THE CHART OF NUCLEI

Kai Schweda

Outline

Deformation - rotational bands

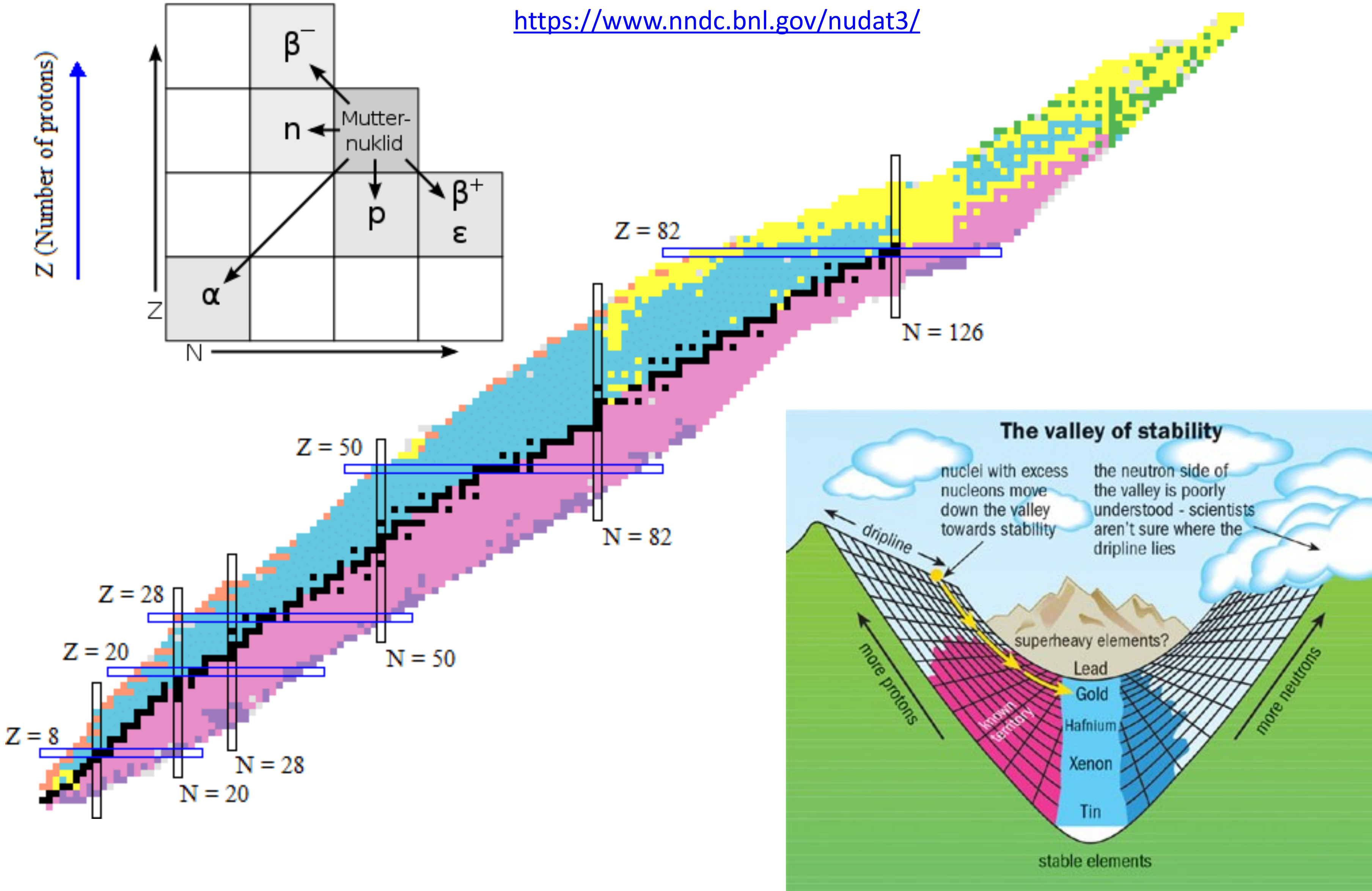
Double beta decay - neutrino mass

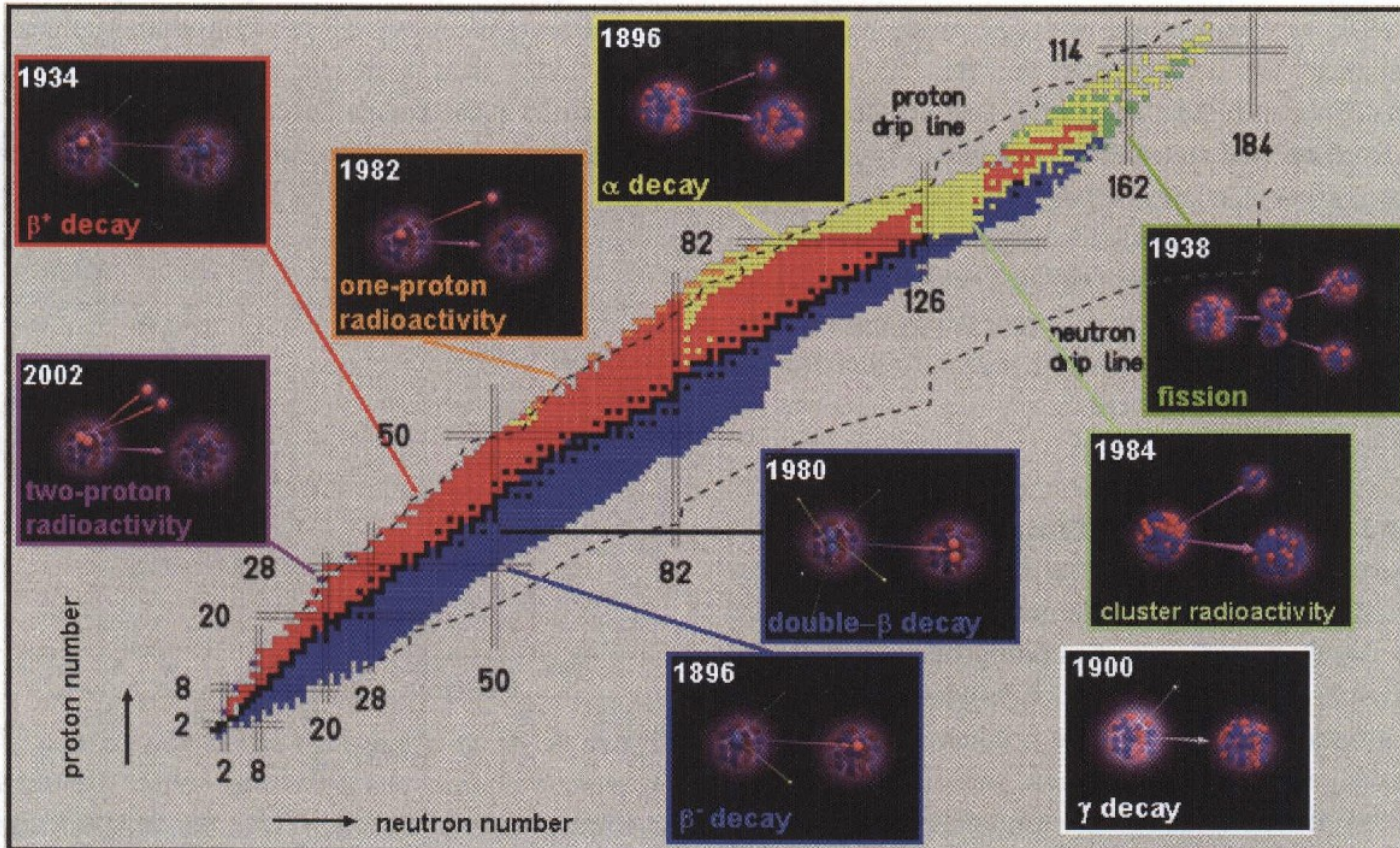
Alpha decay - cluster radioactivity

Fission - synthesis of heaviest elements

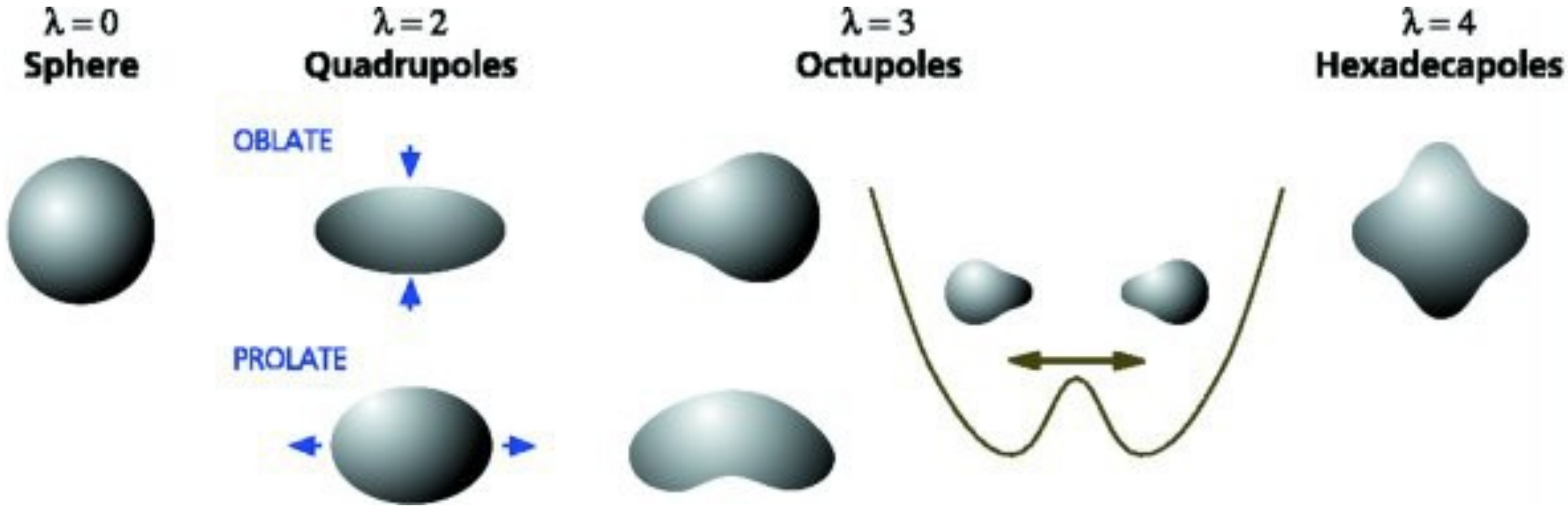
NUCLEAR CHART - VALLEY OF STABILITY

https://www.nndc.bnl.gov/nudat3/




1934
β⁺ decay
1982
one-proton
radioactivity
1896
α decay
1938
fission
2002
two-proton
radioactivity
1980
double–β decay
1984
cluster radioactivity
1896
β⁻ decay
1900
γ decay
proton
drip line
neutron
drip line
114
184
162
82
126
50
28
20
8
2
proton number
neutron number

NUCLEAR DEFORMATION



$$R(\theta, \phi) = c(\beta) R_0 \left(1 + \sum_{\lambda=2}^{\lambda_{\max}} \beta_\lambda Y^*_{\lambda 0}(\theta, \phi) \right)$$

