Lecture 2

the nucleon, static properties

brief reminder of nucleon properties: (from PDG, see there for references)

N BARYONS
$(S = 0, I = 1/2)$

$p,\ N^+ = uud$; $n,\ N^0 = udd$

p $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: $****$

p MASS (MeV)	**938.272046±0.000021**
p MAGNETIC MOMENT (μ_N)	**2.792847356±0.000000023**
p CHARGE RADIUS	*VALUE* (fm) **0.8751 ±0.0061** **0.84087±0.00026±0.00029**

This is the rms electric charge radius, $\sqrt{\langle r_E^2 \rangle}$.

this discrepancy is the subject of lecture 3

n $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: $****$

n MASS (MeV)	**939.5654133±0.0000058**
n MEAN LIFE	*VALUE* (s) **879.4± 0.6** 886.3± 1.2± 3.2
n MAGNETIC MOMENT	*VALUE* (μ_N) **−1.91304273±0.00000045**

N BARYONS
($S = 0$, $I = 1/2$)

p, $N^+ = uud$; n, $N^0 = udd$

p

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

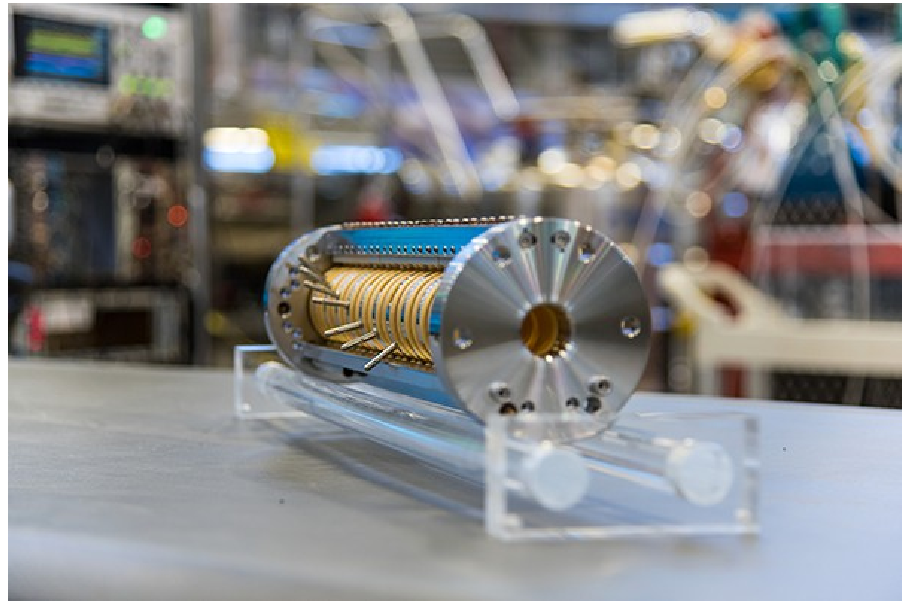
Mass $m = 1.00727646663 \pm 0.00000000009$ u (S = 2.9)
Mass $m = 938.272081 \pm 0.000006$ MeV [a]
$\left|m_p - m_{\overline{p}}\right|/m_p < 7 \times 10^{-10}$, CL = 90% [b]
$\left|\frac{q_{\overline{p}}}{m_{\overline{p}}}\right|/(\frac{q_p}{m_p}) = 1.00000000000 \pm 0.00000000007$
$\left|q_p + q_{\overline{p}}\right|/e < 7 \times 10^{-10}$, CL = 90% [b]
$\left|q_p + q_e\right|/e < 1 \times 10^{-21}$ [c]
Magnetic moment $\mu = 2.7928473446 \pm 0.0000000008\ \mu_N$
$(\mu_p + \mu_{\overline{p}})\ /\ \mu_p = (0.002 \pm 0.004) \times 10^{-6}$
Electric dipole moment $d < 0.021 \times 10^{-23}\ e\,\mathrm{cm}$
Electric polarizability $\alpha = (11.2 \pm 0.4) \times 10^{-4}\ \mathrm{fm}^3$
Magnetic polarizability $\beta = (2.5 \pm 0.4) \times 10^{-4}\ \mathrm{fm}^3$ (S = 1.2)
Charge radius, μp Lamb shift $= 0.84087 \pm 0.00039$ fm [d]
Charge radius $= 0.8409 \pm 0.0004$ fm [d]
Magnetic radius $= 0.851 \pm 0.026$ fm [e]
Mean life $\tau > 3.6 \times 10^{29}$ years, CL = 90% [f] ($p \rightarrow$ invisible mode)
Mean life $\tau > 10^{31}$ to 10^{33} years [f] (mode dependent)

