Lecture 23: Extreme Matter: from Cold Quantum Gases to the Quark-Gluon Plasma via Black Holes

- introductory remarks: physics in the strongly coupled limit
- the counter-intuitive world of viscosity
- how does gravity get into the game?
- the world of cold quantum gases
- hot and dense matter, quark-gluon plasma, flow at RHIC and LHC

a new paradigm: strongly coupled systems

strongly coupled systems arise when the scattering cross section becomes very large

the viscosity/entropy density ratio η/s of such matter is then 'universal', i.e. independent of its detailed structure

from purely dimensional considerations

$$\frac{F}{A} = \eta \frac{v}{d} \quad \rightarrow \quad \eta$$ has dimensions of kg/(m s)

the entropy density s has dimension 1/m^3 ($k_B = 1$)

→ η/s has dimension kg m^2 s^{-1}, i.e. the dimension of $\hbar$

kinetic theory, viscosity, and the Heisenberg uncertainty relation

Estimate of viscosity from kinetic theory

$$\eta \sim \rho v \ell, \qquad s \sim n = \frac{\rho}{m}$$

$$\frac{\eta}{s} \sim m v \ell \sim \hbar \frac{\text{mean free path}}{\text{de Broglie wavelength}}$$

Quasiparticles: de Broglie wavelength $\lesssim$ mean free path

Therefore $\eta/s \gtrsim \hbar$

Danielewicz & Gyulassy 1985
Dam Son, QM2006

shear viscosity and cross section

$$\eta = 4/15 \cdot \rho v l = 4/15 \cdot m v n l = 4/15 \cdot \langle p \rangle n l \tag{1}$$

with ρ the mass density, m the particle mass, n the particle density, and l the mean free path

(S.R. de Groot et al., Relativistic Kinetic Theory, North Holland, Amsterdam 1980)

then, with $l = 1/(n\sigma)$ where σ is the scattering cross section

$$\eta = 4/15 \cdot \langle p \rangle / \sigma \tag{2}$$

note: the larger the cross section, the smaller the viscosity

also: viscosity is independent of density!
Maxwell 1865 (Boyle and Hooke 1660)

coefficient in front of $\hbar$

can be derived using AdS/CFT correspondence

Maldacena's conjecture:
some string theories in curved space time are connected to conformal gauge theories in flat (3+1) dimensional space

In the limit of very strong coupling the string theory becomes identical to a version of Einstein's theory of general relativity with negative cosmological constant

In technical terms:
4 dimensional N=4 Super Symmetric
SU(N_c) Yang-Mills theory ($N_c = \infty$) is dual to
Type II B Super String theory in an AdSxS_5

a new paradigm: strongly coupled systems

in strongly coupled systems, η/s is close to $\frac{\eta}{s} = \frac{\hbar}{4\pi}$
(Policastro, Son, Starinets, PRL 87 (2001) 081601)

in weakly coupled systems, η/s is large and diverges for a nearly ideal gas

in the dilute limit, $\frac{\eta}{s} \sim \epsilon\tau >> 1$

with ϵ : mean energy/particle
and τ : mean time between collisions

viscosity of pitch

experiment started in Queensland, Australia in 1927

only eight drops have fallen since then

determine shear viscosity of pitch to be 10^{11} times that of water
η = 2.3 10^8 Pa s

note: for QGP the shear viscosity
η = 5 10^{11} Pa s

it is not the viscosity, but the η/s ratio which determines the properties of a fluid

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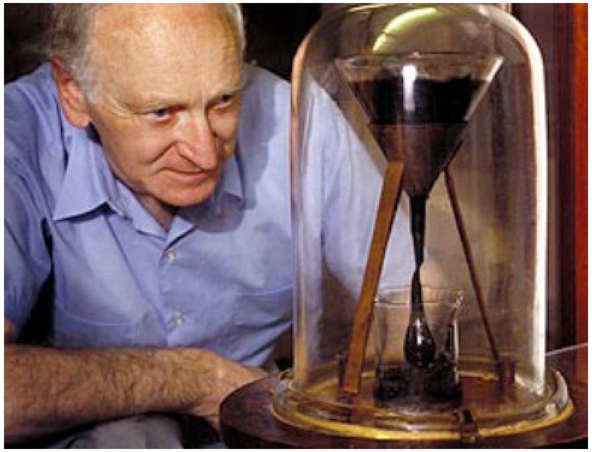


waiting for nr. 8

it is not the viscosity, but the η/s ratio which determines the properties of a fluid

File:Pitch drop experiment with John Mainstone.jpg

The University of Queensland pitch drop experiment, featuring its then-current custodian, Professor John Mainstone (taken in 1990, two years after the seventh drop and 10 years before the eighth drop fell).

Further history of the pitch drop experiment

The ninth drop touched the eighth drop on 17 April 2014. However, it was still attached to the funnel.

On 24 April 2014, Professor White decided to replace the beaker holding the previous eight drops before the ninth drop fused to them. While the bell jar was being lifted, the wooden base wobbled and the ninth drop snapped away from the funnel.

Since mid-March 2018, the live feed was interrupted due to technical problems in the experiment's webpage.

viscosity and QCD

Strong interactions described by Quantum chromodynamics (QCD)

QCD is weakly coupled at high energy (high temperature, small distances), but strongly coupled at low energy ($\lesssim 1$ GeV): “asymptotic freedom”

In weakly coupled QCD:

Dam Son
2005

$$\eta = \frac{\#}{g^4 \ln(1/g)}$$

At experimentally achievable temperatures (few hundreds MeV) $g \gtrsim 1$.
Estimate of error: $> 100\%$

No strong-coupling technique to compute η

Lattice QCD: numerically solves QCD but in imagninary time, while we need real-time Green’s functions.

Gauge/gravity duality (AdS/CFT correspondence) becomes a tool.

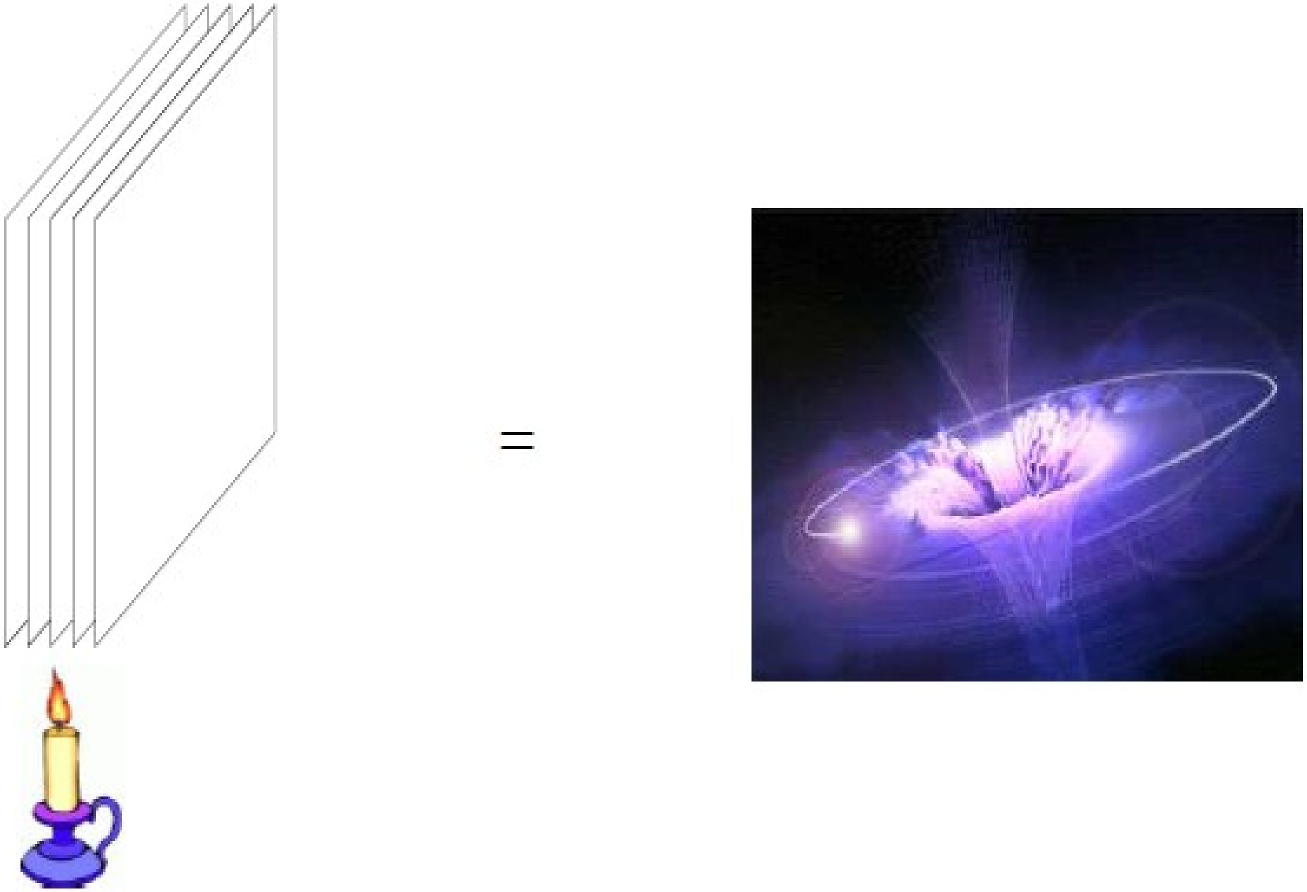
viscosity and entropy density in AdS/CFT

viscosity is obtained by computing the absorption cross section of low energy gravitons by the black hole

$$\eta = \frac{\sigma_{\rm abs}(0)}{16\pi G}$$

entropy is obtained via S = A/(4G) where A is the area of the black hole horizon and G is the gravitational constant

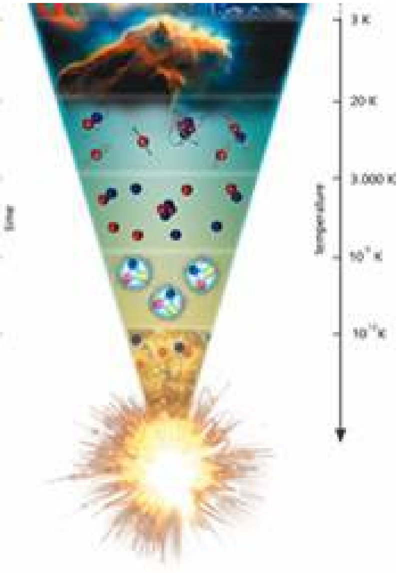
Finite temperature AdS/CFT correspondence

