e) Heat Transport

Absolute value of thermal conductivity is extremely high

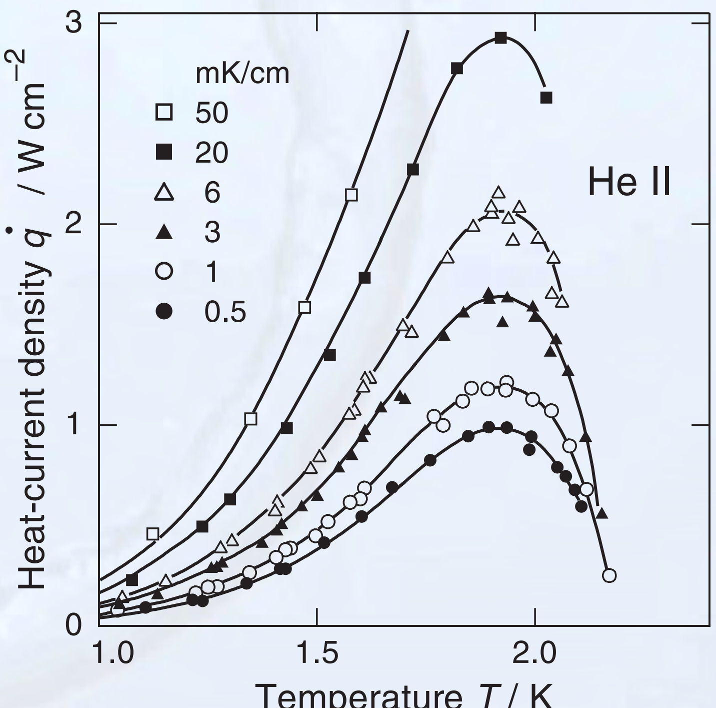
$\Lambda_{\text{He-II}} > 10^6 \, \Lambda_{\text{He-I}}$ at $T \sim 1.8$ K

- ► best condensed matter heat conductor by far
- ► explains why no boiling is observed at $T \leq T_\lambda$ since no temperature gradient

Further unusual properties of the heat transport

heat current density ≙ heat flow per area

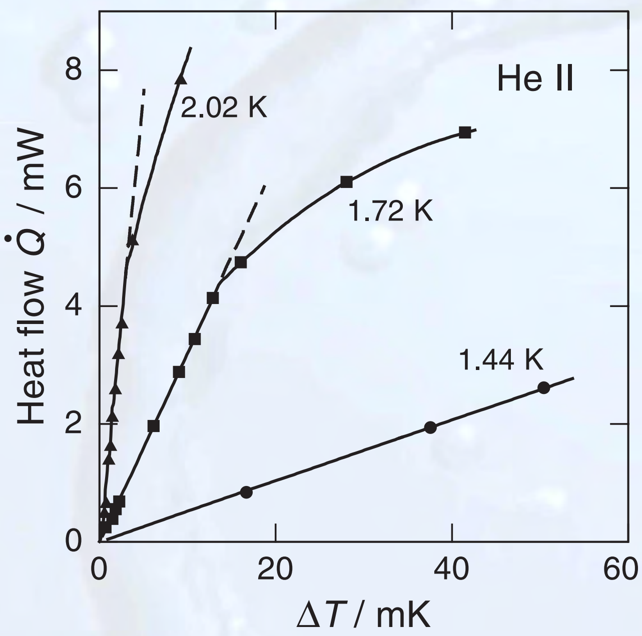
$$d = 0.3 \; .... \; 1.5 \text{ mm}$$
$$L = 2 \; .... \; 40 \text{ cm}$$



