Lecture 4

deep inelastic scattering and the parton model

for a discussion of constituent quarks vs current quarks, see:

Shmuel Nussinov, Robert Shrock
Phys.Rev.D 79 (2009) 016005
arXiv:0811.3404 [hep-ph]

note: we will go quickly at the beginning as the kinematical distributions and expressions of cross sections in terms of x and Q^2 were discussed in detail in the pep4 lecture in 2021

Deep inelastic scattering: preliminaries with some useful formulae on elastic scattering

$$(d\sigma/d\Omega)_{Ruth} = \frac{Z^2\alpha^2}{4E^2\sin^4(\theta/2)}; \qquad \text{or, } \frac{4Z^2\alpha^2E'^2}{q^4}$$

$$(d\sigma/d\Omega)_{Mott} = (d\sigma/d\Omega)_{Ruth}(1-\beta^2\sin^2(\theta/2))$$

As $\beta \rightarrow 1$ then:

$$(d\sigma/d\Omega)_{Mott} = (d\sigma/d\Omega)_{Ruth}(\cos^2(\theta/2))$$

$$(d\sigma/d\Omega)_{point,s=1/2} = (d\sigma/d\Omega)_{Mott}\cdot\Big(1+2\tau\tan^2(\theta/2)\Big)$$

where $\tau = \frac{Q^2}{4M^2}$

elastic electron scattering off a nucleon

Taking into account the internal structure of the protons gives:

$$(d\sigma/d\Omega) = (d\sigma/d\Omega)_{Mott} \cdot \left(\frac{G_E^2(Q^2) + \tau G_M^2(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2)\tan^2(\theta/2) \right)$$

This is known as the *Rosenbluth formula.*

One also often sees:

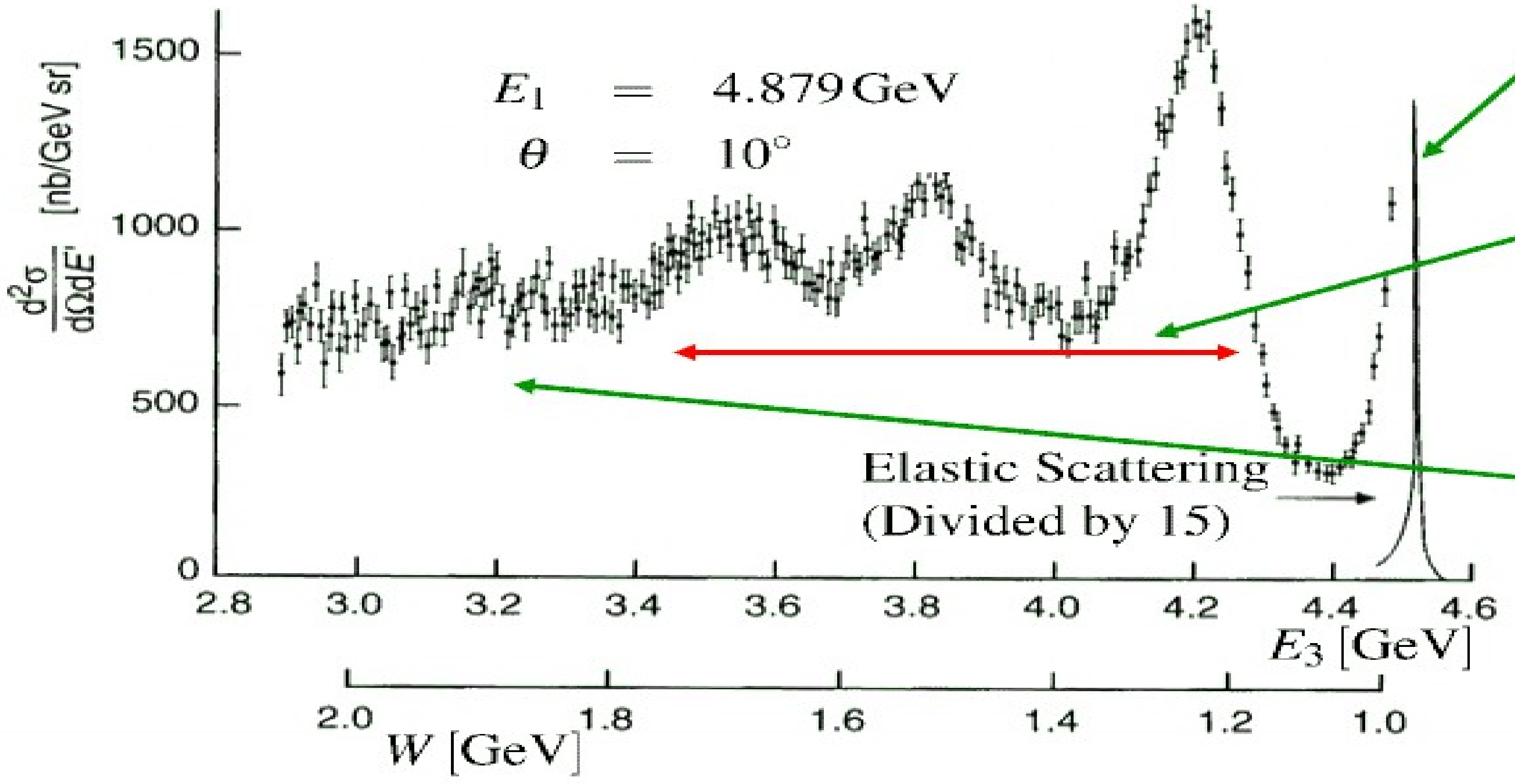
$$(d\sigma/d\Omega) = (d\sigma/d\Omega)_{Mott} \cdot \left((F_1^2 + \kappa\tau F_2^2) + 2\tau(F_1 + \kappa F_2)^2 \tan^2(\theta/2) \right)$$

$G_E \equiv F_1 + \kappa\tau F_2$ and $G_M \equiv F_1 + \kappa F_2$. One advantage of $G_{E,M}$ over $F_{1,2}$ is a simple physical interpretation in the static limit ($q^2 \rightarrow 0$); another is the lack of cross-terms in the cross section expression. The F are used since they are more directly related to the structure of the current terms in the effective Lagrangian density.

nucleon finite size – strong q dependence of formfactor

$$G_M(q^2) \approx \frac{1}{(1+q^2/0.71\mathrm{GeV}^2)^2} \quad \Rightarrow \quad G_M(q^2) \propto q^{-4}$$

at large q, elastic scattering becomes strongly suppressed since it is unlikely that the nucleon survives such a violent collision -- inelastic and break-up reactions will dominate



