HOW MANY GLUONS IN A PROTON?

Kai Schweda

SUM RULES, PROTON

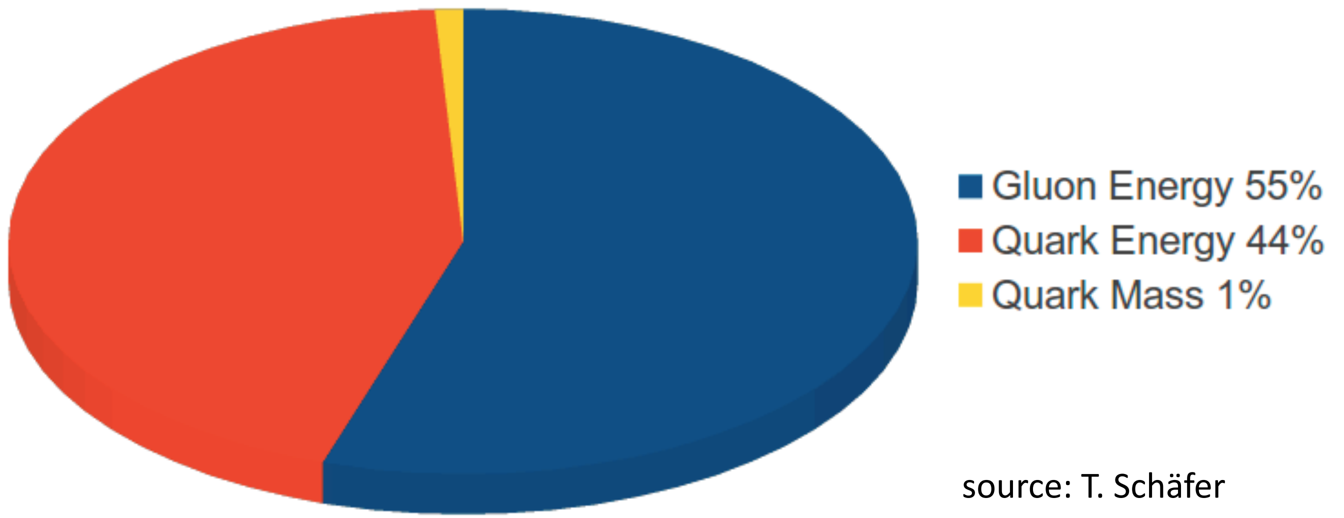
$$\int_0^1 [u(x) - \bar{u}(x)]dx = 2$$

$$\int_0^1 [d(x) - \bar{d}(x)]dx = 1$$

$$\int_0^1 x[u(x) + \bar{u}(x) + d(x) + \bar{d}(x)]dx \approx 0.54 \neq 1$$

- about half of the nucleon momentum is carried by neutral particles

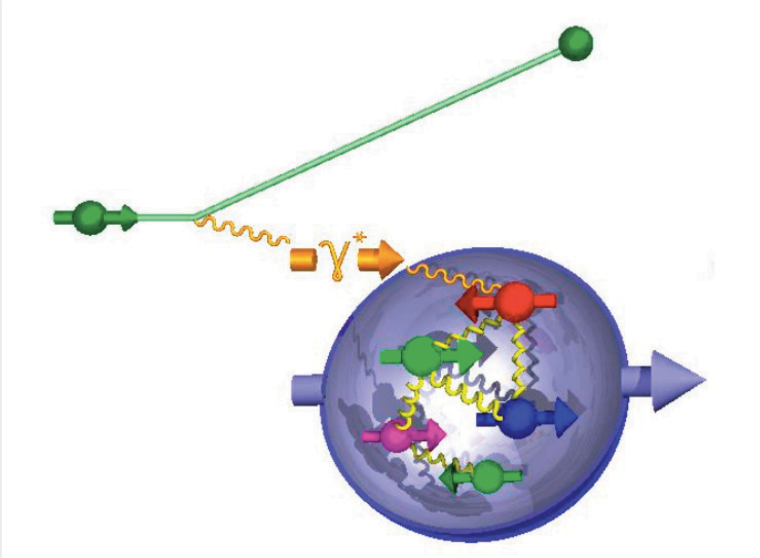
THE MASS OF THE PROTON



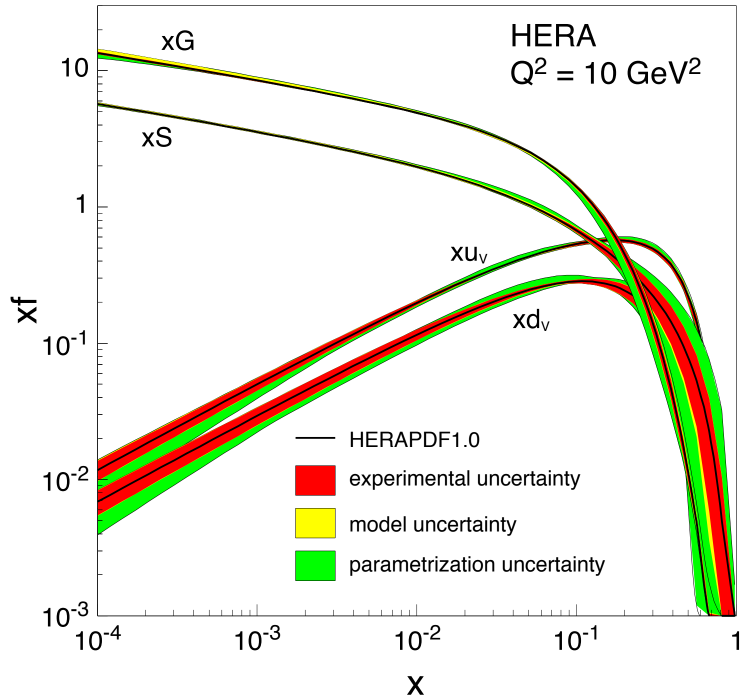
source: T. Schäfer

- proton mass: 938.27 MeV, neutron mass: 939.57 MeV
- up and down quark mass: m_u = 2.2 MeV, m_d = 4.7 MeV
- 99% of protons mass dynamically generated in QCD
- 1% of proton mass from quark bare masses (coupling to Higgs field)