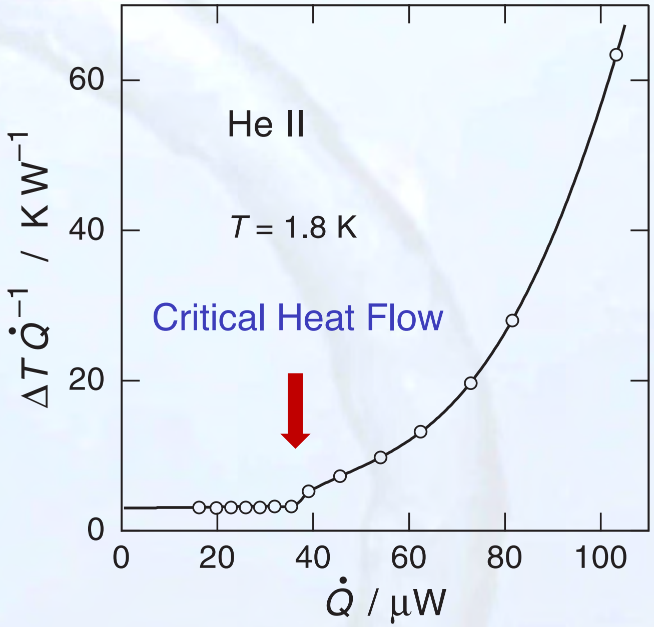
- ► Maximum at 1.8 K
- ► $T < 1.8$ K, $\dot{q} \sim |\text{grad}\, T|^{1/3}$
- ► with $\dot{q} = -\Lambda \, \text{grad}\, T$ ↷ $\Lambda \propto |\text{grad}\, T|^{-2/3}$

e) Heat Transport: Linear Regime

Heat flow in helium II through a 2.4 µm wide slit



Thermal resistance $\Delta T/\dot{Q}$



- $\dot{\boldsymbol{q}} = -\Lambda \,\mathrm{grad}\, T$ for very thin capillaries or small values of $\mathrm{grad}\, T$
- low T, small values of ΔT ➡ linear in ΔT

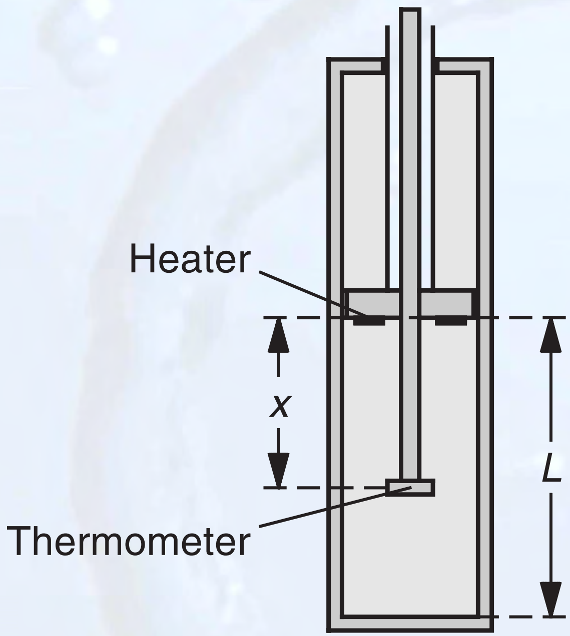
} linear regime

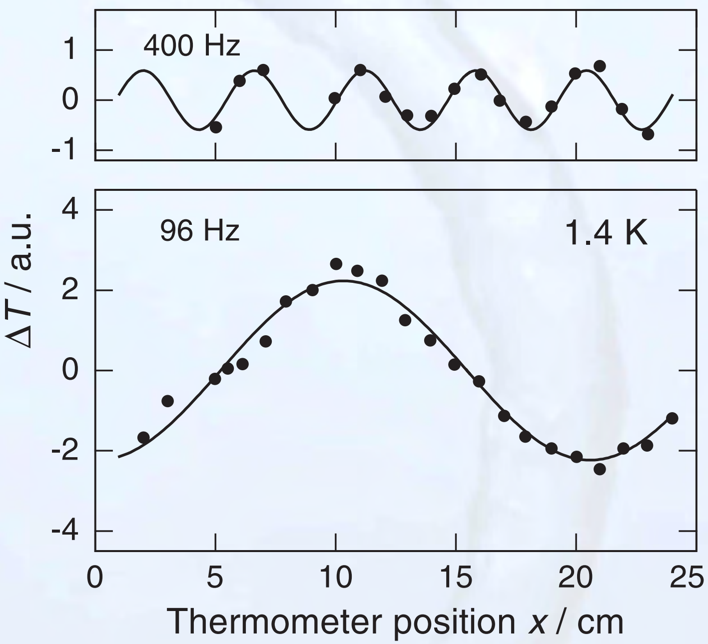
- high T, large values of ΔT ➡ sublinear in ΔT
- critical heat flow ≙ critical velocity