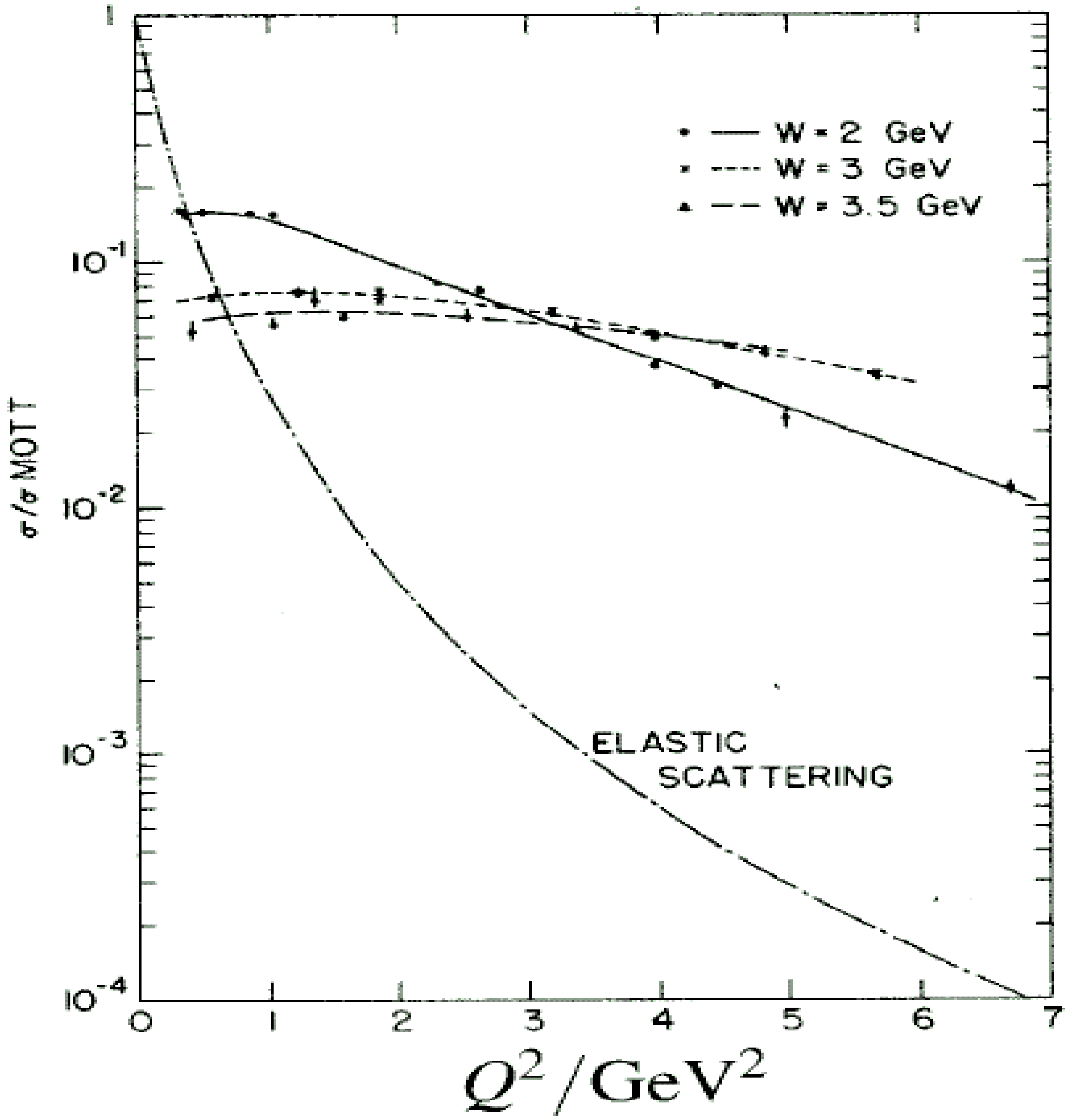
- **Elastic Scattering**
  - proton remains intact
  - $W = M$
- **Inelastic Scattering**
  - produce "excited states" of proton e.g. $\Delta^+(1232)$
  - $W = M_\Delta$
- **Deep Inelastic Scattering**
  - proton breaks up resulting in a many particle final state