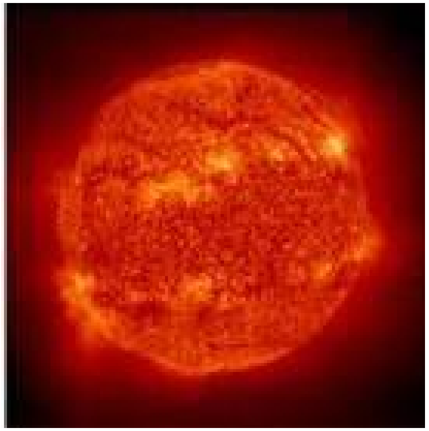


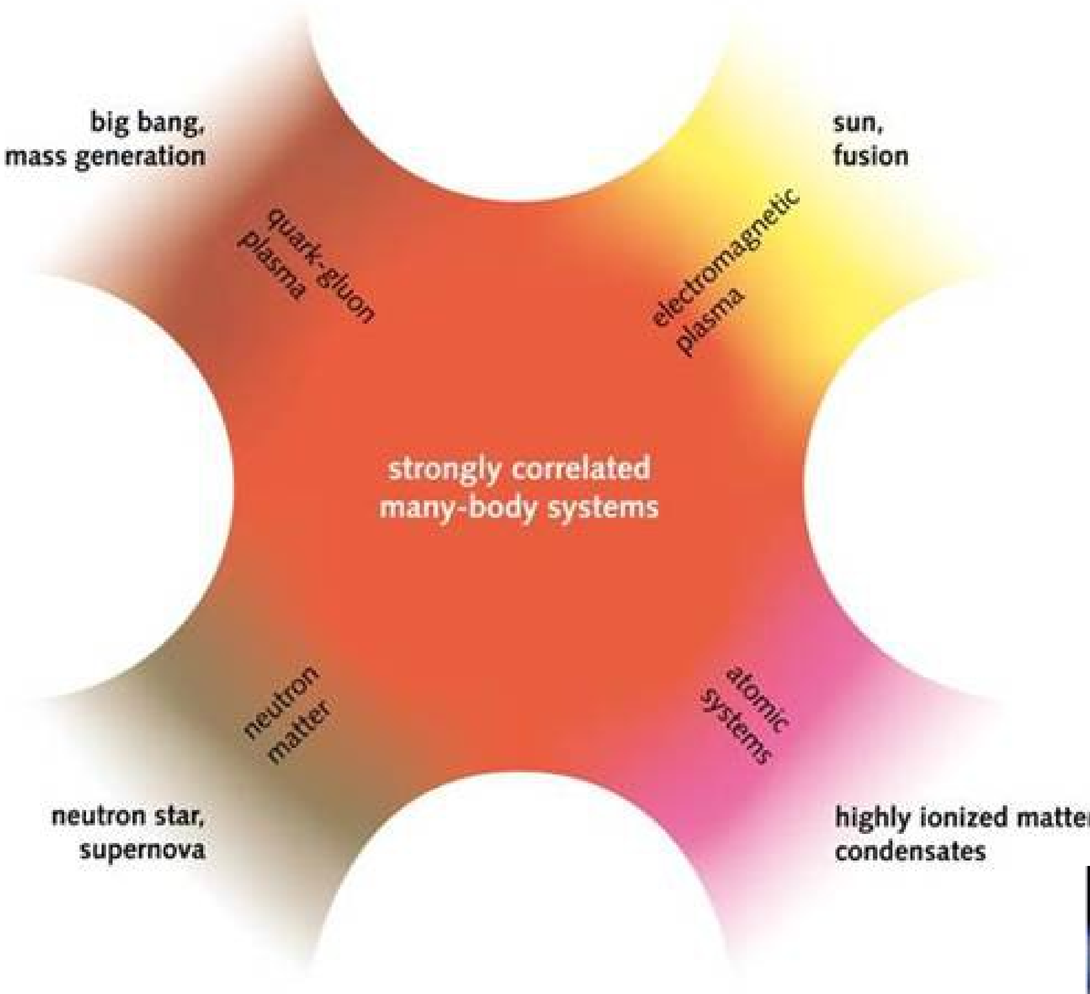
Thermal gauge theory = black hole in anti de-Sitter space

Dam Son, QM2006

strongly coupled systems in nuclear physics and related areas







big bang,
mass generation

quark-gluon
plasma

sun,
fusion

electromagnetic
plasma

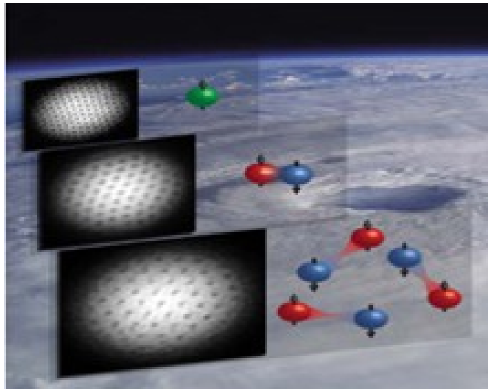
strongly correlated
many-body systems

neutron
matter

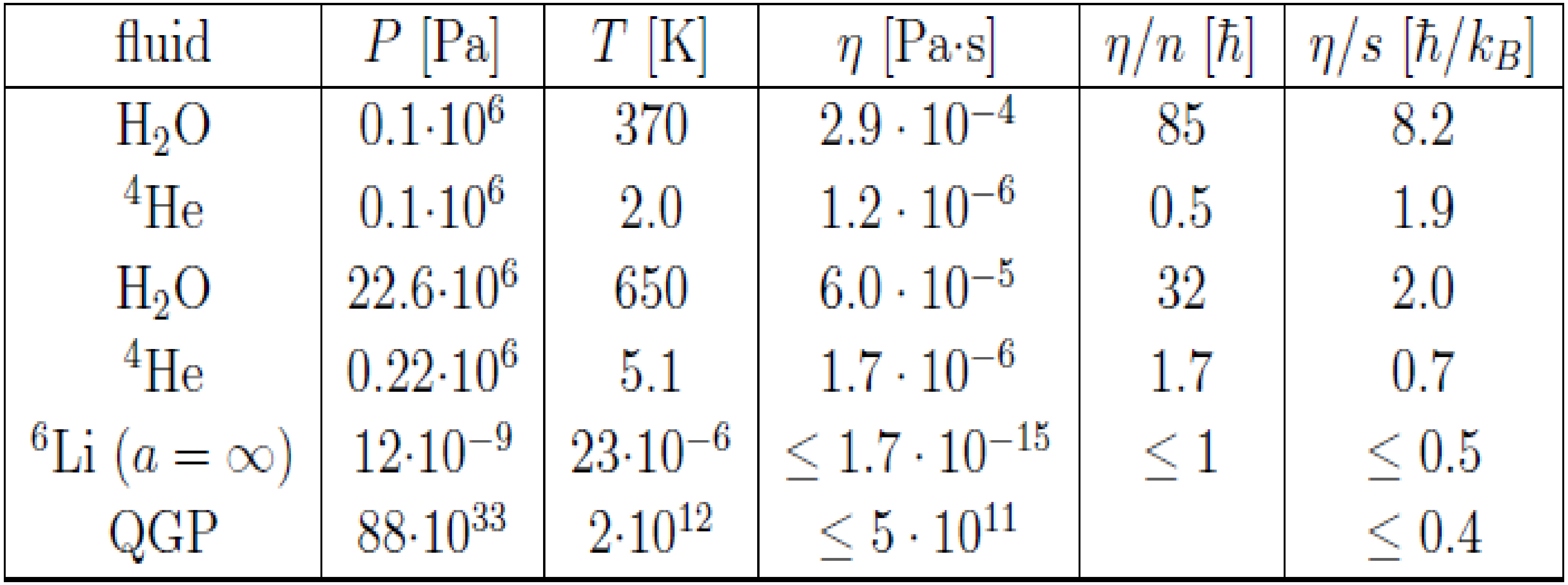
atomic
systems

neutron star,
supernova

highly ionized matter,
condensates



A note on strongly coupled systems



fluid	P [Pa]	T [K]	η [Pa·s]	η/n [$\hbar$]	η/s [$\hbar/k_B$]
H_2O	$0.1 \cdot 10^6$	370	$2.9 \cdot 10^{-4}$	85	8.2
^{4}He	$0.1 \cdot 10^6$	2.0	$1.2 \cdot 10^{-6}$	0.5	1.9
H_2O	$22.6 \cdot 10^6$	650	$6.0 \cdot 10^{-5}$	32	2.0
^{4}He	$0.22 \cdot 10^6$	5.1	$1.7 \cdot 10^{-6}$	1.7	0.7
^{6}Li ($a = \infty$)	$12 \cdot 10^{-9}$	$23 \cdot 10^{-6}$	$\leq 1.7 \cdot 10^{-15}$	≤ 1	≤ 0.5
QGP	$88 \cdot 10^{33}$	$2 \cdot 10^{12}$	$\leq 5 \cdot 10^{11}$		≤ 0.4

Table from: Thomas Schäfer (North Carolina State U.), Derek Teaney (SUNY, Stony Brook & RIKEN BNL). Apr 2009. 69 pp.

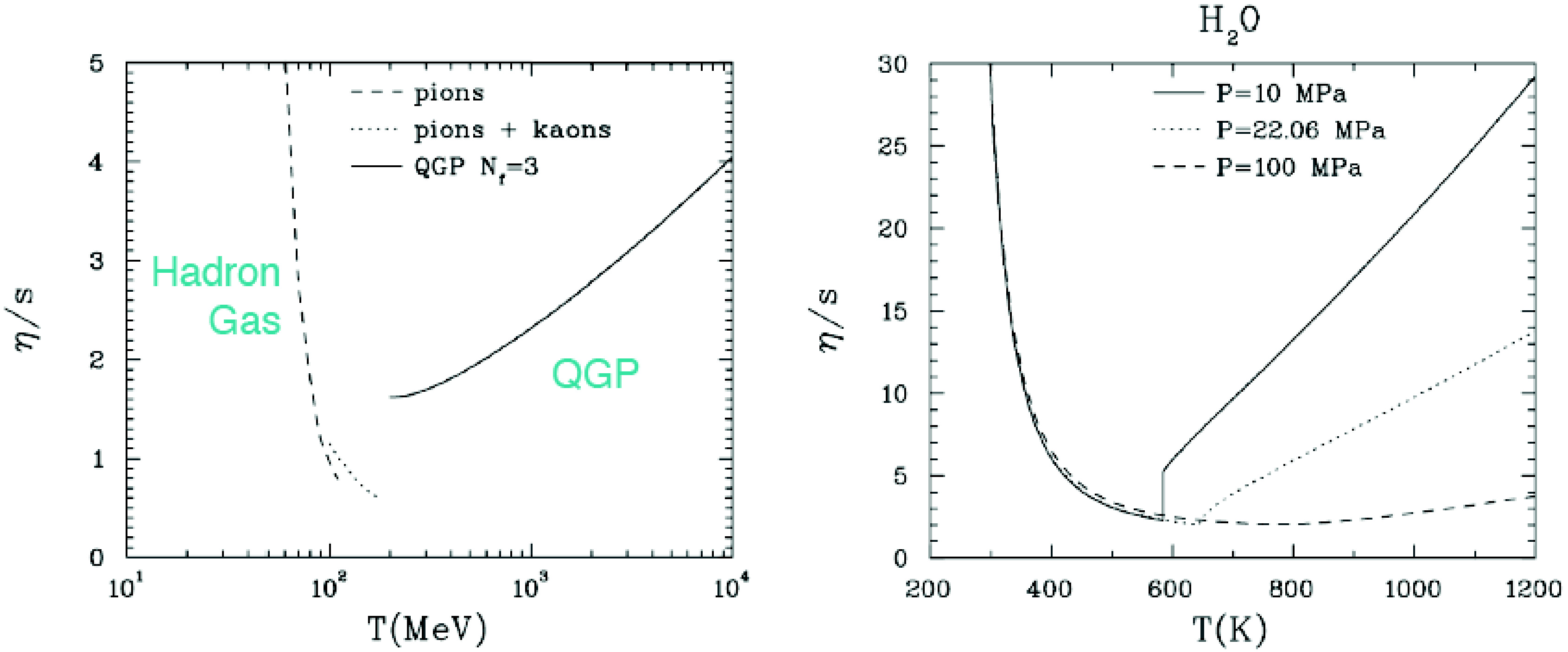
Published in Rept.Prog.Phys. 72 (2009) 126001

