THE STRUCTURE OF ATOMIC NUCLEI

Kai Schweda

OUTLINE

- Masses of nuclei - binding energy
- Size of nuclei - radii
- Spin and parity - shell model

BINDING ENERGY OF AN ATOMIC NUCLEUS

$$E_B(N,Z) = Z \cdot m_p + N \cdot m_n - m_{N,Z}$$

$m_{N,Z}$: mass of atomic nucleus

$M_{N,Z}$: mass of neutral atom with $A = (N + Z)$ nucleons and Z electrons

Atomic masses $M_{N,Z}$ expressed in atomic mass units (u)

1u = 1/12 of the mass of the neutral ^{12}C atom

1u = 931.494 MeV

M_H = 1.0078 u, M_n = 1.087 u

^{4}He : $E_B(2,2) = 2 \cdot 1.078u + 2 \cdot 1.087u - 4.0026u = 0.0304u = 28.3$MeV

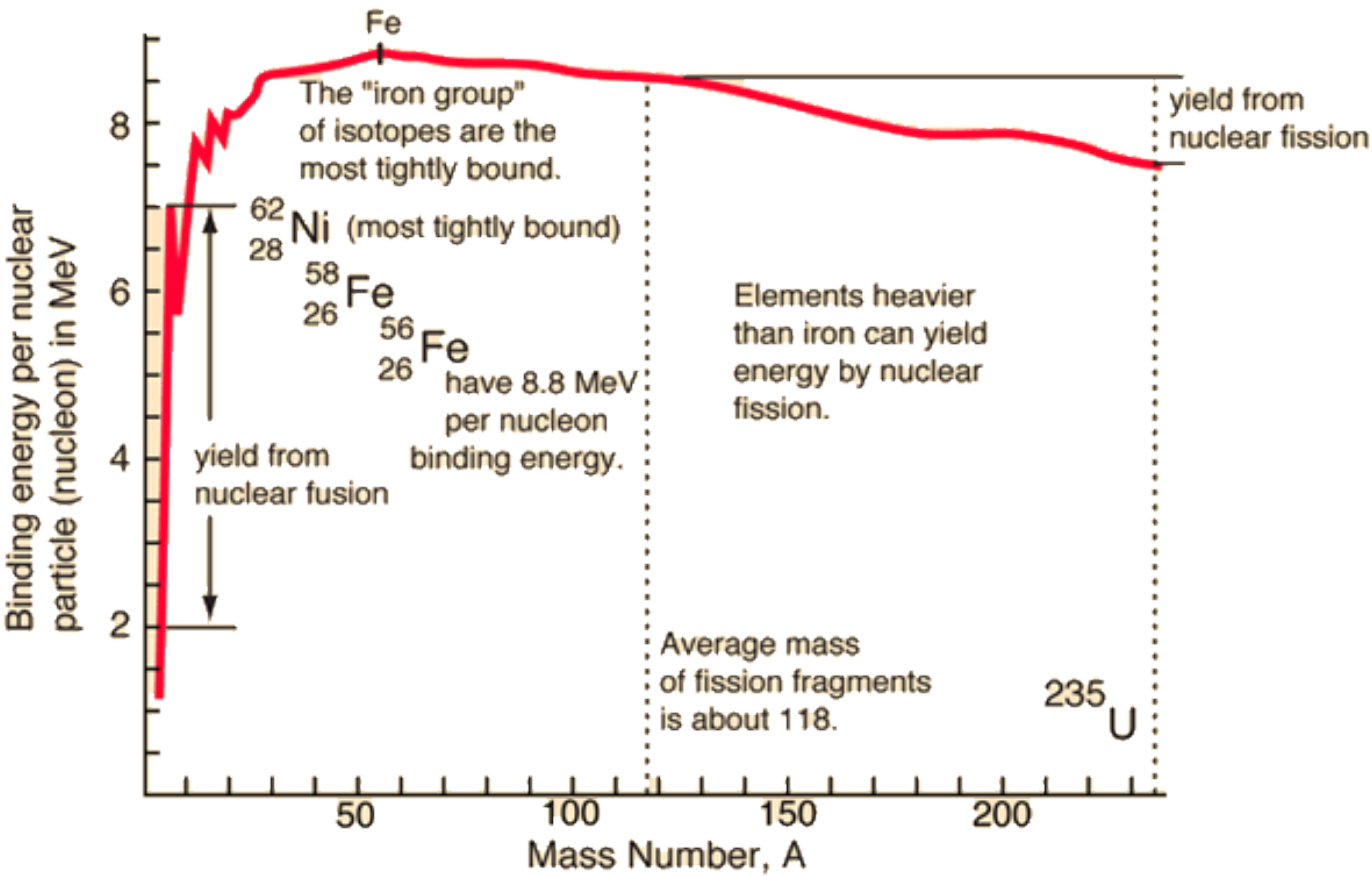
$E_B/A = 7.1$MeV ← Binding energy per nucleon

mass excess: $\Delta(N,Z) = M(N,Z) - A \cdot u$

$\Delta(N,Z) < 0$ nucleus stronger bound than ^{12}C

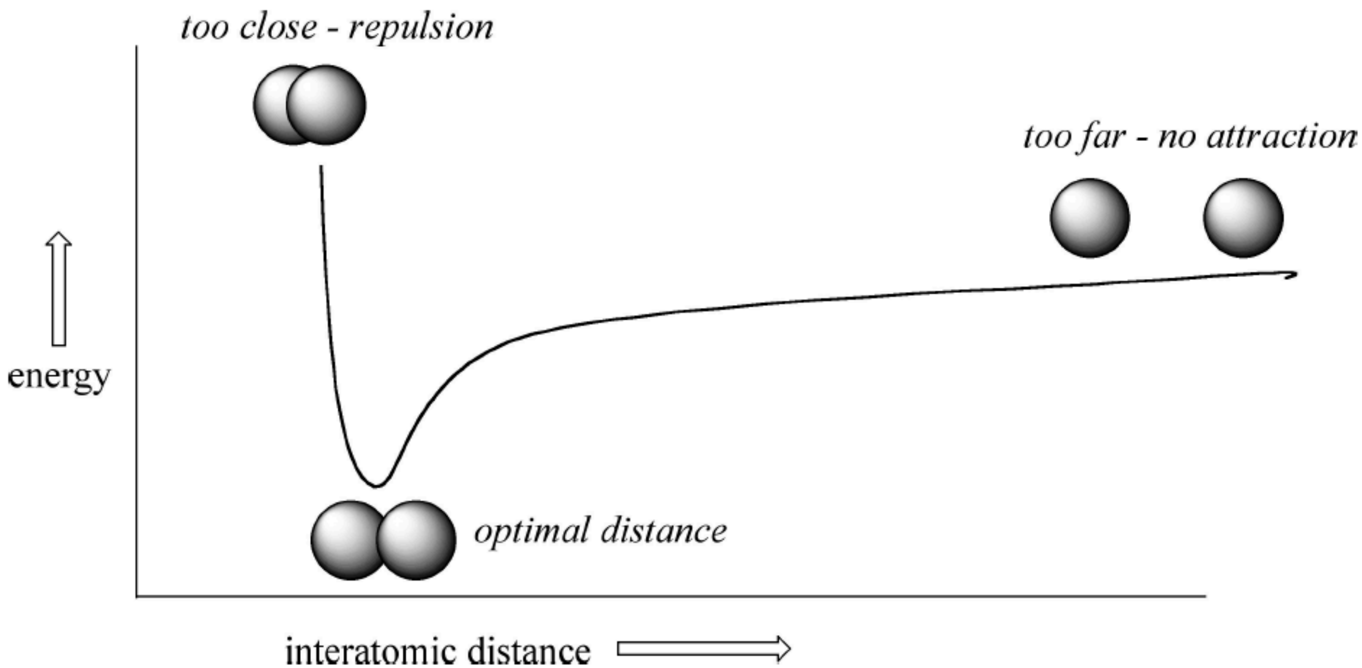
$\Delta(N,Z) > 0$ nucleus weaker bound than ^{12}C

BINDING ENERGY PER NUCLEON - E/A



Source: hyperphysics.phy-astr.gsu.edu

NUCLEON-NUCLEON FORCE



The nuclear force is short-range, but does not allow for compression of nuclear matter. Molecules in a liquid drop have the same basic properties.

