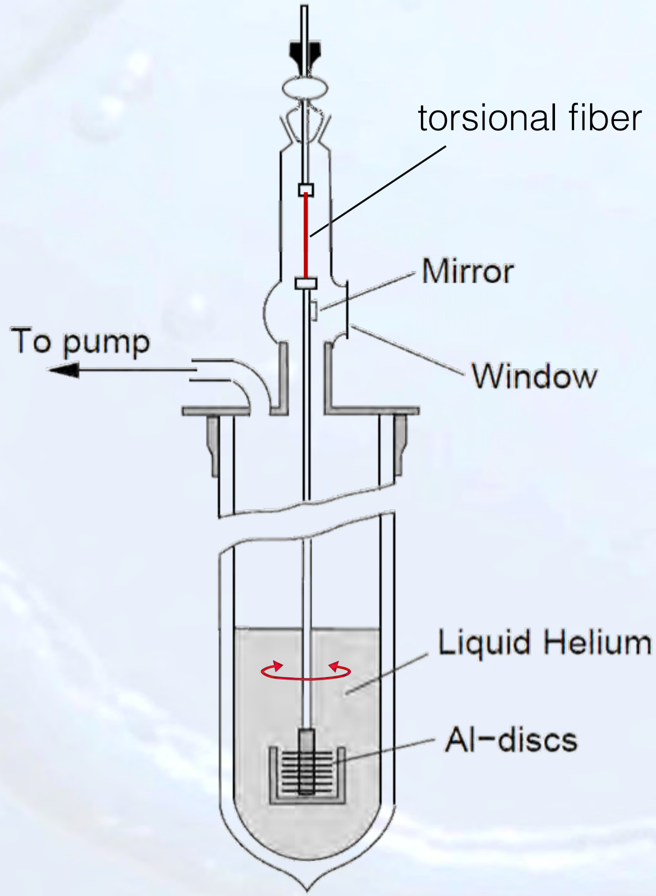
2.3 Properties of He-II described using the two-fluid model

Determination of ϱ_n

Experiment of Andronikasvili (1948)

First direct observation of ϱ_n



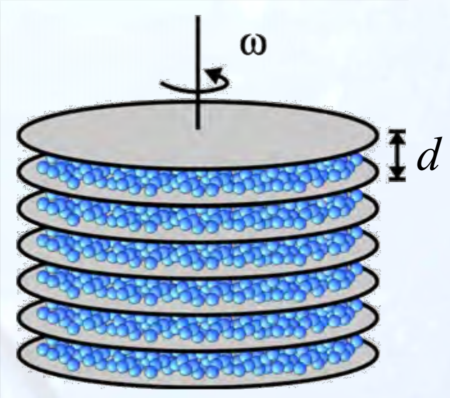
Elepter Luarsabovich Andronikashvili (1910-1989)

50 aluminum discs

thickness 13 μm
diameter 3.5 cm
spacing 210 μm

2.3 Properties of He-II described using the two-fluid model

observation → slow resonant oscillations (mass and torsion fiber)

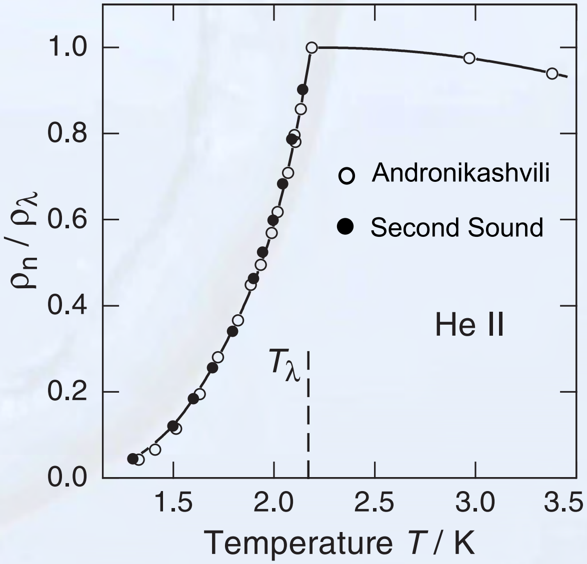


Important parameter is the viscos penetration depth for wave with frequency ω

$$\delta = \sqrt{2\eta_\mathrm{n}/\varrho_\mathrm{n}\omega}$$

$d < \delta$:
- ϱ_n is dragged along with torsion oscillator above and below T_λ
- ϱ_s remains stationary
- period of oscillation determined by mass of torsion oscillator (and spring constant)