n $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Mass $m = 1.0086649159 \pm 0.0000000005$ u
Mass $m = 939.565413 \pm 0.000006$ MeV [a]
$(m_n - m_{\overline{n}})/\ m_n = (9 \pm 6) \times 10^{-5}$
$m_n - m_p = 1.2933321 \pm 0.0000005$ MeV
$= 0.00138844919(45)$ u
Mean life $\tau = 879.4 \pm 0.6$ s (S = 1.6)
$c\tau = 2.6362 \times 10^8$ km
Magnetic moment $\mu = -1.9130427 \pm 0.0000005\ \mu_N$
Electric dipole moment $d < 0.18 \times 10^{-25}$ e cm, CL = 90%
Mean-square charge radius $\langle r_n^2 \rangle = -0.1161 \pm 0.0022$ fm^2 (S = 1.3)
Magnetic radius $\sqrt{\langle r_M^2 \rangle} = 0.864^{+0.009}_{-0.008}$ fm
Electric polarizability $\alpha = (11.8 \pm 1.1) \times 10^{-4}$ fm^3
Magnetic polarizability $\beta = (3.7 \pm 1.2) \times 10^{-4}$ fm^3
Charge $q = (-0.2 \pm 0.8) \times 10^{-21}$ e
Mean $n\overline{n}$-oscillation time $> 8.6 \times 10^7$ s, CL = 90% (free n)
Mean $n\overline{n}$-oscillation time $> 4.7 \times 10^8$ s, CL = 90% [g] (bound n)
Mean nn'-oscillation time > 448 s, CL = 90% [h]

the neutron lifetime puzzle

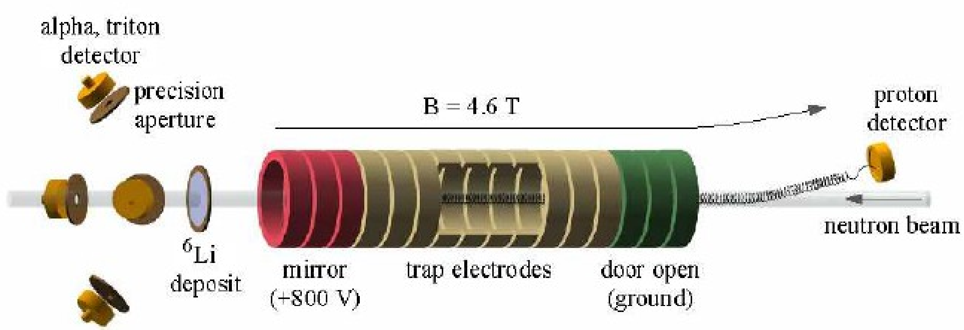
the neutron is the longest-lived of all unstable elementary particle, with an average lifetime of about 880 s.



ultra-cold neutron storage
bottle measurement
878 s

NIST proton trap for measuring neutron lifetime. A free neutron entering the trap as part of a beam will decay into a proton, an electron, and an antineutrino. The number of protons detected can be used to calculate the neutron lifetime.

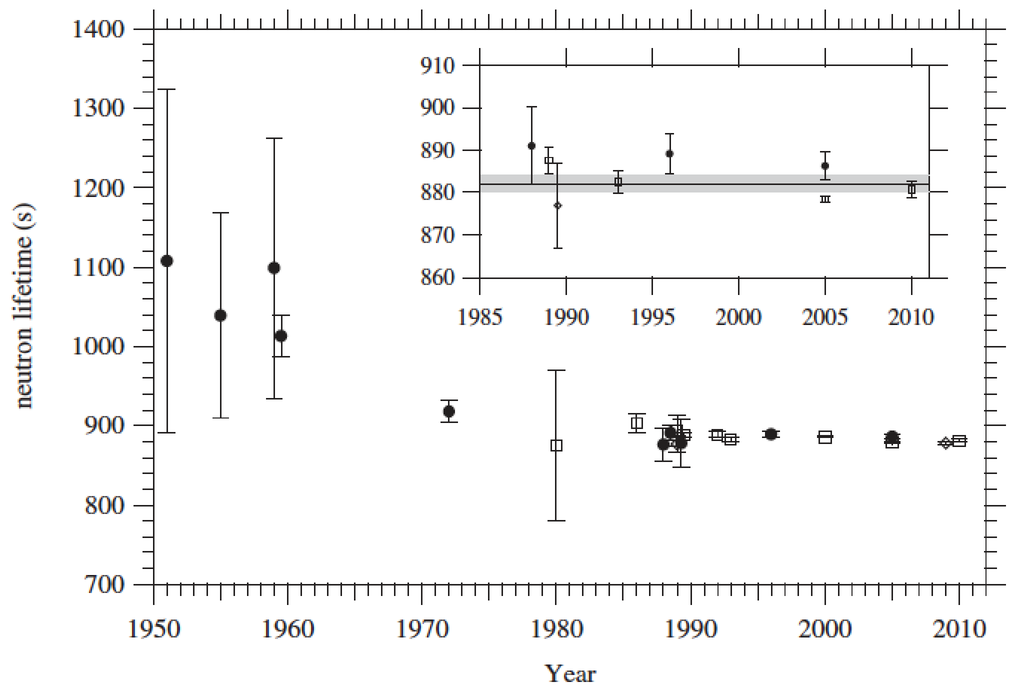
beam neutron lifetime measurement
886 s



Schematic of the beam neutron lifetime experiment using the Sussex-ILL-NIST method. The neutron beam passes through a quasi-Penning trap. Decay protons are trapped by the elevated door and mirror electrode potentials, and counted periodically by lowering the door to ground. Neutrons are counted by detecting the alphas and tritons from the $^6\mathrm{Li}(n,\alpha)$ reaction in a thin $^6\mathrm{LiF}$ deposit.