LIQUID DROP MODEL

treats the nucleus as a drop of incompressible nuclear fluid

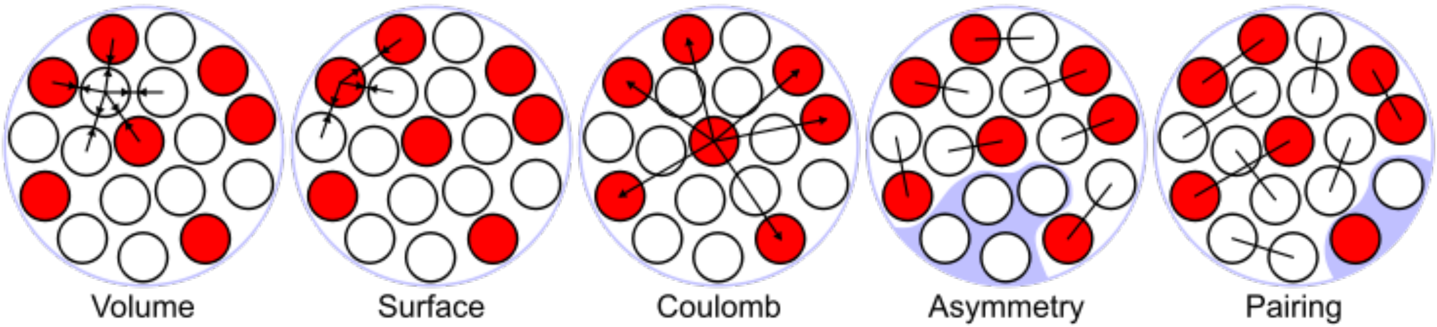
first proposed by George Gamow, further developed by Niels Bohr and John Archibald Wheeler

George Gamow
(1904-1968)

Niels Henrik David Bohr
(1885-1962)

John Archibald Wheeler
(1911-2008)

LIQUID DROP FORMULA



$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(N,Z)$$

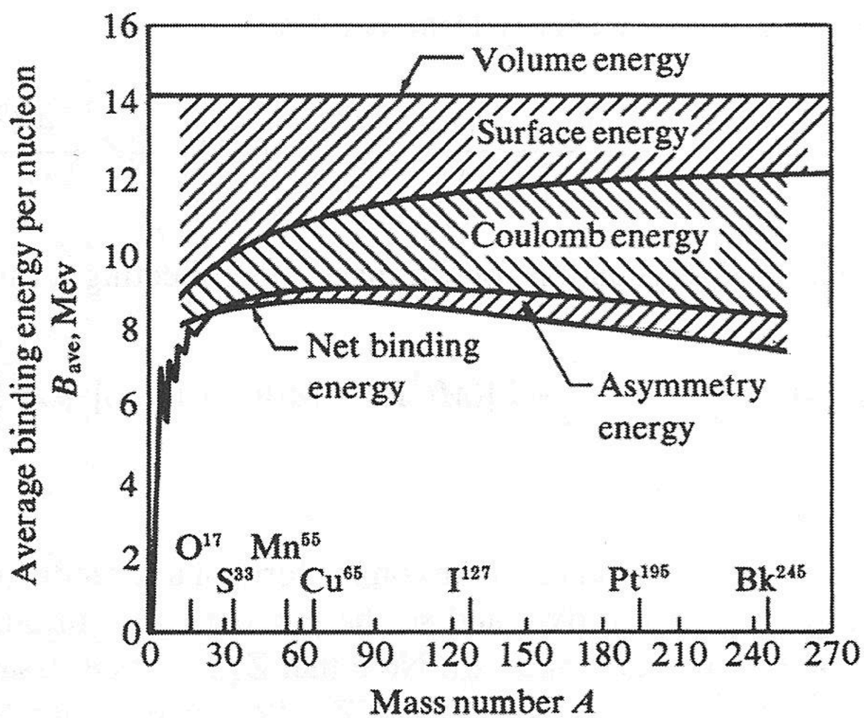
$$\delta(A,Z) = \begin{cases} +\delta_0 & Z, N \text{ even } (A \text{ even}) \\ 0 & A \text{ odd} \\ -\delta_0 & Z, N \text{ odd } (A \text{ even}) \end{cases}$$

$$\delta_0 \approx a_p A^{-1/2}$$

C.F. von Weizsäcker, Z.f. Physik 96 (7-8) 431 (1935)

MASS DEPENDENCE

$$m = Z \cdot m_p + N \cdot m_n - E_B(N, Z)$$

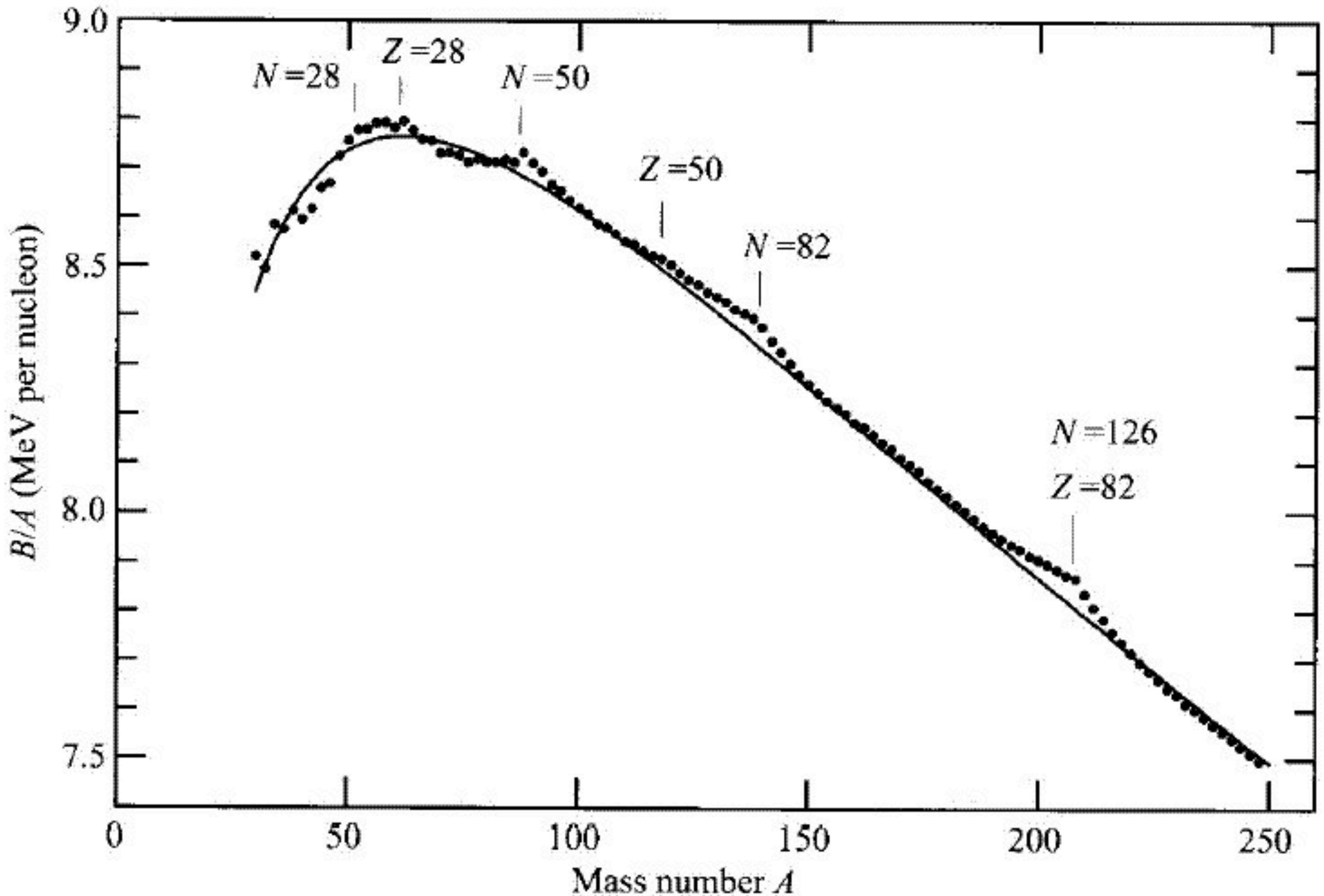


$$a_V = 15.85 \text{ MeV}$$
$$a_S = 18.34 \text{ MeV}$$
$$a_C = 0.71 \text{ MeV}$$
$$a_A = 24.92 \text{ MeV}$$
$$a_p = 11.46 \text{ MeV}$$