DIS = large W

first spectacular results from SLAC in the late 1960ties

form factors depend little
on Q^2 for large inelasticity
$W >> M$

M.Breidenbach et al.,
Phys. Rev. Lett. 23 (1969) 935

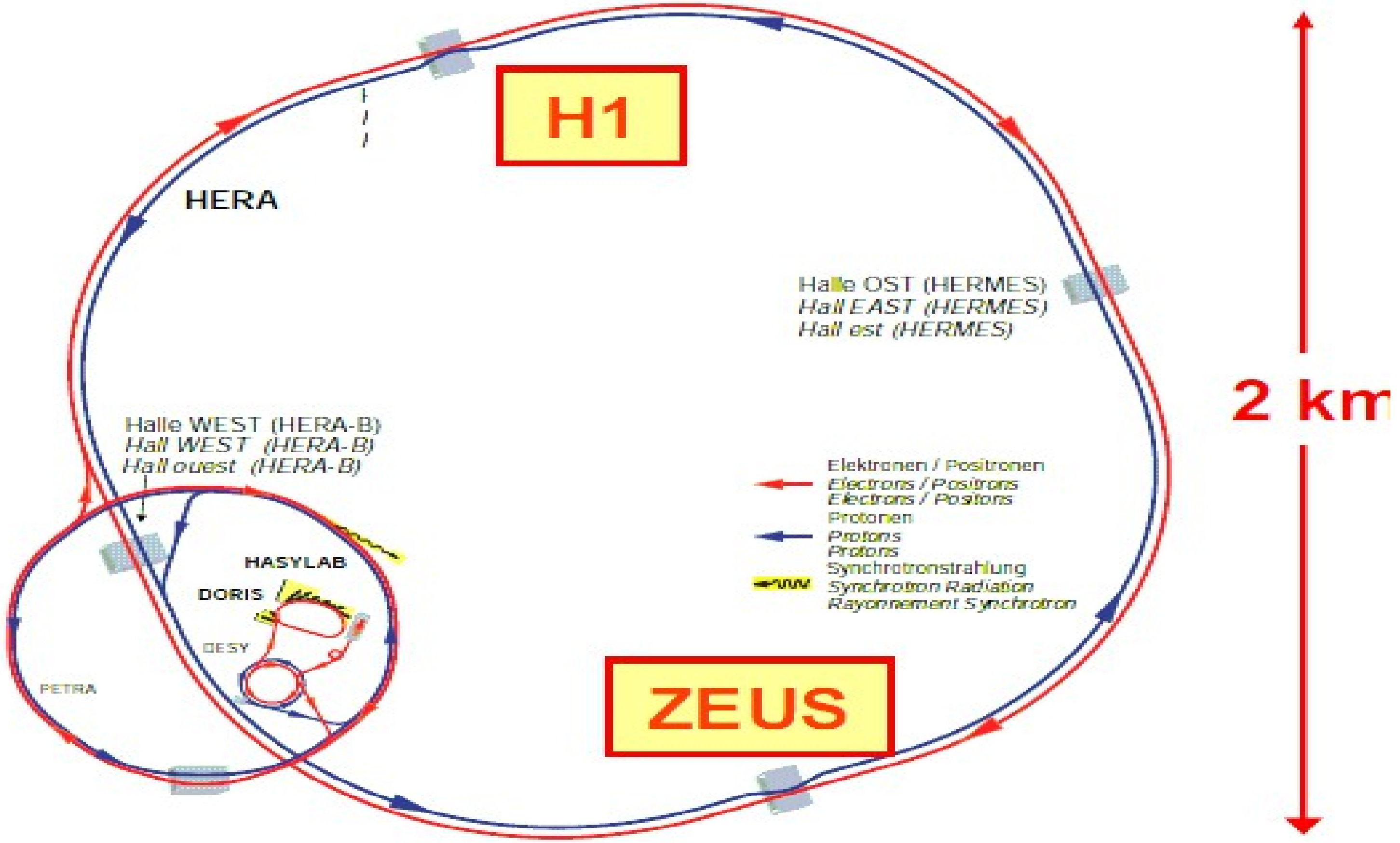


HERA $e^{\pm}p$ Collider

★ DESY (Deutsches Elektronen-Synchroton) Laboratory, Hamburg, Germany

$e^{\pm}$ 27.5 GeV → ← 820 GeV p

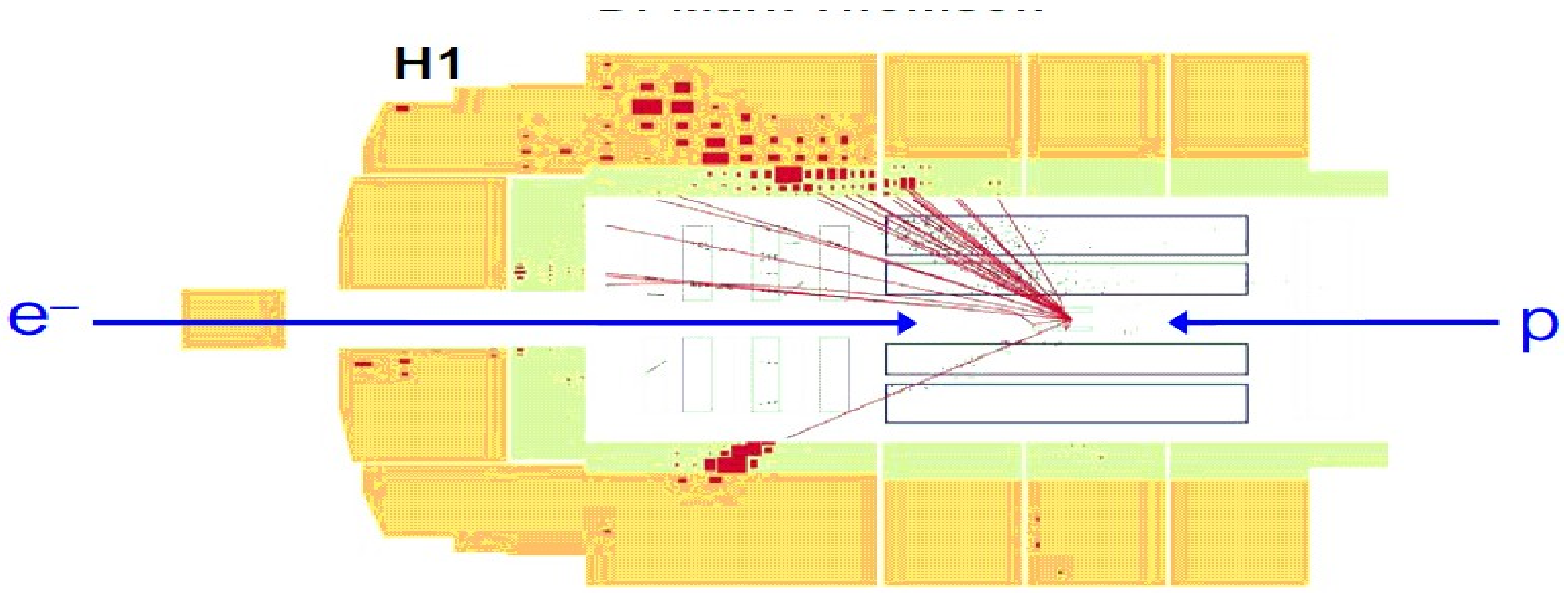
$\sqrt{s}$ = 300 GeV

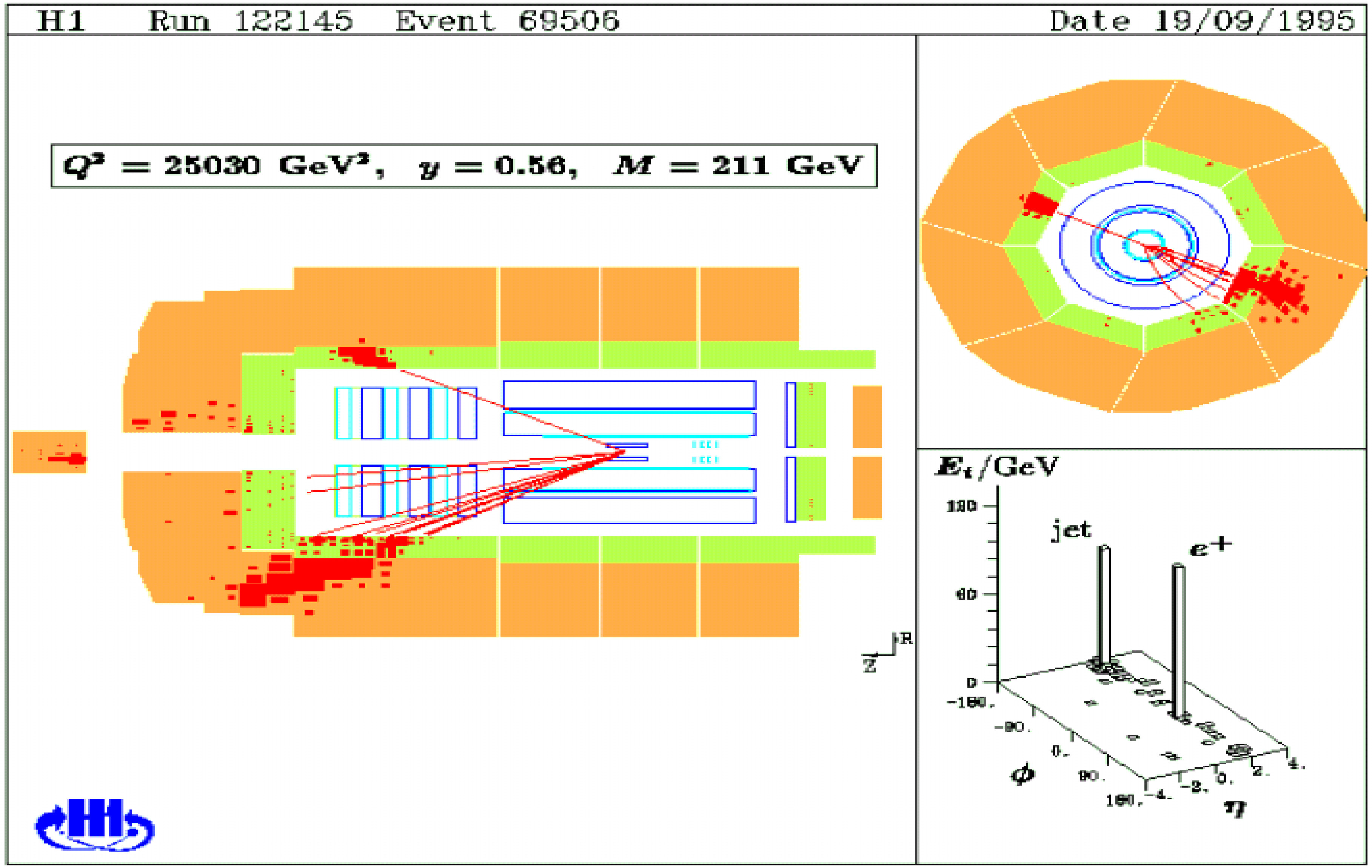


★ Two large experiments : H1 and ZEUS

★ Probe proton at very high Q^2 and very low x

Event display of a deep inelastic event at HERA/H1




H1 Run 122145 Event 69506 Date 19/09/1995
Q^2 = 25030 GeV^2, y = 0.56, M = 211 GeV
E_t/GeV
jet
e^+
ϕ
η

description of inelastic scattering

in analogy to Rosenbluth formula, but with 2 independent variables:

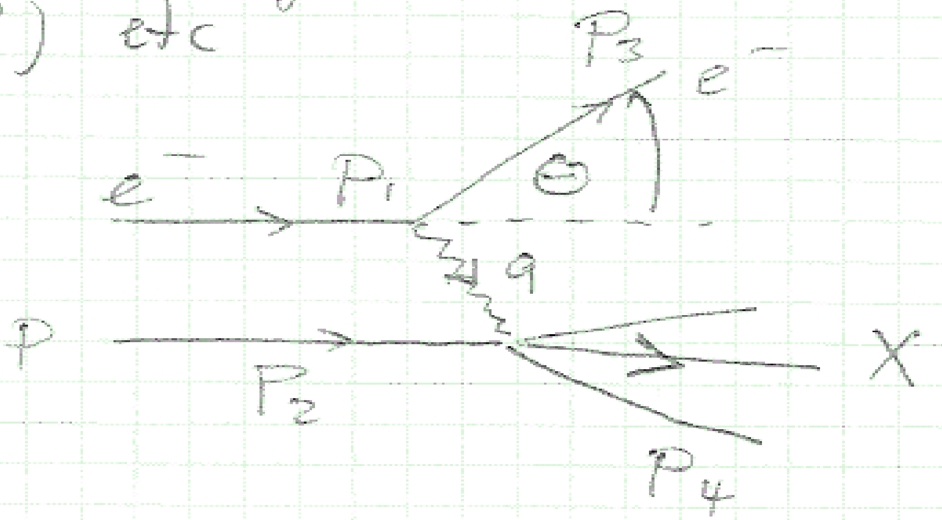
$$\frac{d^2\sigma}{d\Omega\, dE'} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} \left[W_2(Q^2,\nu) + 2W_1(Q^2,\nu)\tan^2(\theta/2)\right]$$

with $\nu \equiv P \cdot q/M$.

We usually discuss the dimensionless structure functions:
$F_1(x, Q^2) = MW_1(Q^2, \nu)$
$F_2(x, Q^2) = \nu W_2(Q^2, \nu)$

Deep inelastic lepton scattering

$p_1 = (E_1, \vec{p}_1)$ etc



$e^- p \to e^- X$

nucleon mass M

recoil mass $M_X \equiv W$

Kinematics : neglect electron mass

$$q = p_1 - p_3 = p_4 - p_2$$ momentum transfer (4 vector)

in terms of electron variables

$$q^2 = (p_1 - p_3)^2 = -2 p_1 p_3 = -4 E_1 E_3 \sin^2 \theta/2$$

note : $q^2 < 0$ "space-like"

center of mass energy of target system

$$W^2 = M_X^2 = (P_2 + q)^2 = M^2 + q^2 + 2 P_2 \cdot q$$

introduce new variables

$$Q^2 = -q^2 > 0$$

$$\nu = P_2 \cdot q / M$$

$$\nu = E_1 - E_3$$

electron energy loss in lab

$$\Rightarrow M_X^2 = M^2 - Q^2 + 2 M \cdot \nu$$

note: Q^2 and ν are Lorentz invariants

for elastic scattering, $M_X = M$

$$\Rightarrow \frac{Q^2}{2M\nu} = 1$$

for inelastic scattering, $M_X > M \Rightarrow \frac{Q^2}{2M\nu} < 1$

for elastic scattering, there is only 1 variable, Q^2

for inelastic scattering, Q^2 and ν are 2 independent variables

to proceed further, we introduce 4 new Lorentz-invariant kinematic variables

$$x = \frac{Q^2}{2 P_2 \cdot q} \qquad \text{Bjorken } x$$

$$Q^2 = -q^2$$

$$\nu = \frac{P_2 \cdot q}{M}$$

$$y = \frac{P_2 \cdot q}{P_2 \cdot P_1}$$

in the lab:

$$x = \frac{Q^2}{2M\nu}, \quad 0 \le x \le 1$$

$$\nu = E_1 - E_3$$

$$Q^2 = 4 E_1 E_3 \sin^2 \theta/2$$

$$y = 1 - \frac{E_3}{E_1} \quad 0 \le y \le 1$$

relations between these kinematic variables

$$s = (p_1 + p_2)^2 = M^2 + 2 p_1 p_2$$

$$x = \frac{Q^2}{2M\nu} \qquad y = \frac{2M\nu}{s - M^2}$$

$$x \cdot y = \frac{Q^2}{2 p_2 \cdot p_1} = \frac{Q^2}{s - M^2}$$

a brief aside on elastic scattering

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\frac{G_E^2 + \tau G_M^2}{(1+\tau)}\left(1 - y - \frac{M^2 y^2}{Q^2}\right) + \frac{1}{2} y^2 G_M^2\right]$$

$$\tau = \frac{Q^2}{4M^2}$$

elastic e-p scattering in terms of Lorentz invariant quantities

note that, since x = 1 for elastic scattering, $y = Q^2/(s - M^2)$

Lorentz-invariant formula for deep inelastic scattering

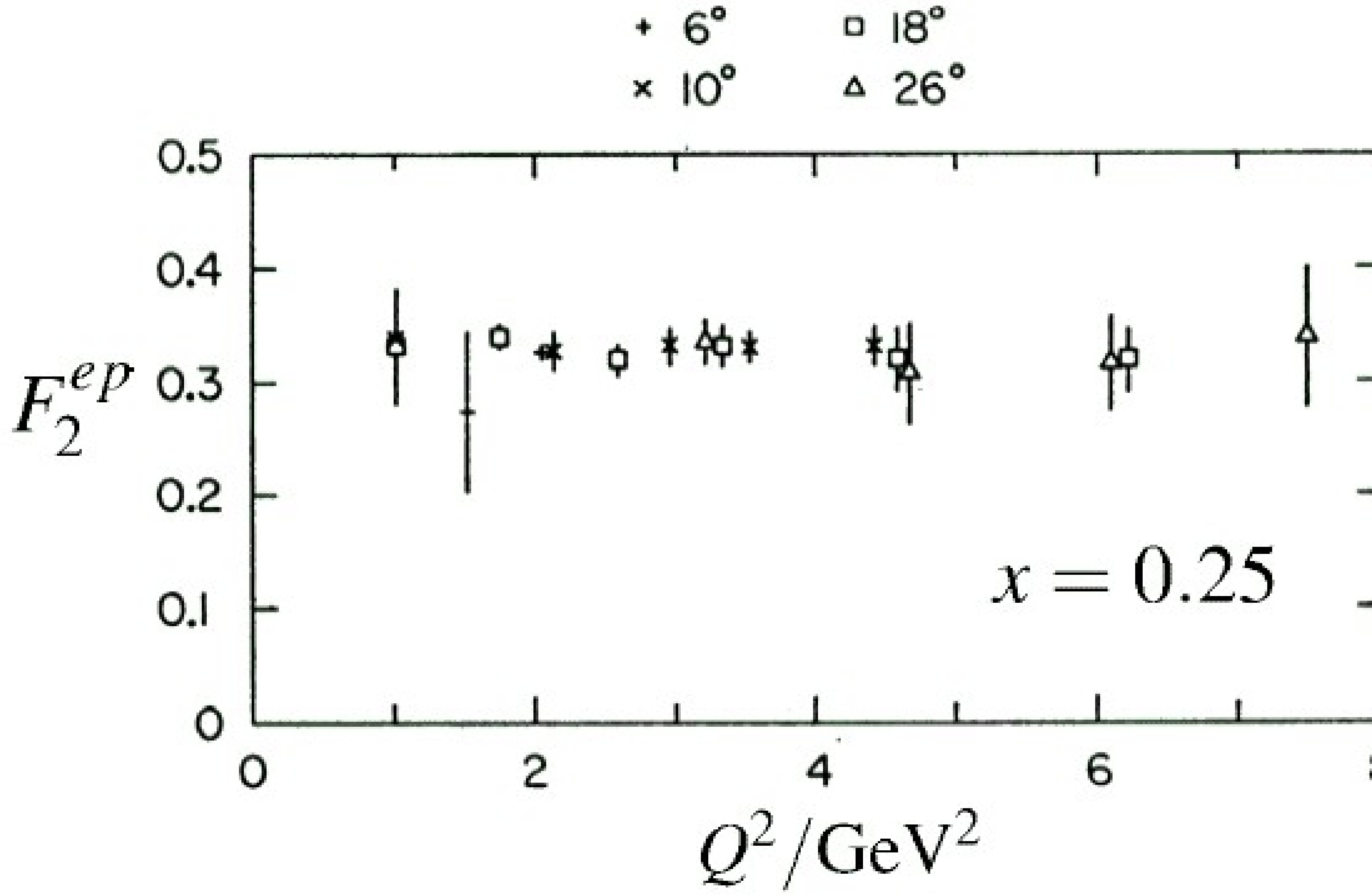
$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{M^2y^2}{Q^2}\right)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right]$$

in the lab frame, the kinematic quantities are:

$$Q^2 = 4E_1E_3\sin^2\theta/2; \quad x = \frac{Q^2}{2M(E_1-E_3)}; \quad y = 1 - \frac{E_3}{E_1}; \quad \nu = E_1 - E_3$$

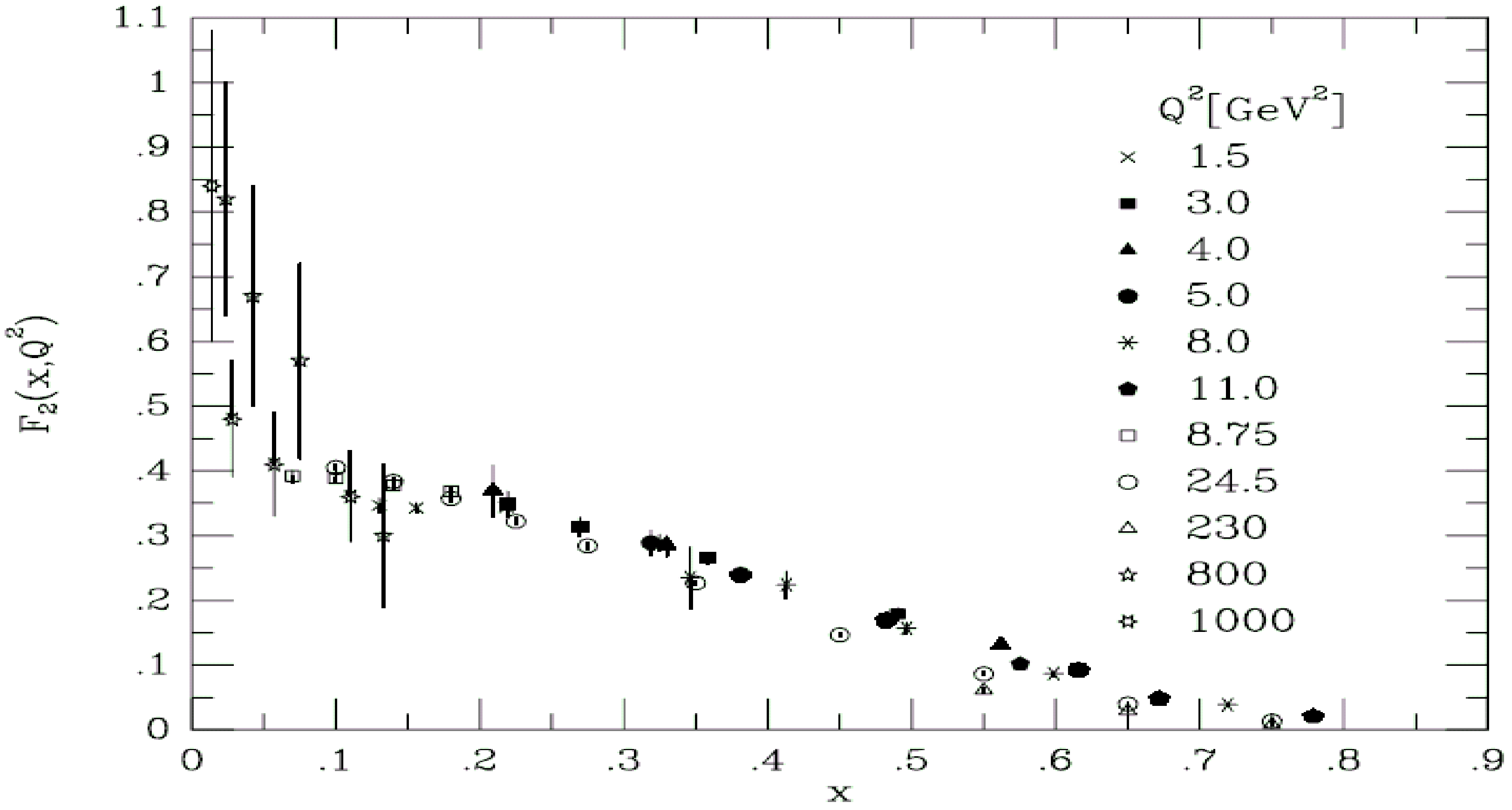
only 2 of the 4 quantities are independent, here we choose x and Q^2