QUADRUPOLE MOMENTS

$$E_J = \frac{1}{2\Theta} J(J+1),\ J = 0,2,4,\ldots$$

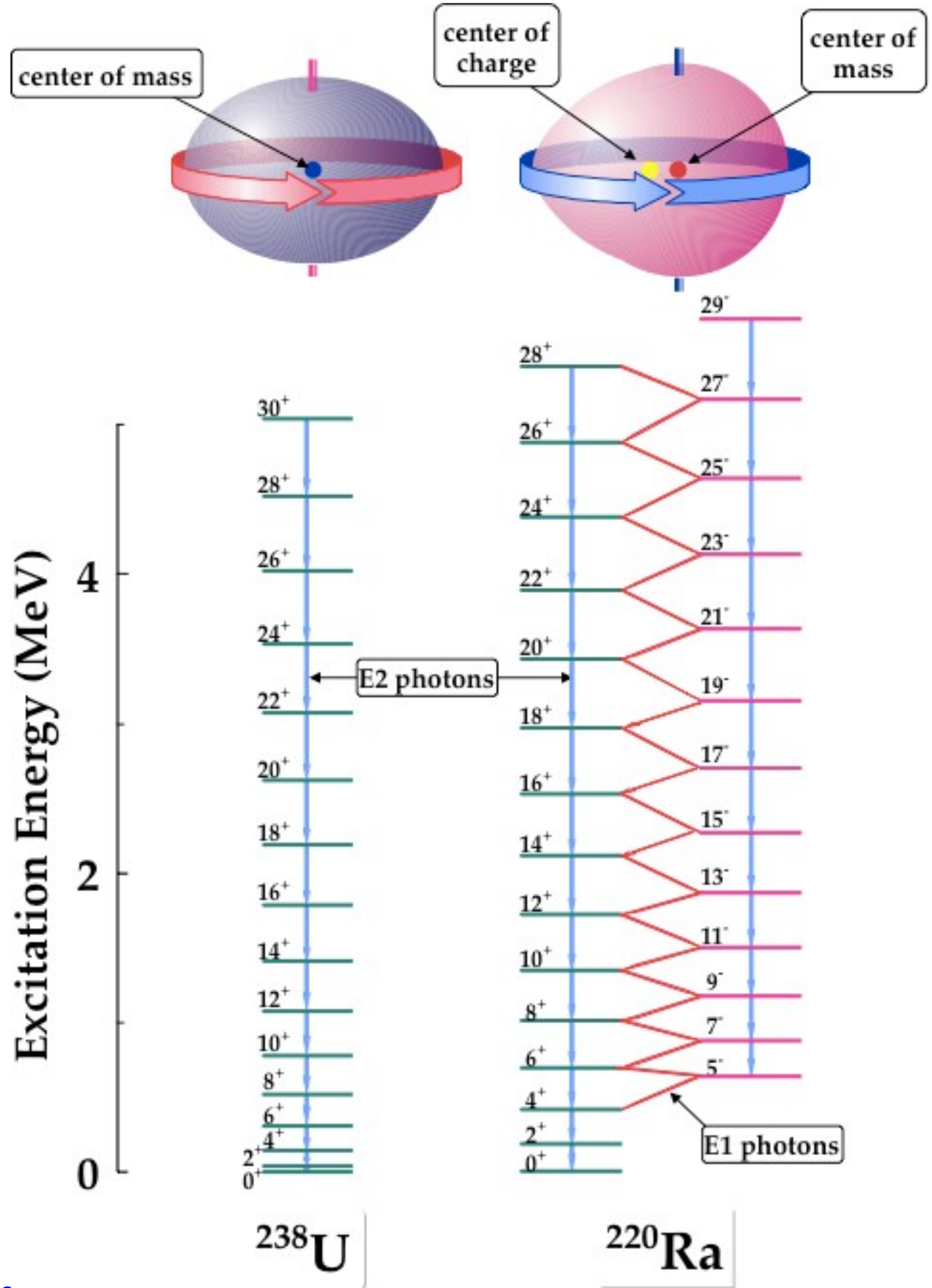
$$\Delta E = E_J - E_{J-2} = \frac{4J}{2\Theta} - \frac{2}{2\Theta}$$

$$\Delta(\Delta E) = \frac{8}{2\Theta}$$

$$^{238}\text{U} : \Delta(\Delta E) \approx 50 \text{ keV}$$

$$\frac{1}{2\Theta} = 7.5 \text{ keV}$$

$$\Theta_{\text{rigid}} = \frac{2}{5} R^2 (1 + 0.31\beta) \approx 6 \text{ keV}$$

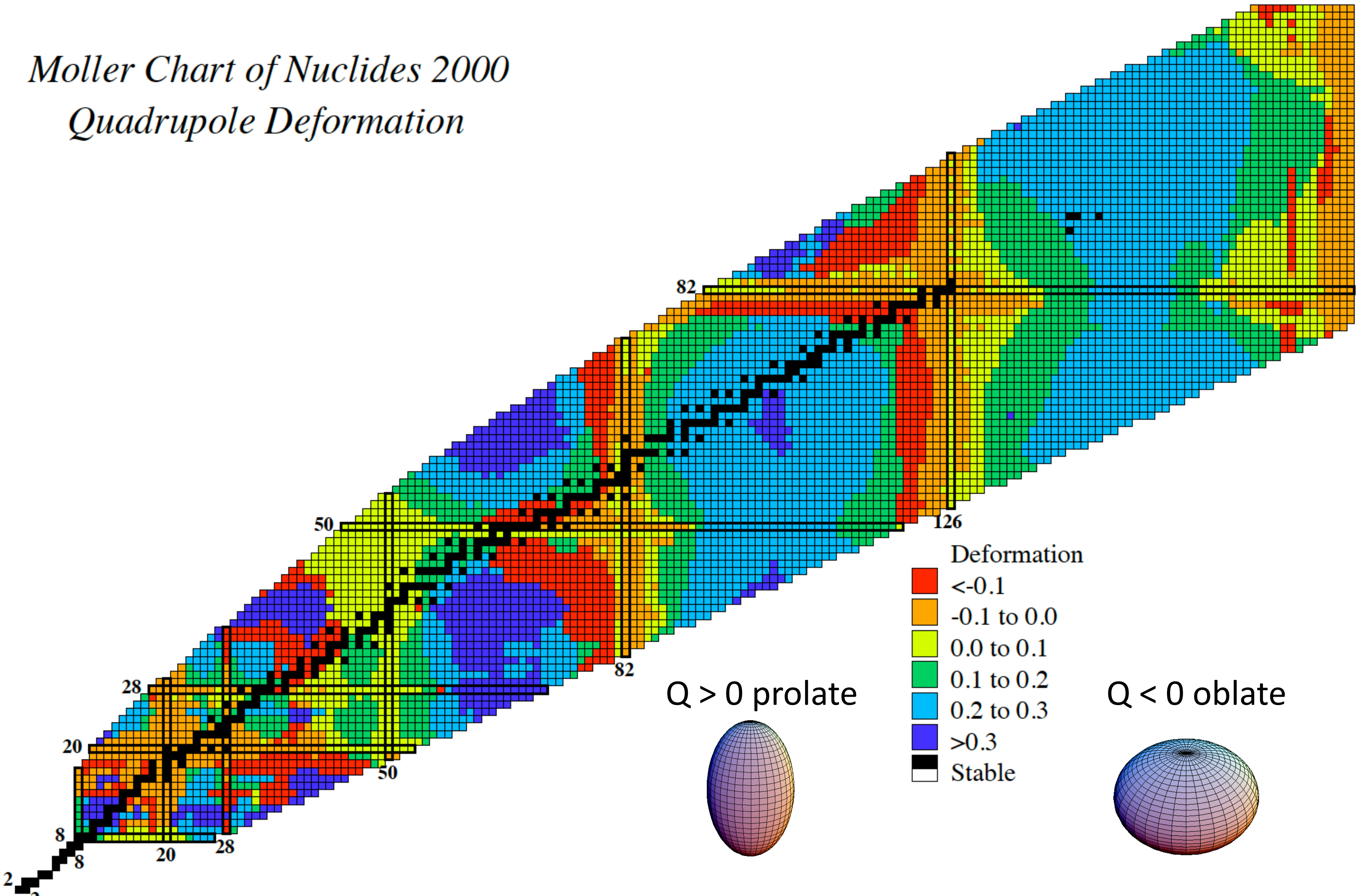


Nature volume 497, pages199–204(2013),

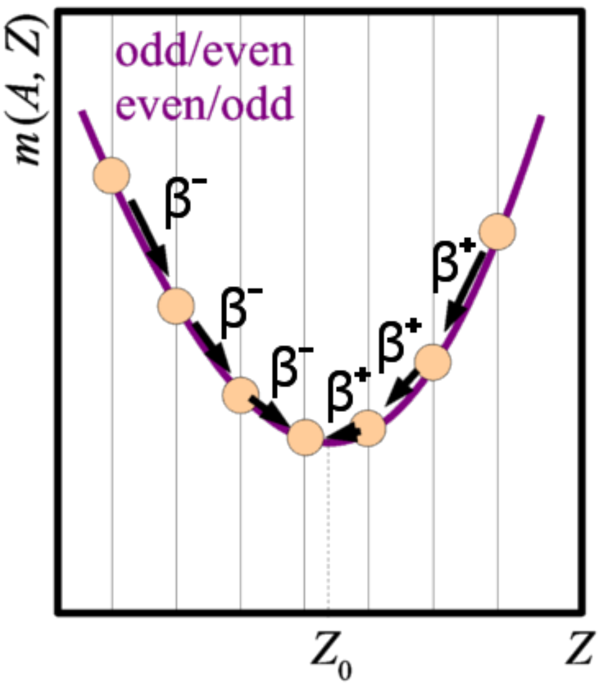
Review: https://royalsocietypublishing.org/doi/epdf/10.1098/rspa.2020.0202

QUADRUPOLE MOMENTS

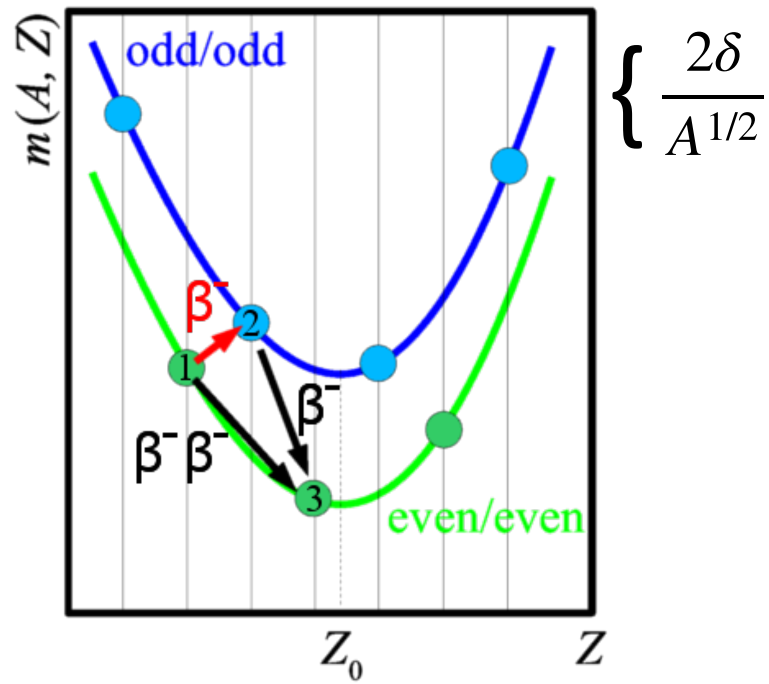
Moller Chart of Nuclides 2000
Quadrupole Deformation