THE STRUCTURE OF THE PROTON

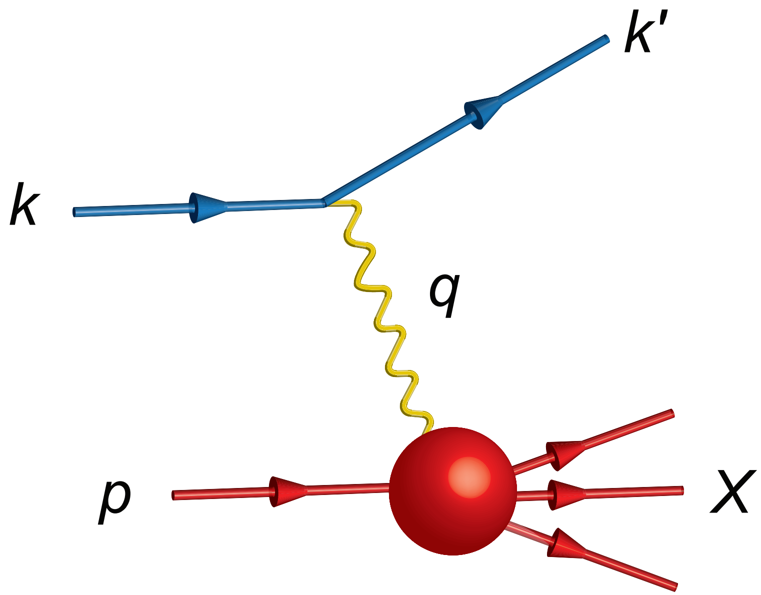


source: U. Elschenbroich (HERMES)



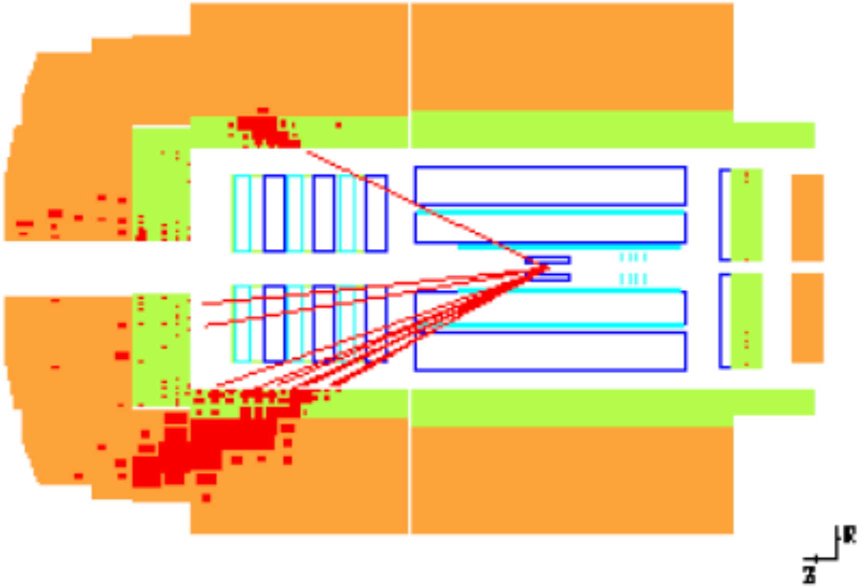
- In 1st order, lepton scattering is sensitive to the charge and magnetic distribution in the proton
- Virtual photon does not directly couple to electrically neutral gluons
- access to gluons through scaling violations in structure functions (higher-order corrections in α_s, DGLAP formalism)
- gluons dominate at low Bjorken x

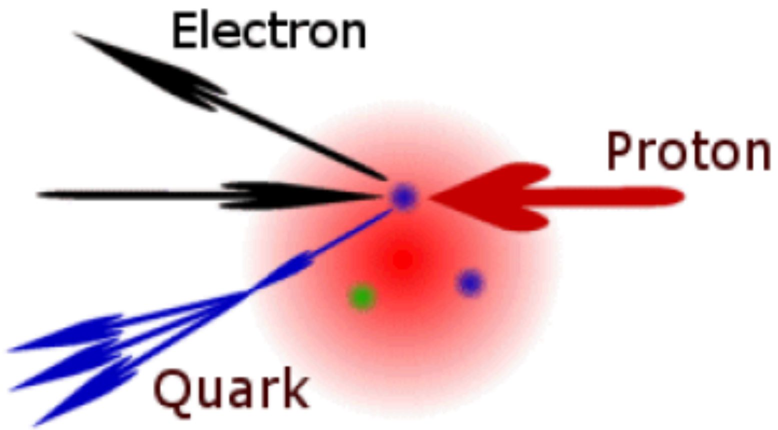
DEEP INELASTIC SCATTERING



- k, k': four momenta of the incoming and outgoing lepton (e,μ)
- p: four momentum of the nucleon (proton, neutron)
- squared momentum transfer: $Q^2 = -q^2 = (k-k')^2$
- Bjorken variable: $x = Q^2/(2p\cdot q)$, $0 < x < 1$
- inelasticity $y = (q\cdot p) / (k\cdot p)$, $0 < y < 1$
- squared invariant mass of hadronic system X: $W^2 = (p+q)^2$

ELECTRON-PROTON SCATTERING AT HERA IN H1





Plot: courtesy of Joachim Meyer, DESY

EXPERIMENTAL OBSERVABLES

Cross section:

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

At large values of Q^2 and leading order in α_s:

$$F_2(x,Q^2) = 2xF_2(x,Q^2) = x\sum e_q^2\left[q(x,Q^2) + \bar{q}(x,Q^2)\right]$$

unpolarized quark and antiquark distributions: $q, \bar{q}$

HERA @ DESY - THE ONLY LEPTON-HADRON COLLIDER TO DATE

- 27.5 GeV polarized electrons on 820 (920) GeV protons
- centre-of-mass energy 318 GeV
- 6.3km circumference
- operated from 1992 to 2007
- 4 experiments: H1, ZEUS, HERMES, HERA-B
- proton structure, QCD, CP violation, search for leptoquarks, …

HERA KINEMATICS - AN EXAMPLE

E_e = 27.5 GeV, E_p = 920 GeV, ultra-relativistic limit: $E = |\overrightarrow{p}|$

four vector: k = (E_k, $\overrightarrow{k}$)

s = $(k+p)^2$ = $(27.5+920)^2$ - $(27.5 - 920)^2$ = 101200 GeV^2

centre-of-mass energy $\sqrt{s} =$ 318 GeV

cms-velocity in lab frame: $\beta_{\mathrm{CMS}} = \frac{v_{\mathrm{CMS}}}{c} = \frac{p_{\mathrm{CMS}}}{E_{\mathrm{CMS}}} = \frac{E_p^{\mathrm{lab}} - E_e^{\mathrm{lab}}}{E_p^{\mathrm{lab}} + E_e^{\mathrm{lab}}} = 0.935$

energy of scattered electron $E_{k'}$ = 73 GeV, scattering angle θ = 135 degs.

