Recitation 4 – Moderns Aspects of Nuclear Physics – SS22

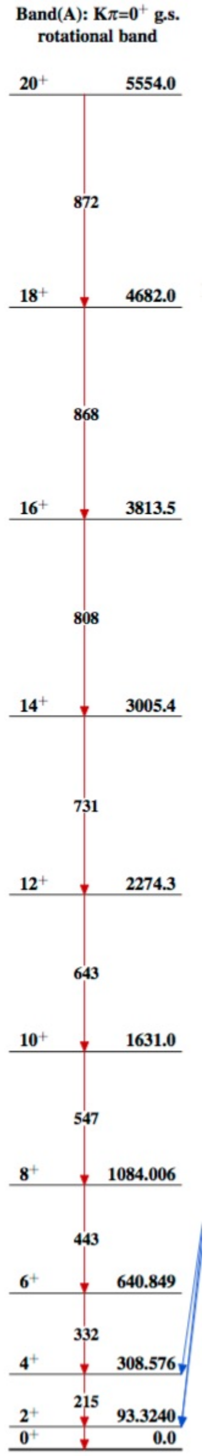
Discussion on Fr., June 17, 2022, 11:15

4.1 Nuclear Rotation (1 Point)

The Figure to the right shows energy levels (in units of keV) of a rotational band of ^{180}Hf. (Source: National Nuclear Data Center, http://www.nndc.bnl.gov)

a) Calculate the value $1/A = 2\Theta$ for this rotational band for each neighboring pair of total angular momentum J, with Θ being the moment of inertia. Plot these values as a function of the total angular momentum. Discuss your result.

b) Calculate the corresponding value $1/A$ for a rigid sphere with the half density radius and mass of ^{180}Hf. Compare with your result from a). If there is a difference, what it the reason for it?



Solution:

a) Energy levels in a rotational band are described by the relation:

$$E = \frac{J(J+1)}{2\Theta}$$

$$\Delta E = E_J - E_{J-2} = \frac{4J-2}{2\Theta}$$

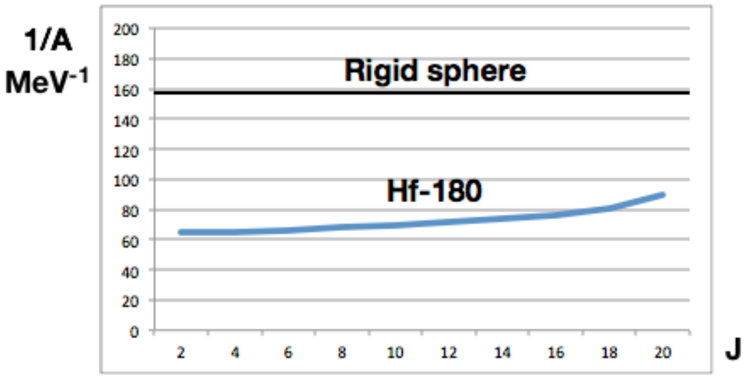
$$1/A = 2\Theta = \frac{4J-2}{\Delta E}$$

Put the values from figure in a tabulated form.

Table 1:

J	2	4	6	8	10	12	14	16	18	20
E [MeV]	0.093	0.309	0.641	1.084	1.631	2.274	3.005	3.814	4.682	5.554
ΔE [MeV]	0.093	0.216	0.332	0.443	0.547	0.643	0.731	0.809	0.868	0.872
$1/A$ [MeV^{-1}]	64.5	64.8	66.3	67.7	69.5	71.5	73.9	76.6	80.6	89.4

The corresponding figure is below.



The slow increase in the moment of inertia can be understood in the liquid droplet model as due to slow growth of deformation with increasing in rotational speed.

b) The moment of inertia for a rigid sphere is given as

$$\Theta = \frac{2}{5} M \cdot R^2$$

Hence

$$1/A = \frac{2\Theta}{\hbar^2} = \frac{4}{5} M R^2$$

The mass can be taken as $M = 180 \times 931.494 = 167669$ MeV;

The radius - $R = 1.2 \times 180^{1/3} = 6.77$ fm;

$$1/A = \frac{4}{5} \cdot \frac{167669 \times 6.77^2}{197.327^2} \frac{\mathrm{MeV} \cdot \mathrm{fm}^2}{\mathrm{MeV}^2 \cdot \mathrm{fm}^2} = 158\ \mathrm{MeV}^{-1}$$

This value is about 2 times larger than the one estimated in a). This can qualitatively be understood as if the paired nucleons do not take part in the rotation and do not contribute to the momentum of inertia - the superfluid rotor.

c) Discussion: What happens if one further increases J? At some point the Coriolis force wins over the pairing force and the pair breaks up. The orbital momenta then line up. This leads to a sudden change of the moment of inertia (Backbending).

4.2 Nuclear Fusion in the Sun (1 Point)

In the sun, the fusion of hydrogen to helium is dominated by the *pp*-cycle.

2×	$p + p$	$\rightarrow$	$d + e^+ + \nu_e$
2×	$d + p$	$\rightarrow$	$^3He + \gamma$
	$^3\text{He} +^3 \text{He}$	$\rightarrow$	$^4\text{He} + 2p$
	$4p$	$\rightarrow$	$^4\text{He} + 2e^+ + 2\nu_e + 2\gamma$

a) Which of these three reaction determines the life time of the sun?

b) Show that the energy release in the *pp*-cycle is 26.72 MeV. Take into account that the released positrons annihilate in matter with electrons.

Hint: Inside the sun, atoms are fully ionized. The mass excess considers the atomic masses, i.e. including the electrons. Assume neutrinos as massless.

c) Assume that fusion occurs when both nuclei touch each other, i.e. $d = r_1 + r_2$. When both nuclei are approaching each other, the Coulomb barrier must be overcome. Calculate for each fusion reaction the minimum kinetic energy and compare it to the average kinetic energy in the centre of the sun.