BETA DECAY



even-odd/odd-even nuclei
in general 1 stable isobar

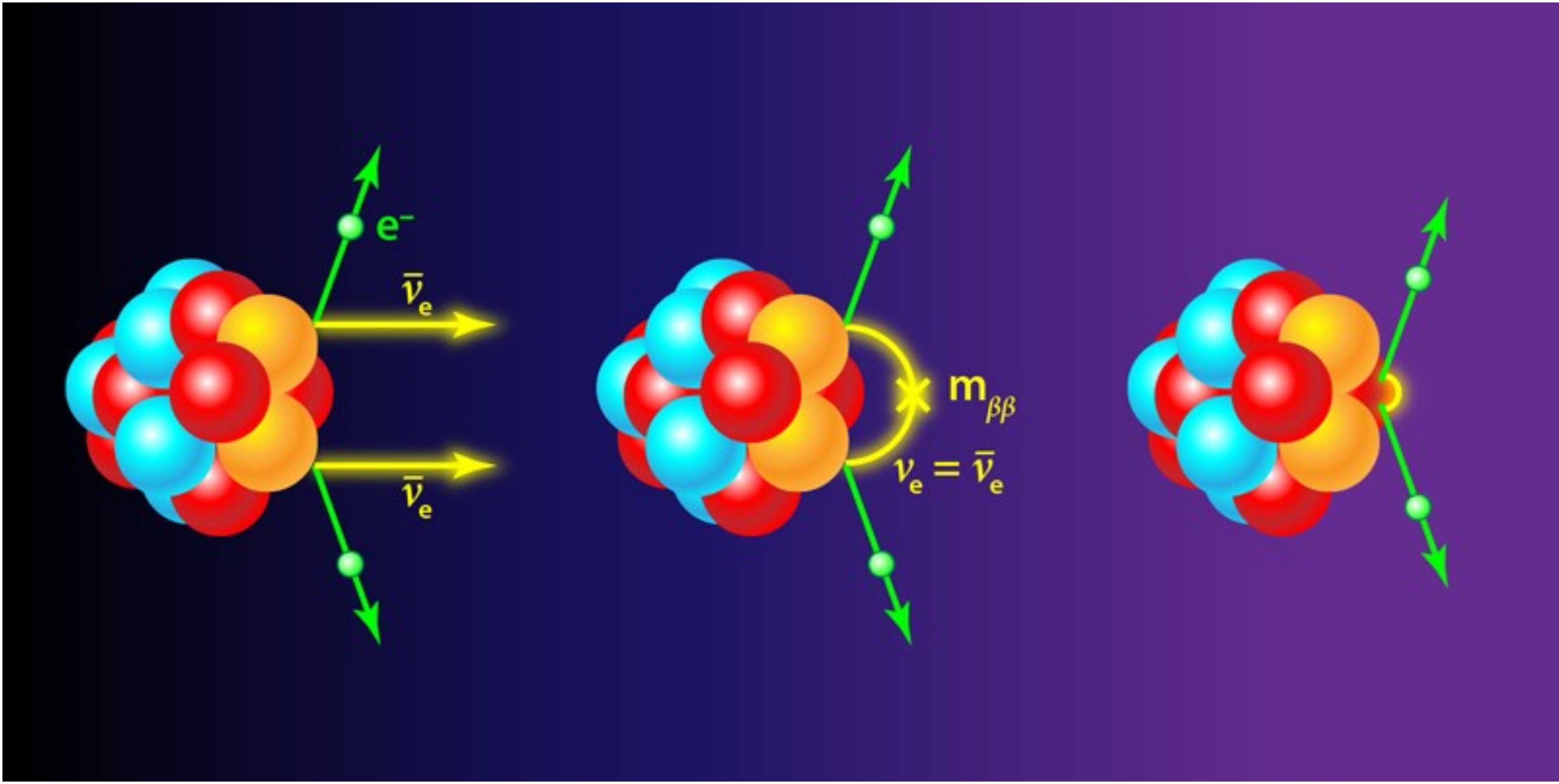


Z>7, only 1 stable odd-odd nucleus

even-even nuclei:
at least 2 stable isobars

double-beta-decay:
possible, strongly suppressed

DOUBLE BETA DECAY



Source: APS

NEUTRINO MIXING

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Pontecorvo, Maki, Nakagawa, Sakata (PMNS) matrix

$$\Gamma^{0\nu\beta\beta} = 1/T_{1/2}^{0\nu\beta\beta} = G^{0\nu\beta\beta}(Q,Z)\,|M^{0\nu\beta\beta}|^2 \langle m_{\beta\beta}\rangle^2 \approx \frac{1}{10^{27-28}\ \text{years}} \frac{\langle m_{\beta\beta}\rangle^2}{(0.01\text{eV})^2}$$

$$\langle m_{\beta\beta}\rangle = |\sum_{i=1}^{3} U_{ei}^2 m_i|$$

neutrino oscillations:

$$\mathrm{P}(\nu_\alpha \rightarrow \nu_\beta) = \left|\sum_{\mathrm{i}} \mathrm{U}_{\alpha \mathrm{i}}\, \mathrm{e}^{-\mathrm{i}\mathrm{E}_\mathrm{i}\mathrm{t}}\, \mathrm{U}_{\beta\mathrm{i}}^*\right|^2 = \underbrace{\sin^2(2\theta) \cdot \sin^2 \frac{|m_2^2 - m_1^2| \cdot L}{4E}}_{\text{2 flavor mixing}}$$

KINEMATICS

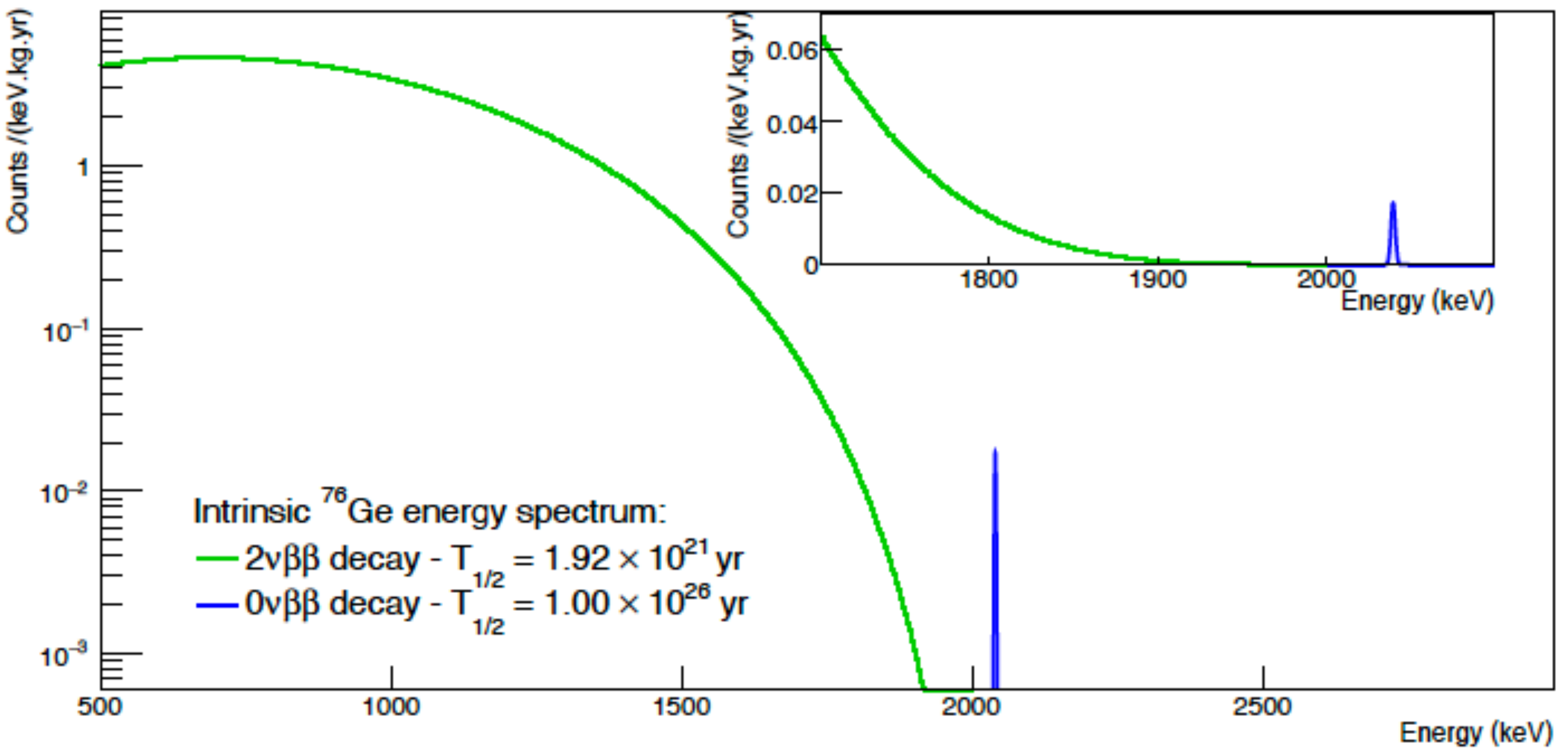
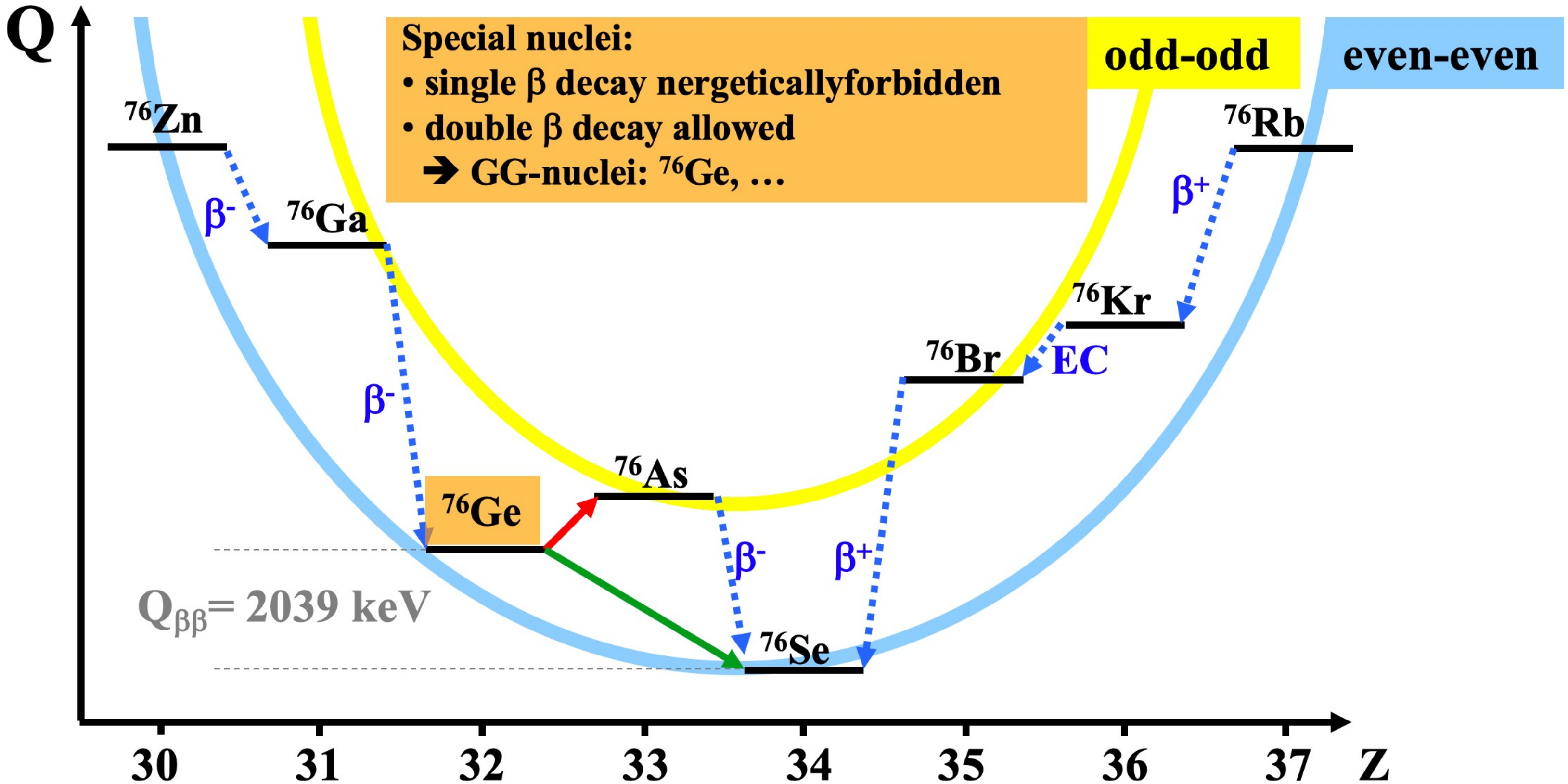