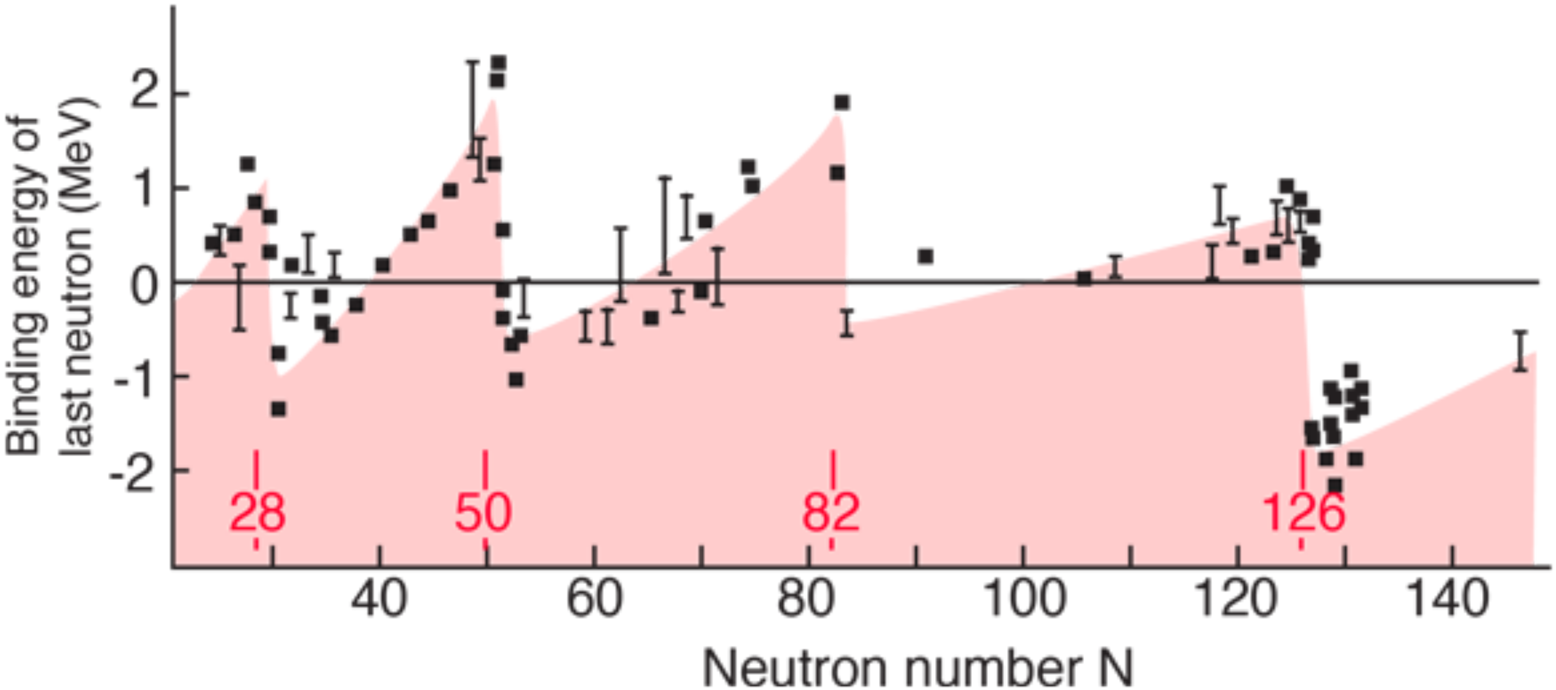
source: T .Mayer Kuckuk, Kernphysik

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(N, Z)$$

COMPARISON TO DATA



BINDING ENERGY OF THE LAST NEUTRON

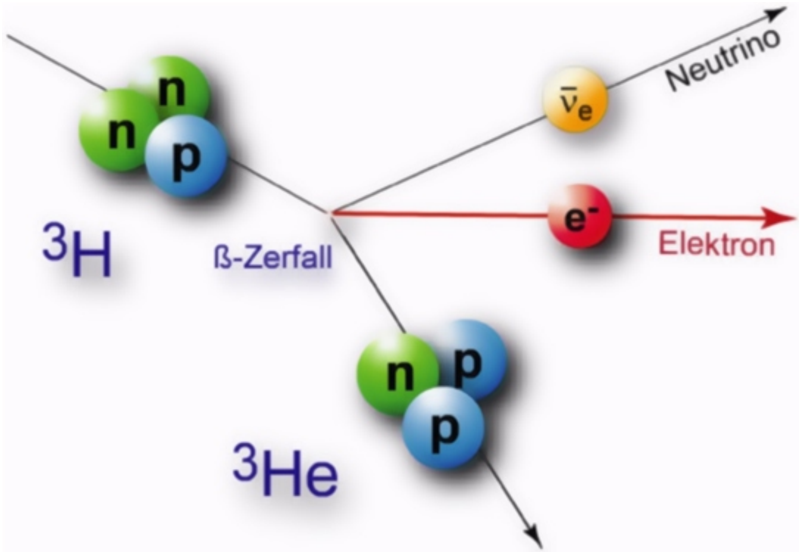


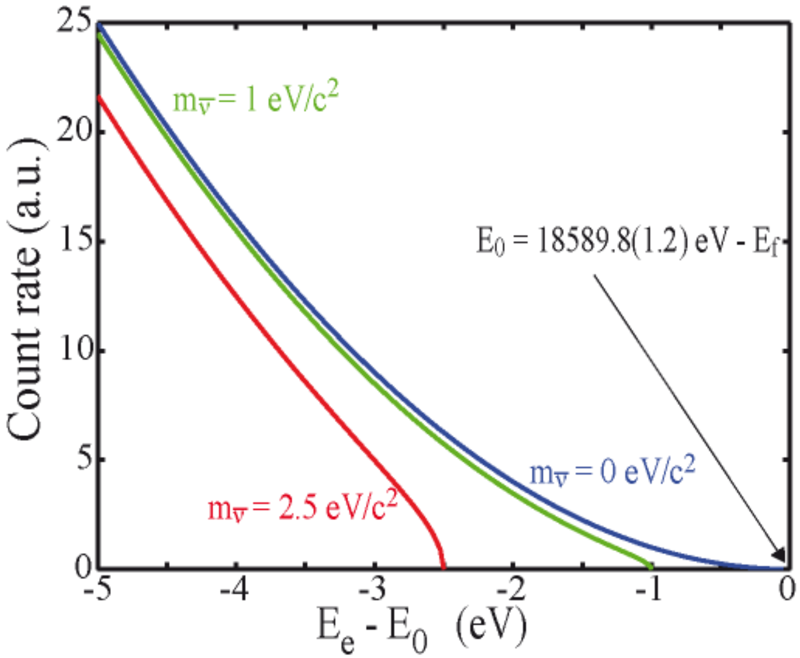
Source: hyperphysics.phy-astr.gsu.edu

MAGIC NUMBERS

- Average binding energy per nucleon is relatively high
- Large neutron (proton) separation energy
- Elements with magic proton numbers have large abundances in the universe
- Excitation energy of the first excited state in particularly high
- At magic proton number, especially large number of isotopes, Sn has 10 stable isotopes
- Cross section for neutron capture small
- Quadrupole moment has minimum, points to spherical symmetric nucleus
- ${}^{4}_{2}He$, ${}^{16}_{8}O$, ${}^{40}_{20}Ca$, ${}^{48}_{20}Ca$, ${}^{56}_{28}Ni$, ${}^{208}_{82}Pb$

NEUTRINO MASS





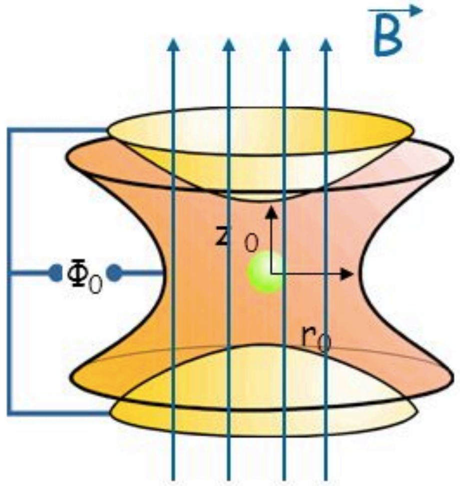
$m(\nu_e) < 2$ eV (95% CL)