Parameters of a number of fluids

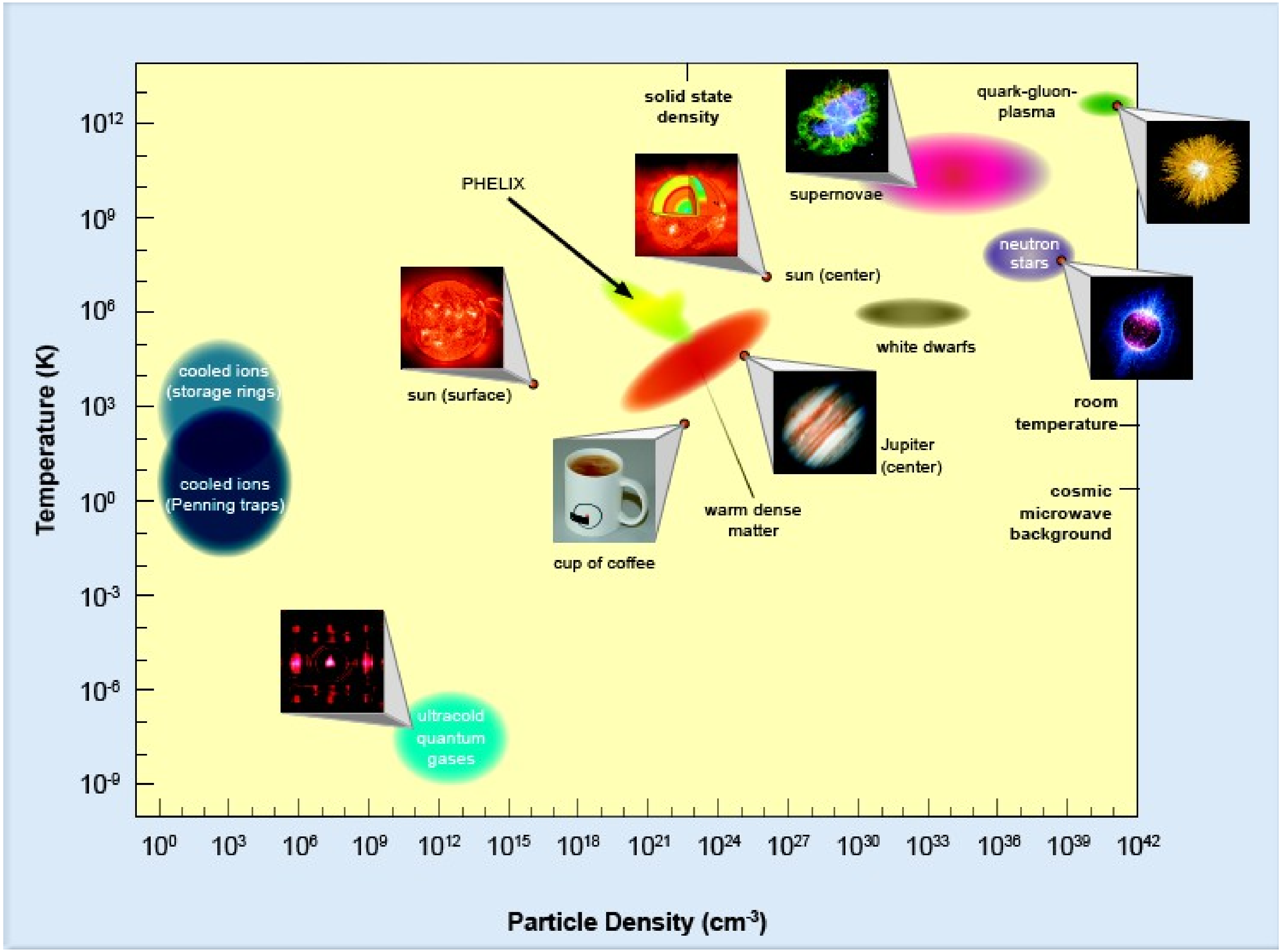
'ideal' fluids are characterized by minimal values for the ratio of
shear viscosity/entropy density
they behave similarly even if other parameters are widely different

Comparing water with QGP

Csernai, Kapusta, McLerran, nucl-th/0604032



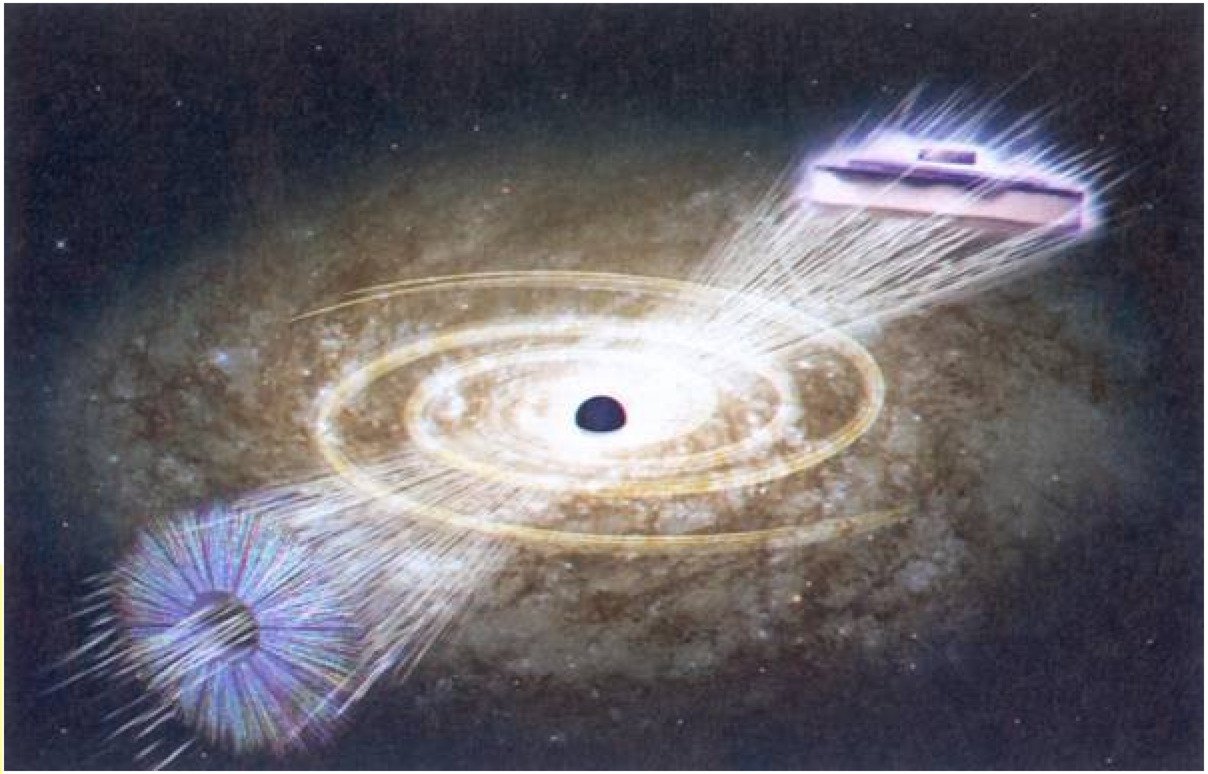
from ultra-cold to ultra-hot



Black holes, strings, QGP and high-T_c superconductors

Nature 448 Aug. 29 (2007) 1001

Nernst effect in 2-d cuprates high T_c super-conductivity



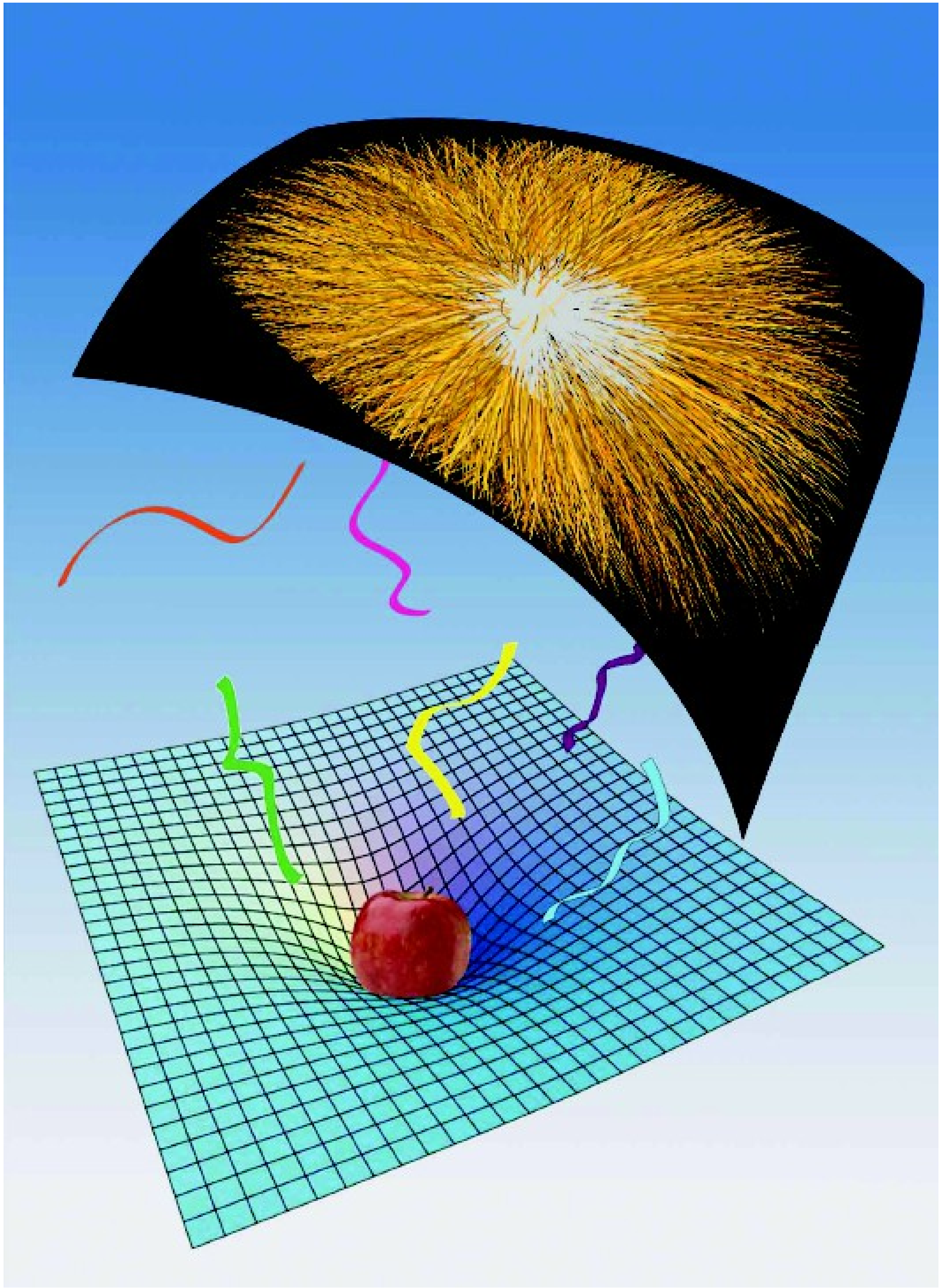
viscosity in the quark-gluon plasma (QGP)

connections between QGP and other areas of physics

QGP and the 'gauge-gravity' dual

See, e.g. E. Witten, 'Quantum Mechanics of Black Holes, Science 337 (3 August 2012)