Figure 2. The expected $2\nu\beta\beta$ (green) and $0\nu\beta\beta$ (blue) for half-lifes of $1.93 \cdot 10^{21}$ yr and $1.0 \cdot 10^{25}$ yr, respectively.

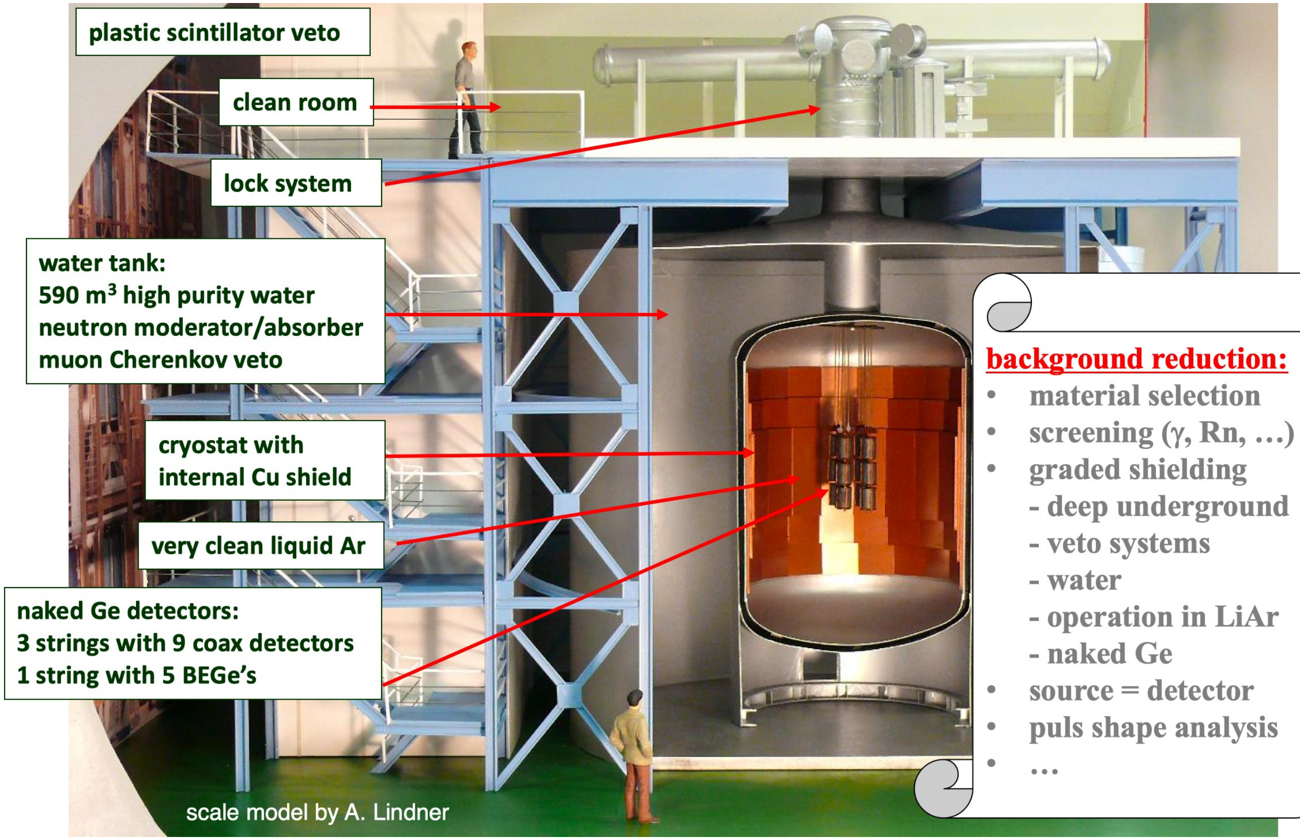
B DECAY



Q
Special nuclei:
• single β decay nergeticallyforbidden
• double β decay allowed
➔ GG-nuclei: 76Ge, …
odd-odd
even-even
76Zn
76Ga
76Ge
76As
76Se
76Br
76Kr
76Rb
β-
β-
β-
β+
β+
EC
Qββ= 2039 keV
30
31
32
33
34
35
36
37
Z

GERDA EXPERIMENT

Final result: GERDA @ Gran Sasso
under construction: Upgrade to LEGEND200



→ **127.2 kg x year exposure**

→ **achieved background < 5.2x10^{-4} cts/(keV*kg*yr)**

→ **record sensitivity: $T^{0\nu}_{1/2}$ > 1 . 8 x 10^{26}yr**

limit on Majorana mass:

m_ν < (0.08-0.180) eV

(depending on NME)

Phys. Rev. Lett. 125 (2020) 25, 252502 https://arxiv.org/abs/2009.06079

GERDA SPECTRUM

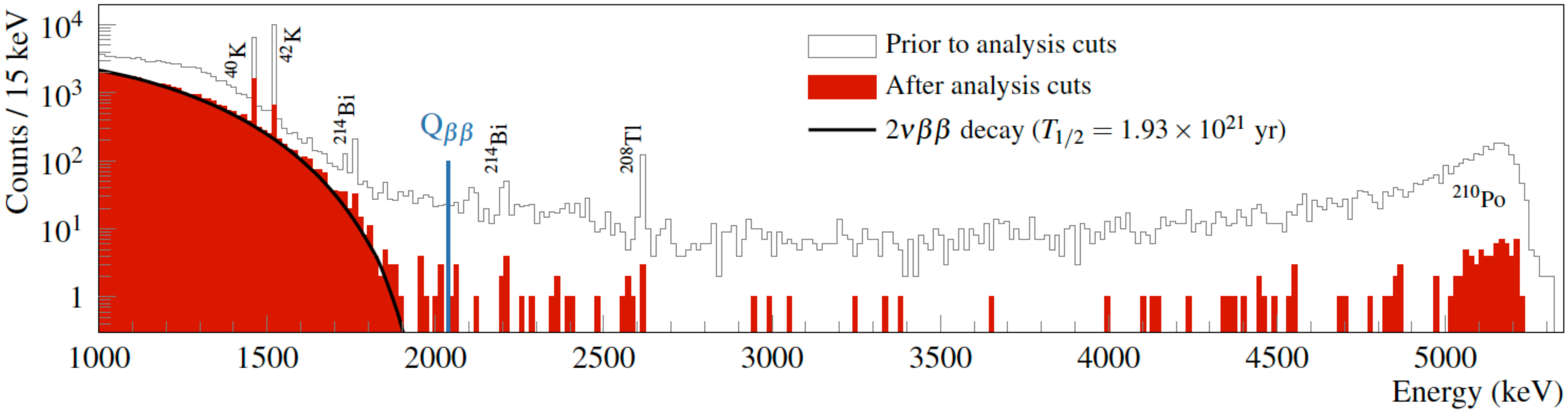
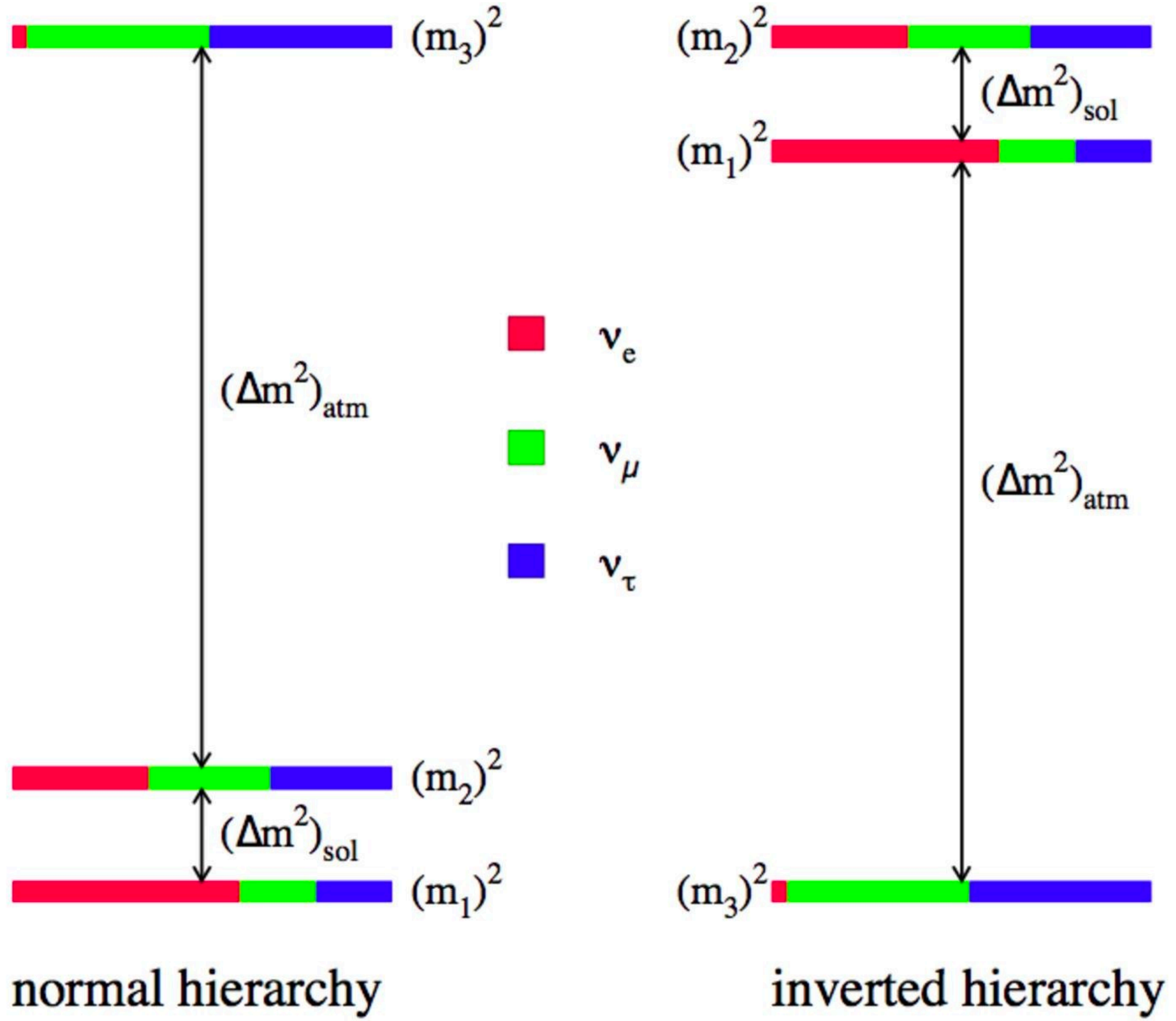