60 years of history of neutron lifetime

Wietfeldt and Greene, Rev. Mod. Phys. 83 (2011) 4, 1173-1192



A summary of neutron lifetime measurements. Solid circles are beam experiments, open squares are bottle experiments, and diamonds are magnetic trap experiments. The inset shows the eight experiments included in our global averages.

current status (Jan. 2020)

$$\tau_{n,beam} = 887.7 \pm 1.2 \pm 1.9s$$

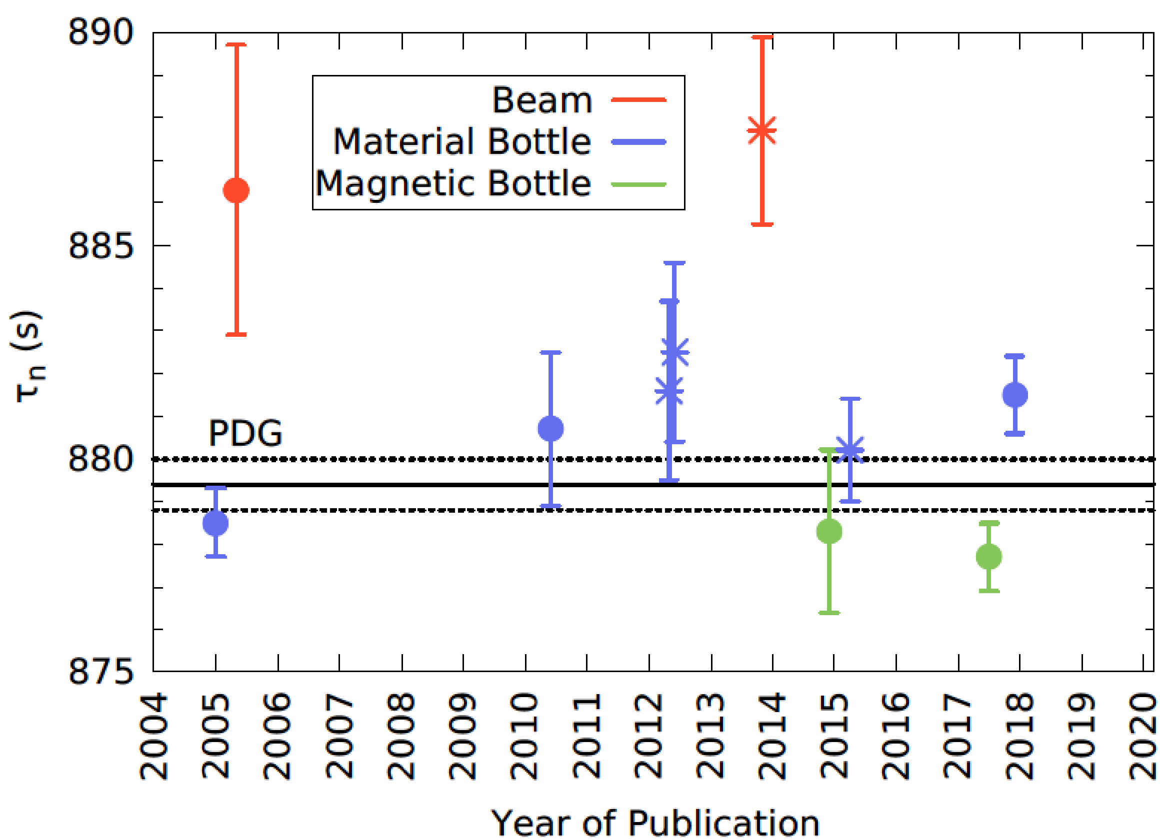
[A.T. Yue et al., PRL 111 222501 (2013)]

$$\neq$$

$$\tau_{n,stored} = 877.7 \pm 0.7 \pm 0.3s$$

[R.W. Pattie Jr. et al., SCI 360 627 (2018)]

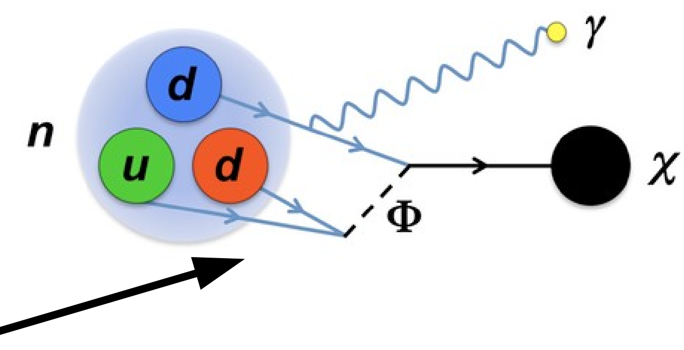
The Lifetime Puzzle



new experiments are planned, but no obvious solution of the puzzle is in sight. There were proposals that the puzzle may be connected to a 'dark matter particle in the neutron decay. Only the beam method would be sensitive to such 'dark' decays, thereby possibly explaining the discrepancy.