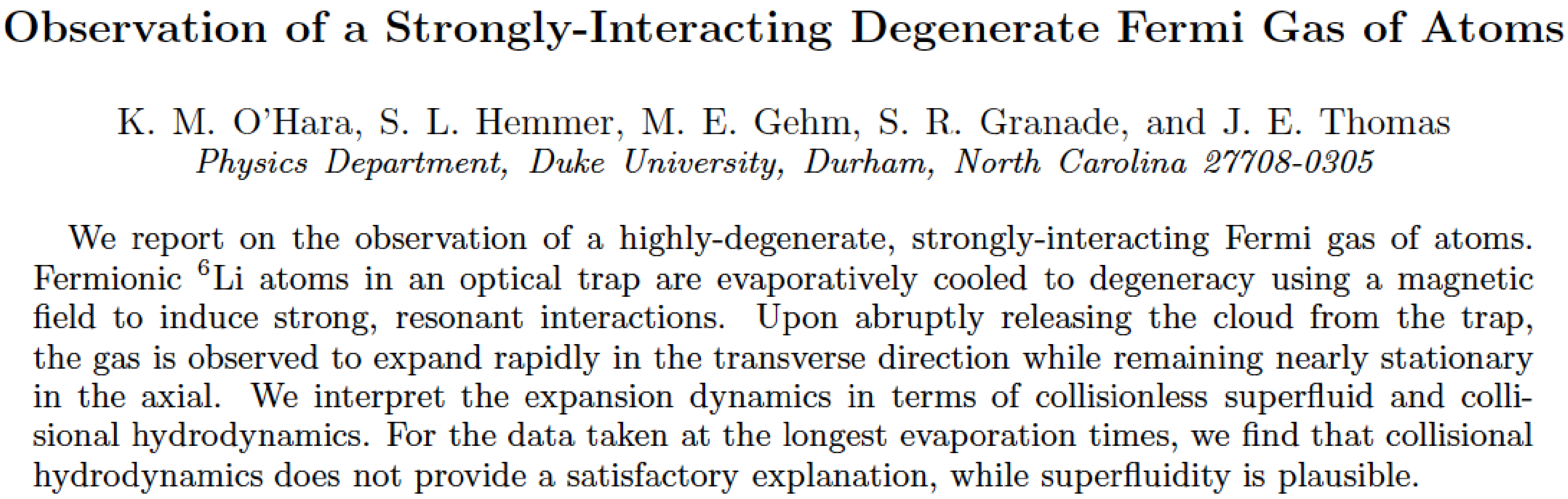
The strongly coupled QCD-like gauge theory is dual to weakly coupled gravitation with a large, negative cosmological constant, Kovtun,Son, Starinets, PRL 94 (2005) 111601



ideal hydrodynamics works also for cold quantum gases

Science 298 (2002) 2179-2182

cond-mat/0212463 [cond-mat.supr-con]



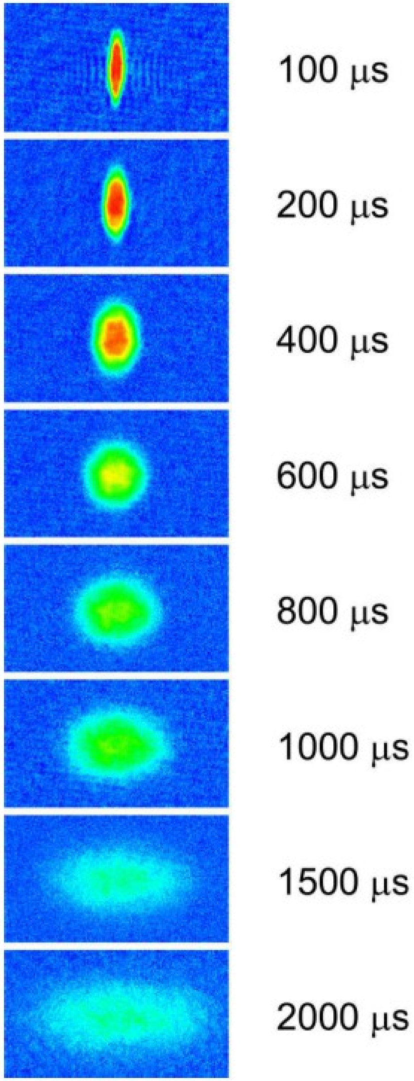
Observation of a Strongly-Interacting Degenerate Fermi Gas of Atoms

K. M. O'Hara, S. L. Hemmer, M. E. Gehm, S. R. Granade, and J. E. Thomas
Physics Department, Duke University, Durham, North Carolina 27708-0305

We report on the observation of a highly-degenerate, strongly-interacting Fermi gas of atoms. Fermionic ^{6}Li atoms in an optical trap are evaporatively cooled to degeneracy using a magnetic field to induce strong, resonant interactions. Upon abruptly releasing the cloud from the trap, the gas is observed to expand rapidly in the transverse direction while remaining nearly stationary in the axial. We interpret the expansion dynamics in terms of collisionless superfluid and collisional hydrodynamics. For the data taken at the longest evaporation times, we find that collisional hydrodynamics does not provide a satisfactory explanation, while superfluidity is plausible.

ultra-cold fermionic ^{6}Li released from a trap expands like an ideal liquid when the interaction strength among the atoms is tuned to 'very large' near a Feshbach resonance

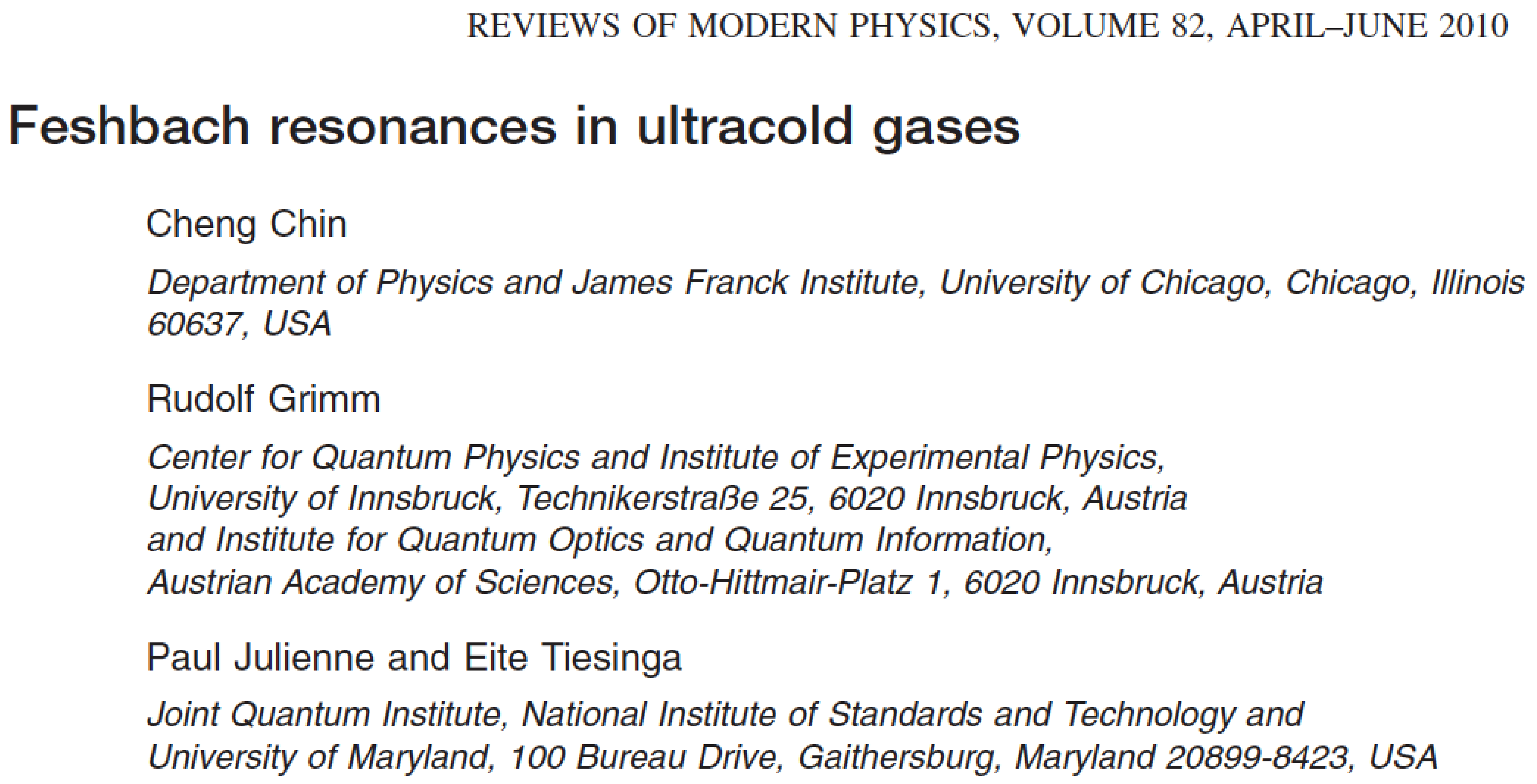
$T_F = 7.9^{+0.3}_{-0.2}\,\mu$K at full trap depth



asymmetric initial condition in the trap leads to strong gradients. the expansion follows the direction of the gradients. very similar to the relativistic case below, where the maximum gradients are induced by the momentum space asymmetry in the plane perpendicular to the beam(s) direction, see below

FIG. 1: False-color absorption images of a strongly interacting, degenerate Fermi gas as a function of time t after release from full trap depth for $t = 0.1 - 2.0$ ms, top to bottom. The axial width of the gas remains nearly stationary as the transverse width expands rapidly.