Bjorken scaling for F2



J.T.Friedman + H.W.Kendall, Ann. Rev. Nucl. Sci. 22 (1972) 203

Scaling: F2 does not depend much on Q^2 when x is reasonably large, x > 0.1



Bjorken scaling

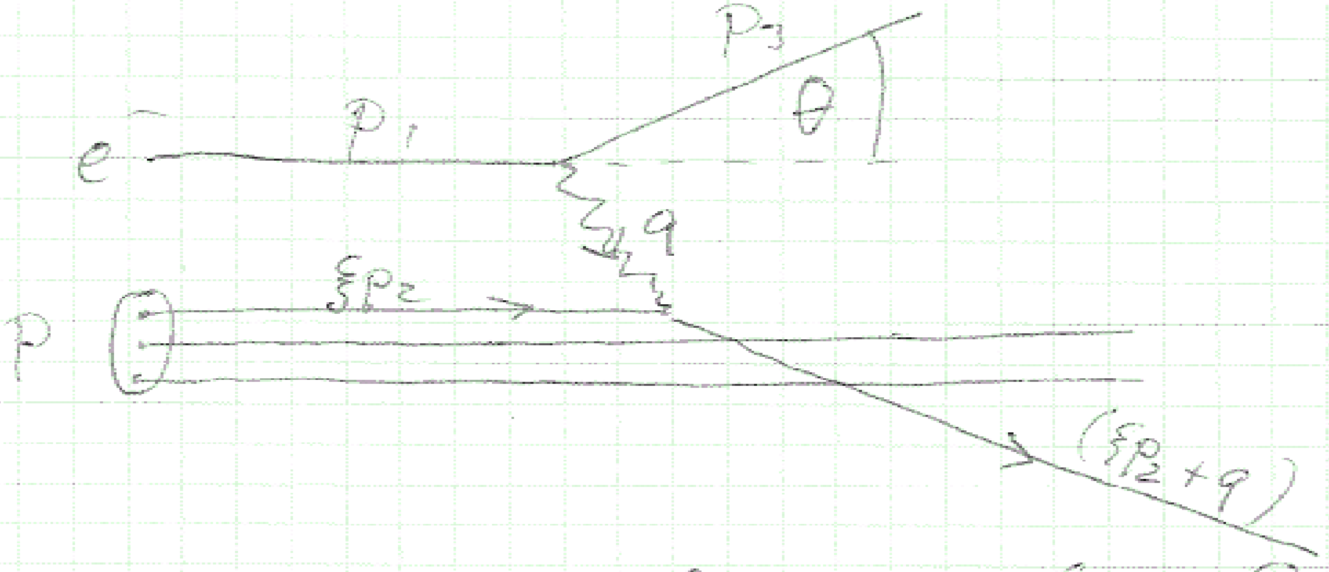
$$\lim_{Q^2\to\infty} F_1(Q^2, x) = F_1(x)$$

$$\lim_{Q^2\to\infty} F_2(Q^2, x) = F_2(x)$$

in practice true already for $Q^2 >$ a few GeV^2

The parton model interpretation

(Bjorken 1969, Feynman 1969)

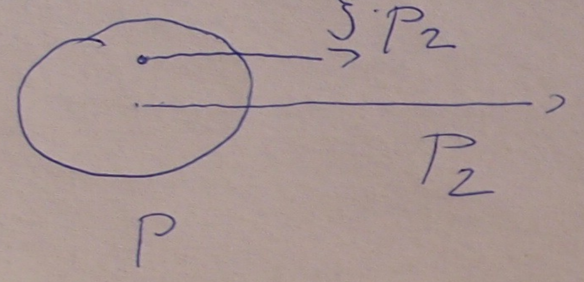


elastic scattering off a pointlike parton

use Breit frame (infinite momentum frame) for the discussion. In this frame, the proton has very high energy $p_2 = (E_2, 0, 0, E_2)$ M negligible

also neglect proton mass and transverse momentum

The Feynman argument (1969)



protons has (large) 4 momentum P_2

$(P_2)^2 >> M^2$

Parton 4 momentum $P_q = \xi P_2$

$(\xi \cdot P_2)^2 = m_q^2$ m_q = parton mass

$m_q^2 = (\xi \cdot P_2)^2$

momentum conservation

after interaction with the virtual photon, parton q carries 4-momentum $\xi \cdot P_2 + q$ =>

