Recitation 6 – Moderns Aspects of Nuclear Physics – SS22

Discussion on Fr., July 15, 2022, 11:15

6.1 Synchrotron radiation (1 Point)

At LHC protons circulate with a design energy of 7 TeV.

a) What is the energy loss of a proton due to synchrotron radiation per turn?

b) What is the energy loss of an electron due to synchrotron radiation per turn, assuming a maximum electron energy of 104 GeV in the same tunnel (LEP)?

c) In total, $5.5 \cdot 10^{14}$ protons circulate in the LHC. What is the total power of emitted synchrotron radiation in units of Watt?

Hint: Assume the LHC being a perfect ring with a circumference of 27km.

Solution:

a)

$$W = P \cdot t_{\text{turn}} = \frac{2}{3}\frac{e^2}{R^2}\gamma^4 \cdot \frac{2\pi R}{v=c} = \frac{2}{3}\frac{e^2}{R^2}\left(\frac{E}{m}\right)^4 \cdot \frac{2\pi R}{v=c} \tag{1}$$

$$W = P \cdot t_{\text{turn}} = \frac{4\pi}{3}\frac{e^2}{R}\left(\frac{E}{m}\right)^4 \tag{2}$$

with $e^2 = 1.44$ MeVfm

$$W = \frac{4\pi}{3}\frac{1.44}{2805}\left(\frac{7000}{0.938}\right)^4 \cdot \text{MeV} \cdot 10^{-15} = 6.7 \text{ keV} \tag{3}$$

b)

$$W = \frac{4\pi}{3}\frac{1.44}{2805}\left(\frac{104}{0.0005}\right)^4 \cdot \text{MeV} \cdot 10^{-15} = 3.4 \text{ GeV} \tag{4}$$

c)

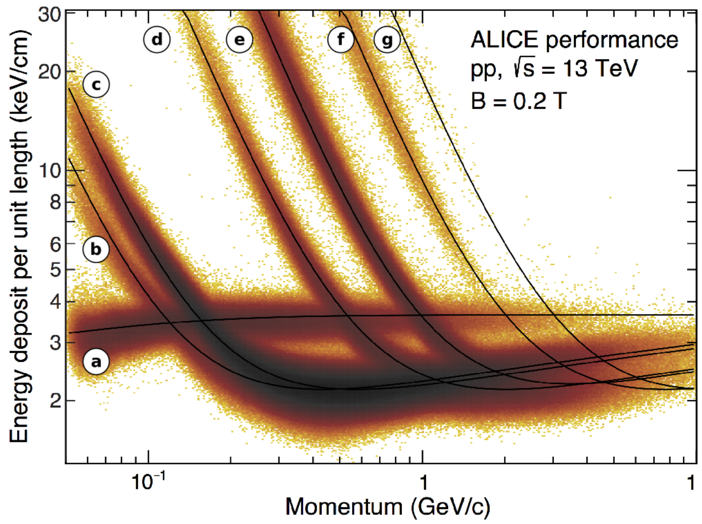
$$P_{\text{total}} = N_p \cdot P = N_p \cdot W \cdot \frac{c}{U} = 5.5 \cdot 10^{14} \cdot 6.7 \cdot \frac{3 \cdot 10^8}{26800} \cdot 10^3 \cdot 1.602 \cdot 10^{-19} \text{ W} = 6.0 \text{ kW} \tag{5}$$

N.B.: The synchrotron photons get absorbed in the liquid helium and nitrogen of the superconducting magnets. This power must be cooled off.

N.B.: The LHC is not a perfect ring but has straight sections where the experiments are located. The LHC bending radius in the arcs is 2805m. This leads to an about 10% larger energy loss than calculated above.

6.2 Particle Identification in a Time Projection Chamber (1 Point)

The ALICE Time Projection Chamber (TPC) ist a gaseous detector where propagating charged particles ionize the gas and freed electrons drift towards the readout chambers. The drift time is a measure of the distance of the originating particle trajectory to the readout plane (time projection). Besides the drift time, also the generated primary charge is recorded as a measure of the particle energy loss. The figure shows the measurement of the energy loss of charged particles in the gas volume of the ALICE TPC.



The particle energy deposited per unit path length versus particle momentum is shown on a double logarithmic scale, plot taken from PDG, Fig. 35.15. Various bands corresponding to individual particle species are visible.

(i) Assign the following particle species to the bands indicated by labels from a to g: kaon, pion, electron, muon, proton, deuteron, triton. Give a reason.