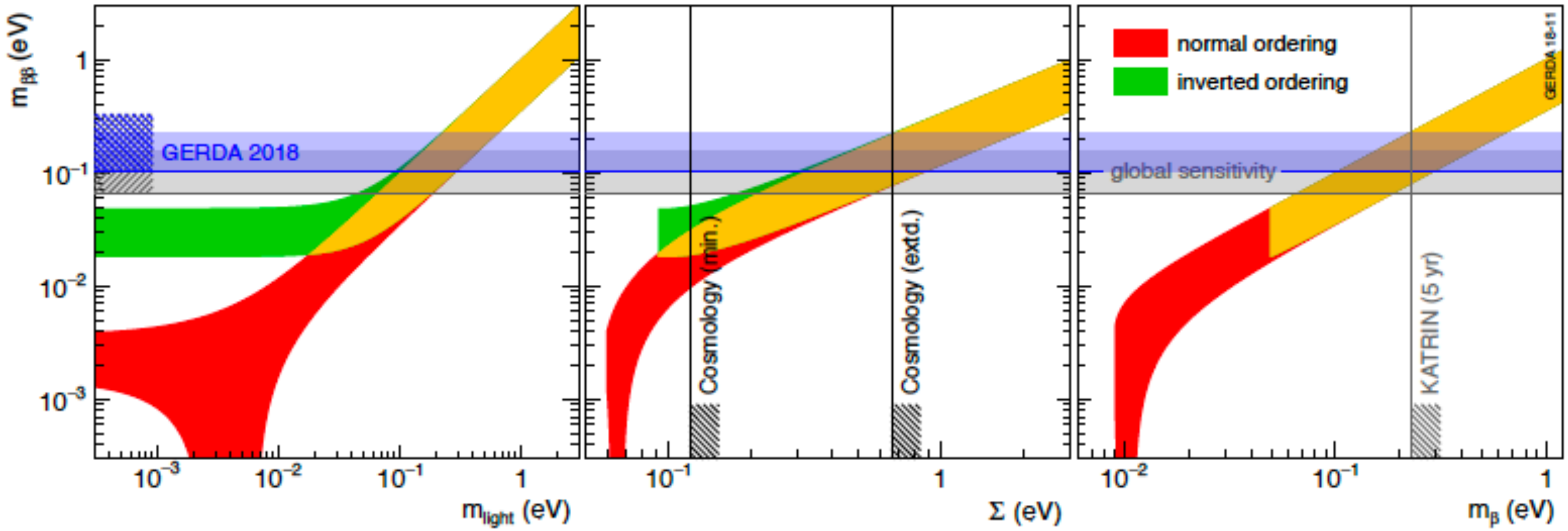
FIG. 1. Energy distribution of GERDA Phase II events (exposure of 103.7 kg yr) before and after analysis cuts. The expected distribution of $2\nu\beta\beta$ decay events is shown assuming the half-life measured by GERDA [31]. The prominent γ lines and the α population around 5.3 MeV are also labeled.

NEUTRINO MASS HIERARCHY

Source: Wikipedia

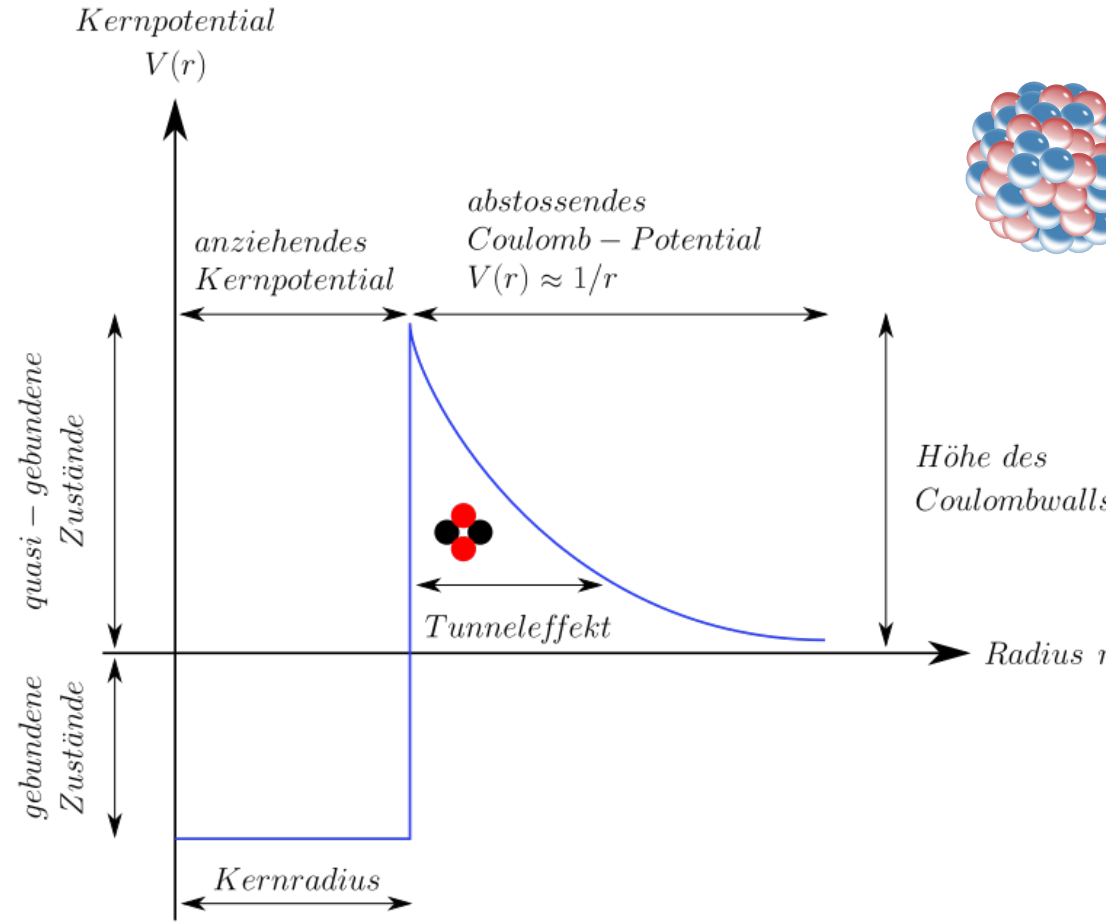


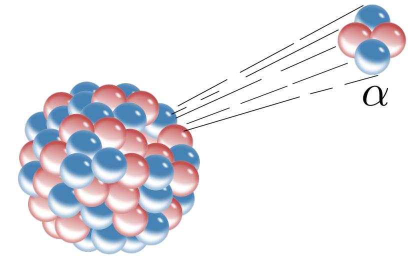
NEUTRINO MASS LIMITS



GERDA:https://arxiv.org/abs/1909.02726

ALPHA DECAY





Transmission Probability

$T = e^{-2G}$ Gamov factor G

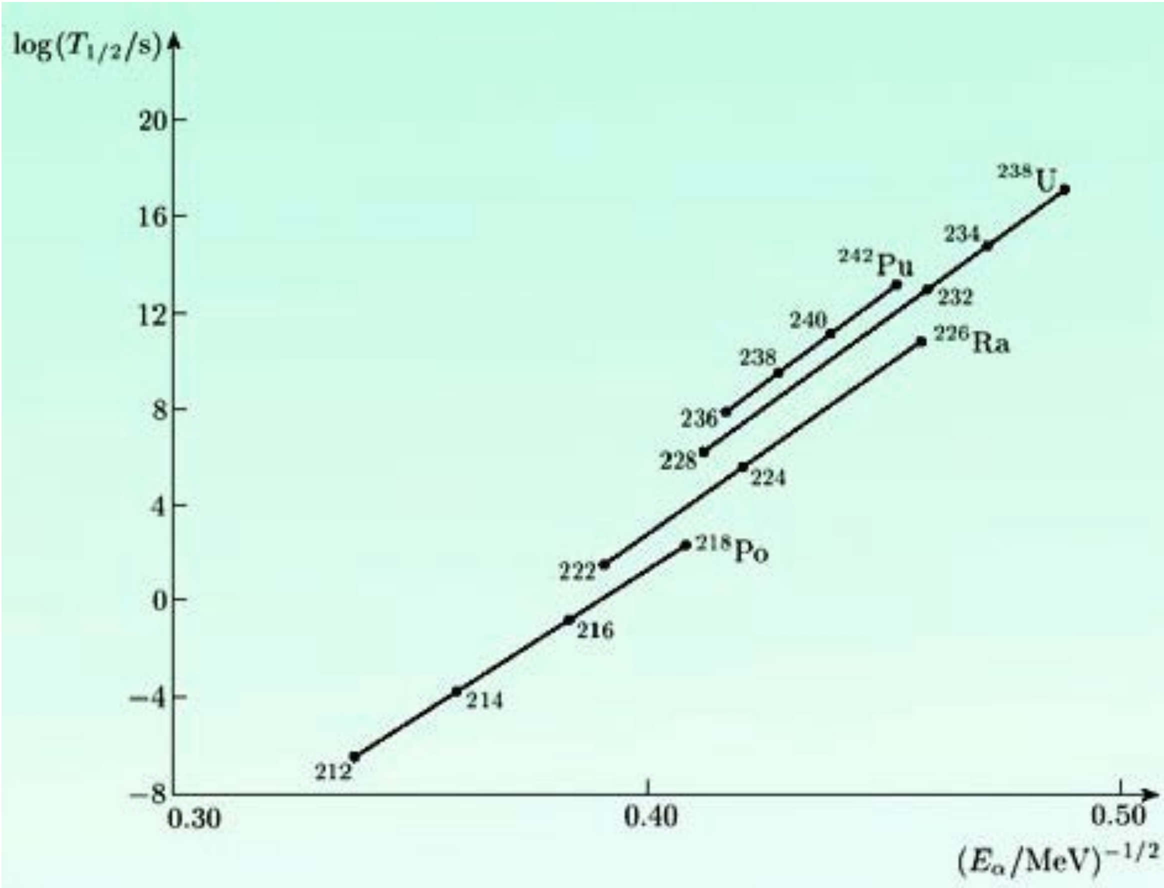
$$G = \int_R^{r_2} \sqrt{2m\,|E_\alpha - V(r)|}\,dr \sim \frac{1}{\sqrt{E_\alpha}}$$