f) Second Sound

Propagation of temperature waves similar to sound waves

suggested by Kapitza
first seen by Peshkov 1944





resonance condition $v_2 = 2L\nu/n$

- Seen up to 100 kHz (experimental limit)
- v_2 independent of frequency

2.2 Two-Fluid Model

Basic idea: He-II has to components → normalfluid, superfluid

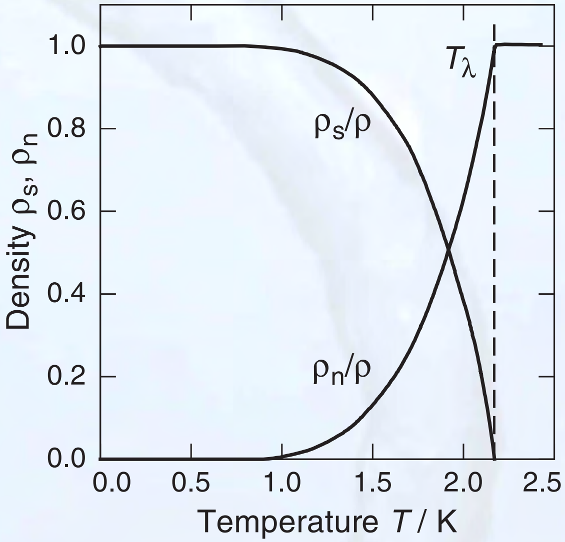
Tisza 1938
London 1938
Landau 1941, 1947
Feynman 1953

Assumptions and Properties:

$$\varrho = \varrho_\mathrm{n} + \varrho_\mathrm{s} \qquad (1)$$

$T = T_\lambda$: $\varrho_\mathrm{s} = 0$ and $\varrho_\mathrm{n} = \varrho$

$T = 0$: $\varrho_\mathrm{s} = \varrho$ and $\varrho_\mathrm{n} = 0$



	density	viscosity	entropy
normal-fluid component	ϱ_n	$\eta_\mathrm{n} = \eta$	$S_\mathrm{n} = S$
superfluid component	ϱ_s	$\eta_\mathrm{s} = 0$	$S_\mathrm{s} = 0$

In addition: no turbulence associated with ϱ_s → rot $\boldsymbol{v}_\mathrm{s} = 0$

Two-Fluid Hydrodynamics

density $$\varrho = \varrho_{\mathrm{n}} + \varrho_{\mathrm{s}} \qquad (1)$$

mass flow $$\boldsymbol{j} = \varrho_{\mathrm{n}} \boldsymbol{v}_{\mathrm{n}} + \varrho_{\mathrm{s}} \boldsymbol{v}_{\mathrm{s}} \qquad (2)$$

continuity eqn.
(mass conservation) $$\frac{\partial \varrho}{\partial t} = -\mathrm{div}\, \boldsymbol{j} \qquad (3)$$

He-II is ideal fluid $\eta_{\mathrm{n}} < 10^{-5}\,\mathrm{P} \sim 0$

➡ Euler eqn. (Newton's 2nd law of motion for continua)

$$\frac{\partial \boldsymbol{j}}{\partial t} + \underbrace{\varrho \boldsymbol{v} \cdot \mathrm{grad}\, \boldsymbol{v}}_{\approx 0} = -\mathrm{grad}\, p$$

for small velocities since quadratic in v
(approximation for linear regime)

$$\frac{\partial \boldsymbol{j}}{\partial t} = -\mathrm{grad}\, p \qquad (4)$$

$$\varrho \frac{d\vec{v}}{dt} = -\mathrm{grad}\, p \quad \text{(pressure)}$$

with $\frac{d\vec{v}}{dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v}\, \mathrm{grad}\, \vec{v} \;\Big|\; \frac{dv}{dx} \cdot \frac{dx}{dt}$