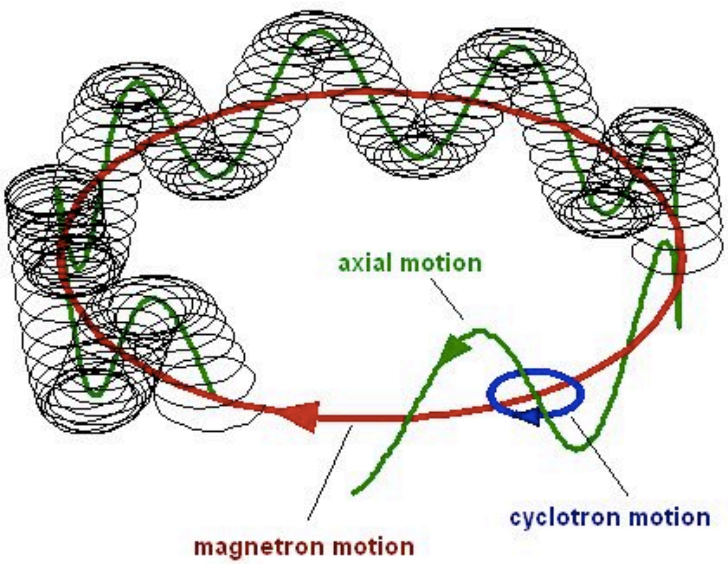
Q_{lit} =18 592.01(7) eV [E. Myers, PRL114, 013003 (2015)]

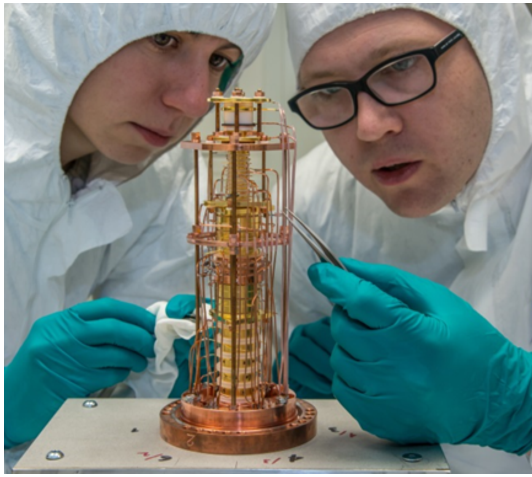
aiming for: $\delta Q(^3T \rightarrow {}^3He) = 20$ meV

$\delta m/m = 7 \cdot 10^{-12}$

PENNING TRAP MASS MEASUREMENT







Axial motion
oscillation in E-field

$$\omega_z = \sqrt{\frac{qV_0}{md^2}}$$

Magnetron motion
E x B drift

$$\omega_- = \frac{\omega_c}{2} - \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}$$

Reduced Cyclotron motion

$$\omega_+ = \frac{\omega_c}{2} + \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}$$

Relevant for measurement Phys. Rev. A 25 (1982) 2423(R)

$$\omega_c = \sqrt{\omega_+^2 + \omega_z^2 + \omega_-^2} = \frac{q}{m}B$$

courtesy of Klaus Blaum

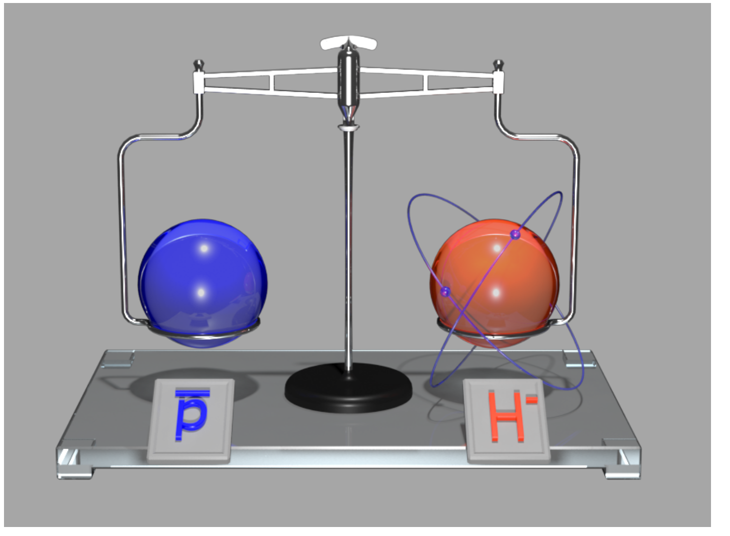
MOST STRINGENT BARYONIC CPT TEST

At CERN antiproton decelerator
2T superconducting magnet

Compare charge-to-mass ratios *R* of p and $\bar{p}$:

$$-(q/m)_p/(q/m)_{\bar{p}} = 1.000000000003(16)$$

S. Ulmer *et al*., Nature 524, 196 (2015)
M.J. Borchert *et al*., Nature 601, 53 (2022)

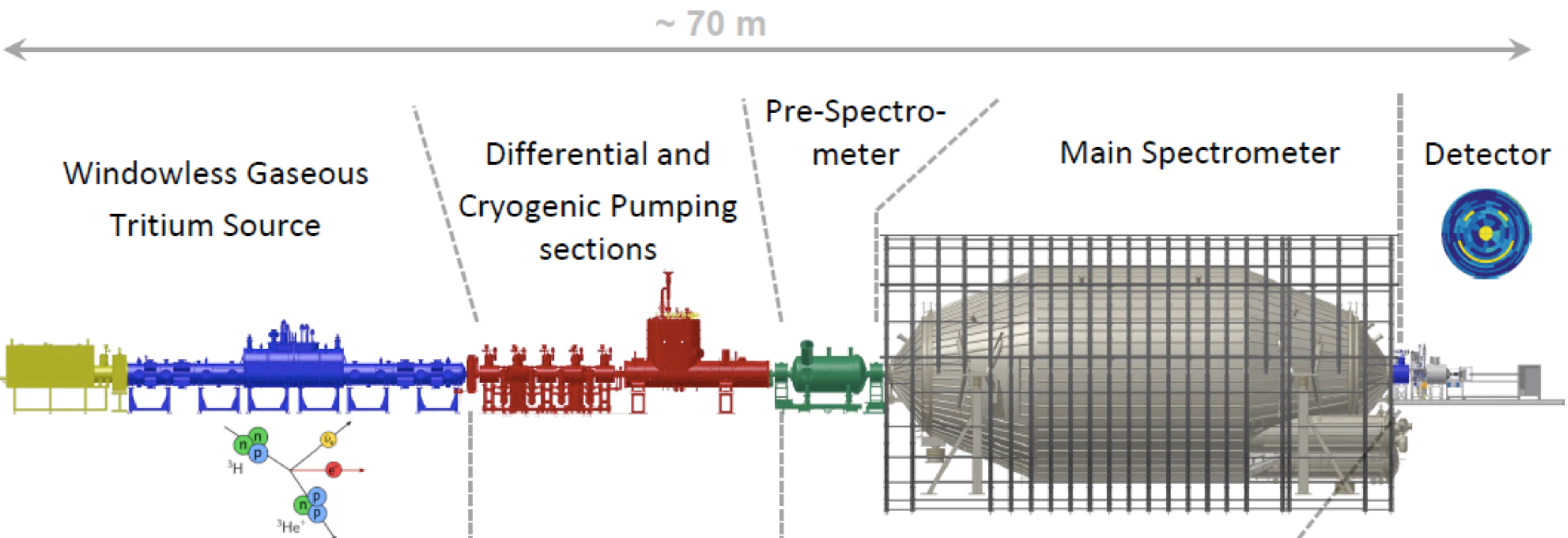


It is not that easy!

$$m_{\mathrm{H}^-} = m_p\left(1 + 2\frac{m_e}{m_p} + \frac{\alpha_{\mathrm{pol,H}^-} B_0^2}{m_p} - \frac{E_\mathrm{b}}{m_p} - \frac{E_\mathrm{a}}{m_p}\right)$$

adapted from Klaus Blaum, Hirschegg 2017.