$$1/\tau = \Gamma = W(\alpha)\nu e^{-2G} \sim e^{-2/\sqrt{E_\alpha}}$$

$W(\alpha)$: Probability to form α particle

$\nu = \dfrac{v}{2R}$: Assault rate

ALPHA DECAY AND CLUSTER RADIOACTIVITY



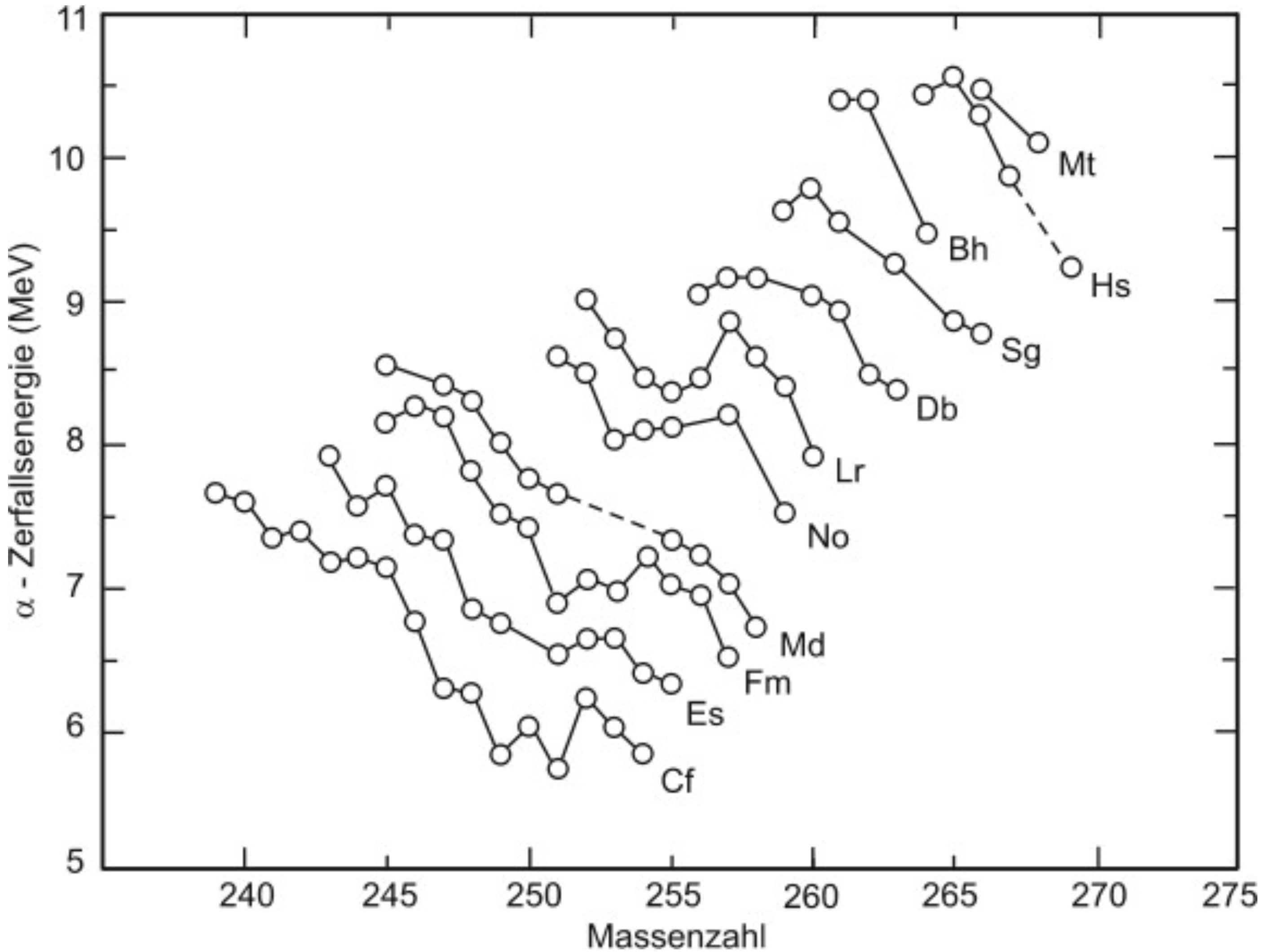
lifetimes range
from µs
up to 10^{19} years (^{209}Bi)

cluster radioactivity: $^{223}_{88}\text{Ra} \rightarrow ^{14}_{6}\text{C} + ^{209}_{82}\text{Pb}$

now: also, e.g., $^{20}_{8}\text{O}$, $^{28}_{12}\text{Mg}$, up to $^{34}_{14}\text{Si}$, suppressed by 10^9 to 10^{16}

prediction: Sandulescu A., Greiner W., Sov. J. Part. Nucl. **11**: 528–541 (1980)., discovery: Rose, H. J.; Jones, G. A., Nature 307 (5948): 245 (1984).

ALPHA ENERGIES



α-energy is fingerprint of mother nucleus

FISSION

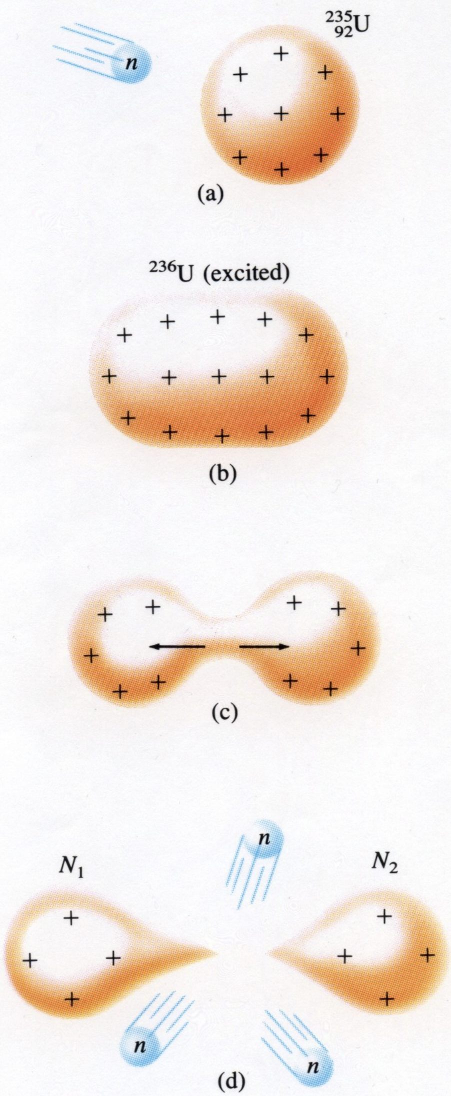
A —> A/2 + A/2

gain ~ 1 MeV/nucleon

α-decay dominant

Coulomb barrier

FIGURE 31–2 Fission of a $^{235}_{92}U$ nucleus after capture of a neutron.



GIANCOLI'S PHYSICS:
PRINCIPLES WITH APPLICATIONS, 3/E

NUCLEAR FISSION

1938: discovered by Otto Hahn, Lise Meitner and Friedrich Strassmann

bombarding ${}^{235}_{92}$U with neutrons, looking for heavier elements

chemical analysis $\rightarrow {}^{138}_{56}$ Ba

$${}^{235}_{92}\text{U} \rightarrow {}^{236}_{92}\text{U}^* \rightarrow {}^{96}_{36}\text{Kr} + {}^{138}_{56}\text{Ba} + 2n + \text{few } \gamma$$

$$\Delta E_B = m({}^{235}_{92}\text{U}) + m_n - m({}^{236}_{92}\text{U}) = 6.4 \text{ MeV} > E_{\text{fission}} \approx 5.8 \text{ MeV}$$