$$\leadsto \varrho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\, \mathrm{grad}\, \vec{v} \right) = -\mathrm{grad}\, p$$

$$\varrho \frac{\partial \vec{v}}{\partial t} + \varrho \vec{v}\, \mathrm{grad}\, \vec{v} = -\mathrm{grad}\, p$$

with $\vec{j} = \varrho \vec{v}$

$$\leadsto \frac{\partial \vec{j}}{\partial t} + \varrho \vec{v}\, \mathrm{grad}\, \vec{v} = -\mathrm{grad}\, p$$

Two-Fluid Hydrodynamics

entropy conservation

motion is reversible since no dissipative processes ➡ He-II is an ideal fluid (in first approximation)

entropy/mass

only ϱ_{n} contributes

$$\frac{\partial(\varrho S)}{\partial t} = -\mathrm{div}(\underbrace{\varrho S}_{\text{entropy density}} \boldsymbol{v}_{\mathrm{n}}) \qquad (5)$$

One more equation is needed ➡ an equation of motion for ϱ_{s} (or ϱ_{n})

this is difficult to derive ➡ see R.B. Dingle, Proc. Phys. Soc. A62, 648 (1949) (40 pages)

here: Gedankenexperiment according to Landau

idea: Superfluid component is added at “constant” volume in the system

Two-Fluid Hydrodynamics

Consider change of internal energy

$$\mathrm{d}U = T\,\mathrm{d}S - p\,\mathrm{d}V + G\,\mathrm{d}m$$

$\mathrm{d}S = 0$ reversible

$\mathrm{d}V = 0$, V = constant

G: Gibbs free energy per unit mass

$$\mathrm{d}U = G\,\mathrm{d}m$$

Gibbs free energy is potential energy of superfluid component / mass

➡ $-\mathrm{grad}\, G$ is corresponding force

$$\frac{\mathrm{d}\boldsymbol{v}_\mathrm{s}}{\mathrm{d}t} = -\mathrm{grad}\,\mu$$

$\mu = G/m$: Chemical potential

and

$$\mathrm{d}\mu = -S\,\mathrm{d}T + \frac{1}{\varrho}\,\mathrm{d}p$$

➡ $$\frac{\partial \boldsymbol{v}_\mathrm{s}}{\partial t} = S\,\mathrm{grad}\,T - \frac{1}{\varrho}\,\mathrm{grad}\,p \qquad (6)$$

Two-Fluid Hydrodynamics

Navier-Stokes equation for normalfluid component

$$\underbrace{\varrho_\mathrm{n}\frac{\mathrm{d}v_\mathrm{n}}{\mathrm{d}t}}_{\text{inertia}} = -\underbrace{\frac{\varrho_\mathrm{n}}{\varrho}\,\mathrm{grad}\,p}_{\text{pressure gradient}} - \underbrace{\varrho_\mathrm{s} S\,\mathrm{grad}\,T}_{\text{temperature gradient}} - \underbrace{\frac{\varrho_\mathrm{s}\varrho_\mathrm{n}}{2\varrho}\,\mathrm{grad}\,(\boldsymbol{v}_\mathrm{n}-\boldsymbol{v}_\mathrm{s})^2}_{\text{additional term due to compressibility}} + \underbrace{\eta_\mathrm{n}\nabla^2 v_\mathrm{n}}_{\text{viscosity}}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} \equiv \frac{\partial v}{\partial t} + \boldsymbol{v}\,\mathrm{grad}\,\boldsymbol{v}$$

and

$$\boldsymbol{v}\,\mathrm{grad}\,\boldsymbol{v} = \mathrm{grad}\left(\frac{|\boldsymbol{v}|^2}{2}\right) + \underbrace{(\mathrm{curl}\,\boldsymbol{v})}_{\text{vorticity}} \times \boldsymbol{v}$$

vorticity (for irrotational flow $\mathrm{curl}\,\boldsymbol{v} = 0$)