→ ϱ_n can be determined



temperature dependence (empirical relation)

$$\varrho_\mathrm{n} = \varrho_\lambda \left(\frac{T}{T_\lambda}\right)^{5.6}$$

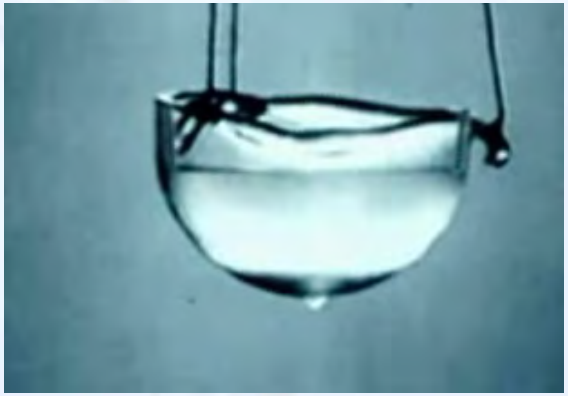
comparison with 2nd Sound → fits well

2.3 Properties of He-II described using the two-fluid model

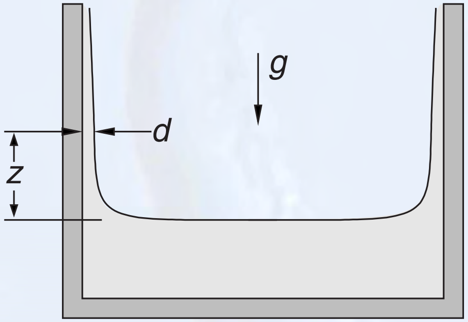
b) Beaker experiments

films are formed with a thickness of ~ 200 Å in saturated vapor pressure also against gravity

let us understand how
comment: the film formation is a “classical” phenomenon



(i) Film formation in saturated vapor



In thermal equilibrium

$$\mu_\mathrm{f} = \mu_\mathrm{g} = \mu_\ell$$

chemical potential for film (gas and liquid)

gravitational force is compensated by v. Waals forces

$$\rightarrow \quad \mu_\mathrm{f} = \mu_\ell + \underbrace{\mu_\mathrm{grav} + \mu_\mathrm{vdW}}_{=0} = \mu_\ell$$

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film thinkness:

$$\left.\begin{aligned}\mu_{\text{grav}} &= gz \\ \mu_{\text{vdW}} &= -\alpha/d^3\end{aligned}\right\} \quad gz - \alpha/d^3 = 0 \quad \curvearrowright \quad \boxed{d = \sqrt[3]{\frac{\alpha}{gz}}}$$

α: atomic polarisability of helium + wall (Hamaker constant)

d^3: depends on film thickness:

$$\mu_{\text{vdW}} = -\alpha/d^3 \quad \text{for} \quad d < 30\,\text{nm}$$
$$\mu_{\text{vdW}} = -\alpha/d^4 \quad \text{for} \quad d > 80\,\text{nm}$$

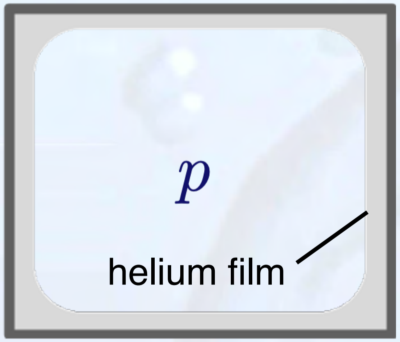
(d^4): retardation of potential

typical value: $d \sim 20$ nm at $z = 10$ cm

comment: property of superfluidity is unimportant for the film formation and thickness, but for the film flow

2.3 Properties of He-II described using the two-fluid model

(ii) film formation in unsaturated vapor



How does d depend on p ?