Feshbach resonance, see:



REVIEWS OF MODERN PHYSICS, VOLUME 82, APRIL–JUNE 2010

Feshbach resonances in ultracold gases

Cheng Chin

Department of Physics and James Franck Institute, University of Chicago, Chicago, Illinois 60637, USA

Rudolf Grimm

Center for Quantum Physics and Institute of Experimental Physics,
University of Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria
and Institute for Quantum Optics and Quantum Information,
Austrian Academy of Sciences, Otto-Hittmair-Platz 1, 6020 Innsbruck, Austria

Paul Julienne and Eite Tiesinga

Joint Quantum Institute, National Institute of Standards and Technology and
University of Maryland, 100 Bureau Drive, Gaithersburg, Maryland 20899-8423, USA

see also: Daniel Kleppner, Physics Today 57, 8, 12 (2004) atom atom scattering leading to a (resonant) molecular state near threshold, with the interaction strength controlled by Zeeman tuning of the resonance from an unbound to a bound state

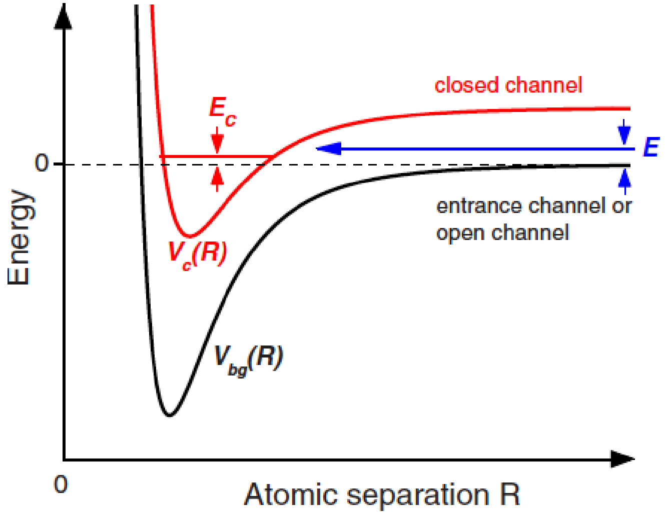


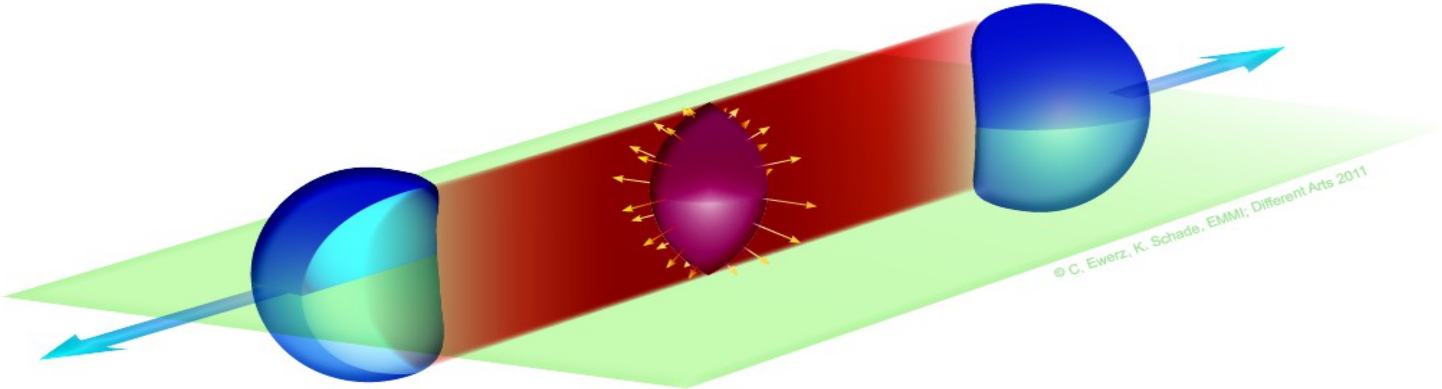
FIG. 1. (Color online) Basic two-channel model for a Feshbach resonance. The phenomenon occurs when two atoms colliding at energy E in the entrance channel resonantly couple to a molecular bound state with energy E_c supported by the closed channel potential. In the ultracold domain, collisions take place near zero energy, $E \rightarrow 0$. Resonant coupling is then conveniently realized by magnetically tuning E_c near 0 if the magnetic moments of the closed and open channels differ.

Scattering length diverges near the resonance described by B_0

$$a(B) = a_{\text{bg}}\left(1 - \frac{\Delta}{B - B_0}\right)$$

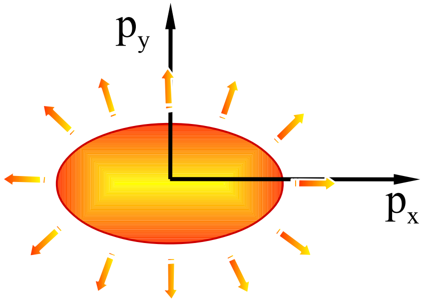
hydrodynamic expansion of QGP fireball with nearly viscous-free flow

fireball expands collectively like an ideal fluid



© C. Ewerz, K. Schade, EMMI; Different Arts 2011

momentum space



$$dN/d\Phi = 1 + 2\ V_2 \cos 2\ (\Phi - \Psi) + \dots$$

hydrodynamic flow characterized by azimuthal anisotropy coeffient v_2
+ higher orders

the angle is measured relative to the reaction plane

heavy ion collisions and hydrodynamics

for $T > 200$ MeV in 2-flavor QGP $n_{\text{parton}} > 4/\text{fm}^3$ and with typical perturbative cross sections $\lambda < 0.8$ fm
rescattering between particles formed in primary collisions may lead to local thermal equilibrium rapidly
treat system as particle fluid using language and tools of hydrodynamics

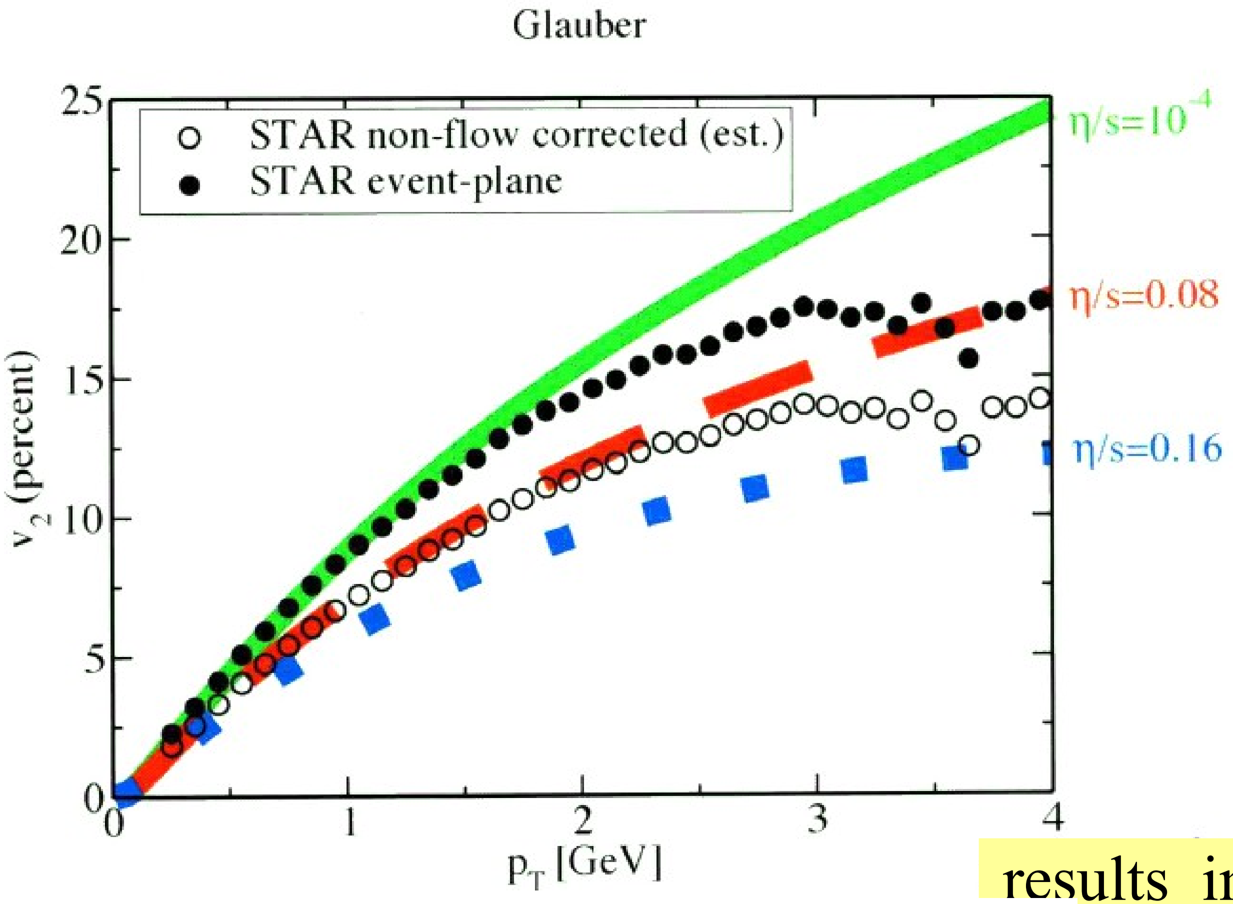
$\partial_\mu T^{\mu\nu} = 0$ $\partial_\mu j^\mu = 0$ with energy-mom tensor $T^{\mu\nu}$ and 4-current of cons. charge j^μ

for ideal fluid: $T^{\mu\nu} = (\varepsilon+p)u^\mu u^\nu - pg^{\mu\nu}$ and $j_i^\mu = n_i u^\mu$

ε: energy density p: pressure u^μ: flow 4 velocity
generally all fields functions of x

generally only EoS and initial condition needed to calculate evolution
EoS: $p = p(\varepsilon, n_1, \ldots , n_n)$ connection pressure – densities
initial cond.: in ideal fluid expansion isentropic, final state multiplicity gives initial entropy, pick volume → system completely determined

the QGP as a nearly perfect fluid



results imply that
η/s < (3 YAdS/CFT limit)

Luzum & Romatschke, PRC 78 (2008) 034915

Note: viscosity of QGP is 25 orders of magnitude larger than that of ultra-cold Li - it is η/s that counts!

RHIC results on flow and quark number scaling

scaled flow coefficient for mesons and baryons coincide

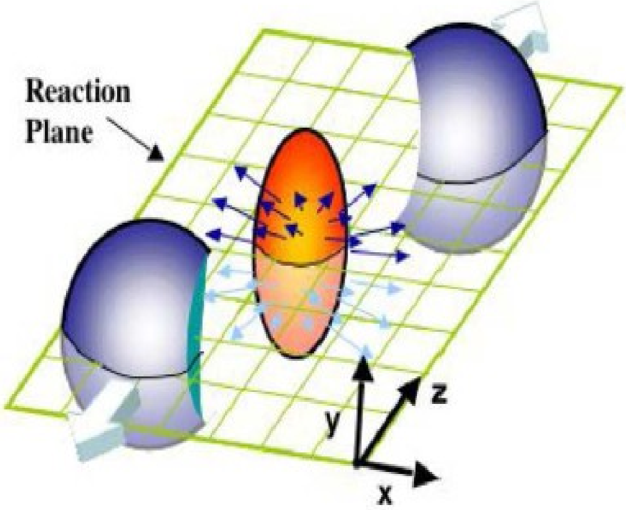
is flow determined in the partonic phase?