$$m_q^2 = (\xi \cdot P_2 + q)^2 = (\xi \cdot P_2)^2 + 2\xi P_2 \cdot q + q^2$$

$$\Rightarrow \xi = -\frac{q^2}{2P_2 \cdot q} = \frac{Q^2}{2P_2 \cdot q} = x$$

$x = \frac{Q^2}{2M \cdot \nu}$ is measured on the electron side

~~Alle~~ furthermore: for the proton

$$s = (p_1 + p_2)^2 = 2 p_1 \cdot p_2$$

$$y = \frac{p_2 \cdot q}{p_2 \cdot p_1} \qquad x = \frac{Q^2}{2 p_2 \cdot q}$$

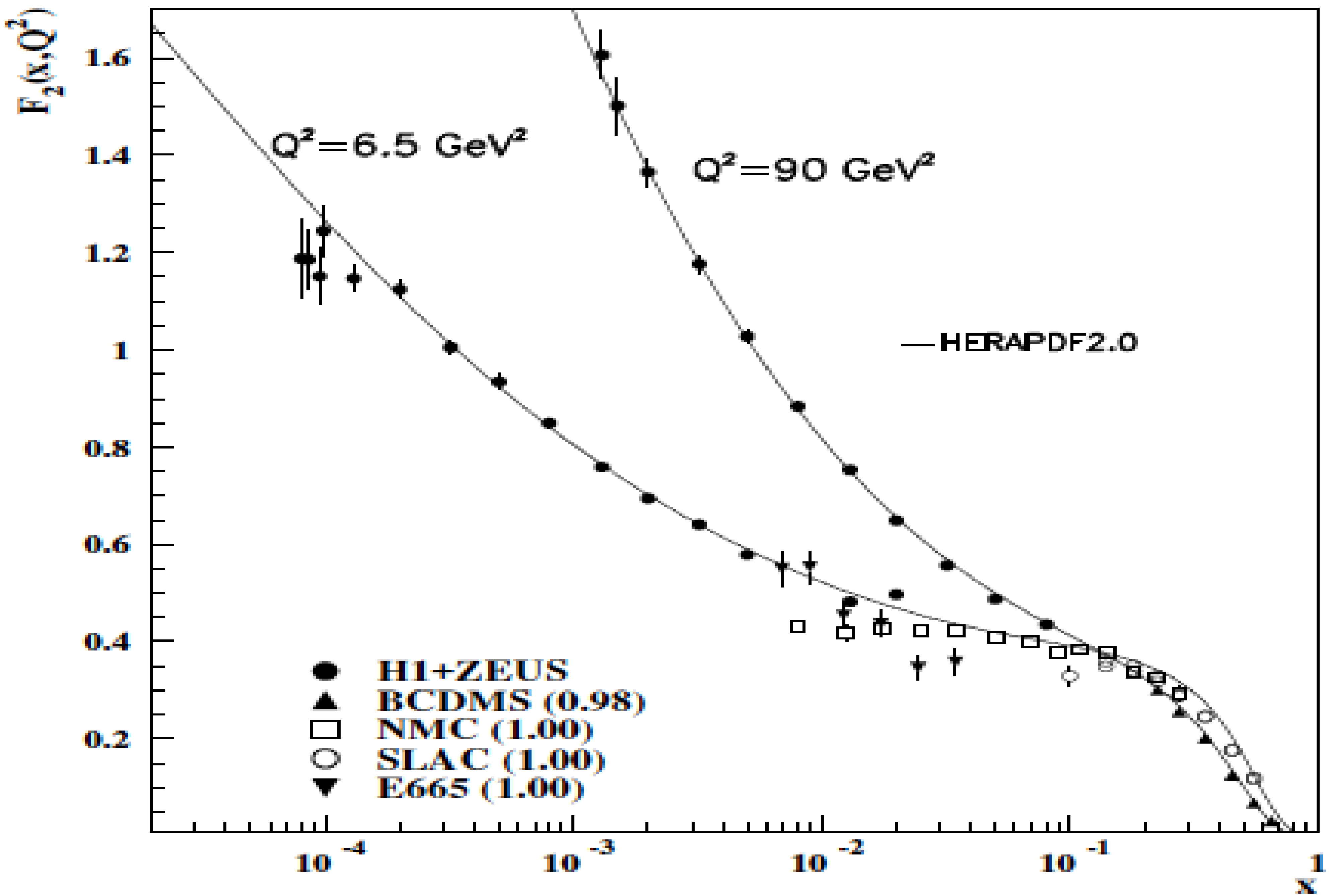
for the parton q we have

$$s^q = (p_1 + x p_2)^2 = 2 x\, p_1 p_2 = x \cdot s$$

$$y^q = \frac{p_q \cdot q}{p_q \cdot p_1} = \frac{x \cdot p_2 \cdot q}{x \cdot p_2 \cdot p_1} = y$$

$$x^q = 1$$ elastic e-q scattering

newer (HERA) data exhibit very strong scaling violations for x < 0.05



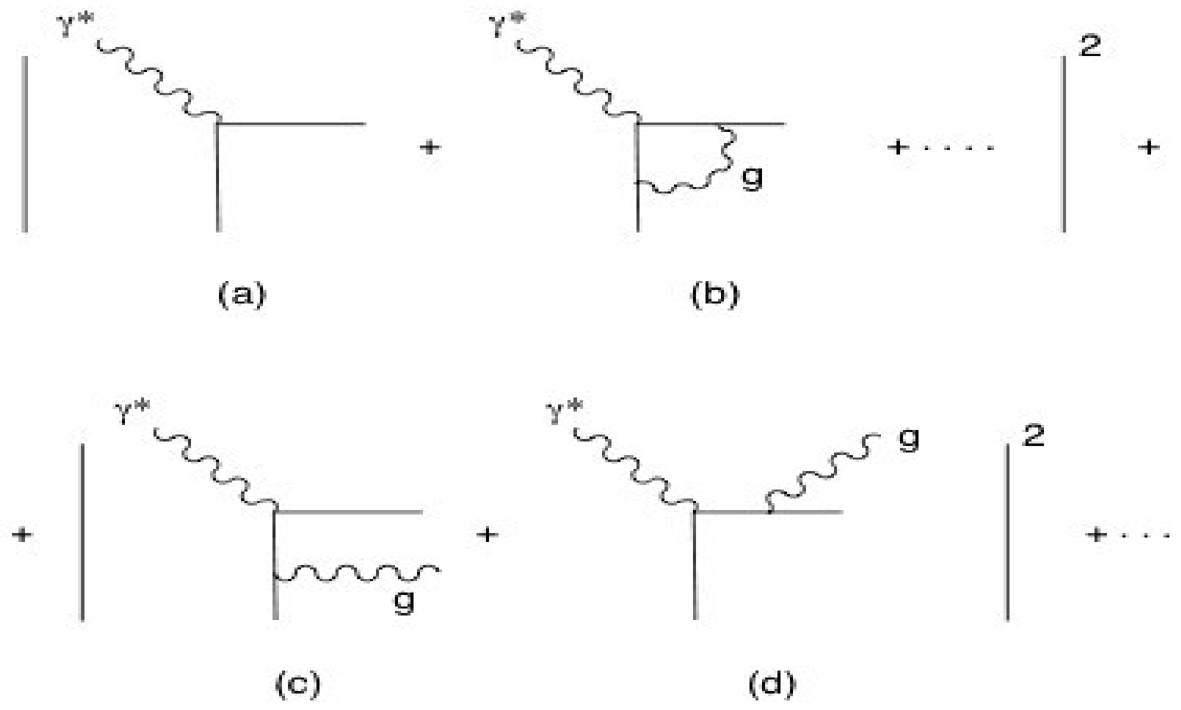
source: PDG and refs. there

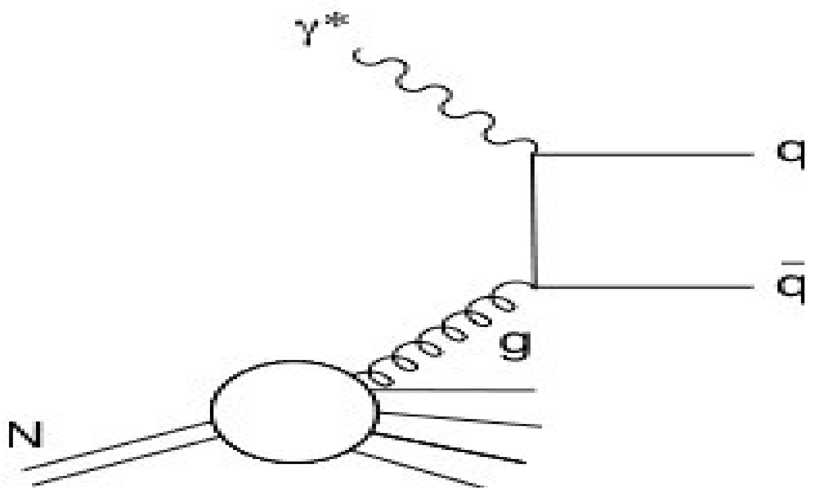
A note on scaling violations

in lowest order, the virtual photon γ^* with 4mom. q interacts only with the charged partons (quarks). At higher orders, there are many other possibilities, many of which involve gluons. As one looks with ever higher resolution, i.e. ever larger Q^2 values, the fine structure of the γ^*-parton interaction becomes more and more prominent. This leads to scaling violations and to the explicit appearance of (uncharged) gluons in the scattering process.

at large Q^2, the virtual photon scatters mostly off low x see quarks and gluons
at small Q^2, it scatters mostly off valence quarks at high x

higher order interactions lead to scaling violations





QCD series calculable in perturbation theory, determines the Q2 dependence

such processes need to be measured at a particular value of Q^2

this leads to DGLAP equations, after Altarelli, Parisi, Doshitzer, Gribov and Lipatov, 1973 – 1977, figure taken from G. Altarelli

cross section for elastic $e^- q$ scattering
(if q is parton with spin 1/2)

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 e_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s_q}\right)^2\right]$$

$$= \frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1 + (1-y)^2\right]$$

here e_q is parton charge.

Note that $-q^2 = Q^2$

also $\frac{q^2}{s_q} = -\frac{Q^2}{x \cdot s} = -y$ in Breit frame final result

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4}\left[(1-y) + \frac{y^2}{2}\right]$$

distribution of parton's momentum in the proton

$f_q(x)\,dx$ is prob. to find parton with momentum fraction $[x, x+dx]$ inside proton

Normalization is usually such that $\int_0 dx\, f_q(x) = n_q$

number of partons

1st compute cross section for e q scattering with q in range x, x+dx

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right] e_q^2 f_q(x)\,dx$$

to get e p cross section, we sum up over all partons in the proton and consider it differential in dx

$$\Rightarrow \frac{d\sigma^{ep}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\sum_q e_q^2 f_q(x)$$

now: in terms of the previously introduced structure functions F_1 and F_2, the ep cross section was: (for the relevant case of $M=0$)