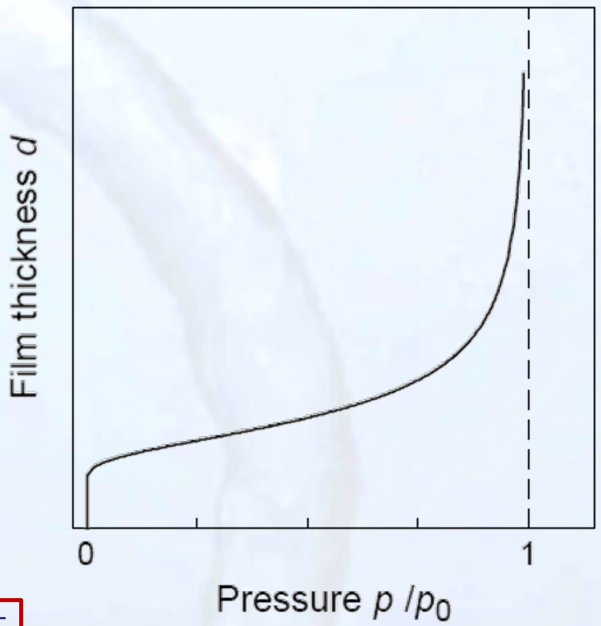
➡ barometric formula

$$\frac{p}{p_0} = \mathrm{e}^{-mgh/k_\mathrm{B}T}$$

$$mgh = k_\mathrm{B}T \ln\left(\frac{p}{p_0}\right)$$

➡ $$\frac{\alpha}{d^3} = \frac{k_\mathrm{B}T}{m_4} \ln\left(\frac{p_0}{p}\right)$$

➡ $$d = \sqrt[3]{\frac{\alpha\, m_4}{k_\mathrm{B}T(\ln p_0 - \ln p)}}$$



- decrease of pressure ➡ decrease of film thickness
- in practice: thinknesses of sub-mono layers are possible and realized

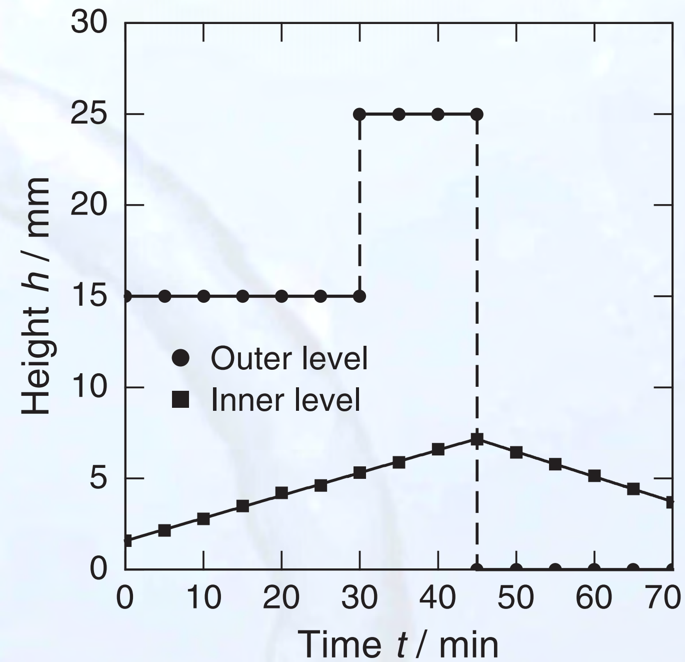
⬇

investigation of superfluidity with third sound: onset of superfluidity at $n > 2.1$ layers

2.3 Properties of He-II described using the two-fluid model

now back to the film flow:

- films are formed
- ϱ_s is moving without friction
- equalizing the chemical potential is driving force
- const. rate $\triangleq$ critical velocity



Interesting question: ϱ_s flows with $S = 0$! ➡ rest should warm up and helium flowing into a vessel should have $T = 0$!

but thermal equilibrium via gas phase

Two-Fluid Hydrodynamics

density $$\varrho = \varrho_\mathrm{n} + \varrho_\mathrm{s} \qquad (1)$$

mass flow $$\boldsymbol{j} = \varrho_\mathrm{n}\boldsymbol{v}_\mathrm{n} + \varrho_\mathrm{s}\boldsymbol{v}_\mathrm{s} \qquad (2)$$

mass conservation
continuity eqn. $$\frac{\partial \varrho}{\partial t} = -\mathrm{div}\,\boldsymbol{j} \qquad (3)$$

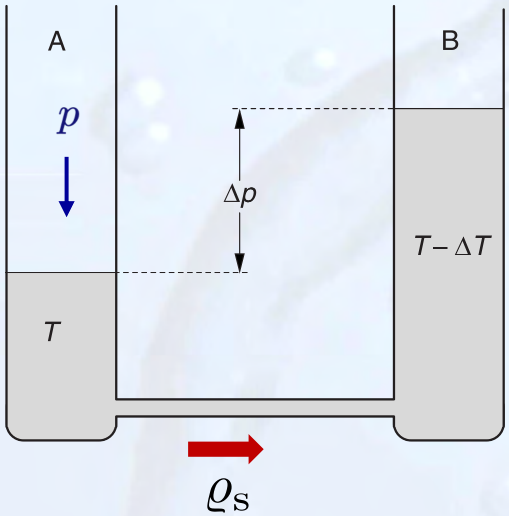
ideal fluid $$\frac{\partial \boldsymbol{j}}{\partial t} = -\mathrm{grad}\,p \qquad (4)$$