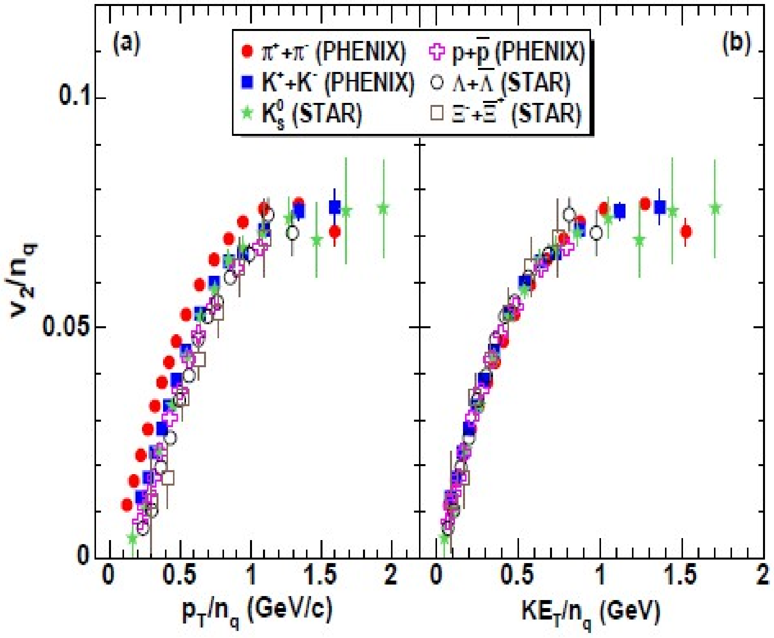
***STAR*:**

PRL ***92***, 052302(04)
PRL ***95***, 122301(05)
PRC77, 54901(08)
PRC81, 44902(10)

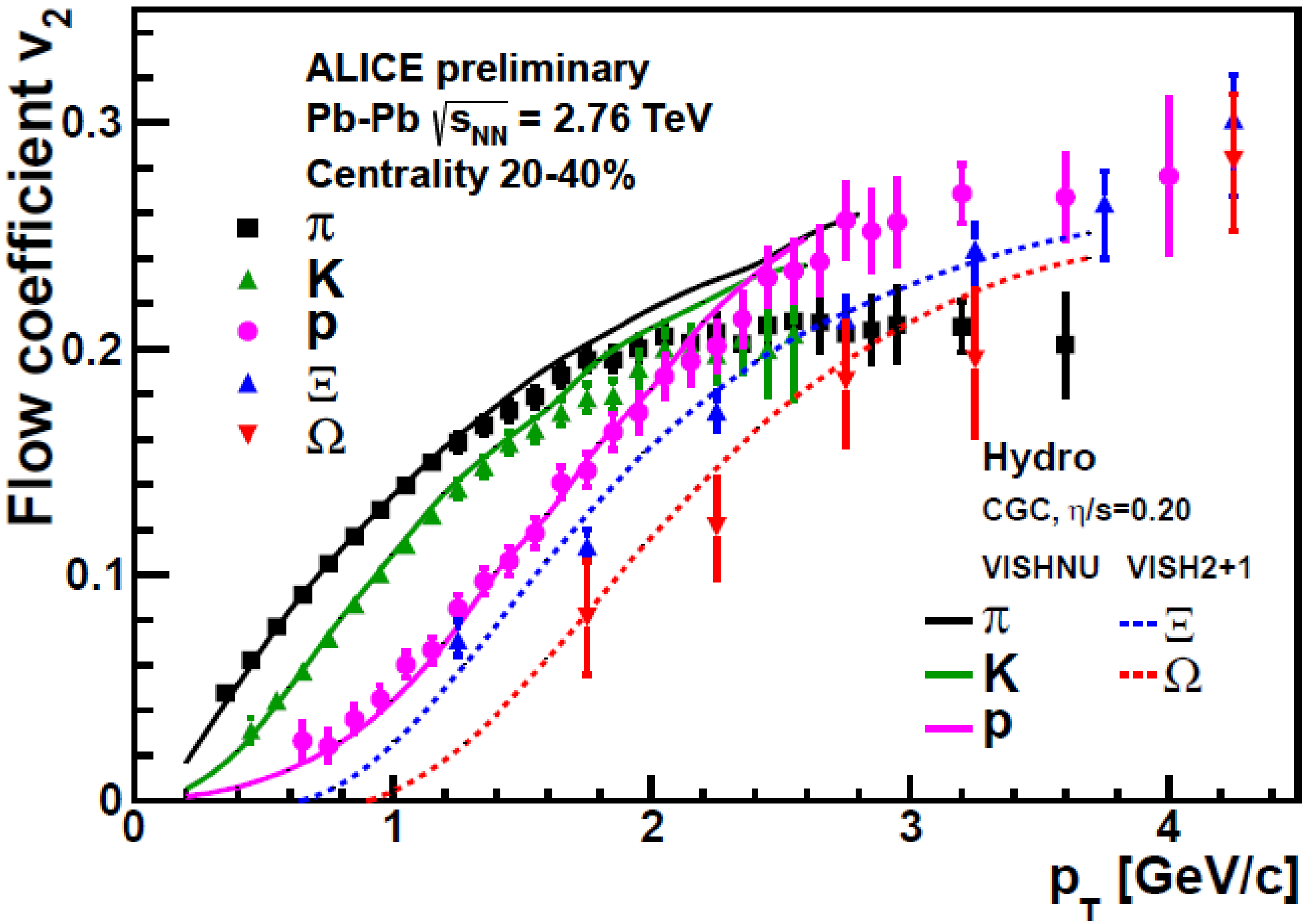
PHENIX:

PRL 98, 162301 (07)





Elliptic Flow in PbPb Collisions at $\sqrt{s_{NN}} = 2.76$ TeV



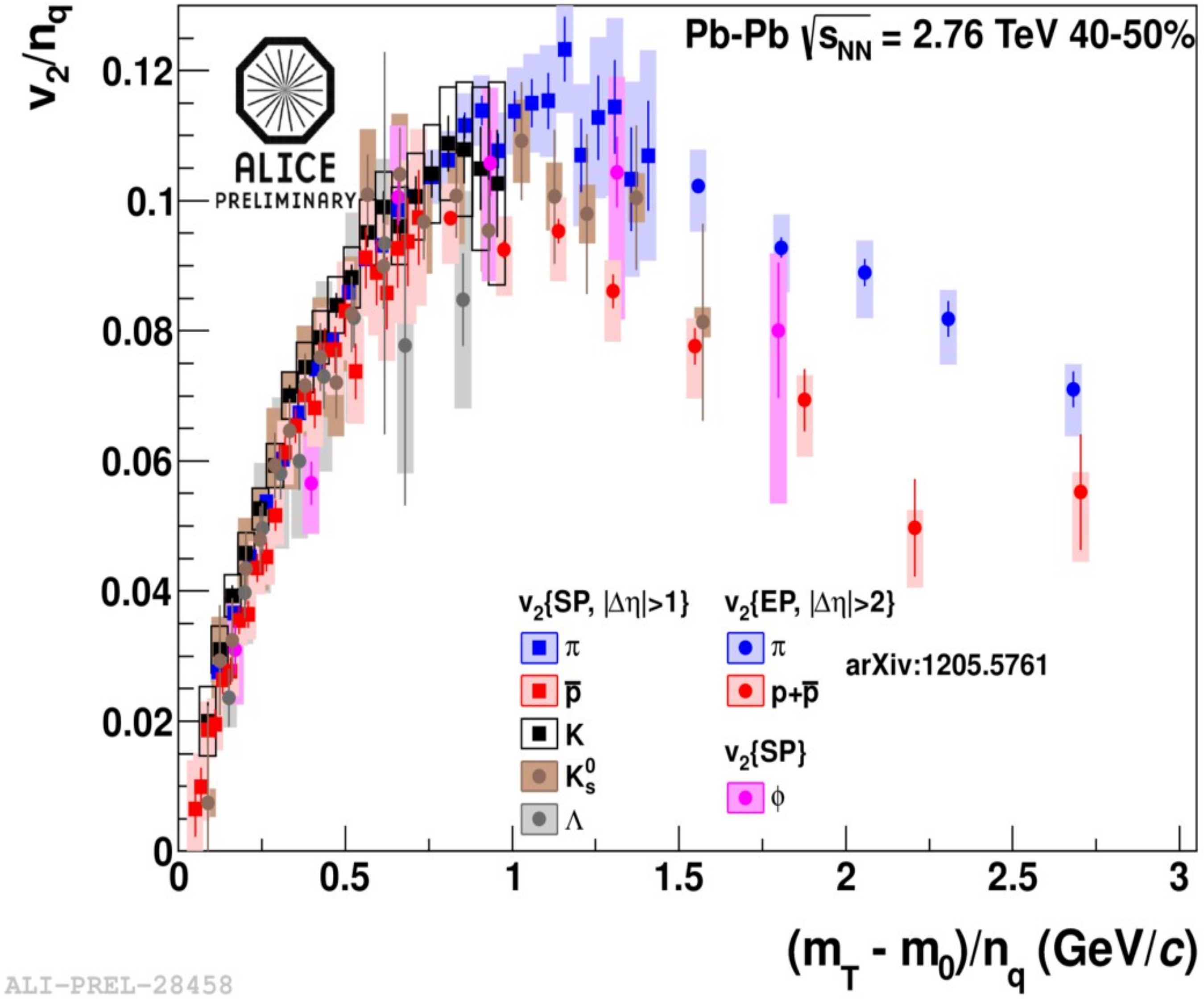
rapidly rising v_2 with p_t and mass ordering are typical features of hydrodyn. expansion

nearly ideal (non-dissipative) hydrodynamics reproduces data, system fairly strongly coupled

Is there valence quark scaling?

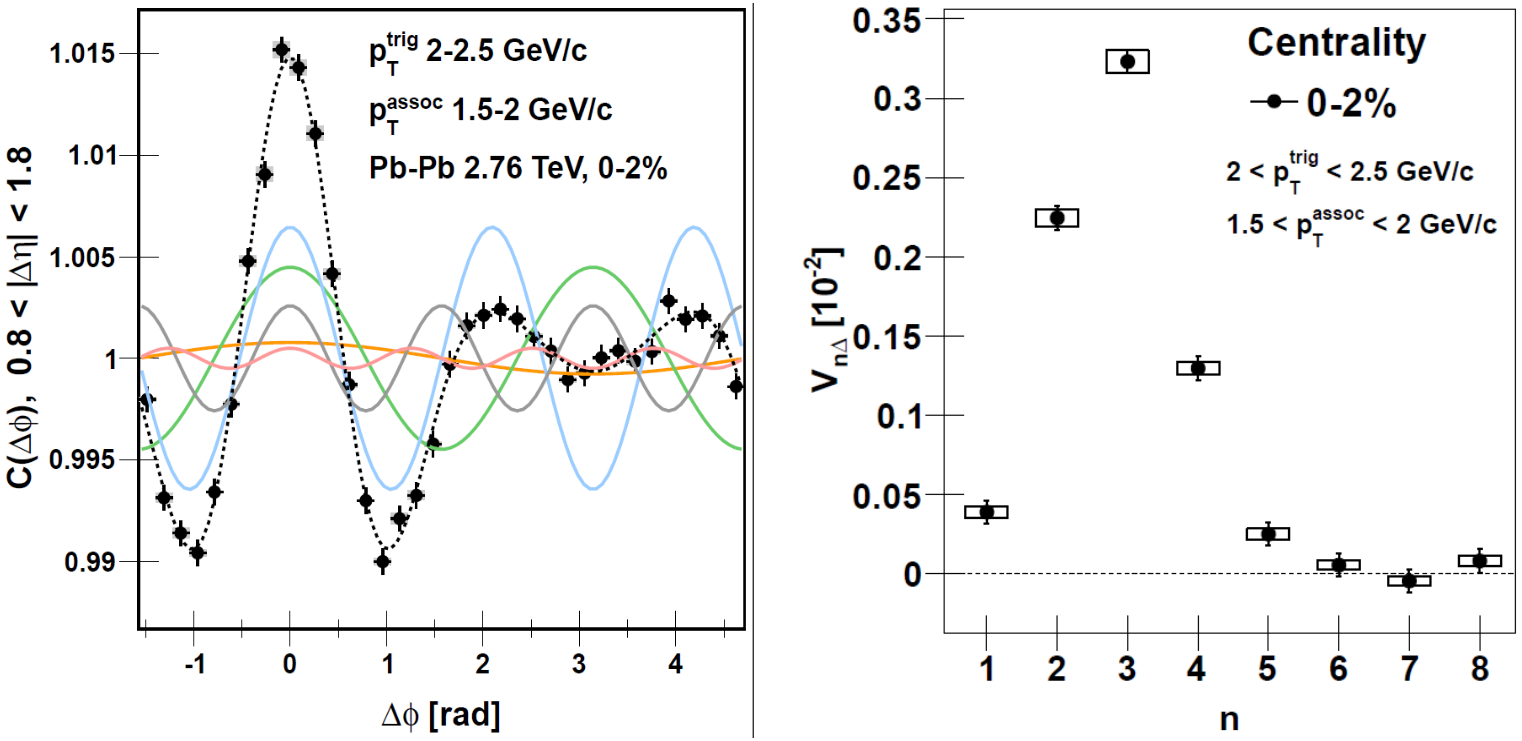
significant scaling violations at LHC energy