KARLSRUHE TRITIUM NEUTRINO EXPERIMENT - KATRIN



- tritium β-decay
- decay rate: 10^{11} 1/s
- T_2 pressure: 10^{-3} to 10^{-6} mbar

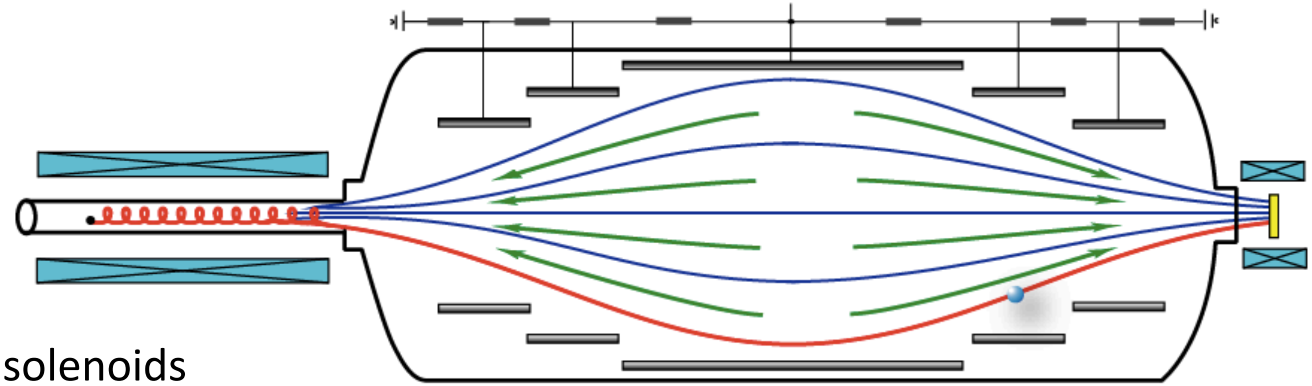
- β-electron transport
- tritium retention (factor > 10^{14})

- energy analysis of β-electrons
- resolution 0.93 eV @ 18.6 keV
- pressure < 10^{-10} mbar

- counting of transmitted β-electrons

TRANSPORT OF THE MAIN SPECTROMETER

MAGNETIC ADIABATIC COLLIMATION COMBINED WITH AN ELECTROSTATIC FILTER - MAC-E



Two supercond. solenoids compose magnetic guiding field

- adiabatic transformation:

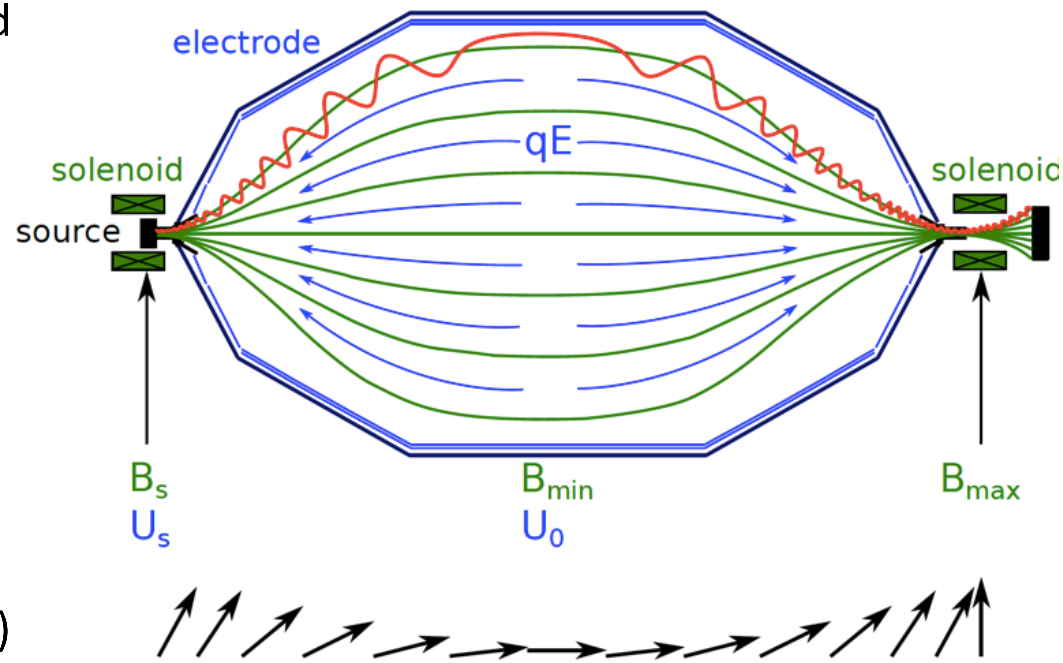
$m = E_T/B = const.$

—> parallel e^- beam

- Energy analysis by electrostat. retarding field

$\Delta E = E \cdot B_{min}/B_{max}$

$= 0.93\ eV$



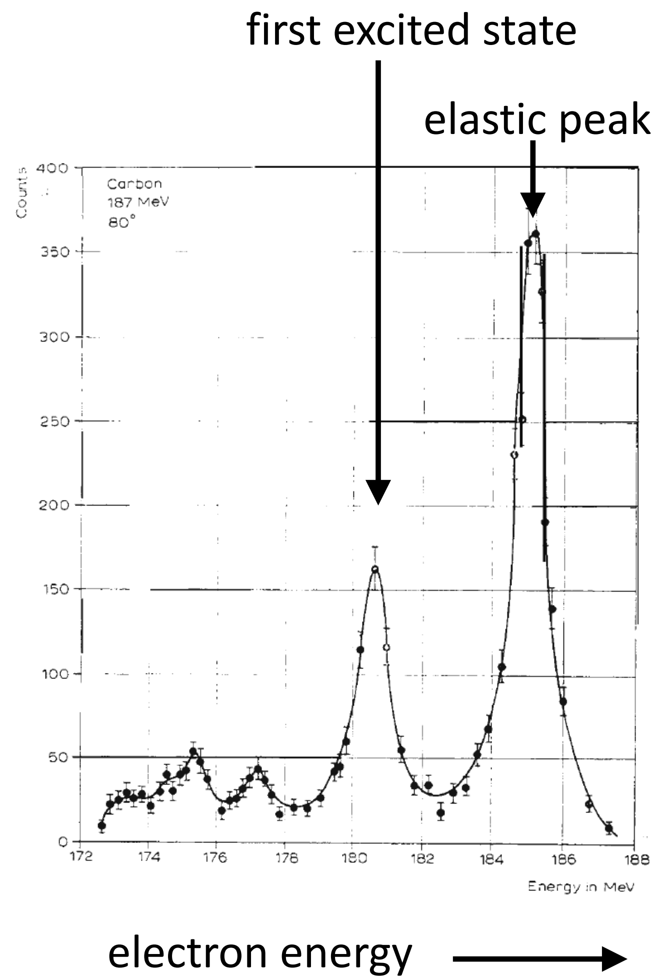
KATRIN: $m(\bar{\nu}_e) < 0.8$ eV (90% CL)

Phys. Rev. Lett. 123, 221802 (2019)

Nature Physics 18, (2022) 160 , the future of KATRIN:https://arxiv.org/pdf/2203.08059.pdf