entropy conservation $$\frac{\partial(\varrho S)}{\partial t} = -\mathrm{div}(\varrho S \boldsymbol{v}_\mathrm{n}) \qquad (5)$$

an equation of motion for
superfluid component $$\frac{\partial \boldsymbol{v}_\mathrm{s}}{\partial t} = S\,\mathrm{grad}\,T - \frac{1}{\varrho}\,\mathrm{grad}\,p \qquad (6)$$

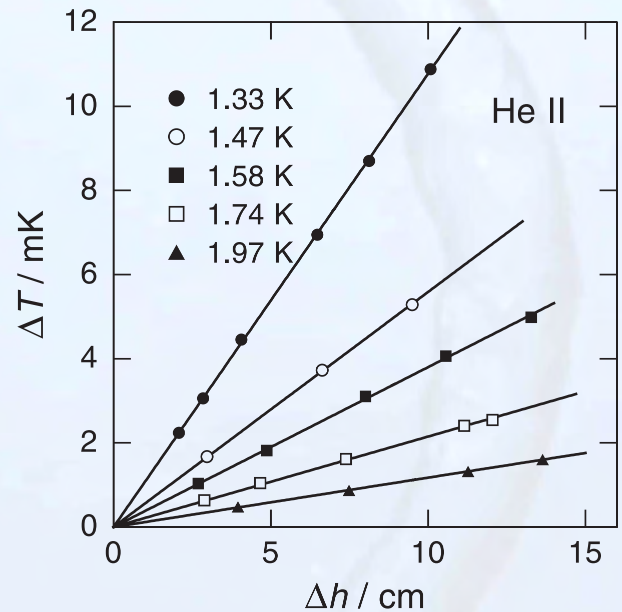
2.3 Properties of He-II described using the two-fluid model

c) Thermomechanical effect



$+\,\varrho_s$ ➡ cooling in B
$-\,\varrho_s$ ➡ warming in A

} $T_B < T_A$



Using (6) in stationary state

$$\frac{\partial \boldsymbol{v}_s}{\partial t} = S \,\mathrm{grad}\, T - \frac{1}{\varrho}\,\mathrm{grad}\, p = 0$$

in equilibrium nothing flows

$$\frac{\Delta p}{\Delta T} = \varrho S$$

London equation
(H. London 1939)