this is not a signal for partonic collectivity



the 2-particle correlation function – higher moments

ALICE, PRL 107 (2011) 032301



measurement of the first 8 harmonic coefficients

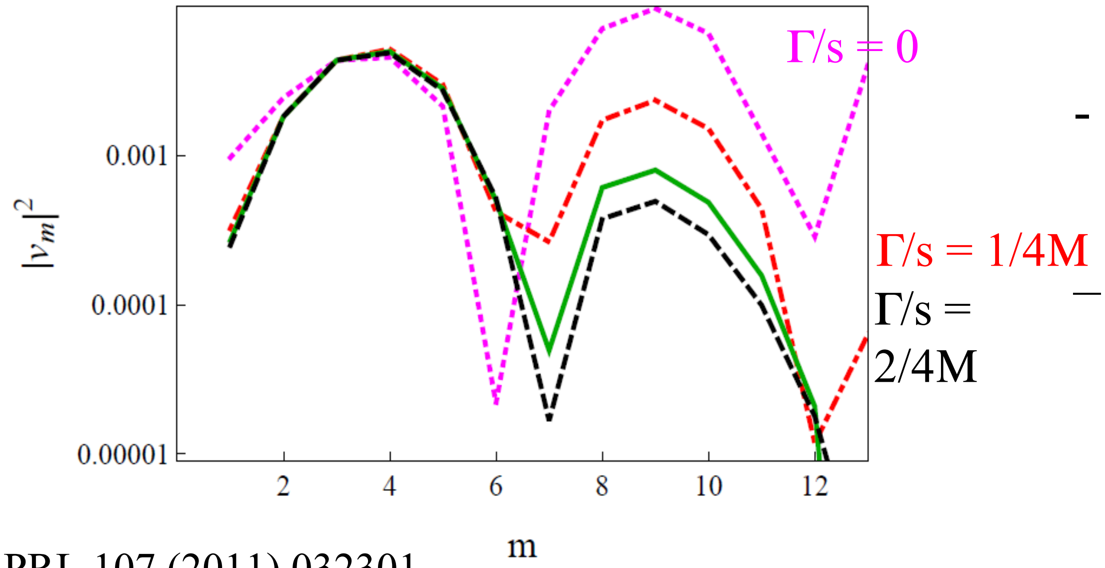
v_1-v_5 significantly larger than 0, maximum at v_3

current understanding: higher harmonics (3,4,5,...) are due to initial inhomogeneities caused by granularity of binary parton-parton collisions

Analogy with early universe power spectrum of CMB

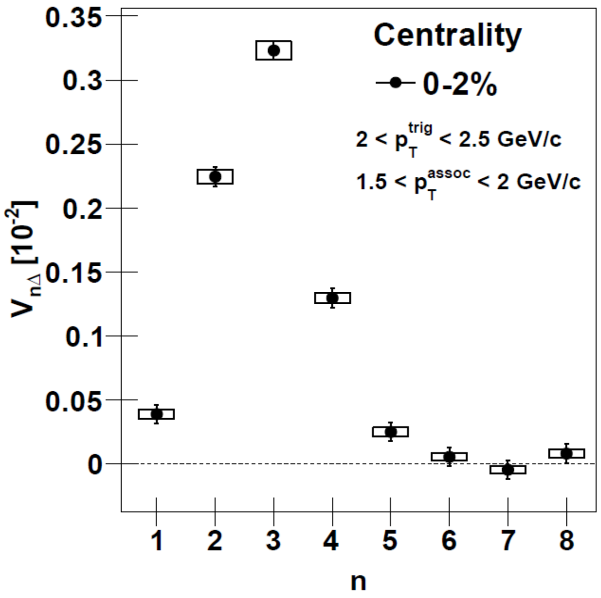
Propagation of sound in the quark-gluon plasma

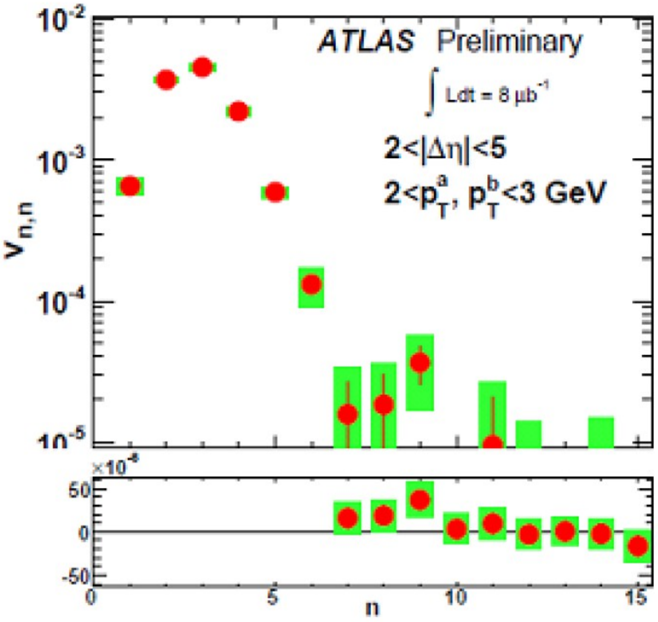
Staig & Shuryak arXiv:1109.6633



- hydrodynamics describes even small perturbations of exploding fireball
- sensitivity to ratio shear viscosity/entropy density and to expansion velocity

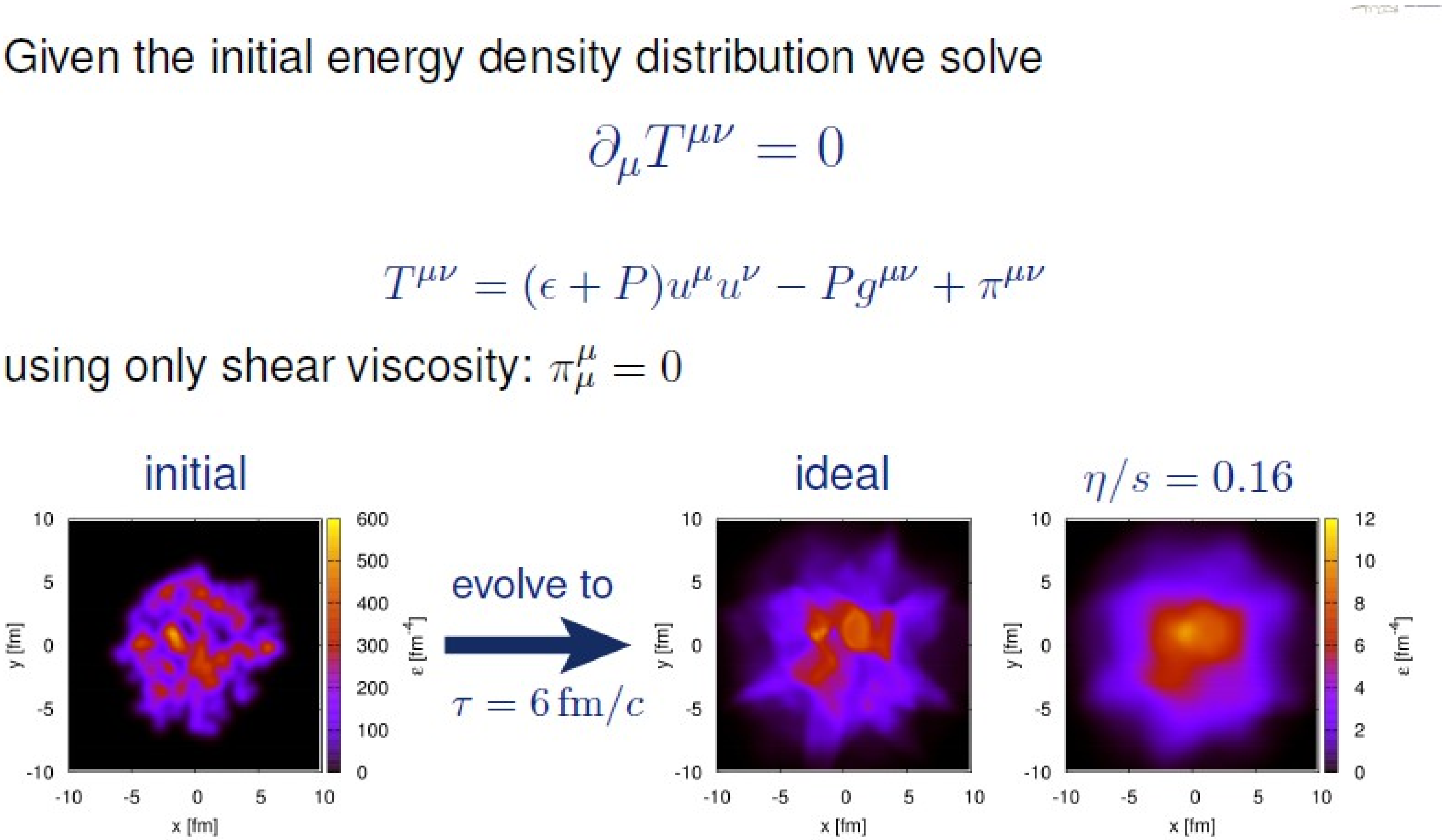
ALICE, PRL 107 (2011) 032301





Introducing initial quantum fluctuations into calculation

B. Schenke, QM2012



Given the initial energy density distribution we solve

$$\partial_\mu T^{\mu\nu} = 0$$

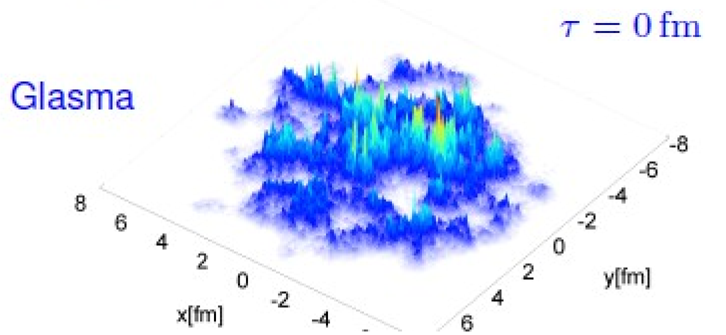
$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \pi^{\mu\nu}$$

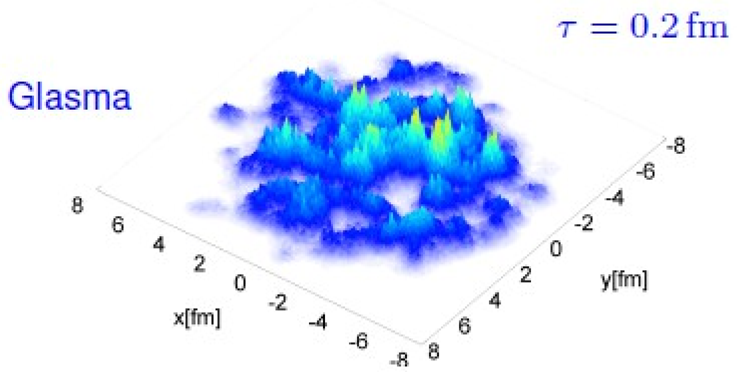
using only shear viscosity: $\pi^\mu_\mu = 0$