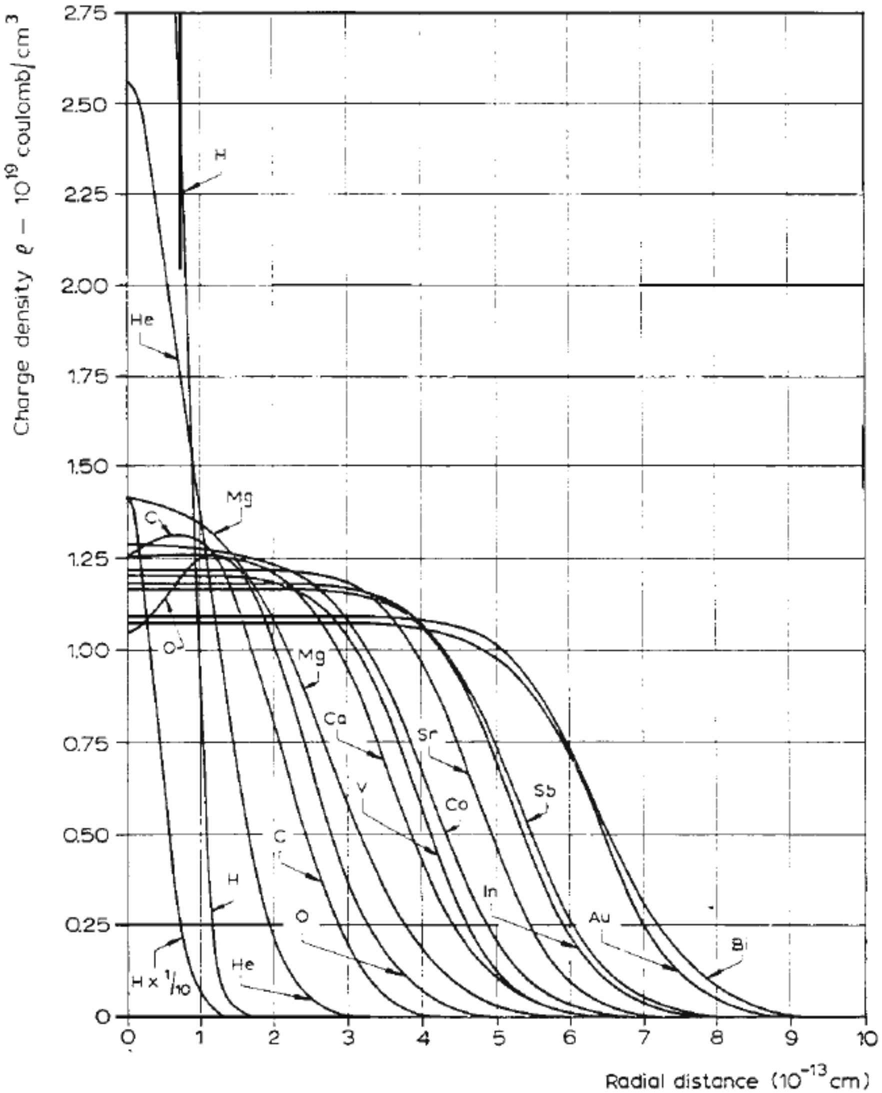
ELASTIC ELECTRON SCATTERING AT SLAC

Abb. 3.3: Das Experiment von Robert Hofstadter (links) bei SLAC; neben ihm seine Kollegen Lance Rogers (oben) und Bud Bunkers (Foto SLAC).

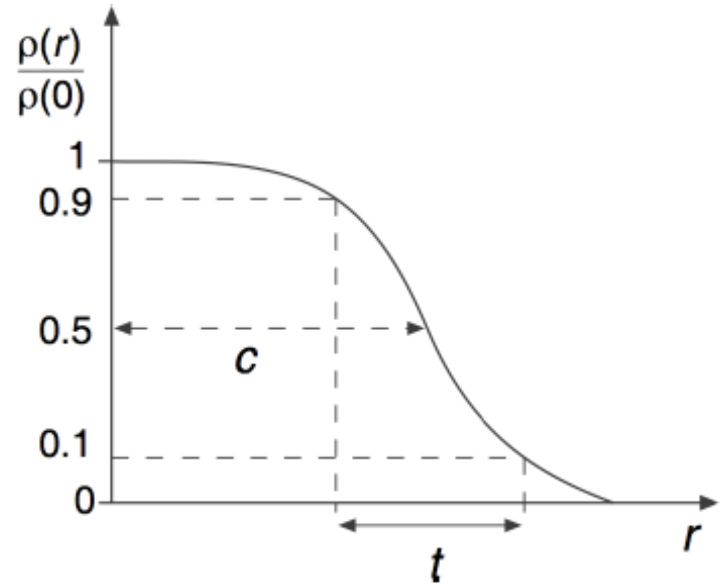


R.Hofstadter, Nobel Lecture 1961.

CHARGE DENSITY OF NUCLEI



R.Hofstadter, Nobel Lecture 1961.

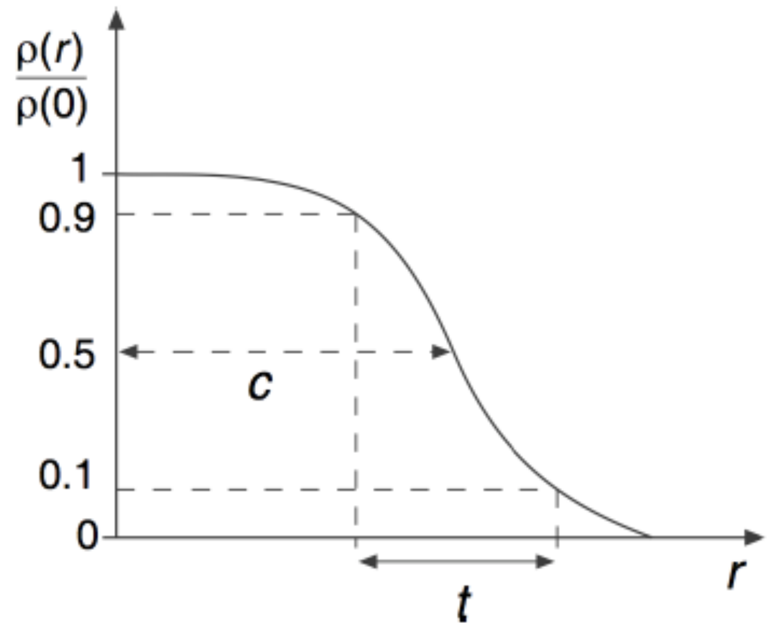


$\rho \approx 0.17$ nucleons/fm^3

R.Hofstadter, Nobel Lecture 1961.

PARAMETRIZATION

$$\rho(r) = \frac{\rho_0}{1 + e^{(r-c)/a}}$$



$$c = 1.07\text{fm} \cdot A^{1/3}, a = 0.54\text{fm}$$

$$\langle r^2 \rangle^{1/2} = r_0 \cdot A^{1/3}, r_0 = 0.94\text{fm}$$

assume homogeneously charged hard sphere, radius R

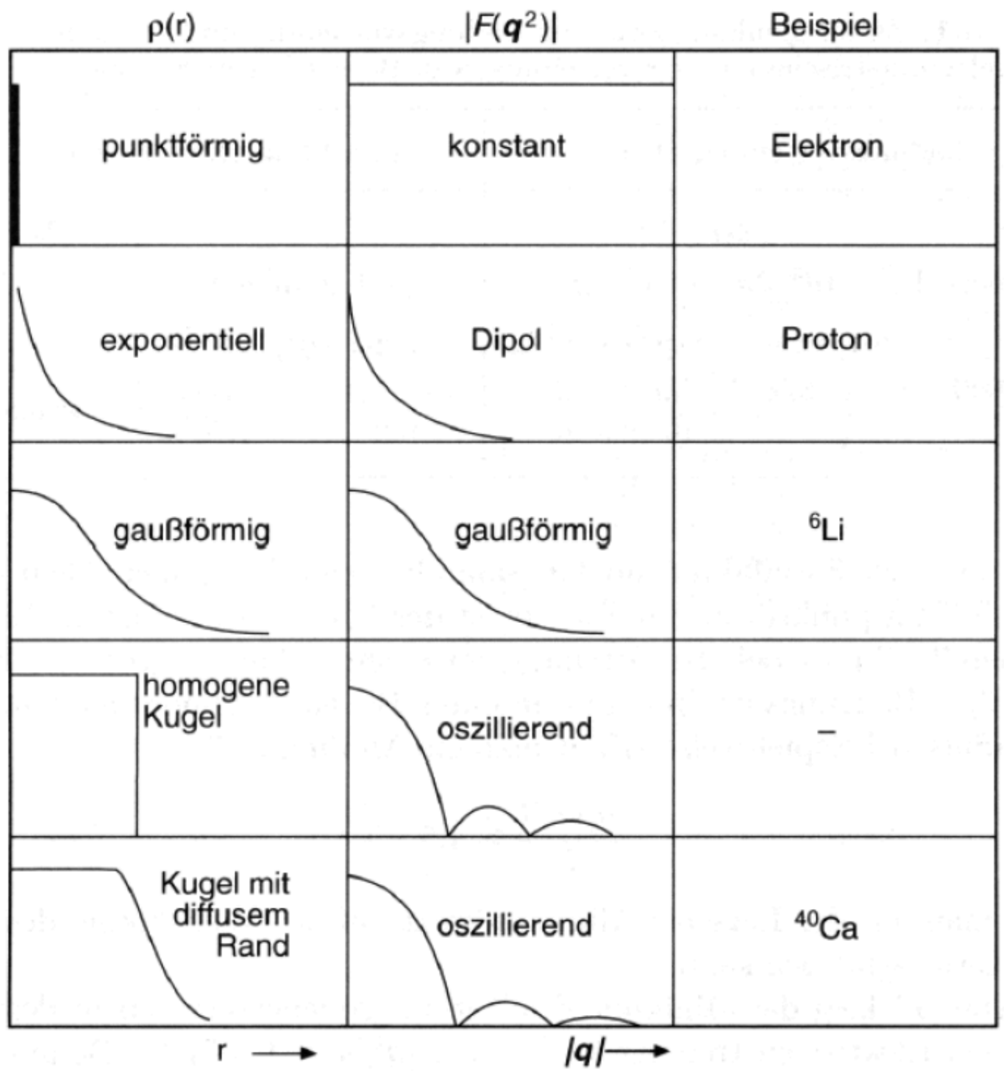
$$\langle r^2 \rangle_{\text{hard sphere}} = \int \vec{r}^2 \rho(r) d\vec{r} = \frac{3}{4\pi R^3} 4\pi \int_0^R r^4 dr = \frac{3}{5} R^2$$