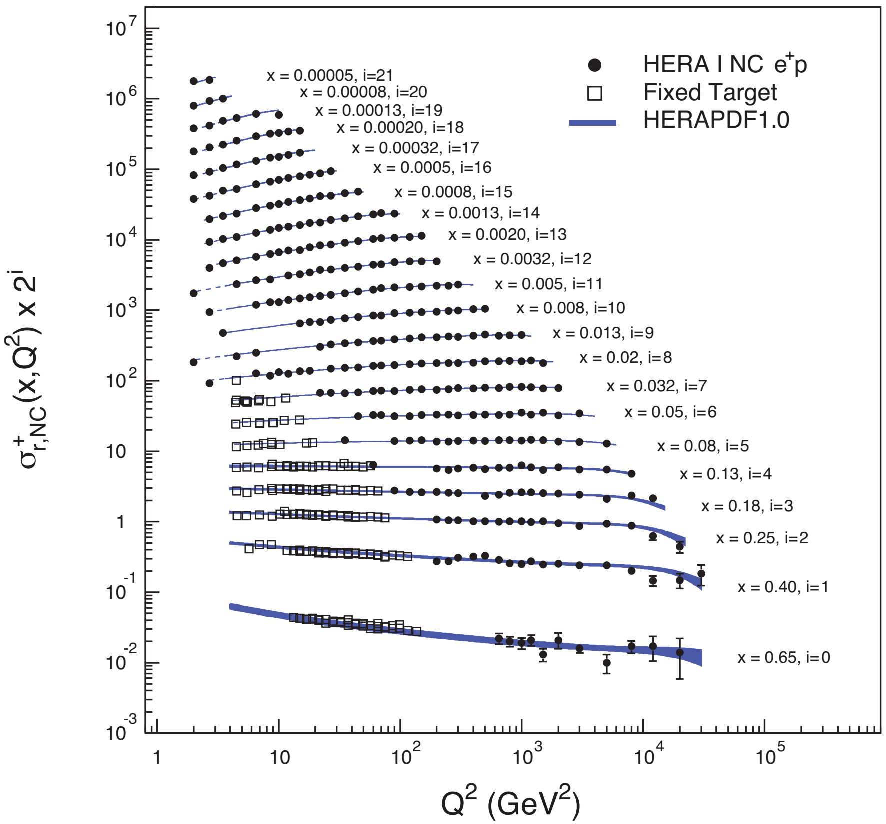
momentum transfer $q^2 = (k-k')^2$ = 2 $E_k E_{k'}$ (1 - cos θ) = 2 x 27.5 x 73 x (1 - cos θ)

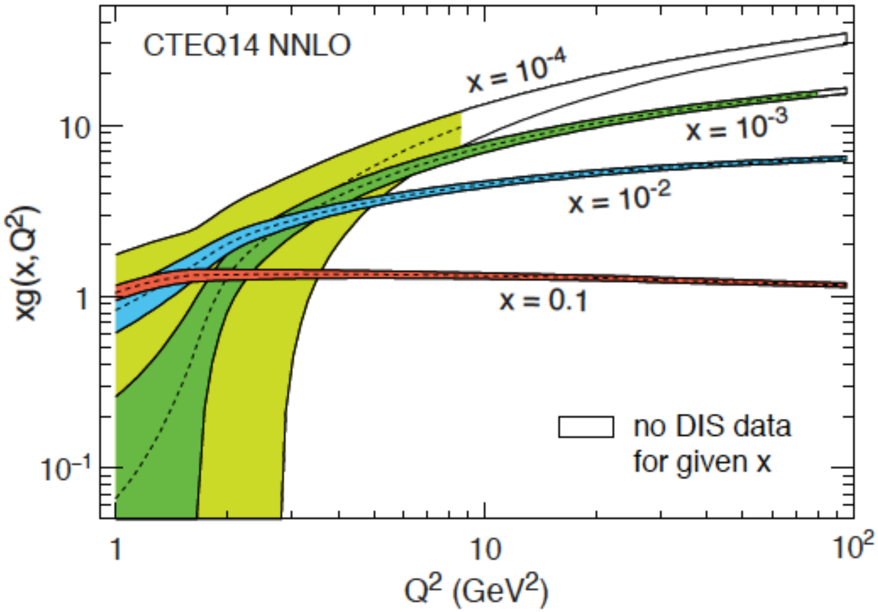
= 6854 GeV^2, |q| = 83 GeV

Resolve structure at $\Delta p \cdot \Delta x \approx 1, \;\; \Delta x \approx 10^{-18}$m, proton $\approx 10^{-15}$m

take snapshot of proton at 1/Q $\approx 10^{-25}$s exposure time

STRUCTURE FUNCTION F_2 AND GLUON DISTRIBUTION





- at large x and moderate Q^2, Bjorken scaling
- At low x, gluon density increases with increasing Q^2: scaling violation
- approximately: $xG(x,Q^2) \approx \dfrac{27\pi}{10\alpha_s(Q^2)} \dfrac{dF_2(x,Q^2)}{d\ln Q^2}$

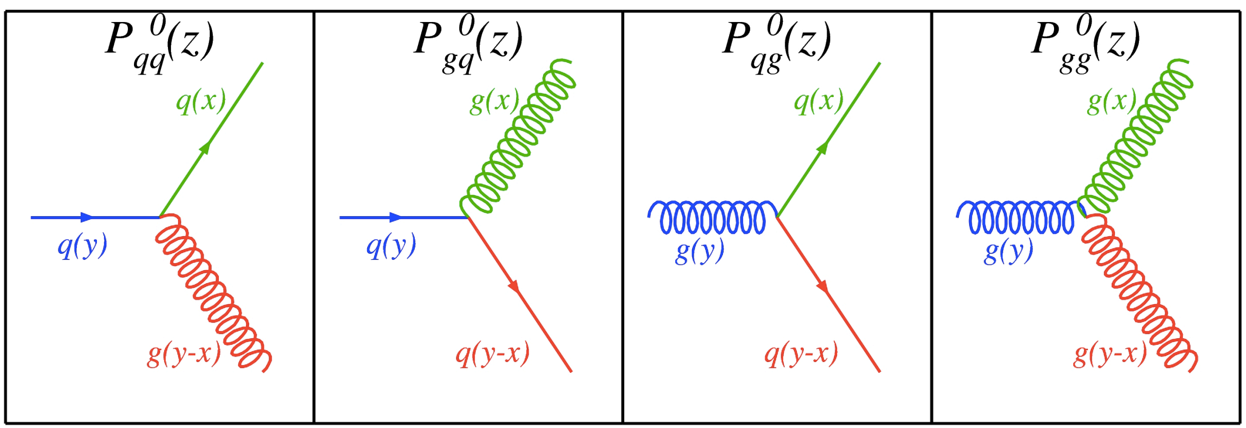
RENORMALIZATION GROUP EQUATIONS — DGLAP

$$Q^2 \frac{\partial}{\partial Q^2} \begin{pmatrix} q_i(x, Q^2) \\ \bar{q}_i(x, Q^2) \\ g(x, Q^2) \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi} \sum_j \int_x^1 \frac{d\xi}{\xi} \begin{pmatrix} P_{q_i q_j}(x/\xi) & 0 & P_{q_i g}(x/\xi) \\ 0 & P_{\bar{q}_i \bar{q}_j}(x/\xi) & P_{\bar{q}_i g}(x/\xi) \\ P_{g q_j}(x/\xi) & P_{g \bar{q}_j}(x/\xi) & P_{gg}(x/\xi) \end{pmatrix} \begin{pmatrix} q_j(\xi, Q^2) \\ \bar{q}_j(\xi, Q^2) \\ g(\xi, Q^2) \end{pmatrix}$$