Note: alternate means to determine eta/s

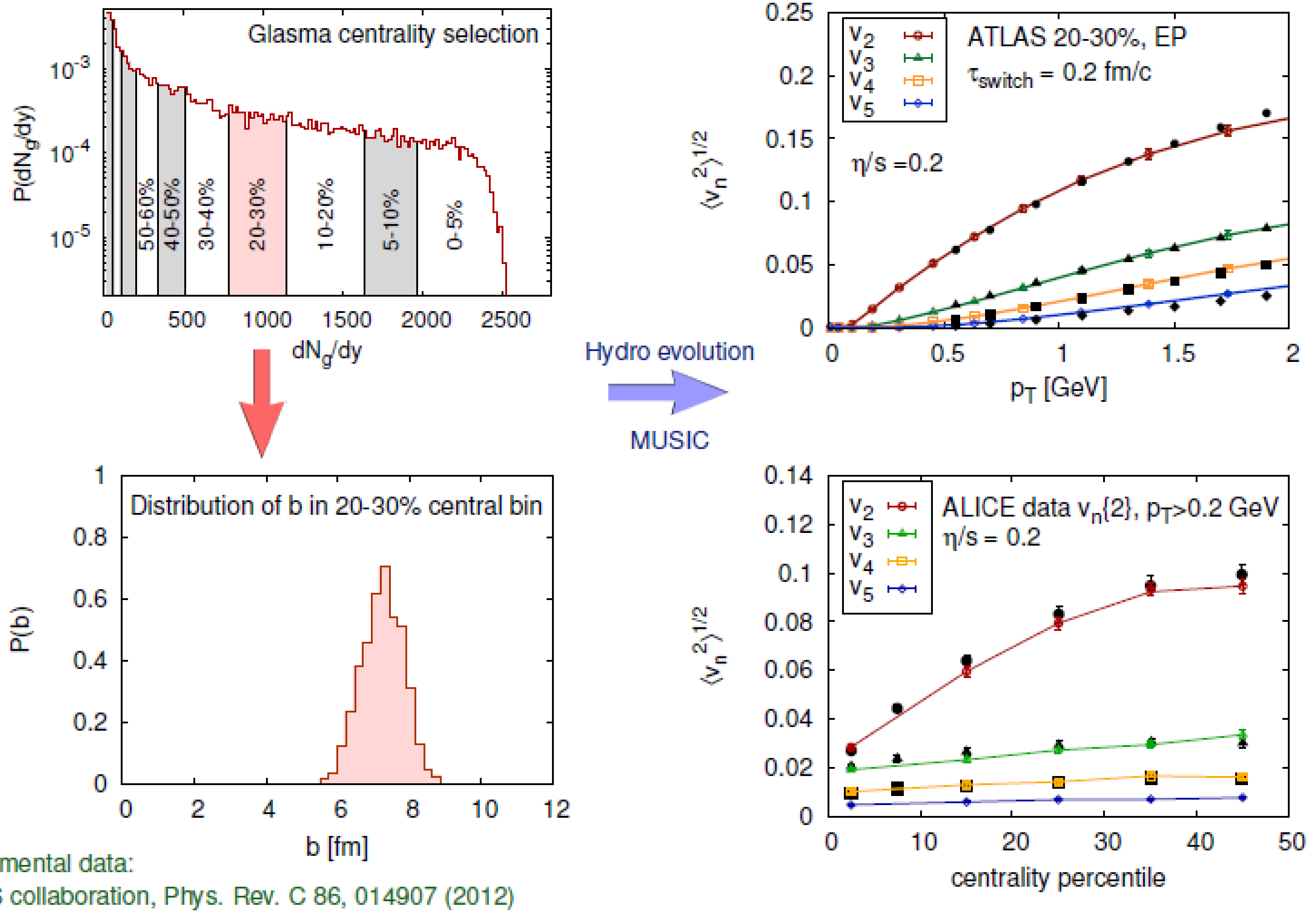
Energy density B.Schenke, P.Tribedy, R.Venugopalan, Phys.Rev.Lett. 108, 252301 (2012)

Solve for gauge fields after the collision in the forward lightcone
Compute energy density in the fields at $\tau = 0$ and later times with CYM evolution
Lattice: Krasnitz, Venugopalan, Nucl.Phys. B557 (1999) 237





Quantitative description of ATLAS and ALICE data



Experimental data:
ATLAS collaboration, Phys. Rev. C 86, 014907 (2012)
ALICE collaboration, Phys. Rev. Lett. 107, 032301 (2011)

calc.: B. Schenke et al., QM2012, eta/s = 0.2

Determination of eta/s of fireball

Latest results from the ALICE collaboration

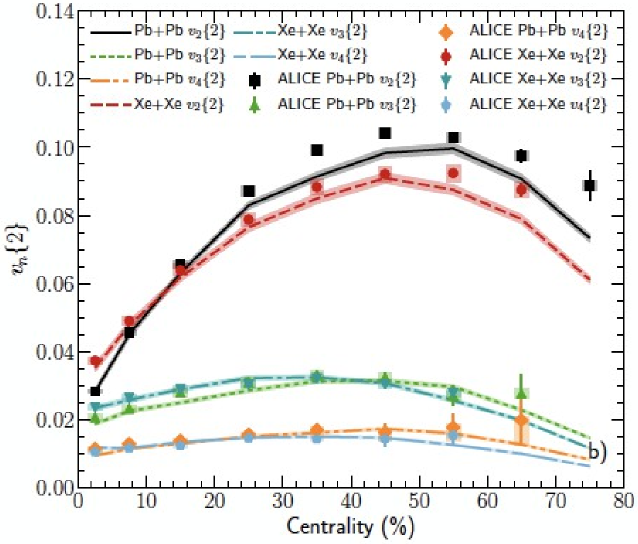
Model-independent determination of eta/s still outstanding

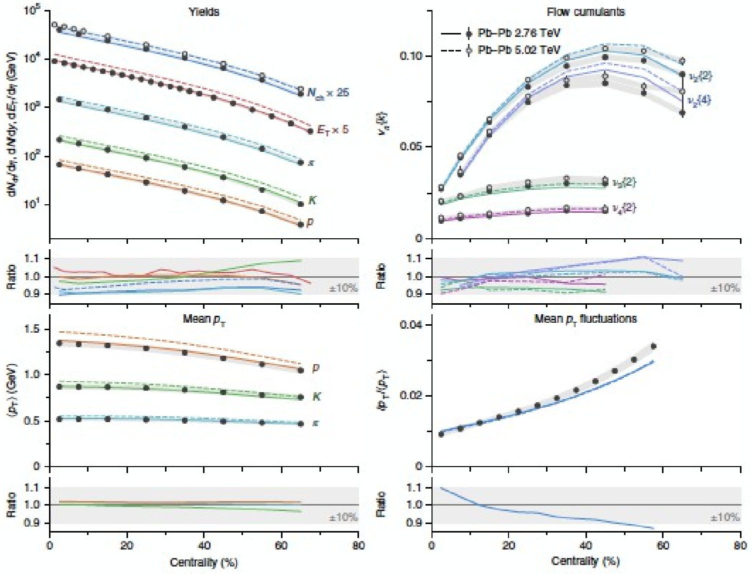
latest analysis of ALICE data on hydrodynamic flow within the framework of relativistic hydrodynamics in including shear and bulk viscosity

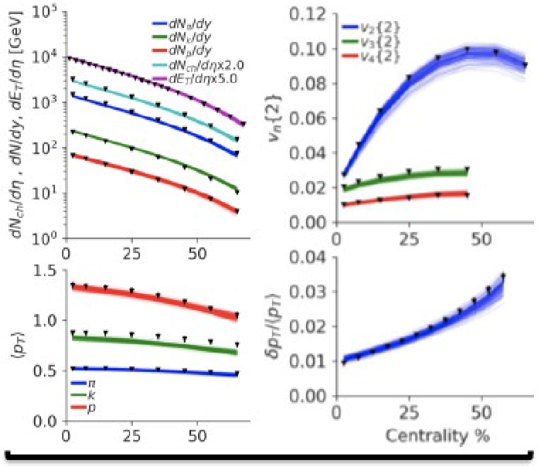
BNL-WSU
arXiv:2005.14682

Duke-OSU
Nature Physics 15 (2019) 1113–1117

JETSCAPE
QM 2019

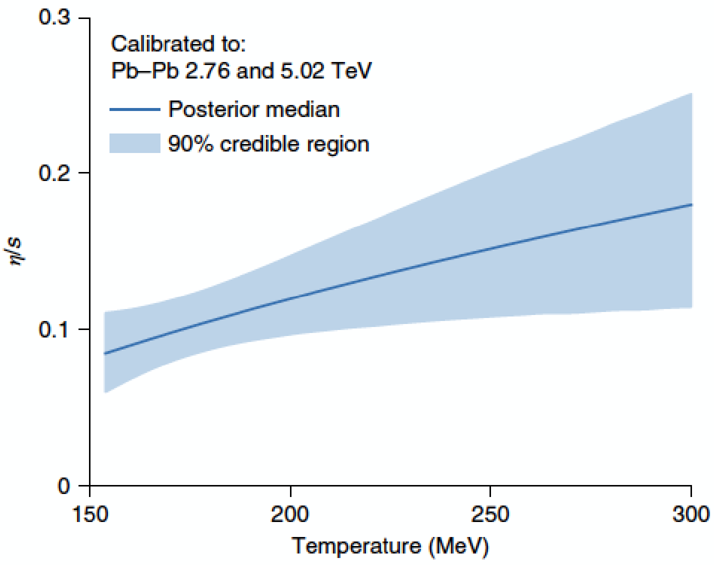




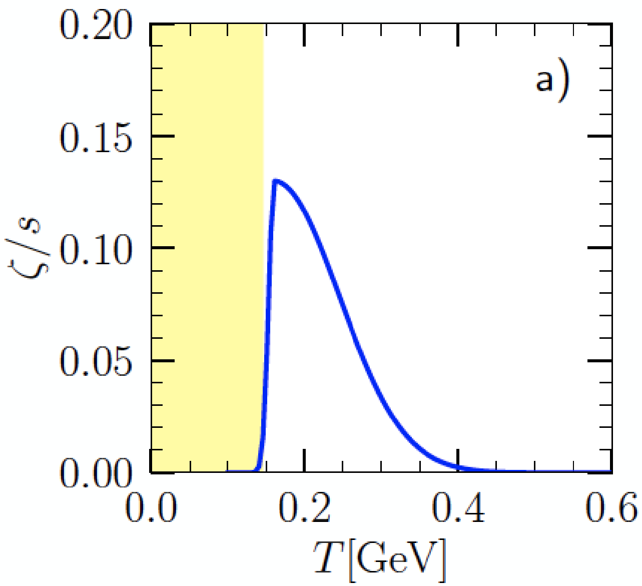


PbPb $\sqrt{s_{NN}}$=2760 GeV
[All measurements from ALICE]

shear viscosity/entropy density



bulk viscosity/entropy density



Summary of hydrodynamical analysis of LHC data

- hydro works well also at LHC energy
- 'ideal fluid' scenario also applies to LHC data
- Universal flow properties of QGP and cold quantum gases
- measurement of higher harmonics allows determination of initial state fluctuations (similar to WMAP analysis for early universe)