(ii) The slope of the bands corresponding to the labels from b to g exhibit a striking similarity in the steepest region. Estimate the slope in this region. Argue why this is the case.

Solution:
The minimum of the Bethe formula (minimum-ionizing particle, MIP) occurs at

$$\beta\gamma \approx 3.5.$$

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And

$$\beta\gamma = \frac{p}{E}\frac{E}{m} = \frac{p}{m}$$

Therefore, the minimum of energy loss versus total momentum p moves to larger values with increasing particle mass. Therefore:

a) Electron, minimum at $\frac{p}{m} = 3.5$, with $m_e = 0.511$ MeV, thus $p \approx 0.002$ GeV$/c$ and thus not visible in the figure. Practically, electrons are always at the Fermi plateau that follows after the relativistic rise (saturation).

b) muon

c) pion

d) kaon

e) proton

f) deuteron

g) triton

The slope follows from Bethe formula according to $dE/dx \propto 1/\beta^2$ ist.

6.3 Pion Decay (1 Point)

Consider the decay of a neutral pion into two photons: $\pi^0 \to \gamma + \gamma$.

a) Assume that the energies E_{γ_1} and E_{γ_2} of both photons and their relative angle α (opening angle) are measured in the laboratory frame. Calculate the invariant mass of the two-photon system.

b) Calculate the maximum (minimum) photon energy in the laboratory frame as a function of the total energy and momentum of the neutral pion.

c) Calculate the minimum opening angle between both photons in the laboratory frame as a function of the total energy and momentum of the neutral pion.
Hint: The minimum opening angle in the laboratory frame corresponds to both photons being emitted perpendicular to the pion momentum in the pion rest frame.

Solution:

P_π, P_{γ_1} und P_{γ_2} are four-vectors, $P^2 = m^2$.

a) $P_\pi = P_{\gamma_1} + P_{\gamma_2} \quad |(^2)$
$m_\pi^2 = m_{\gamma_1}^2 + m_{\gamma_2}^2 - 2P_{\gamma_1} \cdot P_{\gamma_2} \qquad [E_\gamma = p_\gamma]$
$m_\pi^2 = 2E_{\gamma_1}E_{\gamma_2} - 2E_{\gamma_1}E_{\gamma_2} \cdot \cos\alpha$
$m_\pi^2 = 2E_{\gamma_1}E_{\gamma_2}(1-\cos\alpha)$
$m_\pi^2 = 4E_{\gamma_1}E_{\gamma_2}\sin^2\frac{\alpha}{2}$

b) $P_\pi = P_{\gamma_1} + P_{\gamma_2}$
$P_\pi - P_{\gamma_1} = P_{\gamma_2} \quad |(^2)$
$P_\pi^2 + P_{\gamma_1}^2 - 2P_\pi \cdot P_{\gamma_1} = P_{\gamma_2}^2$
$m_\pi^2 + m_{\gamma_1}^2 - 2E_\pi \cdot E_{\gamma_1} + 2p_\pi E_{\gamma_1}\cos\theta = m_{\gamma_2}^2$, $\quad \theta$ is the angle between the photon and the pion

$m_\pi^2 = 2E_\pi \cdot E_{\gamma_1} - 2p_\pi E_{\gamma_1}\cos\theta$
$\frac{m_\pi^2}{2} = E_\pi \cdot E_{\gamma_1} - p_\pi E_{\gamma_1}\cos\theta$
E_{γ_1} is maximal (minimal) when $\cos\theta = 1$ ($\cos\theta = -1$), i.e. the photon is emitted in direction (in opposite direction) of the neutral pion.
$\frac{m_\pi^2}{2} = E_\pi \cdot E_{\gamma_1} \pm p_\pi E_{\gamma_1}$

$E_{\gamma_1} = \frac{m_\pi^2}{2(E_\pi \pm p_\pi)}$
In the laboratory frame, the decay is asymmetric with the difference between both photons increasing with increasing pion energy.

c) In the rest frame of the neutral pion, both photons have identical energy, $E_{\gamma_1} = E_{\gamma_2} = \frac{m_\pi}{2}$. This is also the transverse momentum in the laboratory frame.

When boosting to the laboratory frame, the transverse momentum is not affected, the total momentum/energy is $E_{\gamma_1} = E_{\gamma_2} = \frac{E_\pi}{2}$.
The opening angle is thus $\sin\frac{\alpha}{2} = \frac{m_\pi/2}{E_\pi/2} = \frac{1}{\gamma}$. With increasing pion energy, the opening angle decreases until both photons are not resolved anymore, e.g. in a calorimeter. This restricts pion identification at high momentum.