Independently discovered by Y. Dokshitzer, W. Gribov, L. Lipatov, G. Altarelli, G. Parisi

e.g. Y. Dokshitzer, Sov. Phys. JETP. Band 46, Nr. 4, 1977, S. 641–653.

with P(x/ξ) the splitting functions



Calculate parton density at given x for any value of Q^2 from known parton densities

SPLITTING FUNCTIONS

$$P_{q_i q_j} = P_{\bar{q}_i \bar{q}_j} \equiv \delta_{ij} P_{qq} = \delta_{ij} C_F \left(\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x) \right)$$

$$P_{g q_i} = P_{g \bar{q}_i} \equiv P_{gq} = C_F \left(\frac{1+(1-x)^2}{x} \right)$$

$$P_{q_i g} = P_{\bar{q}_i g} \equiv P_{qg} = T_F \left(x^2 + (1-x)^2 \right)$$

$$P_{gg} = 2C_A \left(\frac{x}{(1-x)_+} + (1-x)\left(x + \frac{1}{x} \right) \right) + \frac{11C_A - 4n_f T_F}{6}\delta(1-x)$$

Plus distribution: $\displaystyle\int_0^1 \frac{f(x)}{(1-x)_+}\mathrm{d}x = \int_0^1 \frac{f(x)-f(1)}{1-x}\mathrm{d}x$

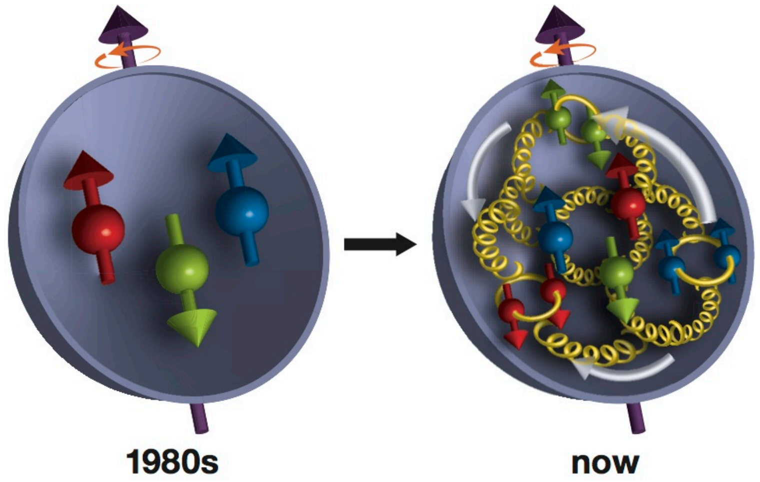
Casimir operators from SU(3) Lie group: $C_F = 4/3$, $C_A = 3$, $T_F = 1/2$

Running coupling constant:

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + b_0 \alpha_s(\mu^2) \ln \frac{Q^2}{\mu^2}} \qquad b_0 = \frac{33 - 2n_f}{12\pi}$$

n_f: number of active quark flavors

NUCLEON SPIN - BRIEF HISTORY



source: Brookhaven National Laboratory

1934, O. Stern and I.I. Rabi, anomalous magnetic moment of the proton

Nature volume 132, pages 169 (1933)

1964, B. W. Lee, A. Pais, proton / neutron ratio is -3/2

O. Greenberg, discovery of color degree of freedom

1988 EMC @ CERN, spin crisis, quarks and anti-quarks carry ~30% of nucleon spin

1990s - 2000s, confirmation of spin crisis, disentangle flavors, search for gluon contribution

SMC & COMPASS @ CERN

HERMES @ DESY

E142, E143, E154, E155, @ SLAC

HALL A & CLAS @ CEBAF/Jefferson Lab.

2012, STAR @ RHIC, finite contribution from gluons

EXPERIMENTAL OBSERVABLES

Double-spin asymmetry:

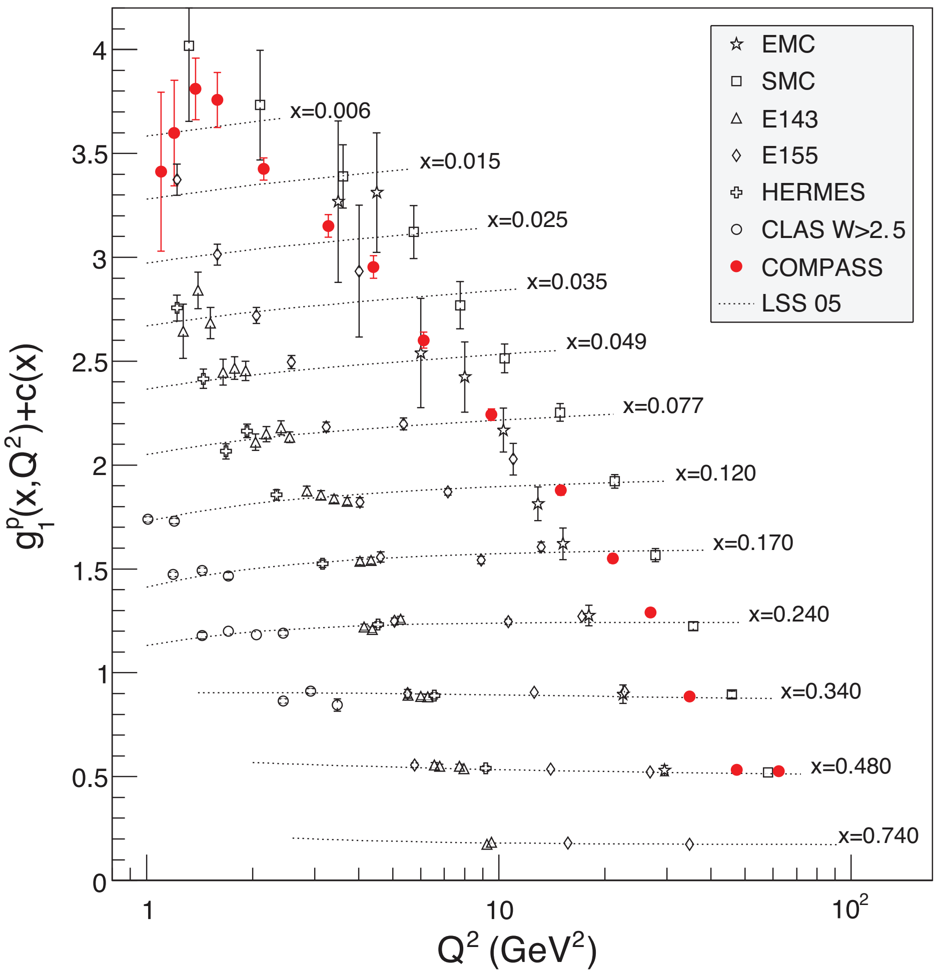
$$\frac{1}{2}\left[\frac{\mathrm{d}^2\sigma^{\rightleftarrows}}{\mathrm{d}x\mathrm{d}Q^2}-\frac{\mathrm{d}^2\sigma^{\rightrightarrows}}{\mathrm{d}x\mathrm{d}Q^2}\right]\simeq\frac{4\pi\alpha^2}{Q^4}y(2-y)g_1(x,Q^2)$$