$$R^2 = \frac{5}{3} \langle r^2 \rangle^{1/2} \rightarrow R = 1.21 \cdot A^{1/3}$$

surface thickness

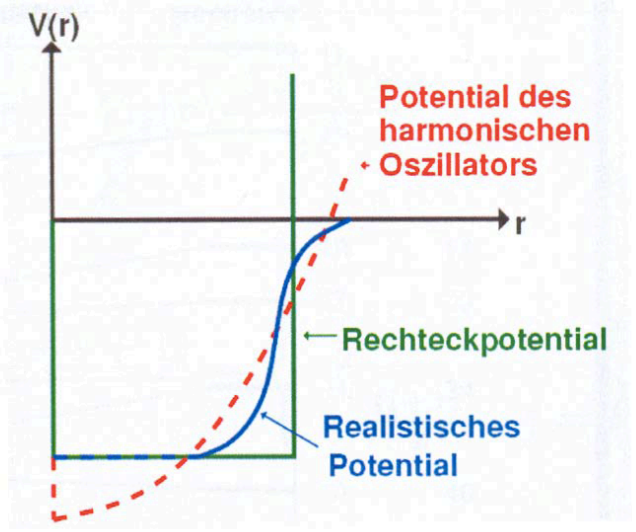
$$t = r_{\rho/\rho_0=0.1} - r_{\rho/\rho_0=0.9} \qquad t = 2a \ln 9 \approx 2.40\text{fm}$$

CHARGE DENSITIES AND FORM FACTORS



NUCLEAR SHELL MODEL

assume spherically symmetric potential



number of nodes in radial wavefunction : $n = 1,2,3,4,...$

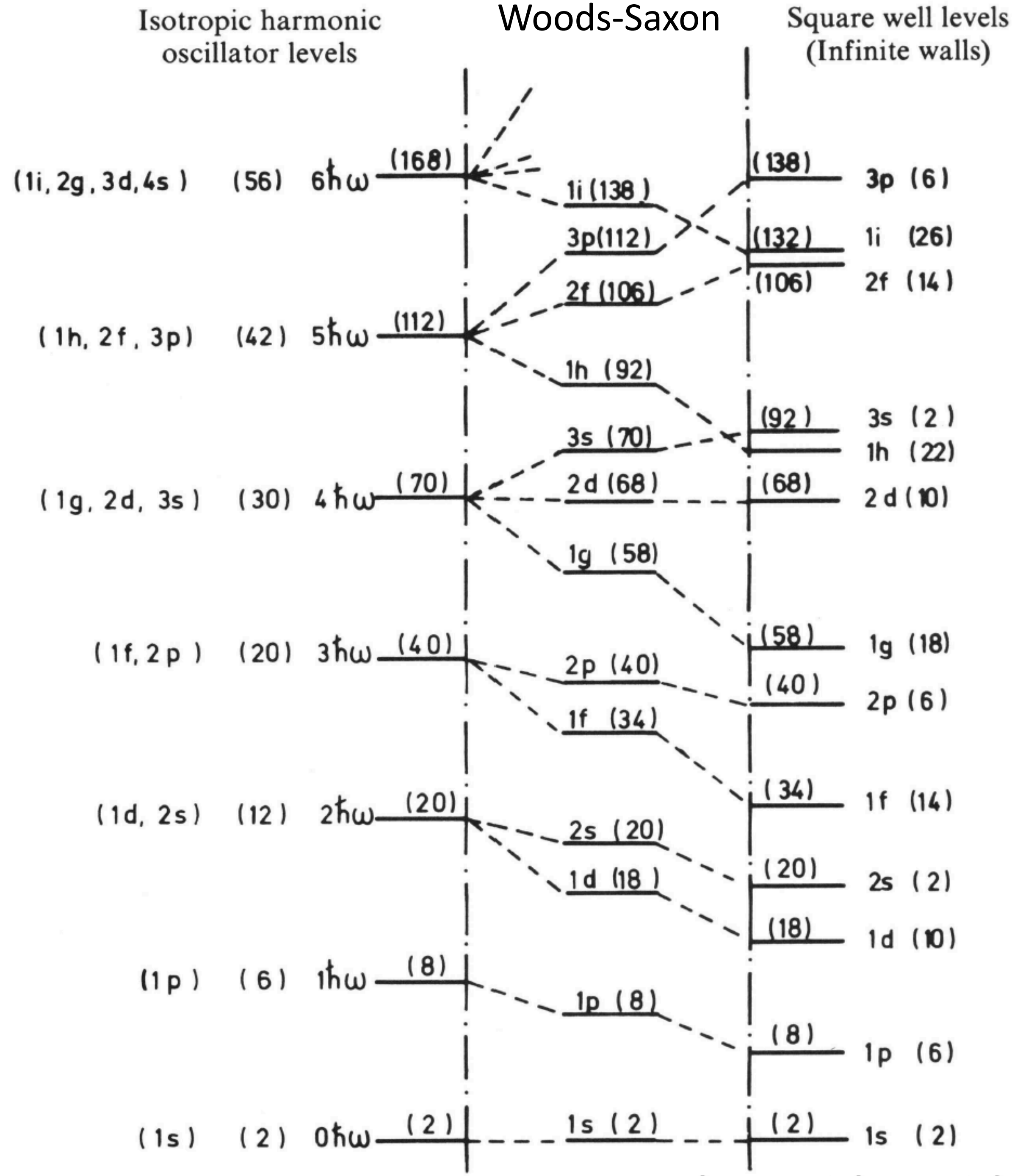
angular dependence : $l =$ s,p,d,f,g,h,...

spherical harmonics: Y_l^m, parity $P = (-1)^l$

degeneracy: $2 \cdot (2l + 1)$

harmonic oscillator: $E_{\text{harm.osc.}} = (N + 3/2) \cdot \hbar\omega$

$N = 2(n - 1) + l$


Isotropic harmonic oscillator levels
Woods-Saxon
Square well levels (Infinite walls)
(1i, 2g, 3d, 4s) (56) 6ħω (168)
(1h, 2f, 3p) (42) 5ħω (112)
(1g, 2d, 3s) (30) 4ħω (70)
(1f, 2p) (20) 3ħω (40)
(1d, 2s) (12) 2ħω (20)
(1p) (6) 1ħω (8)
(1s) (2) 0ħω (2)
1i (138)
3p (112)
2f (106)
1h (92)
3s (70)
2d (68)
1g (58)
2p (40)
1f (34)
2s (20)
1d (18)
1p (8)
1s (2)
(138) 3p (6)
(132) 1i (26)
(106) 2f (14)
(92) 3s (2)
1h (22)
(68) 2d (10)
(58) 1g (18)
(40) 2p (6)
(34) 1f (14)
(20) 2s (2)
(18) 1d (10)
(8) 1p (6)
(2) 1s (2)