Hint: The temperature in the centre of the sun is $T \approx 1.55 \cdot 10^7$ K.

d) Each neutrino of the *pp*-cycle carries 300 keV on average. The remaining energy is emitted by electromagnetic radiation. The luminosity of the sun in terms of electromagnetic radiation is $L = 3.8 \cdot 10^{26}$ W.

How many fusion reactions take place each second? What is the mass decrease of the sun per second?

e) Estimate the total flux of solar neutrinos on earth. The average distance sun-earth is $1.49 \cdot 10^8$ km.

Solution:

a) The first reaction determines the lifetime of the sun since it is a weak reaction.

b) $p + p \rightarrow d + e^+ + \nu_e$:
$\Delta E_1 = 2m_pc^2 - m_dc^2 - m_ec^2 = 2 \cdot$ 938.27 MeV -1875.61 MeV - 0.511 MeV = 0.42 MeV.
Since positron annihilation frees an additional $2m_ec^2$ = 1.022 MeV, a total of $\Delta E_1' = 1.44$MeV is released.
$d + p \rightarrow^3 He + \gamma$:
$\Delta E_2 = m_p + m_d - m(^3He^{++}) = 938.27\text{MeV} + 1875.61\text{MeV} - 2808.35\text{MeV} = 5.53\text{MeV}$
$^3He +^3 He \rightarrow^4 He + 2p$
$\Delta E_3 = 2m(^3He^{++}) - m_\alpha - 2m_p = 2 \cdot 2808.35\text{MeV} - 3727.38\text{MeV} - 2 \cdot 938.27\text{MeV} = 12.78\text{MeV}.$

In total, one gets for the reaction $4p \rightarrow^4 He + 2e^+ + 2\nu_e$:
$\Delta E_{pp} = 2\Delta E_1' + 2\Delta E_2 + \Delta E_3 = 26.72\text{MeV}$.
Or, simpler ... :
$4m_pc^2 + 2m_ec^2 - m_{^4He}c^2 = 4 \cdot 938.27\text{MeV} + 2 \cdot 0.511\text{MeV} - 3727.38\text{MeV} = 26.72\text{MeV}$.

c) $E_C = \frac{Z_1Z_2e^2}{4\pi\epsilon_0(r_1+r_2)} = \frac{1}{137}\frac{Z_1Z_2 \cdot 197MeVfm}{r_1+r_2} = 1.44\frac{Z_1Z_2}{r_1+r_2}\text{MeVfm}$.
$p + p : r_p = 0.88fm; E_C = 0.82\text{MeV}$,
$d + p : r_d = 2.14fm; E_C = 0.47\text{MeV}$
$^3He +^3 He : r_{^3He} = 1.96\text{fm}; \text{E} = 1.47\text{MeV}$. The thermal energy of particles in the centre of the sun ($T \approx 1.55 \cdot 10^7$ K) amounts to 2.0 keV.
The thermal energy is 2-3 orders of magnitude less than the Coulomb barrier.

For discussion: Why does nuclear fusion still takes place inside the sun? Due to quantum mechanical tunneling.

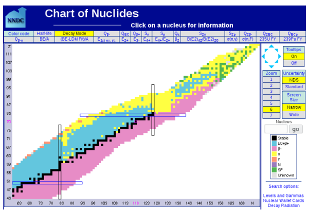
d) The number of fusion reactions equals the total electromagnetic radiation power divided by the electromagnetic radiation released per fusion cycle.
$N = \frac{L}{26.1\text{MeV}} = 9.1 \cdot 10^{37}\text{s}^{-1}$.

The mass loss of the sun corresponds to the total energy released (including the neutrinos) $\Delta M = N \cdot 26.7\text{MeV} = 2.5 \cdot 10^{45}\text{eV} = 0.43 \cdot 10^{10}\text{kg} = 4 \cdot 10^6\text{t}$ per second.
This is negligible in comparison to the total mass of the sun, ($M_{\text{sun}} = 2 \cdot 10^{30}\text{kg}$).

e) The emission of neutrinos is isotropic, i.e. the flux on earth equals the total number of neutrinos emitted per time, divided by the visible surface of a sphere with radius $R = d_{\text{sun}-\text{earth}}$.
For each radiated energy of 26.1 MeV, 2 neutrinos are emitted, i.e.
$\Phi_\nu = \frac{1}{4\pi R^2}\frac{L}{26.1\text{MeV}} \cdot 2 = 6.5 \cdot 10^{10}\text{cm}^{-2}\text{s}^{-1}$.

4.3 Heavy-element synthesis in the universe (1 Point)

In the s-process in massive stars (red giants), elements with high proton number can be synthesized at low neutron densities as long as the half life of neutron capturing isotopes is larger than about 10^3 years.



Answer the following questions by using the interactive nuclear chart (→ Decay Mode) at https://www.nndc.bnl.gov/nudat3/

a) Can gold nuclei be produced in the s-process, under the assumption made above?
Start your considerations with the nucleus ^{22}Ne that provides a neutron source via the reaction $^{22}\text{Ne} + \alpha \rightarrow ^{25}\text{Mg} + \text{n}$.

b) Which naturally occurring isotopes can not be produced via the s-process? Give three examples.

c) Which other astrophysical events exist that can produce these isotopes? Give a reason.

Solution:

a) In general possible, yield can not be determined.

b) ^{238}U : r - process,
^{110}Pd : r - process,
^{92}Mo : rp - Process.

c) r - Prozess: Supernovae, Neutron Star Merger
rp - Prozess: Binary star systems with accreting neutron star

---