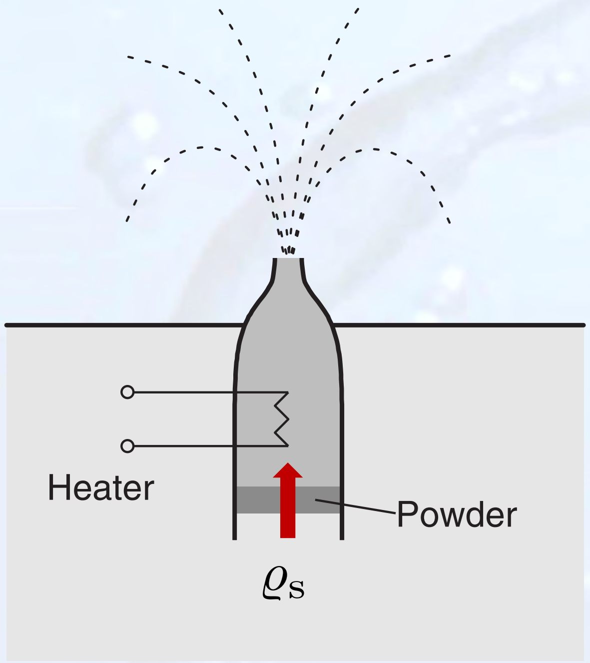
➡ Linear relation between Δp and ΔT

$\Delta h = 2\,\mathrm{cm}$
$T = 1.5\,\mathrm{K}$
} $\Delta T = 1\,\mathrm{mK}$

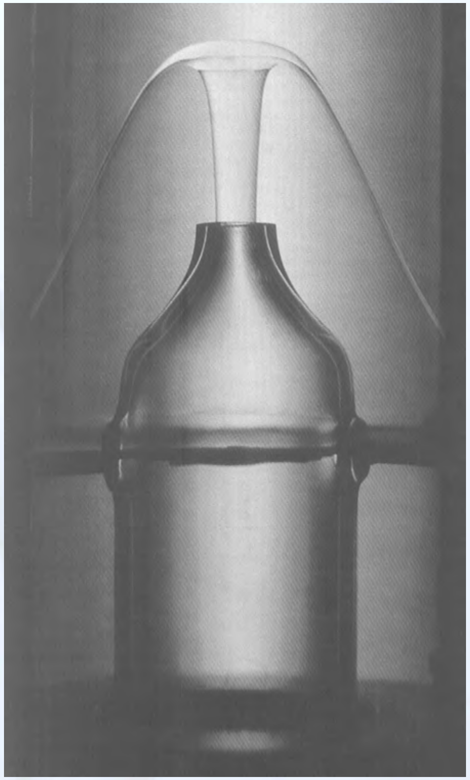
not very effective cooling

2.3 Properties of He-II described using the two-fluid model

Reverse thermomechanical effect: Fountain effect



$$\Delta T \curvearrowright \Delta p$$



- heating of helium inside vessel ⟶ ratio of ϱ_n/ϱ_s increases inside the vessel
- the temperature inside is higher than outside
- to equalize the system ϱ_s flows through superleak (compressed powder)
- pressure rises and fountain starts to flow (and flows as long as heater is on)

2.3 Properties of He-II described using the two-fluid model

d) Heat Transport

- in not too small capillaries $v_n \neq 0$
- even in equilibrium ($\Delta p = \varrho S \Delta T$) there is a constant flow of ϱ_n from the warm end to the cold end and ϱ_s in the opposite direction by "convection"

$\varrho_n \longrightarrow$ cold end
$\varrho_s \longrightarrow$ warm end

} entropy transport $\triangleq$ heat transport

heat transport maximum at 1.8 K where $\varrho_n \approx \varrho_s$

- limited only by the mobility of ϱ_n and therefore η_n
- viscos mass flow of ϱ_n :

$$\dot{V}_n = \frac{\beta}{\eta_n} \frac{\Delta p}{L} \qquad (*)$$

volume rate ($\dot{V}_n$)

$\beta \propto r^4$ for capillaries

$\beta \propto d^3$ for slits

- entropy flow $\dot{V}_n \varrho S \longrightarrow$ heat flow $\dot{Q} = T \dot{V}_n \varrho S \qquad (**)$

$\delta Q = T \delta S$

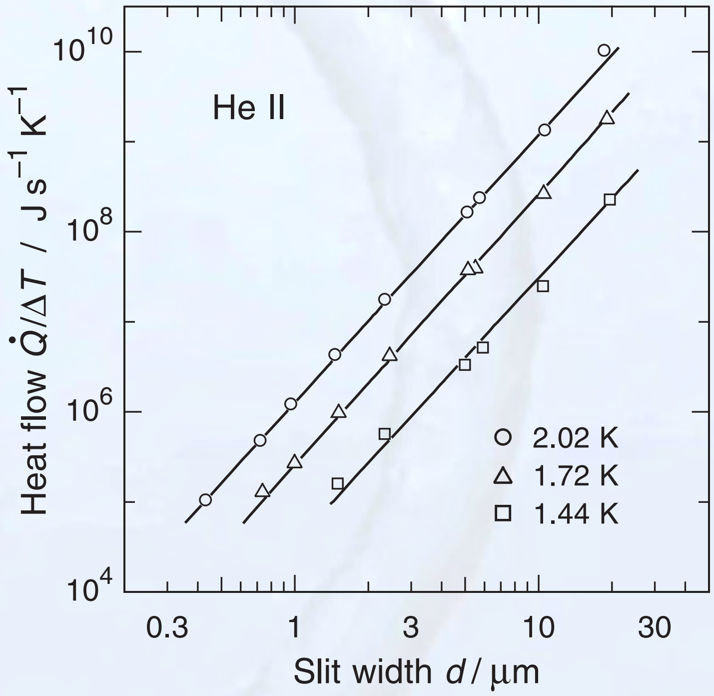
2.3 Properties of He-II described using the two-fluid model

$(*)$ insert in $(**)$ and London equation $(\Delta p = \varrho S \Delta T)$

$$\dot{Q} = \frac{\beta T (\varrho S)^2}{\eta_n L} \Delta T$$

linear regime

heat flow log $\dot{Q}/\Delta T$ vs log d



experimental results:

$\dot{Q} \propto \beta \propto d^3$ (as expected)

$\dot{Q}$ rises with T (as expected)