Fornal and Grinstein, PRL 120 (2018) 191801

but many controversial discussions in the community



dark matter decay

instead of the n-> p + e^- +v_e(bar)

likely ruled out,
see, e.g. D. Dubbers et al, 'Exotic decay channels are not the cause of the neutron lifetime anomaly', Phys. Lett. B791 (2019) 6-10

Baryons Magnetic Moments in the Quark Model

with 3 quarks, can make 27 different combinations

10 states are symmetric
e.g. uuu ddd sss or ddu+udd+dud or
dsu+uds+sud+sdu+dus+usd

1 state is completely antisymmetric, namely:
dsu+uds+sud-usd-sdu-dus

the remaining 16 states are classified into two octets of mixed symmetry

Baryon wave functions

color wave function is antisymmetric
space-spin-flavor part is symmetric

$$|\,qqq\,\rangle_A = |\,\mathrm{color}\,\rangle_A \times |\,\mathrm{space,\ spin,\ flavor}\,\rangle_S$$

dramatic difference to nuclear wave functions
each nucleon is color neutral

example: 3 nucleons

$$|\,NNN\,\rangle_A = |\,\mathrm{space,\ spin,\ isospin}\,\rangle_A$$

Color wave function of baryons

totally antisymmetric:

$$\sum_{a,b,c} \varepsilon_{abc} q_a q_b q_c = q_r q_g q_b - q_r q_b q_g + \dots$$

(RGB–RBG–GRB–BGR+BRG+GBR)/√6 (anti-symmetric in color)

note: at least 3 colors are needed, since this is the minimum number needed to build a totally antisymmetric wave function

also: contrast to color symmetric color wave function for mesor $(R\bar{R}+B\bar{B}+G\bar{G})/\sqrt{3}$ (symmetric in color)

Proton flavor wave function with spin up

$$|\psi_{p\uparrow}>=\frac{1}{\sqrt{18}}\Big[\;2u\uparrow d\downarrow u\uparrow+2d\downarrow u\uparrow u\uparrow+2u\uparrow u\uparrow d\downarrow$$
$$-u\downarrow d\uparrow u\uparrow-d\uparrow u\downarrow u\downarrow-u\uparrow u\downarrow d\uparrow$$
$$-u\uparrow d\uparrow u\downarrow-d\uparrow u\uparrow u\downarrow-u\downarrow u\uparrow d\uparrow\Big]$$

note: this must be symmetric because color is antisymmetric, remember uuu is symmetric for J = 3/2.

Proton Wave Function

$$|p^{\uparrow}\rangle = |u^{\uparrow}u^{\uparrow}d^{\downarrow}\rangle \qquad |n^{\uparrow}\rangle = |u^{\downarrow}d^{\uparrow}d^{\uparrow}\rangle\,.$$

$$\chi_p\left(J=\frac{1}{2}, m_j=\frac{1}{2}\right)=\sqrt{2/3}\,\chi_{uu}(1,1)\,\chi_d\left(1/2,-1/2\right)-\sqrt{1/3}\,\chi_{uu}(1,0)\,\chi_d\left(1/2,1/2\right)$$

$$\chi(1,0)=\left(\uparrow\downarrow+\downarrow\uparrow\right)/\sqrt{2}$$

this wave function is symmetric w.r.t. exchange of 2 u quarks

$$|p^{\uparrow}\rangle = \sqrt{2/3}\,|u^{\uparrow}u^{\uparrow}d^{\downarrow}\rangle - \sqrt{1/6}\,|u^{\uparrow}u^{\downarrow}d^{\uparrow}\rangle - \sqrt{1/6}\,|u^{\downarrow}u^{\uparrow}d^{\uparrow}\rangle\,.$$

note: the detailed fractions are obtained with Clebsch-Gordan coefficients

Proton and Neutron Wave Functions

$$|p^{\uparrow}\rangle = \frac{1}{\sqrt{18}} \{ \; 2\,|u^{\uparrow}u^{\uparrow}d^{\downarrow}\rangle + 2\,|u^{\uparrow}d^{\downarrow}u^{\uparrow}\rangle + 2\,|d^{\downarrow}u^{\uparrow}u^{\uparrow}\rangle - |u^{\uparrow}u^{\downarrow}d^{\uparrow}\rangle$$

$$-|u^{\uparrow}d^{\uparrow}u^{\downarrow}\rangle - |d^{\uparrow}u^{\uparrow}u^{\downarrow}\rangle - |u^{\downarrow}u^{\uparrow}d^{\uparrow}\rangle - |u^{\downarrow}d^{\uparrow}u^{\uparrow}\rangle - |d^{\uparrow}u^{\downarrow}u^{\uparrow}\rangle \; \} . \; ($$

$$|n^{\uparrow}\rangle = \frac{1}{\sqrt{18}} \{ \; 2\,|d^{\uparrow}d^{\uparrow}u^{\downarrow}\rangle + 2\,|d^{\uparrow}u^{\downarrow}d^{\uparrow}\rangle + 2\,|u^{\downarrow}d^{\uparrow}d^{\uparrow}\rangle - |d^{\uparrow}d^{\downarrow}u^{\uparrow}\rangle$$

$$-|d^{\uparrow}u^{\uparrow}d^{\downarrow}\rangle - |u^{\uparrow}d^{\uparrow}d^{\downarrow}\rangle - |d^{\downarrow}d^{\uparrow}u^{\uparrow}\rangle - |d^{\downarrow}u^{\uparrow}d^{\uparrow}\rangle - |u^{\uparrow}d^{\downarrow}d^{\uparrow}\rangle \; \}$$