Navier-Stokes equation for superfluid component

$$\varrho_\mathrm{s}\frac{\mathrm{d}v_\mathrm{s}}{\mathrm{d}t} = -\frac{\varrho_\mathrm{s}}{\varrho}\,\mathrm{grad}\,p - \varrho_\mathrm{s} S\,\mathrm{grad}\,T - \frac{\varrho_\mathrm{s}\varrho_\mathrm{n}}{2\varrho}\,\mathrm{grad}\,(\boldsymbol{v}_\mathrm{n}-\boldsymbol{v}_\mathrm{s})^2 + \cancel{\eta_\mathrm{s}\nabla^2 v_\mathrm{s}}$$

$\eta_\mathrm{s} = 0$

Euler-type equation for superfluid

if vorticity is included ➡ Gross-Pitaevskii equation

Two-Fluid Hydrodynamics

density $$\varrho = \varrho_\mathrm{n} + \varrho_\mathrm{s} \qquad (1)$$

mass flow $$\boldsymbol{j} = \varrho_\mathrm{n}\boldsymbol{v}_\mathrm{n} + \varrho_\mathrm{s}\boldsymbol{v}_\mathrm{s} \qquad (2)$$

mass conservation
continuity eqn.
$$\frac{\partial \varrho}{\partial t} = -\mathrm{div}\,\boldsymbol{j} \qquad (3)$$

ideal fluid $$\frac{\partial \boldsymbol{j}}{\partial t} = -\mathrm{grad}\,p \qquad (4)$$

entropy conservation $$\frac{\partial (\varrho S)}{\partial t} = -\mathrm{div}(\varrho S \boldsymbol{v}_\mathrm{n}) \qquad (5)$$

an equation of motion for
superfluid component
$$\frac{\partial \boldsymbol{v}_\mathrm{s}}{\partial t} = S\,\mathrm{grad}\,T - \frac{1}{\varrho}\,\mathrm{grad}\,p \qquad (6)$$

2.3 Properties of He-II described using the two-fluid model

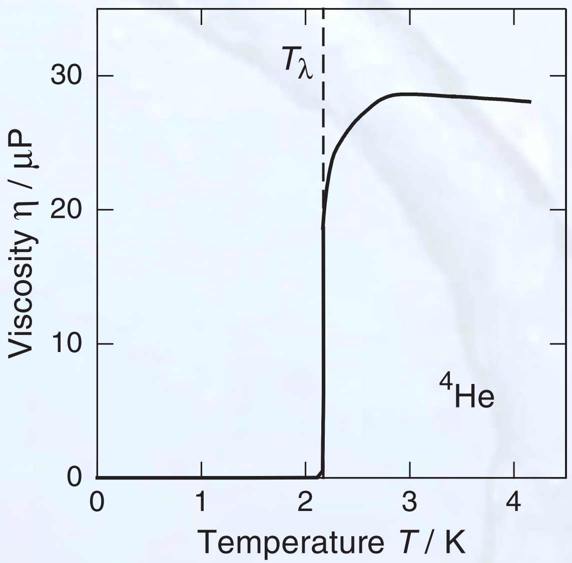
a) Viscosity

(i) capillaries (extremely thin)