Energy release: 200 - 250 MeV

83% kinetic energy carried by the fission fragments

2.5% kinetic energy of neutrons

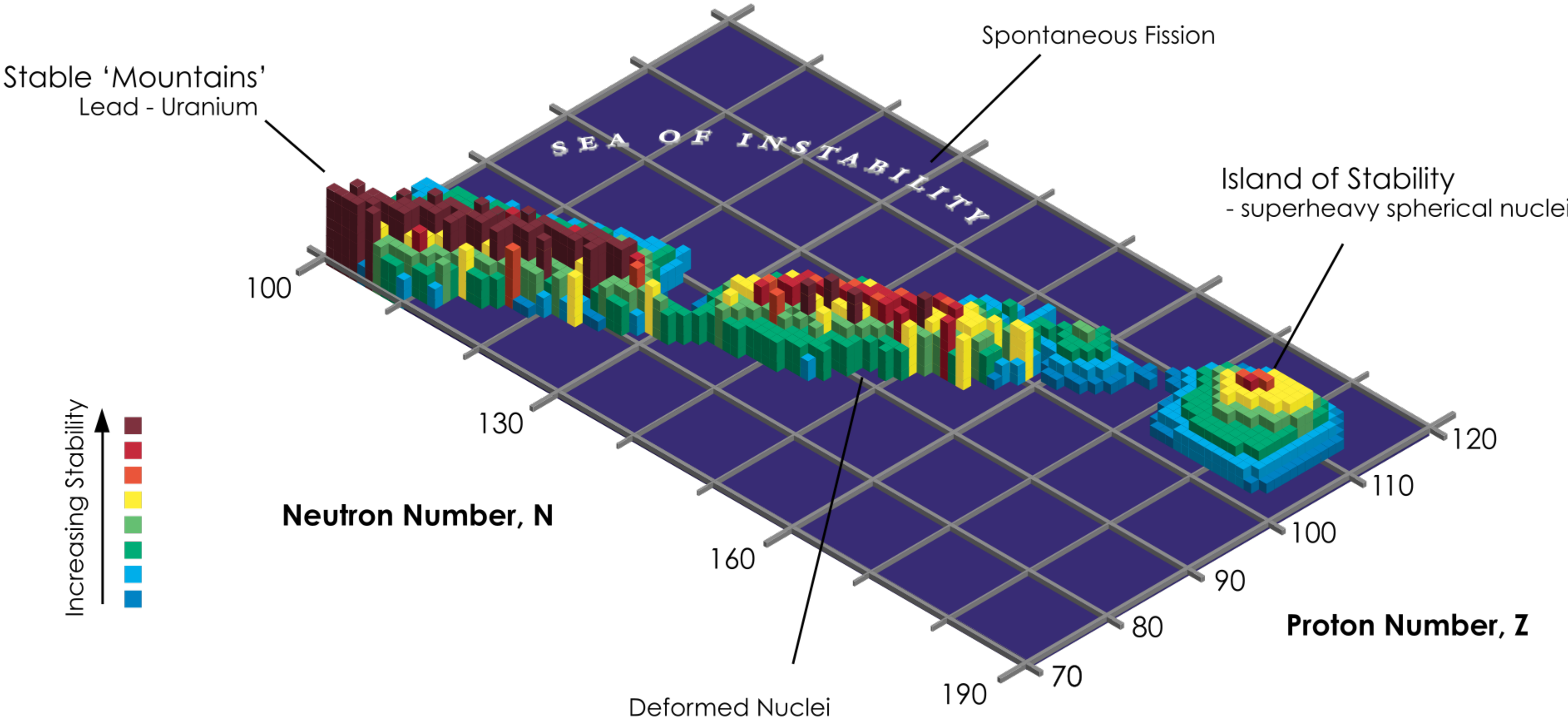
3.5% prompt γ radiation

11% excitation energy

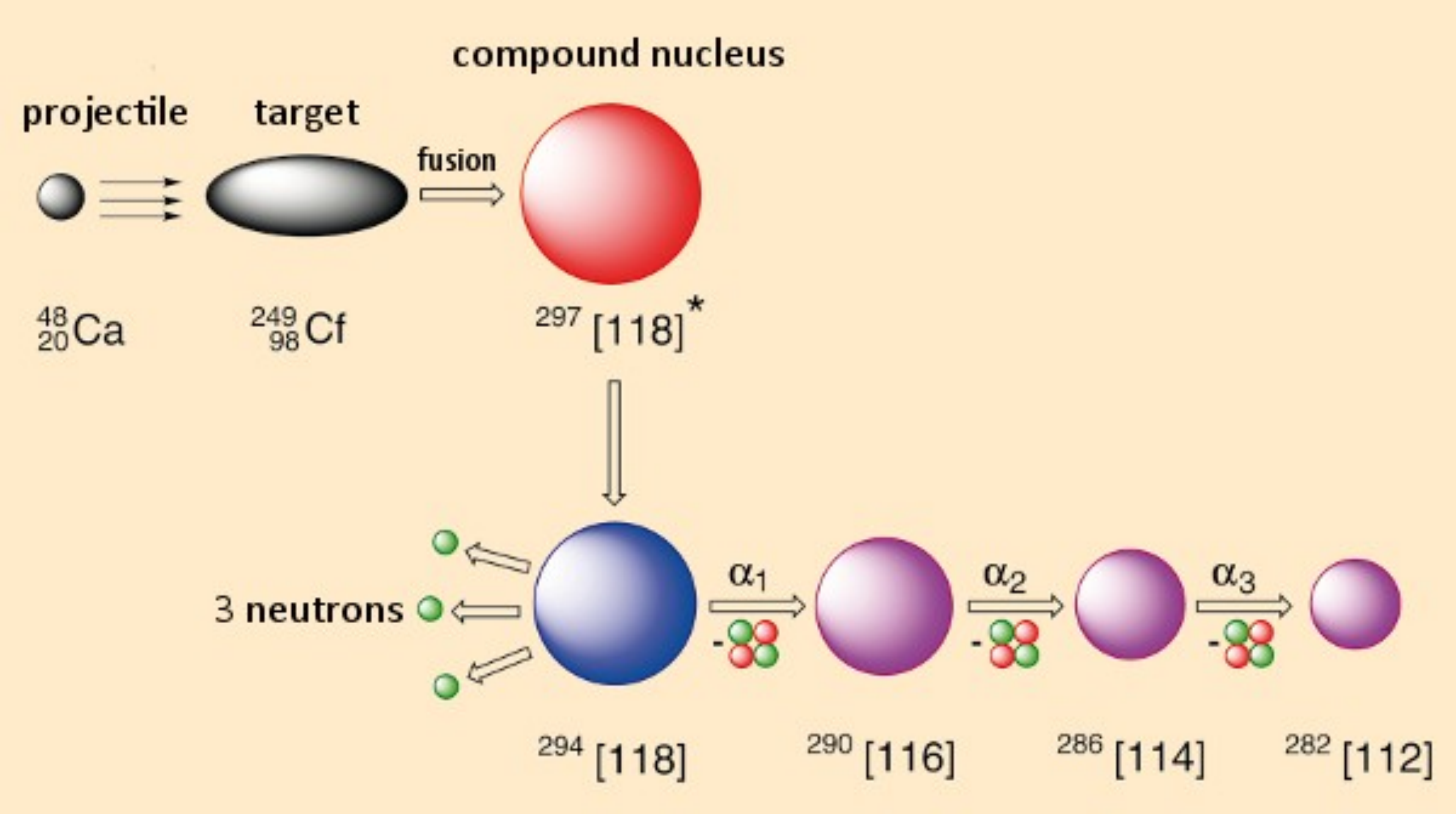
delayed neutrons *(after β-decay)

β- and γ-decays

SYNTHESIS OF THE HEAVIEST ELEMENTS



SYNTHESIS OF ELEMENT Z = 118



source:ChemistryNews.org

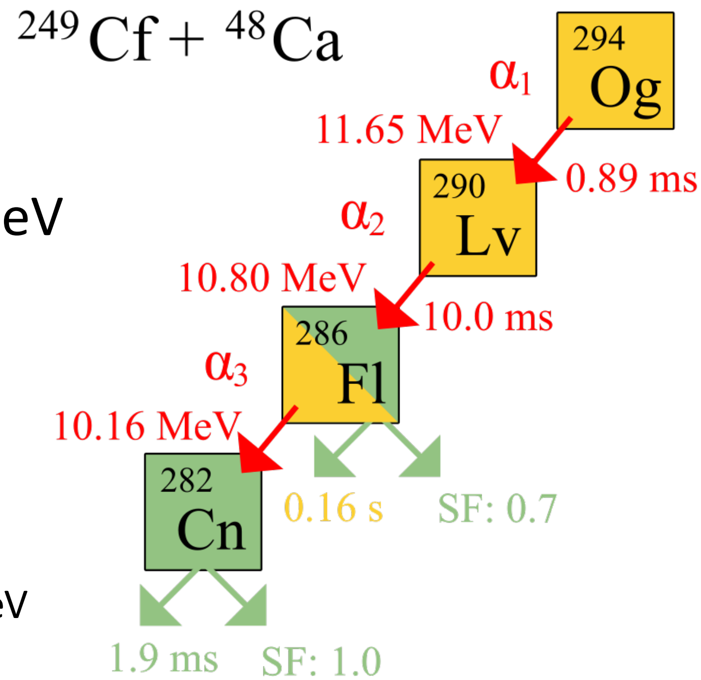
DISCOVERY OF ELEMENT Z = 118

Coulomb barrier $V_C \approx \frac{Z_P Z_T}{A_P^{1/3} + A_T^{1/3}} = 196$ MeV

Q-value:

$\Delta(^{48}\mathrm{Ca}) + \Delta(^{249}\mathrm{Cf}) = \Delta(^{294}\mathrm{Og}) + Q$

$Q = -44.2\mathrm{MeV} + 249.1\mathrm{MeV} - 196.5\mathrm{MeV} = -171\mathrm{MeV}$



Expect about 30 MeV excitation energy of the compound nucleus ^{297}Og
about 3 neutrons must be evaporated

then alpha decays until fission

production cross section: ~0.5 pb

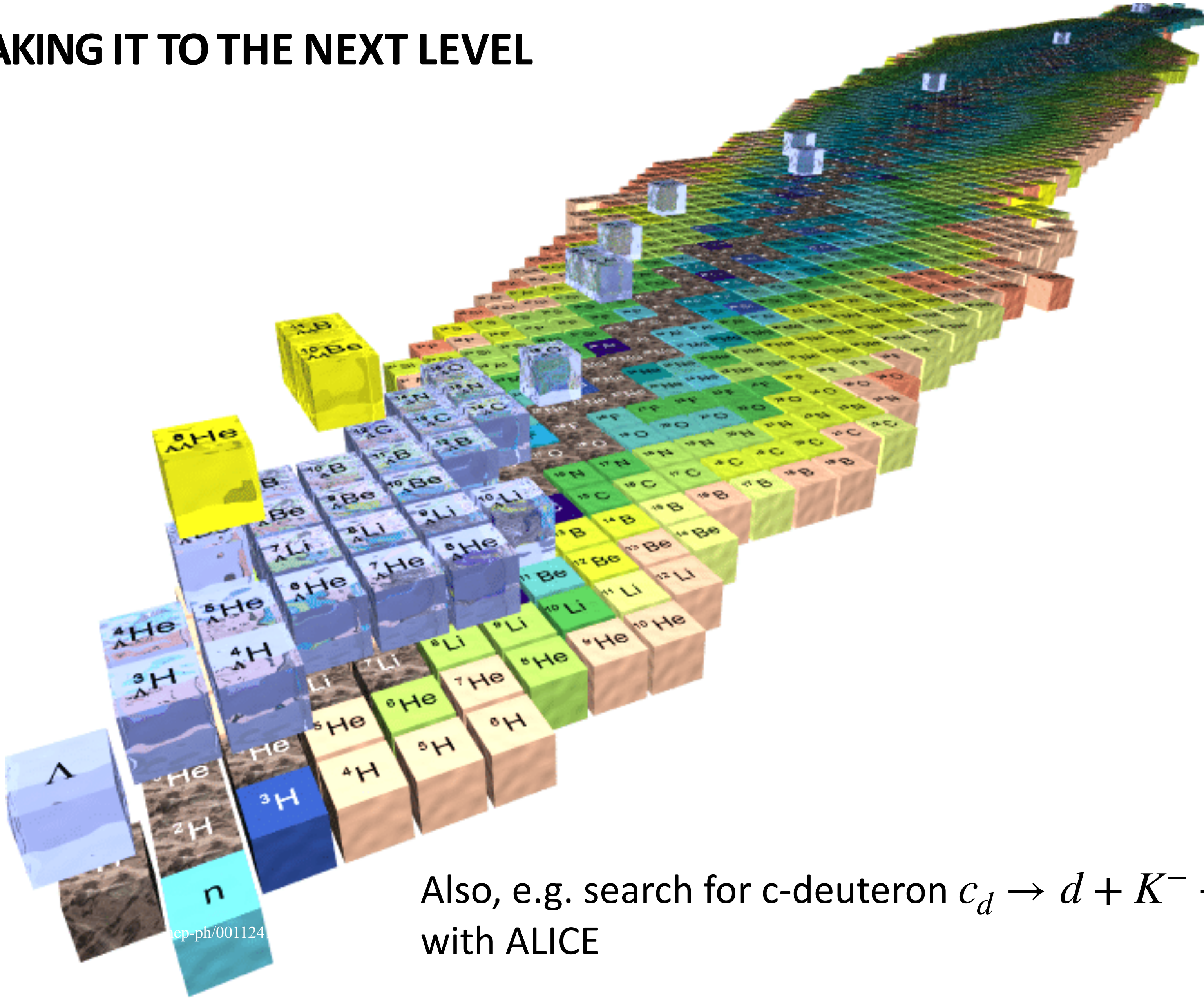
Yu. Oganessian et al., Phys. Rev. C 74 (2006) 044602.