- For **longitudinally polarized** proton and electron beams
- Final hadronic state X not detected

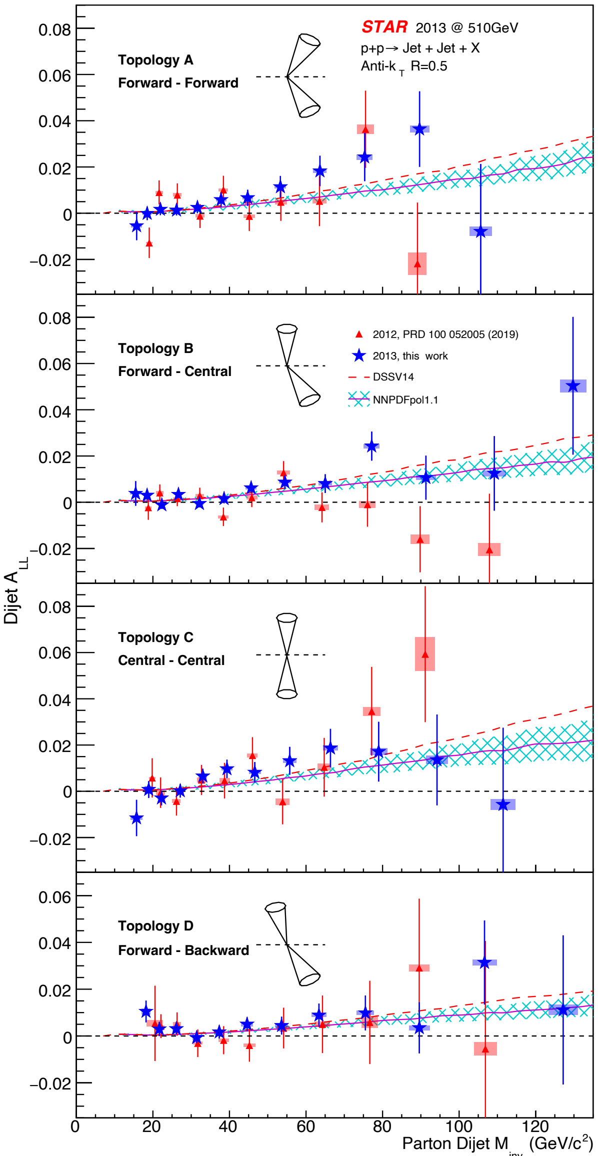
$$g_1(x,Q^2)=\frac{1}{2}\sum e_q^2\left[\Delta q(x,Q^2)+\Delta\overline{q}(x,Q^2)\right]$$

longitudinally polarized quark and antiquark distributions: $\Delta q, \Delta\bar{q}$

SPIN DEPENDENT STRUCTURE FUNCTION G1



GLUON SPIN CONTRIBUTION - POLARIZED PROTON COLLISIONS



- Collisions of polarized protons at RHIC
- Double-spin asymmetry of di-jet production

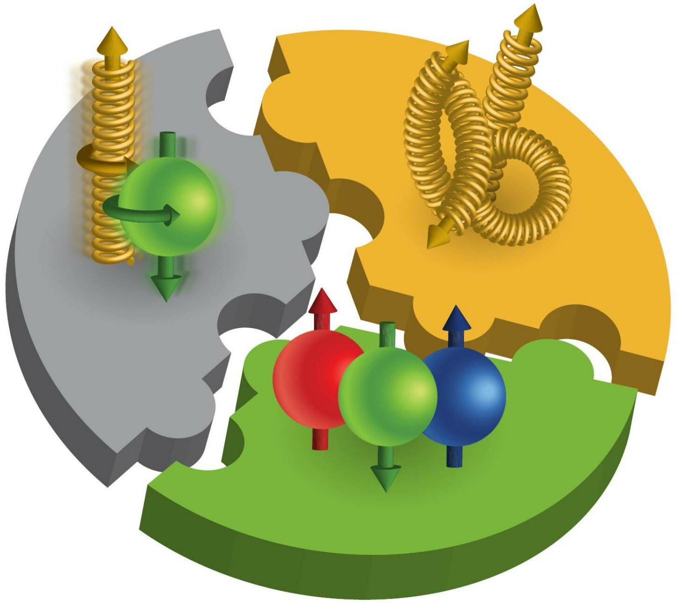
$$A_{LL} = \frac{\sigma^{++} - \sigma^{+-}}{\sigma^{++} + \sigma^{+-}}$$

$$M_{inv} = \sqrt{s x_1 x_2}$$

- Confirms evidence for finite gluon contribution to proton spin down to x ≃ 0.001

Dijet measurement: arXiv:2110.11020
Neutral pion measurement: STAR,PRD 98, 032013 (2018), arXiv:1805.09745

PROTON SPIN PUZZLE



Spin sum rule:

$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

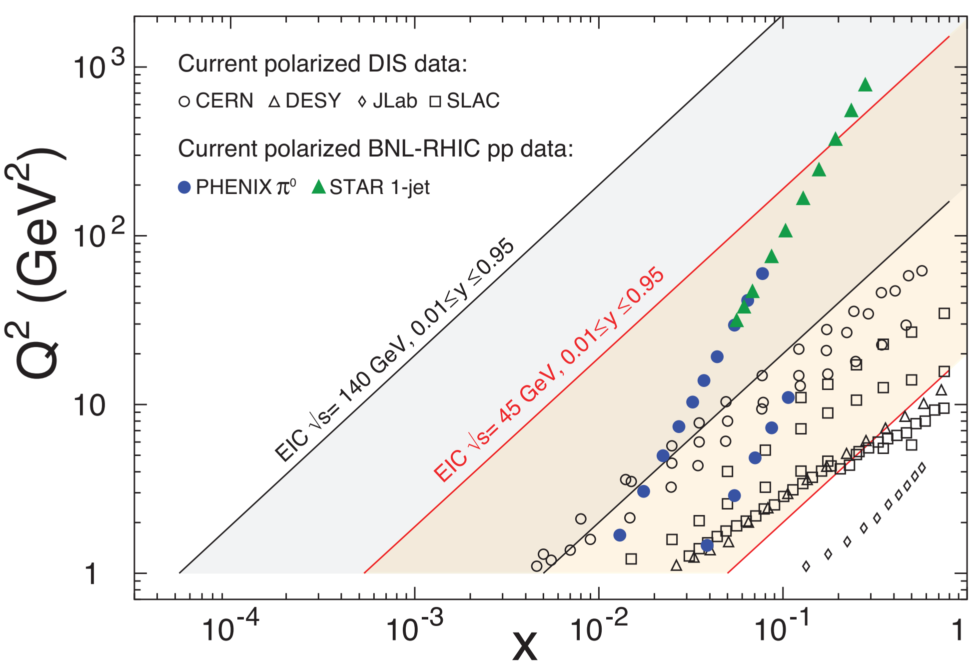
$$S_q(Q^2) = \frac{1}{2}\int_0^1 \Delta\Sigma(x,Q^2)dx \equiv \frac{1}{2}\int_0^1 \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s}\right)(x,Q^2)dx$$

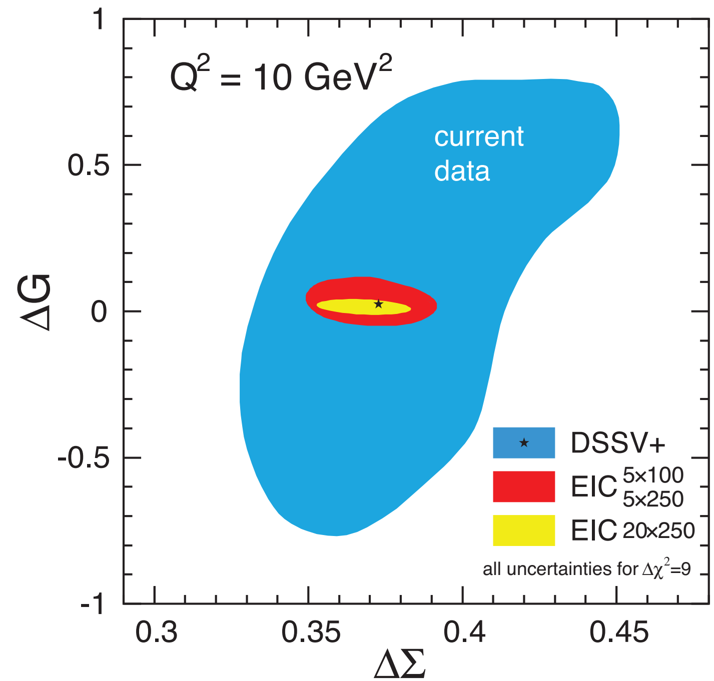
$$S_g(Q^2) = \int_0^1 \Delta g(x,Q^2)dx$$

contribution from orbital momentum of quarks and gluons, L_q and L_g, probably small

challenging to address experimentally

THE CASE FOR AN ELECTRON PROTON COLLIDER





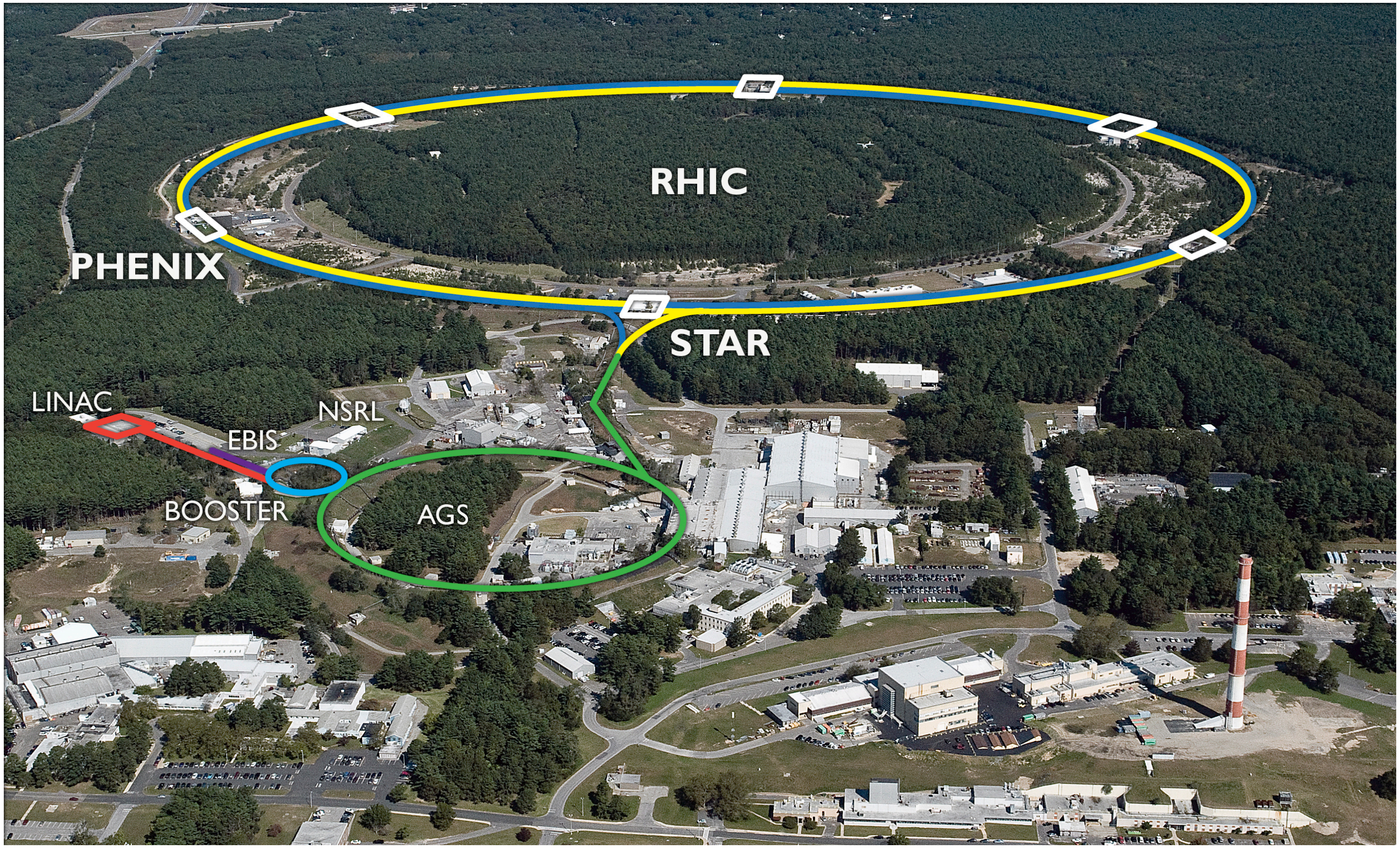
quarks and antiquarks carry ~30% of nucleon spin

need access to low-x regime of gluons

—> **collide** longitudinally polarized electrons on longitudinally polarized protons

