_{T\to 0}\left(\frac{1}{1 - \mu/(k_B T) + \ldots - 1}\right) \approx -\frac{k_B T}{\mu}$$

$$\boxed{\mu = -\frac{k_B T}{N_0}}$$ close to $T = 0$

➡ at $T = 1\ \mathrm{K} \longrightarrow \mu/k_B \approx 10^{-22}\ \mathrm{K}$!

2.4 Bose-Einstein Condensation

Calculation of N_0 and N_e :

number of particles in excited states

$$\sum_i f(E_i,T) = N = N_0(T) + N_\mathrm{e}(T)$$

$$= N_0(T) + \int_0^\infty D(E)\, f(E,T)\,\mathrm{d}E$$

density of states for free particles without $D(0)$

density of states for free particles $E_k \propto k^2$

$$D(E) = \frac{V\,(2m)^{3/2}\,\sqrt{E}}{4\pi^2\hbar^3}$$

with $E/k_\mathrm{B}T = x$ and $|\mu| \ll \Delta E$ $\longrightarrow$ $\exp[(E-\mu)/k_\mathrm{B}T] \approx \exp(E/k_\mathrm{B}T)$

$$\Rightarrow \quad N = N_0 + \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}(k_\mathrm{B}T)^{3/2}\underbrace{\int_0^\infty \frac{\sqrt{x}}{\mathrm{e}^x - 1}\,\mathrm{d}x}_{\Gamma(5/2)\times\zeta(5/2)\approx 1.783}$$

2.4 Bose-Einstein Condensation

with $V_Q = \left(\frac{h}{\sqrt{2\pi m k_B T}} \right)^3 = \lambda_B^3$

$$N \approx N_0 + 2.6 \frac{V}{V_Q}$$

$$\boxed{N_0 = N - 2.6 \frac{V}{V_Q}}$$

Interpretation

NV_A

as long as $2.6 \frac{V}{V_Q} \ll 10^{22}$, which means that the de Broglie wavelength is significantly larger as an atom ➡ condensation !

factor $\sqrt[3]{2.6} = 1.37$

- $T = 0 \longrightarrow N_0 = N$ trivial !
- $0 < T < T_c \longrightarrow N_0$ still macroscopically large!
- $N_e \triangleq$ normalfluid component

comment:
λ_B^3 must not be as large as the vessel as proposed by London