Magnetic Moments

$$\mu_{\mathrm{Dirac}} = \frac{e\hbar}{2M} .$$

$$\mu_{\mathrm{N}} = \frac{e\hbar}{2M_{\mathrm{p}}} .$$

$$\mu_{\mathrm{p}} = \langle \boldsymbol{\mu}_{\mathrm{p}} \rangle = \langle \psi_{\mathrm{p}} | \boldsymbol{\mu}_{\mathrm{p}} | \psi_{\mathrm{p}} \rangle ,$$

$$\boldsymbol{\mu}_{\mathrm{p}} = \boldsymbol{\mu}_{\mathrm{u}} + \boldsymbol{\mu}_{\mathrm{u}} + \boldsymbol{\mu}_{\mathrm{d}} .$$

$$\mu_{\mathrm{u,d}} = \frac{z_{\mathrm{u,d}}\, e\hbar}{2m_{\mathrm{u,d}}} .$$

detailed derivation

$$\mu_p = \langle \Psi_p | \mu_p^{(z)} | \Psi_p \rangle$$ only z component

for magnetic moment only the spin part of the wave function counts

SO: $$\chi_p(J=1/2, m_J=1/2) = \sqrt{2/3}\,\chi_{uu}(1,1)\,\chi_d(1/2,-1/2) - \sqrt{1/3}\,\chi_{uu}(1,0)\,\chi_d(1/2,1/2)$$

$$\vec{\mu}_p = \vec{\mu}_u + \vec{\mu}_u + \vec{\mu}_d$$

$$\Rightarrow \mu_p = \langle \chi_p | \mu_u(z) + \mu_u(z) + \mu_d(z) | \chi_p \rangle$$

$$= \frac{2}{3}(\mu_u + \mu_u - \mu_d) + \frac{1}{3}\mu_d$$

$$= \frac{4}{3}\mu_u - \frac{1}{3}\mu_d$$

here we treat quarks as elementary fermions with Dirac magnetic moments

$$\mu_u = \frac{2}{3}\frac{e}{2m_u} \quad \mu_d = -\frac{1}{3}\frac{e}{2m_d}$$

note that for μ_n, just exchange u with d

$$\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$$

note that the quark masses are fictitious, they are
not the 'QCD' quarks but called 'constituent quarks'
in this schematic picture : $m_u \approx m_d \approx \frac{1}{3} m_p$

then it follows that $\mu_u = -2\mu_d$

then we get: $\mu_p = \frac{3}{2}\mu_u$ $\mu_n = -\mu_u$

so $\frac{\mu_n}{\mu_p} = -\frac{2}{3}$

experimentally we have -0.685 (see slide 1)
one of the early triumphs of the quark model

History The explanation of $\frac{\mu_n}{\mu_p}$ in the framework of the quark model was first given in:

M.A.B. Beg, B.W. Lee, A. Pais, PRL 13 (1964) 514

note: in the course of these investigations, it turned out that the quark wave function was symmetric for exchange of quarks. This led to the discovery of color by O.W. Greenberg, Compendium of Quantum Physics, Eds. D. Greenberger, K. Hentschel, F. Weinert, Springer, Berlin, p. 109-111, see also arXiv: 0805.0289.

First measurements of μ_p and μ_d, leading to μ_n were made by Otto Stern and his group, and by I. I. Rabi and his group, starting from 1934 on.

more on color:

in addition to the Greenberg paper, there is also a nearly simultaneous paper by M.Y. Han and Noichiro Nambu, Phys.Rev. 139 (1965) B1006-B1010
where the need for color was introduced. We should also recognize that, while the symmetry of the wave function is not obvious for the spin 1/2 proton, the wave function of the $\Delta^{++}(1232)$ with total angular momentum 3/2 and L = 0 consists in the quark model uniquely of the symmetric (uuu) configuration with all quark spins lined up, so this requires, to be consistent with the Pauli principle, also an 'anti-symmetrizer, i.e. Color

for a historical account of the events and findings that led to the invention of color, see O. W. Greenberg, arXiv:0805.0289

magnetic moments summary

$$\mu_p = \frac{2}{3}(\mu_u + \mu_u - \mu_d) + \frac{1}{3}\mu_d = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d ,$$

$$\mu_n = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u$$

$$\mu_p = \frac{3}{2}\mu_u , \qquad \mu_n = -\mu_u .$$

$$\mu_p = 2.79\,\mu_N = 2.79\frac{e\hbar}{2M_p}$$

$$\frac{\mu_n}{\mu_p} = -\frac{2}{3}$$

$$\mu_p = \frac{3}{2}\mu_u = \frac{e\hbar}{2m_u}$$

Exp ~ -0.685

$$m_u = \frac{M_p}{2.79} = 336\ \mathrm{MeV}/c^2 ,$$

magnetic moments and the quark model

source: PDG

: Quark model predictions and measured magnetic dipole moments of the ground state baryons in units of μ_N; $\kappa \equiv \frac{e}{2m} = 2.793\ \mu_N$ and $\kappa_s \equiv \frac{e}{2m_s} = -3\mu_\Lambda = 1.84\ \mu_N$. $^\dagger\Sigma^0 \to \Lambda$ transition magnetic moment.