—> electron - ion collider (EIC)

RELATIVISTIC HEAVY-ION COLLIDER: RHIC

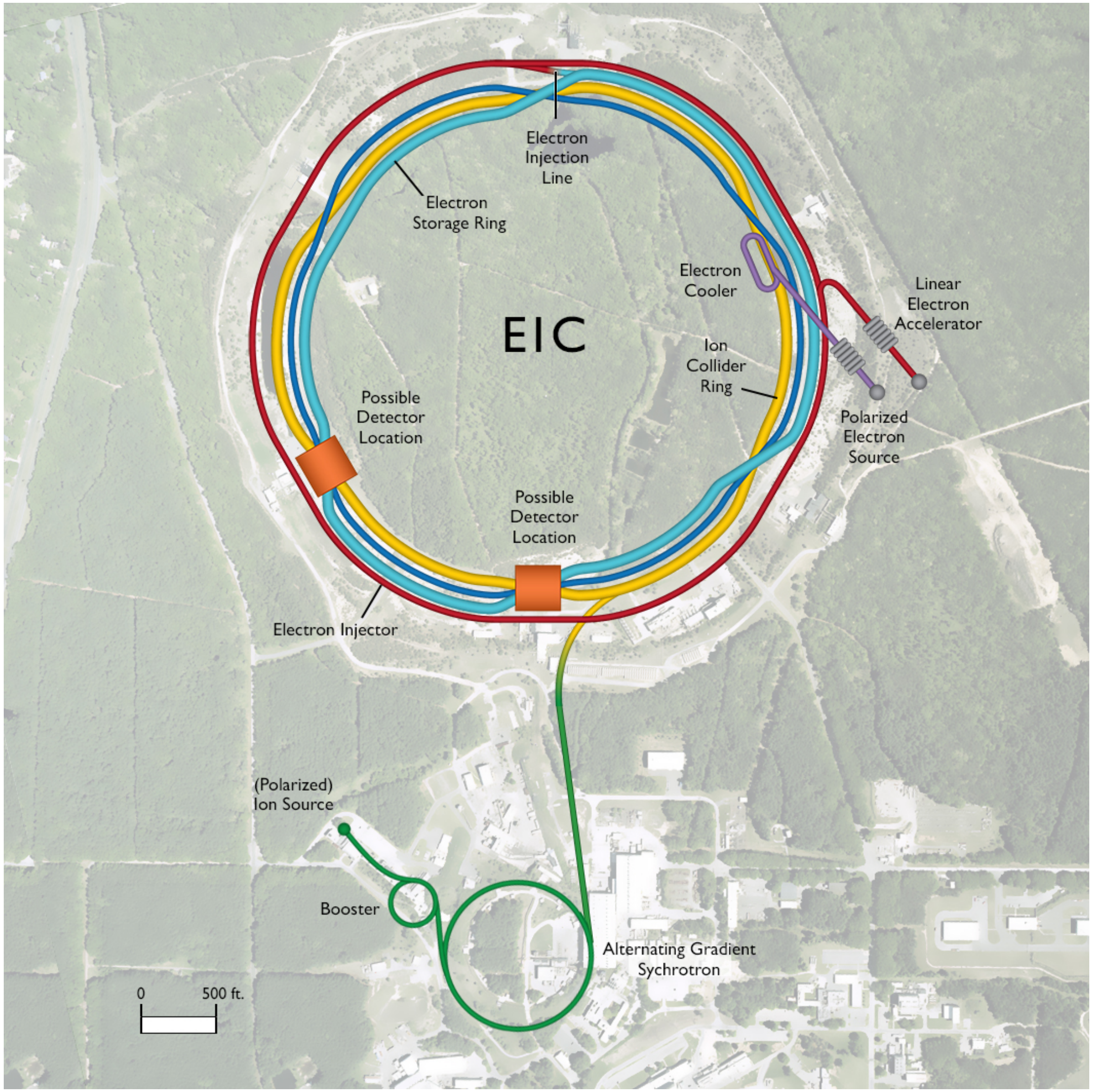


source: Brookhaven National Laboratory, Upton, NY, USA

beams of polarized protons up to 255 GeV

beams of nuclei from d to U up to 100 GeV/A

AN ELECTRON - ION(PROTON) COLLIDER: ERHIC

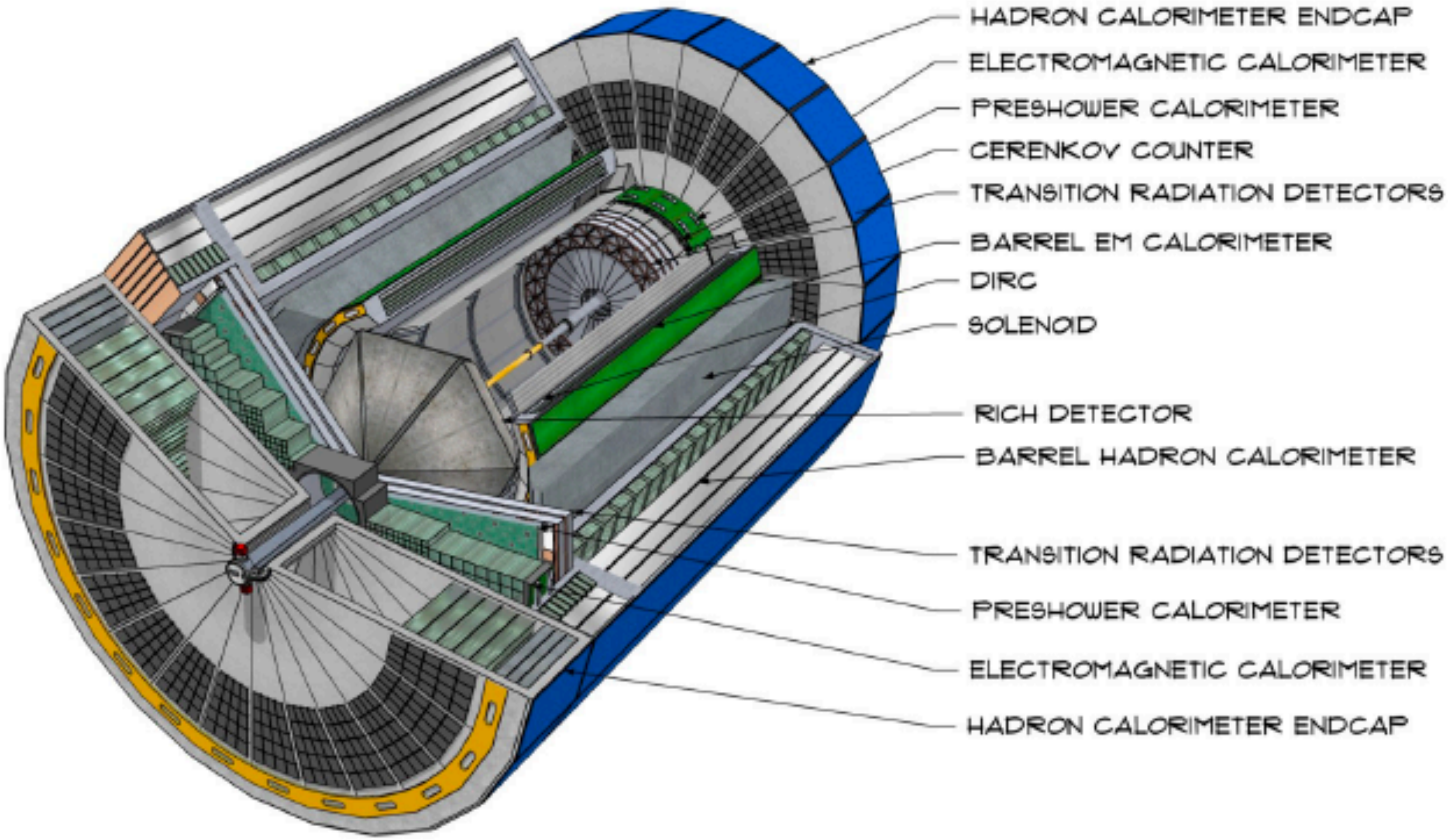


Approved in 2020

scheduled starting date: 2030

cost: 2B US$

AN ELECTRON - ION(PROTON) COLLIDER: ERHIC



ERHIC - BEAM PARAMETERS

Table 1.1: Maximum luminosity parameters.

Parameter	hadron	electron
Center-of-mass energy [GeV]	104.9	
Energy [GeV]	275	10
Number of bunches	1160	
Particles per bunch [10^{10}]	6.9	17.2
Beam current [A]	1.0	2.5
Horizontal emittance [nm]	11.3	20.0
Vertical emittance [nm]	1.0	1.3
Horizontal β-function at IP β_x^* [cm]	80	45
Vertical β-function at IP β_y^* [cm]	7.2	5.6
Horizontal/Vertical fractional betatron tunes	0.228/0.210	0.08/0.06
Horizontal divergence at IP $\sigma_{x'}^*$ [mrad]	0.119	0.211
Vertical divergence at IP $\sigma_{y'}^*$ [mrad]	0.119	0.152
Horizontal beam-beam parameter ξ_x	0.012	0.072