PERIODIC TABLE

Group → Period ↓	1	2	3		4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1 H																		2 He
2	3 Li	4 Be												5 B	6 C	7 N	8 O	9 F	10 Ne
3	11 Na	12 Mg												13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
4	19 K	20 Ca	21 Sc		22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
5	37 Rb	38 Sr	39 Y		40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
6	55 Cs	56 Ba	57 La	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
7	87 Fr	88 Ra	89 Ac	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Nh	114 Fl	115 Mc	116 Lv	117 Ts	118 Og
				*	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu	
				**	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr	

Z>94: Berkeley, Darmstadt, Dubna, Wako, Livermore, Oak Ridge

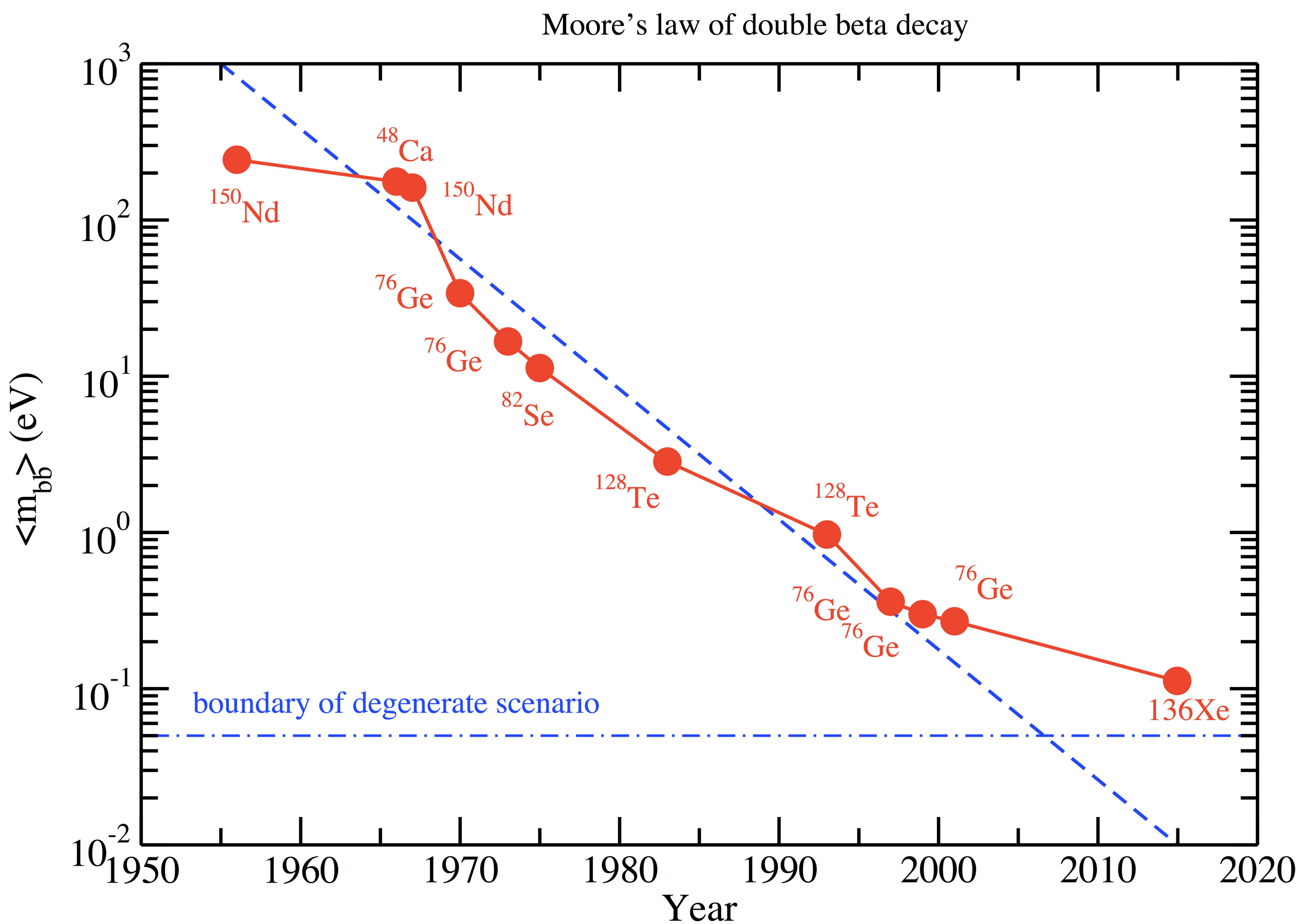
TAKING IT TO THE NEXT LEVEL



Also, e.g. search for c-deuteron $c_d \rightarrow d + K^- + \pi^+$ with ALICE

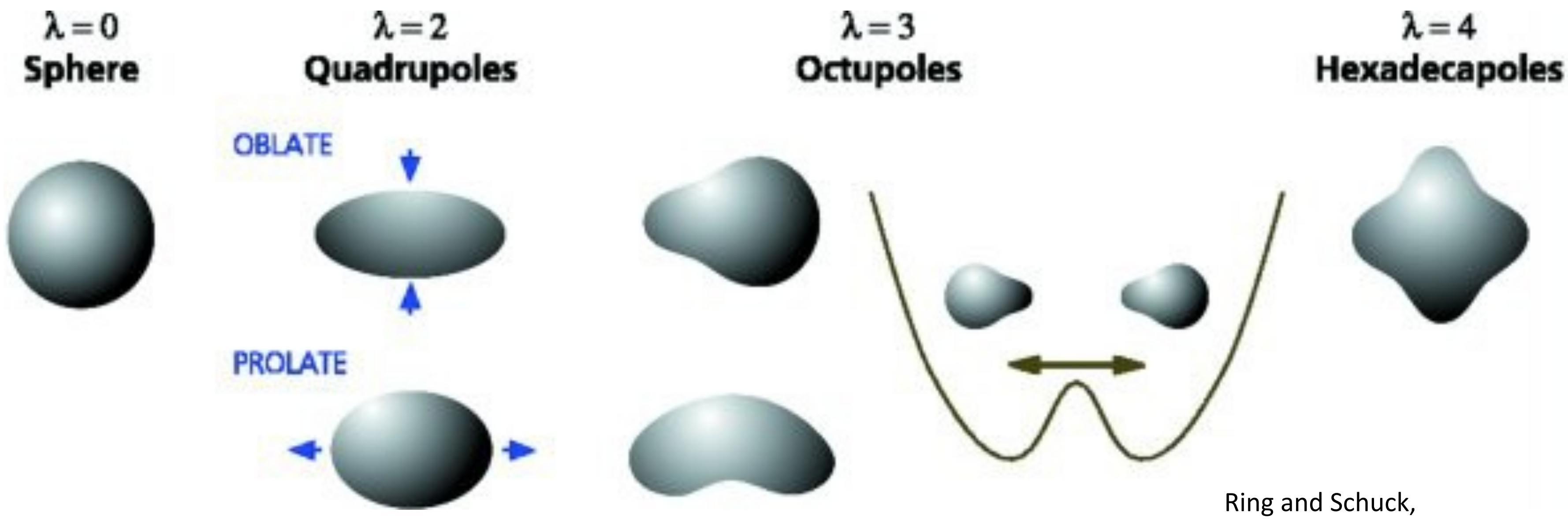
BACKUP

DOUBLE BETA DECAY

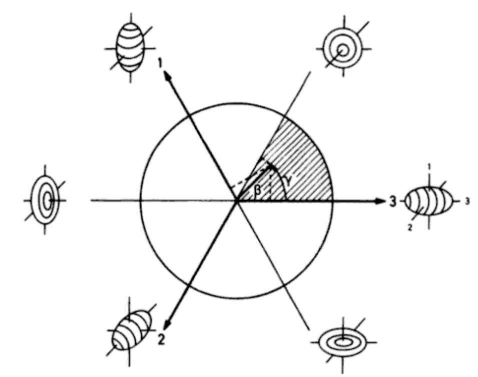


Source: P. Vogl, U Mass, Amherst (2017)

NUCLEAR DEFORMATION



Ring and Schuck,
The nuclear many body problem



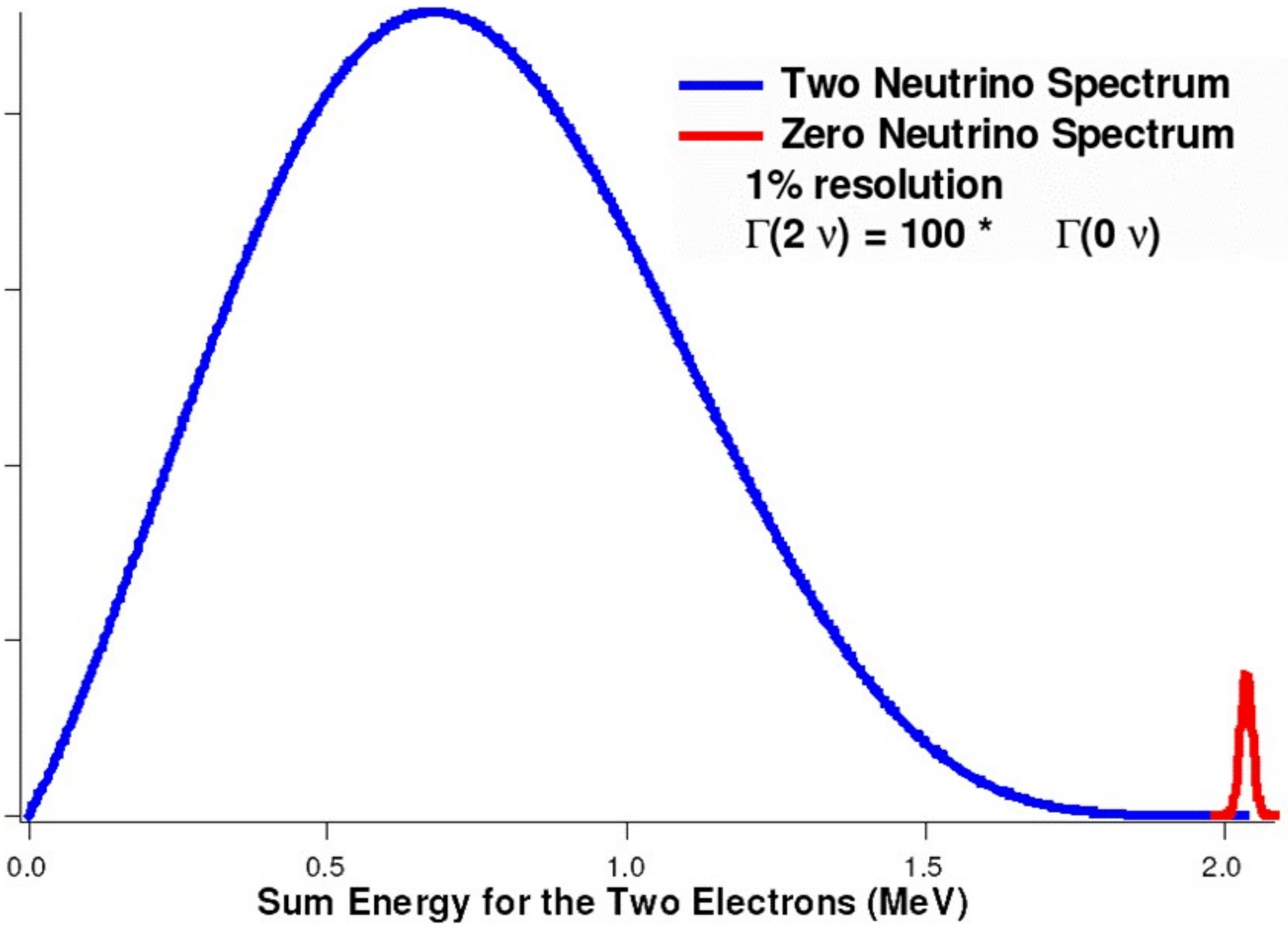
$$R(\theta,\phi) = c(\alpha)R_0\left(1 + \sum_{\lambda=2}\sum_{\mu=-\lambda}^{\lambda}\alpha_{\lambda\mu}Y^*_{\lambda\mu}(\theta,\phi)\right)$$

Axial symmetry gives $\beta_\lambda = \alpha_{\lambda 0},\ \lambda = 2,3,4,\ldots$

$\lambda = 2$, quadrupole: $\alpha_{21} = \alpha_{2-1} = 0,\ \ \alpha_{22} = \alpha_{2-2}$

$\alpha_{20} = \beta\cos\gamma,\ \ \alpha_{20} = \frac{1}{\sqrt{2}}\beta\sin\gamma,\ \ \gamma = 0$ (prolate), $\gamma = \pi/3$ (oblate)

KINEMATICS



Source: COBRA experiment