Baryon	Quark model			Experimental value		
p	κ		input	2.793		
n	$-\frac{2}{3}\kappa$	=	–1.86	–1.913		
Λ	$-\frac{1}{3}\kappa_s$		input	–0.6138	±	0.0047
Σ^+	$\frac{8}{9}\kappa + \frac{1}{9}\kappa_s$	=	2.68	2.458	±	0.010
Σ^0	$\frac{2}{9}\kappa + \frac{1}{9}\kappa_s$	=	0.82			
$\Sigma^{0\dagger}$	$-\frac{1}{\sqrt{3}}\kappa$	=	–1.61	–1.61	±	0.08
Σ^-	$-\frac{4}{9}\kappa + \frac{1}{9}\kappa_s$	=	–1.04	–1.160	±	0.025
Ξ^0	$-\frac{2}{9}\kappa - \frac{4}{9}\kappa_s$	=	–1.44	–1.250	±	0.014
Ξ^-	$\frac{1}{9}\kappa - \frac{4}{9}\kappa_s$	=	–0.51	–0.6507	±	0.0025
Ω^-	$-\kappa_s$	=	–1.84	–2.024	±	0.056
Δ^{++}	2κ	=	5.58	4.52	±	0.67
Δ^+	κ	=	2.79	2.3	-	4.5

quark quantum numbers and hypercharge Y

	d	u	s	c	b	t
Q – electric charge	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$
I – isospin	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0
I_z – isospin z-component	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0	0	0
S – strangeness	0	0	-1	0	0	0
C – charm	0	0	0	$+1$	0	0
B – bottomness	0	0	0	0	-1	0
T – topness	0	0	0	0	0	$+1$

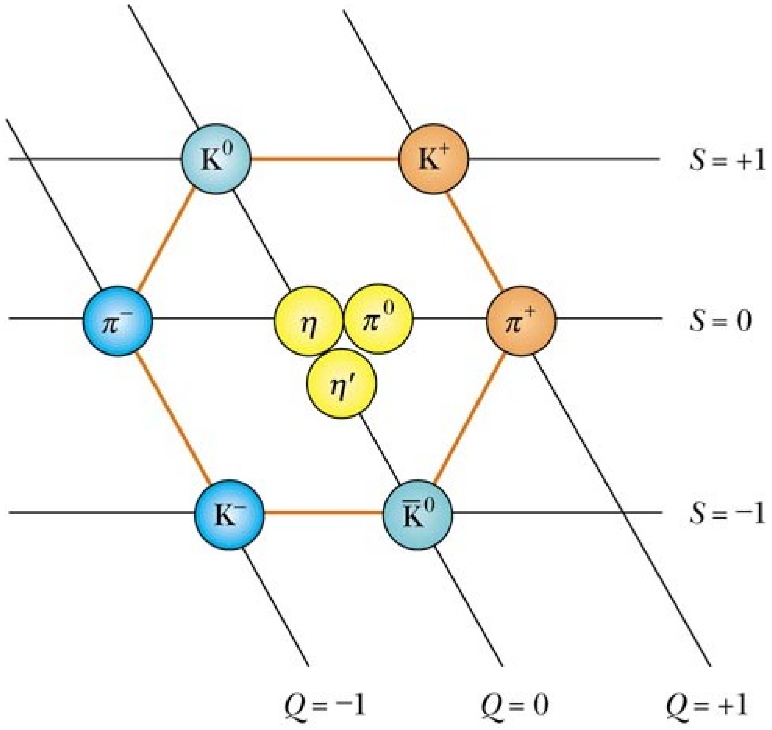
$$Y = \mathcal{B} + S - \frac{C - B + T}{3}$$

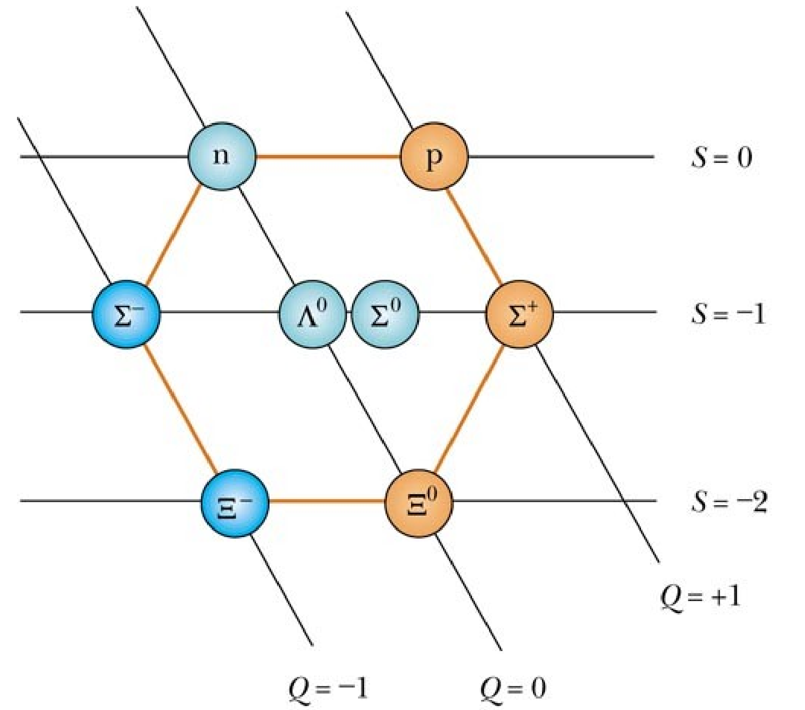
note: Y = 1/3 for u and d quarks, Y = -2/3 for s quarks, and zero otherwise