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

comparing the terms yields

$$F_2(x,Q^2) = x\sum_q e_q^2 f_q(x)$$

$$F_1(x,Q^2) = \frac{1}{2}\sum_q e_q^2 f_q(x)$$

Phys.Rev.Lett. 22 (1969) 156-159

this yields the Callan - Gross relation

$$F_2(x,Q^2) = F_2(x) = 2x \cdot F_1(x)$$

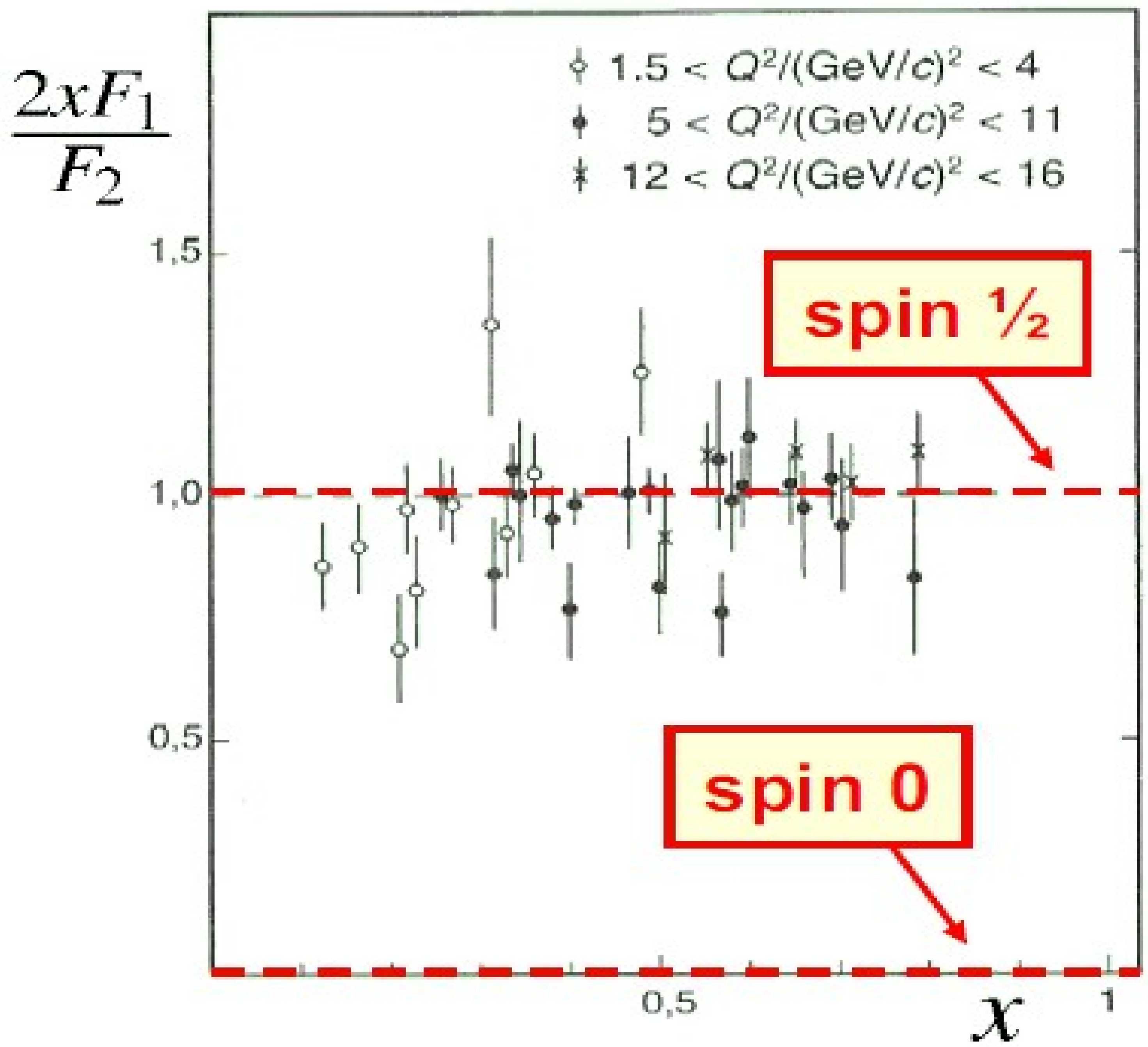
as well as exhibiting (Bjorken) scaling explicitely

Note: the Callan-Gross relation was derived under the assumption that the partons have spin 1/2

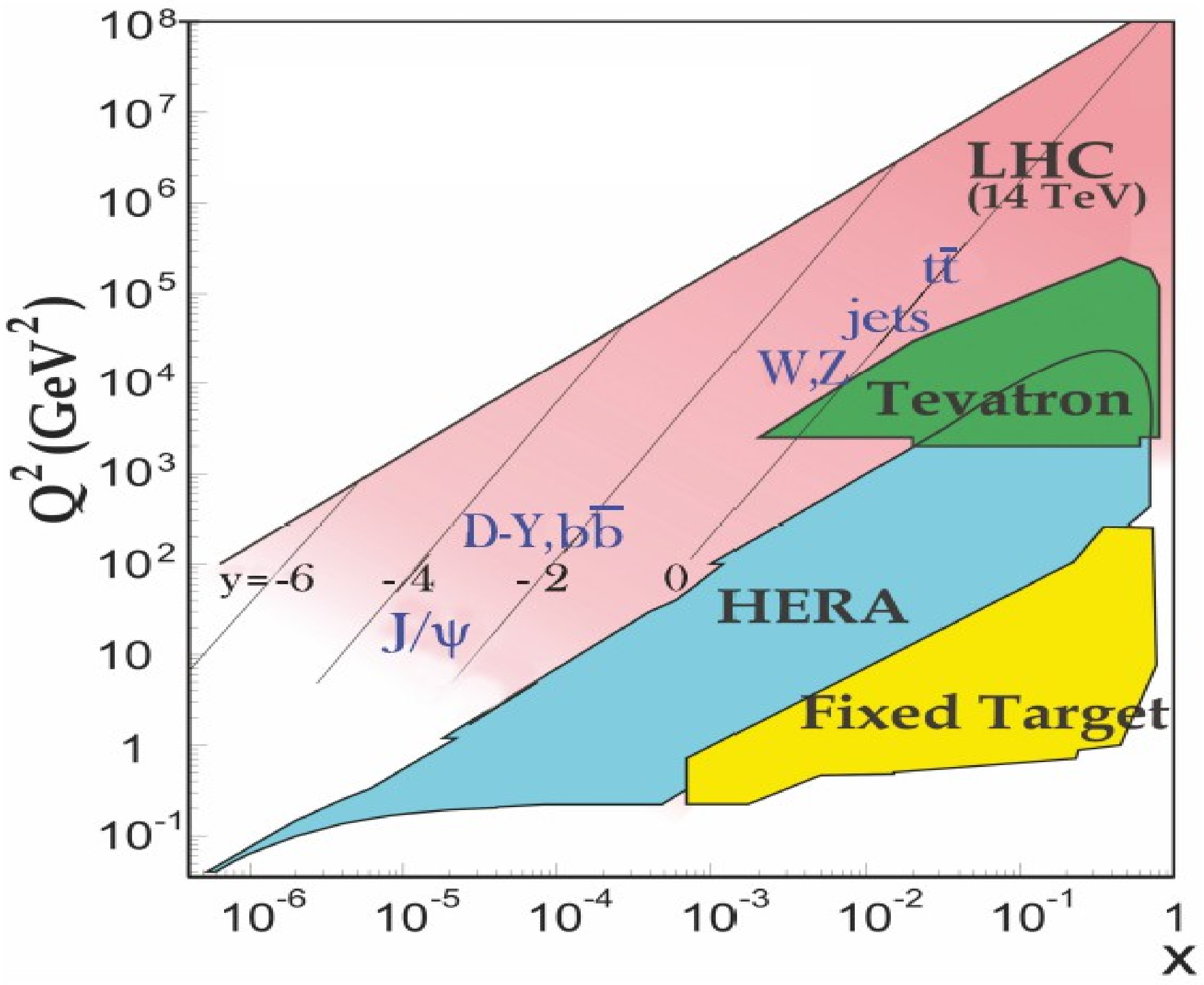
for the case of spin 0 we would only have electric scattering and $F_1 = 0$