SPIN-ORBIT COUPLING

add spin-orbit term to potential

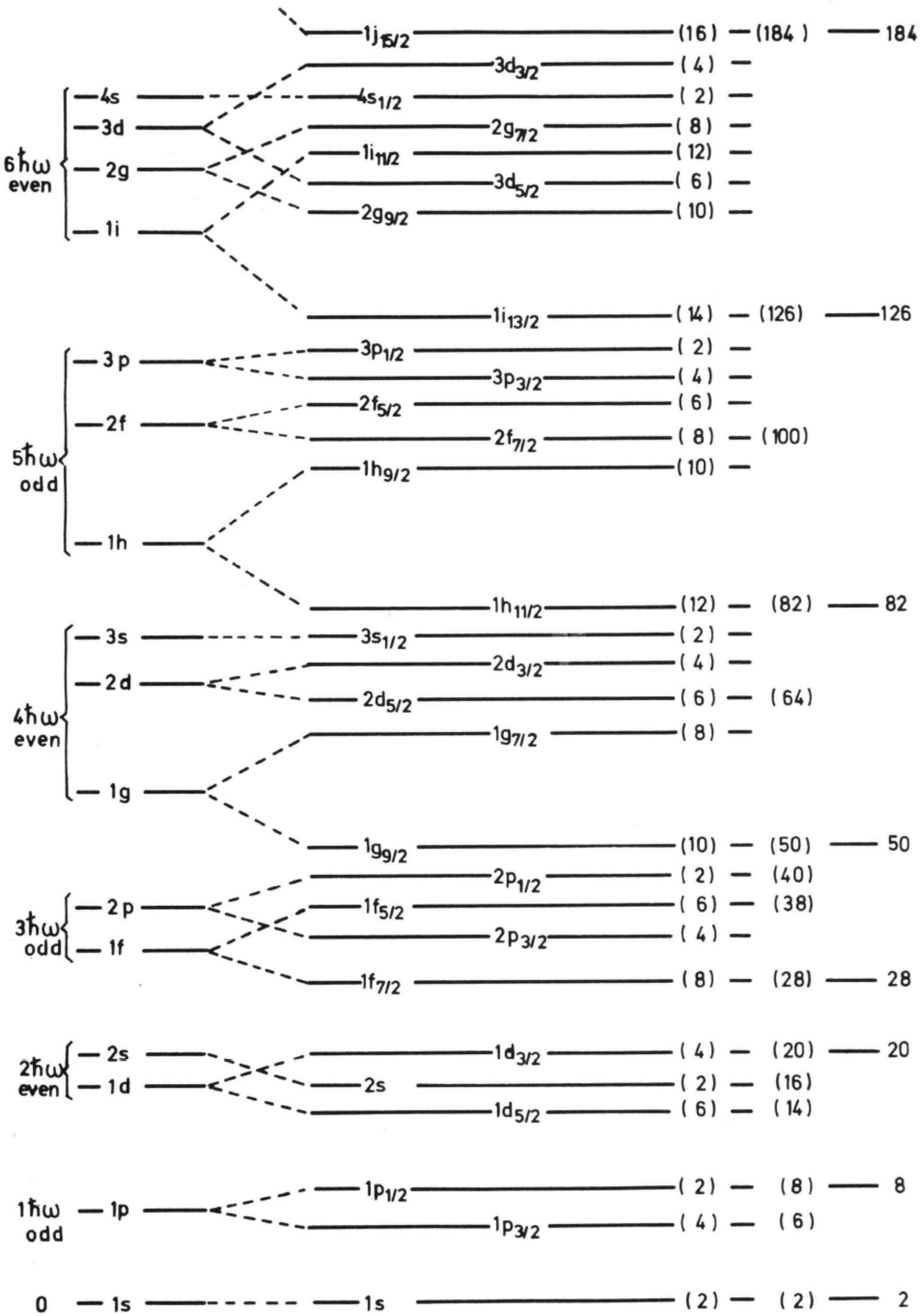
$$V(r) = V_{\text{central}} + V_{ls}(r)\langle \vec{l} \cdot \vec{s} \rangle$$

$$\vec{j} = \vec{l} + \vec{s}$$

$$\langle j^2 \rangle = \langle l^2 \rangle + \langle s^2 \rangle + 2\langle \vec{l} \cdot \vec{s} \rangle$$

$$\langle \vec{l} \cdot \vec{s} \rangle = \frac{1}{2}\langle (j(j+1) - l(l+1) - s(s+1)) \rangle = \begin{cases} l/2, & j = l + 1/2 \\ -(l+1)/2, & j = l - 1/2 \end{cases}$$

spin-orbit coupling leads to changes of a few MeV
compare to fine structure in electron shell $\sim \alpha_{em}^2,\ 10^{-5}$ effect



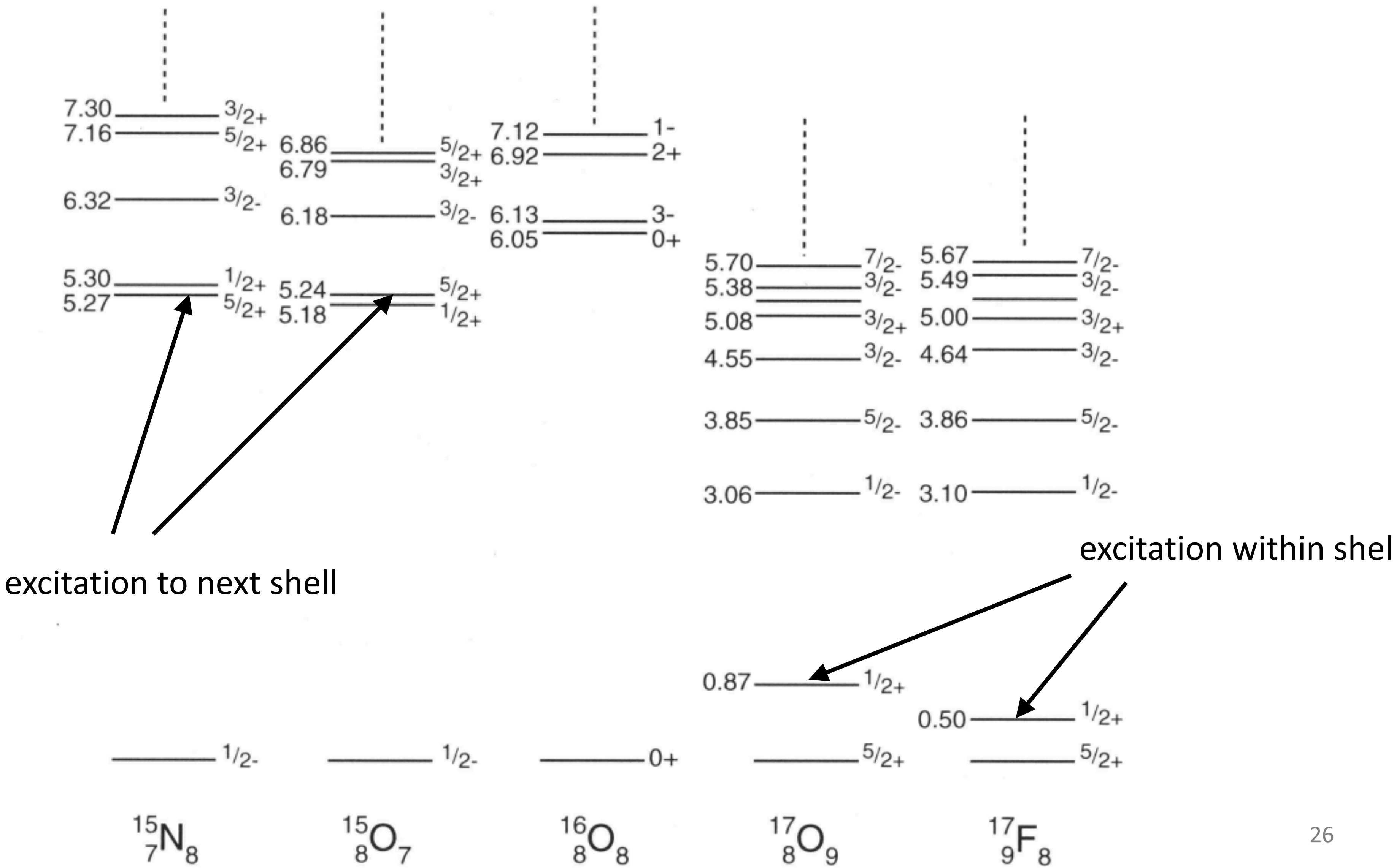
Maria Goeppert-Mayer, Phys. Rev. 75 (1949) 1969,
Otto Haxel, J.Hans D. Jensen, Hans E. Suess, Phys. Rev. 75 (1949) 1766.

$$J^{\pi}\left({}^{209}_{82}\mathrm{Pb}\right) = 9/2^{+}$$

$$J^{\pi}\left({}^{207}_{82}\mathrm{Pb}\right) = 1/2^{-}$$

Shell Model Monte Carlo calculations review
Phys. Rept. 278 (1997) 1
https://arxiv.org/abs/nucl-th/9602006

SINGLE-PARTICLE EXCITATIONS



NUCLEAR CHART - VALLEY OF STABILITY

$$\frac{\partial m(Z,A)}{\partial Z} = 0 \rightarrow N \geq Z$$