The eightfold way

mesons

baryons

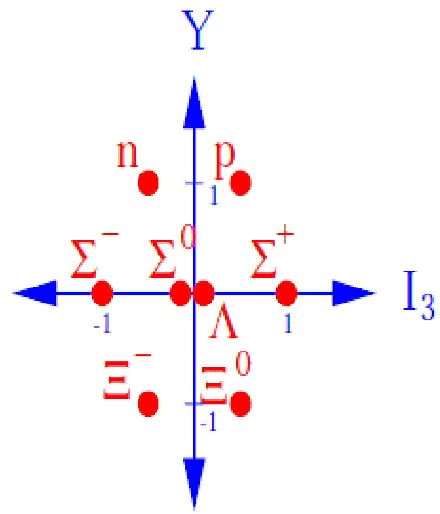




EXAMPLES OF MULTIPLETS

Baryon Octet

$J^P = \frac{1}{2}^+$

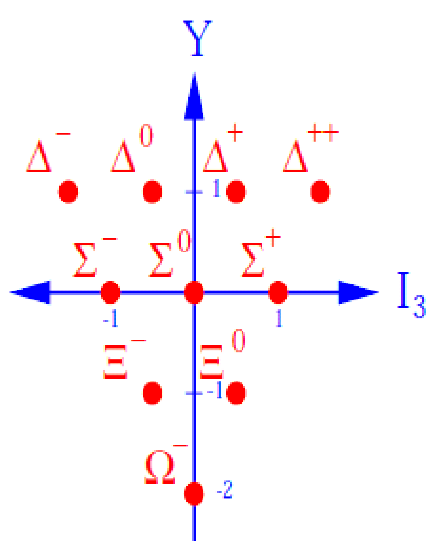


N(939)	I=1/2
Σ(1193)	I=1
Λ(1116)	I=0
Ξ(1318)	I=1/2

$$\frac{Q}{e} = I_3 + \frac{1}{2}(N+S)$$
$$= I_3 + \frac{1}{2}Y$$

Baryon Decuplet

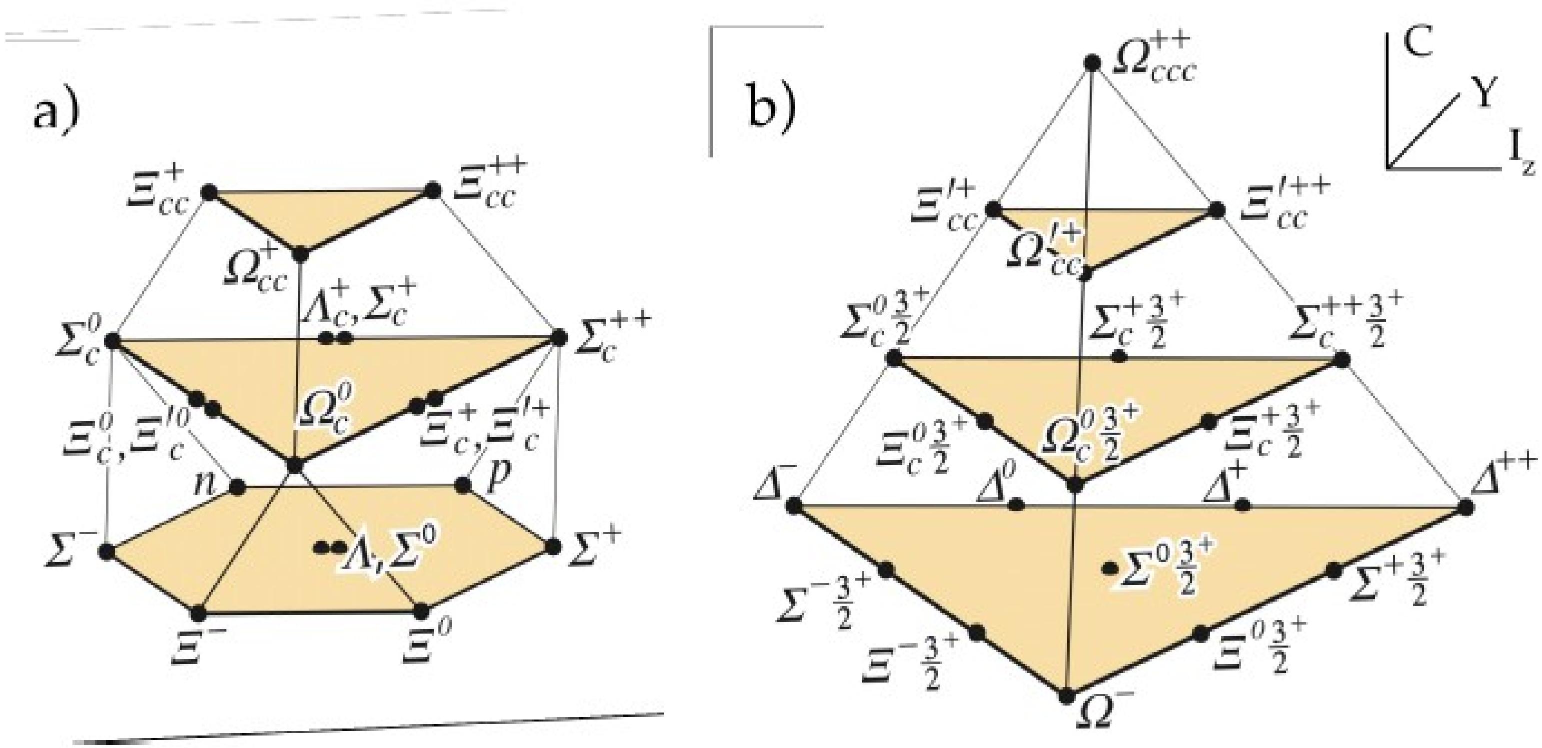
$J^P = \frac{3}{2}^+$



Δ(1232)	I=3/2
Σ(1384)	I=1
Ξ(1533)	I=1/2
Ω(1672)	I=0

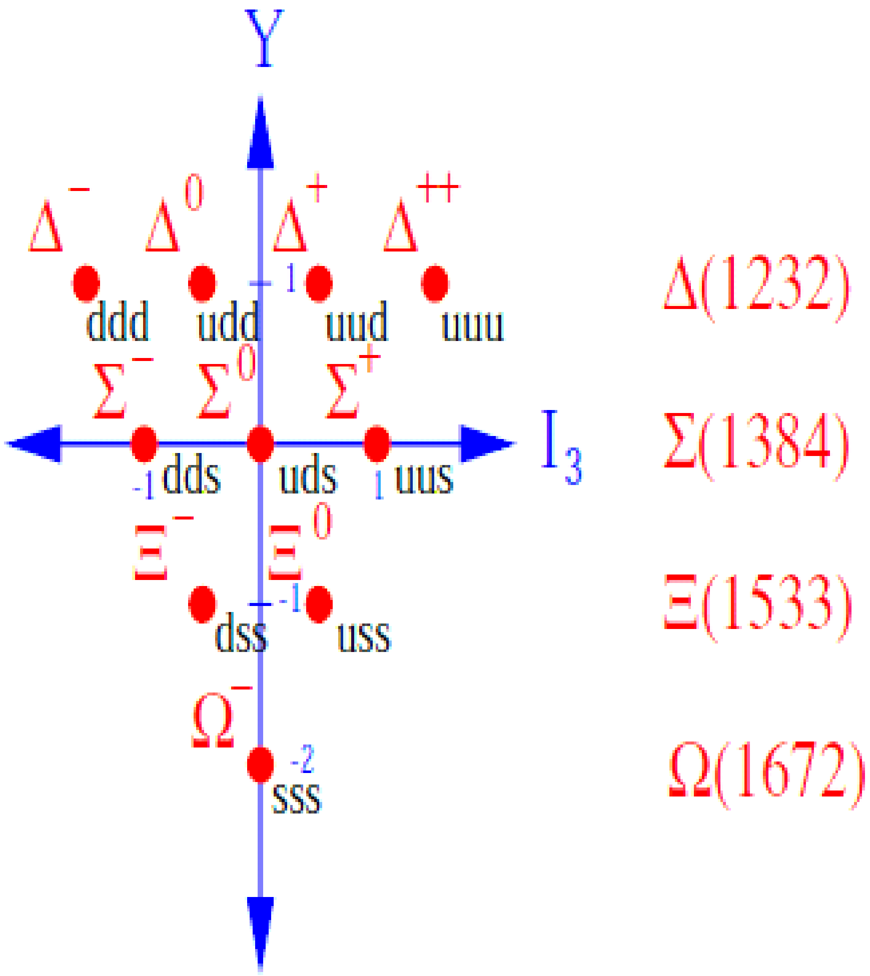
the baryon multiplets including strangeness and charm

source: PDG



SU(4)$_f$ multiplets of ground state baryons made of u, d, s, and c quarks. (a) The spin $\frac{1}{2}$ 20-plet extends the charmless SU(3)$_f$ octet to C = 1,2; (b) the spin $\frac{3}{2}$ 20-plet extends the SU(3)$_f$ decuplet to C = 1, 2, 3.

We can identify the 10 symmetric states with the baryon $J^P = \frac{3}{2}^+$ decuplet.



We now have a $J^P = \frac{3}{2}^+$ decuplet and a $J^P = \frac{1}{2}^+$ octet of baryons. (18 out of the 27 states used).

It turns out that the remaining 8 mixed symmetry flavour states form a $J^P = \frac{3}{2}^-$ octet of baryon resonances of higher mass.

$N(1520)$	$I = \frac{1}{2}$	
$\Lambda(1520)$	$I = 0$	SPINS
$\Xi(1820)$	$I = \frac{1}{2}$	$\uparrow\uparrow\uparrow$
$\Sigma(1670)$	$I = 1$	

The antisymmetric flavour state forms a $J^P = \frac{1}{2}^-$ singlet baryon resonance, the $\Lambda(1405)$.

Mass Differences of Baryons

1. members of isobaric multiplets are (nearly) degenerate

the nucleon, the Delta states, the Cascades, the Sigma's, etc

2. states with different strangeness have different masses

the masses are approximately proportional to strangeness content

consequence:

u and d quarks have equal mass
s quarks are heavier by Delta m

	S	Δm
$\Delta(1232)$	0	
		152 MeV
$\Sigma(1384)$	-1	
		149 MeV
$\Xi(1533)$	-2	
		139 MeV
$\Omega(1672)$	-3	