the data imply that the partons have spin 1/2

important result: partons have spin 1/2



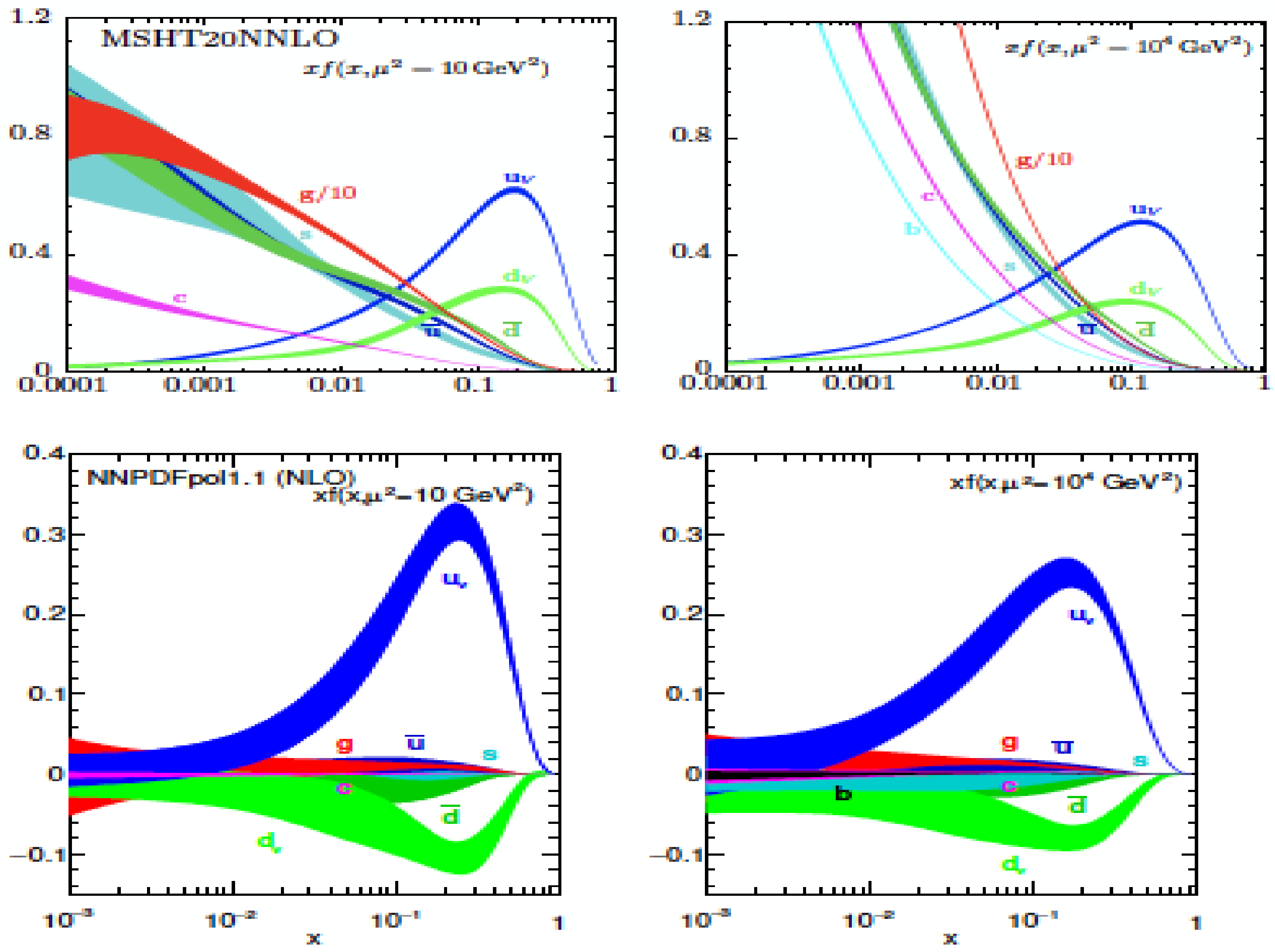
update 2022



source: PDG2022
section 18

Kinematic domains in x and Q^2 probed by fixed-target and collider experiments, where here Q^2 can refer either the literal Q^2 for deep inelastic scattering, or the hard scale of the process in hadron-hadron collisions, e.g. invariant mass or transverse momentum p_T^2. Some of the final states accessible at the LHC are indicated in the appropriate regions, where y is the rapidity. The incoming partons have $x_{1,2} = (Q/14\text{ TeV})e^{\pm y}$ where Q is the hard scale of the process shown in blue in the figure. For example, open charm production [24] and exclusive J/ψ and Υ production [25] at high $|y|$ at the LHC may probe the gluon PDF down to $x \sim 10^{-5}$.

current status of structure functions (parton distributions)



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PDG2022

The bands are x times the unpolarized parton distributions $f(x)$ (where $f = u_v, d_v, \overline{u}, \overline{d}, s \simeq \bar{s}, c = \bar{c}, b = \bar{b}, g$) obtained in the NNLO MSHT20 global analysis [26] (top) at scales $\mu^2 = 10\ \mathrm{GeV}^2$ (left) and $\mu^2 = 10^4\ \mathrm{GeV}^2$ (right), with $\alpha_s(M_Z^2) = 0.118$. The polarized parton distributions $f(x)$ obtained in the NLO NNPDFpol1.1 fit [27] (bottom).

Impact of heavy quark and quarkonium data on nuclear gluon PDFs

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K. F. Muzakka [1] F. I. Olness [3] R. Ruiz [2] I. Schienbein [4] and J. Y. Yu [4]

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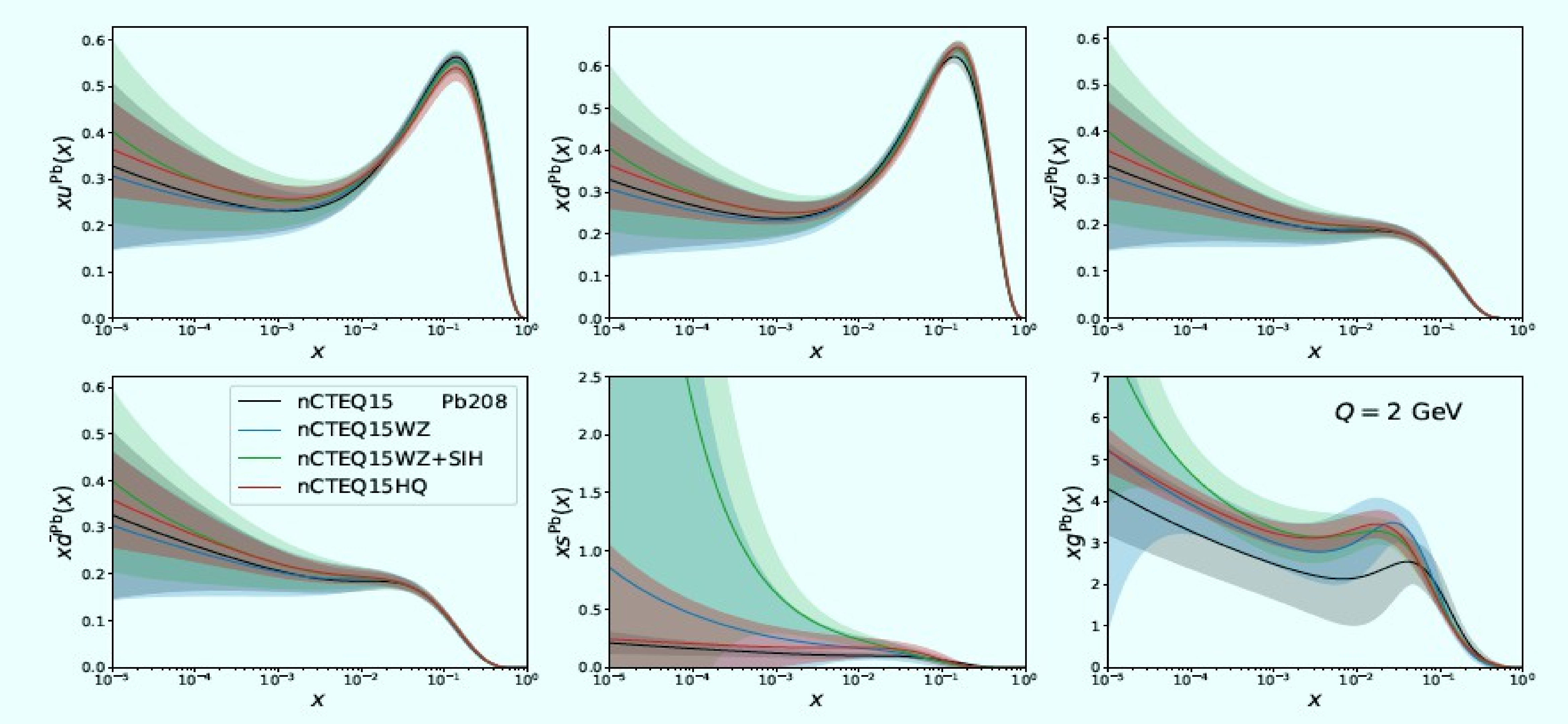

Lead PDFs from different nCTEQ15 versions. The baseline nCTEQ15 fit is shown in black, nCTEQ15WZ in blue, nCTEQ15WZSIH in green, and the new fit in red.