Interpretation: $\boldsymbol{v}_\mathrm{n} \approx 0$

➡ only superfluid phase is observed

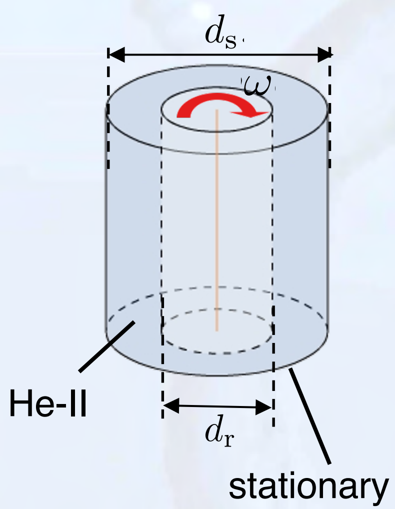
$\eta = \eta_\mathrm{s} = 0$



2.3 Properties of He-II described using the two-fluid model

a) Viscosity

(ii) rotary viscosimeter



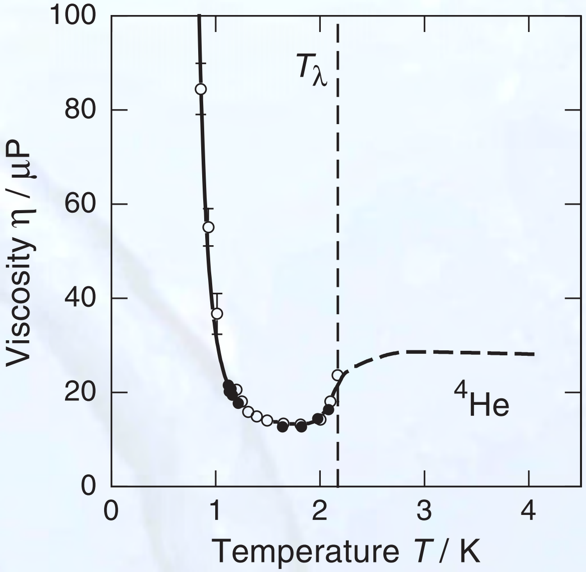
Torque acting on stationary cylinder is measured

$$M_r = \pi\eta\omega d_r^2 d_s^2/(d_s^2 - d_r^2)$$

since $\eta_s = 0$ no torque resulting from ϱ_s

$$\Rightarrow \quad M_r \propto \eta = \eta_n$$

two-fluid model



Temperature dependence

$\eta_n(T)$ at very low temperatures $T < 1.8$ K ?

$\eta_n \propto \ell_n$ $\Rightarrow$ mean free path increases with decreasing temperature because thermal excitations disappear

Viscosity

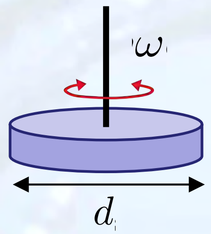
$$\eta = \frac{1}{3}\varrho v \ell$$

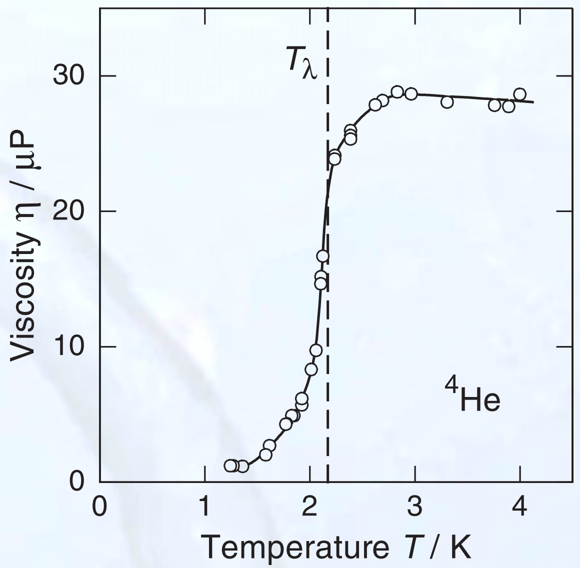
Landau-Chalatnikow Theory

2.3 Properties of He-II described using the two-fluid model

a) Viscosity

(iii) oscillating disc





Torque acting on the disc:

$$M_\mathrm{d} = \pi\sqrt{\varrho\eta}\,\omega^{3/2} r^4\, \Theta(\omega)$$

$$\Theta(\omega) = \Theta_0 \cos(\omega t - \pi/4)$$

$$M_\mathrm{d} \propto \sqrt{\varrho\eta}$$

product is important for M_d

$T < T_\lambda$ → $\eta_\mathrm{s} = 0$ → $\eta_\mathrm{n}\varrho_\mathrm{n}$ is measured

for $T \rightarrow 0$ → $\varrho_\mathrm{n} \rightarrow 0$ → $\varrho_\mathrm{n}\,\eta_\mathrm{n} \rightarrow 0$