Vertical beam-beam parameter ξ_y	0.012	0.1
IBS growth time longitudinal/horizontal [hr]	2.9/2.0	-
Synchrotron radiation power [MW]	-	9.0
Bunch length [cm]	6	0.7
Hourglass and crab reduction factor [17]	0.94	
Luminosity [10^{34} cm^{-2} s^{-1}]	1.0	

Electron Ion Collider, Technical Design Report (2021)
https://www.bnl.gov/ec/files/eic_cdr_final.pdf

HOW MANY GLUONS DO EXIST ?

$$(r\bar{b} + b\bar{r})/\sqrt{2}$$

$$-i(r\bar{b} - b\bar{r})/\sqrt{2}$$

$$(r\bar{g} + g\bar{r})/\sqrt{2}$$

$$-i(r\bar{g} - g\bar{r})/\sqrt{2}$$

$$(b\bar{g} + g\bar{b})/\sqrt{2}$$

$$-i(b\bar{g} - g\bar{b})/\sqrt{2}$$

$$(r\bar{r} - b\bar{b})/\sqrt{2}$$

$$(r\bar{r} + b\bar{b} - 2g\bar{g})/\sqrt{6}$$

Gluon octet, representation of color SU(3)

David Griffiths, Introduction to elementary particles, Wiley

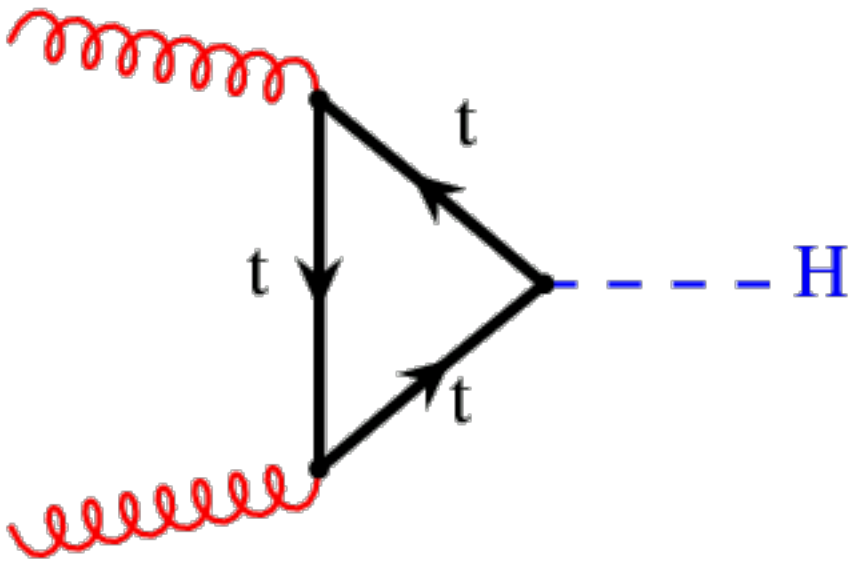
$$(r\bar{r} + b\bar{b} + g\bar{g})/\sqrt{3}$$

Gluon singlet
would not be subject to confinement

Mass = 0 —> infinite range

Does not exist!

NO GLUON - NO (EARLY) HIGGS DISCOVERY



gluon fusion ~90% of Higgs production cross section at LHC !