Recitation 2 – Moderns Aspects of Nuclear Physics – SS20

Discussion on Fr., May 20, 2022, 11:15

2.1 EIC kinematics (1 Point)

At the planned electron-ion collider, electrons with an energy of 18 GeV will collide head-on with protons with an energy of 275 GeV.

a) Calculate the center-of-mass energy $\sqrt{s}$.

b) Calculate the velocity of the center-of-mass system in the laboratory frame.

c) Calculate the dependence of the square of the four-momentum transfer, Q^2, as a function of the Bjorken-x variable at a constant inelasticity $y = 0.95$. Plot the values in the $\ln Q^2 - \ln x$-plane.

Solution:

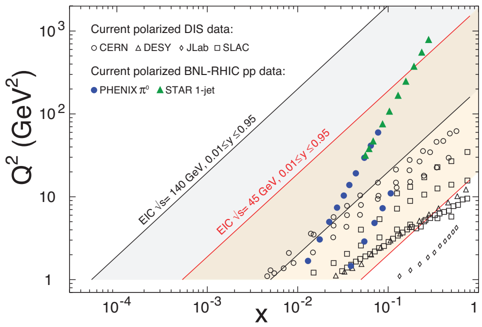
a) $s = (k + p)^2 = (275 + 18)^2 - (275 - 18)^2 = 19800\text{GeV}^2$. $\sqrt{s}$=140 GeV

b) $\beta_{\text{CMS}} = \frac{v_{\text{CMS}}}{c} = \frac{p_{\text{CMS}}}{E_{\text{CMS}}} = \frac{E_p^{\text{lab}} - E_e^{\text{lab}}}{E_p^{\text{lab}} + E_e^{\text{lab}}} = \frac{257}{293} = 0.877$

c) $Q^2 = xys$

In a log-log graph, relationships like $y = a \cdot x^k$ appear as straight lines, with k as the slope.

$\ln y = k \cdot \ln x + \ln a$, with a the intercept on the $\ln y$ axis, e.g. when reversing the logs, a is the value $y(x = 1)$.



2.2 Color Factors (1 Point)

When calculating cross sections in QCD, color factors take into account that 8 gluons exist which can possibly participate in the interaction, in comparison to QED with only one photon, see Fig. 1.

a) Choose your favorite color for the initial quark i. Write down a Feynman diagram for gluon emission off that quark. Keep in mind that color is conserved at the vertex. Indicate the color flow.

b) Consider the following 8 linearly independent wave functions for the gluons.

$$\psi_1 = |r\bar{g}\rangle$$

$$\psi_2 = |r\bar{b}\rangle$$

$$\psi_3 = |g\bar{r}\rangle$$

$$\psi_4 = |g\bar{b}\rangle$$

$$\psi_5 = |b\bar{r}\rangle$$

$$\psi_6 = |b\overline{g}\rangle$$

$$\psi_7 = \frac{1}{\sqrt{2}}(|r\overline{r}\rangle - |g\overline{g}\rangle)$$

$$\psi_8 = \frac{1}{\sqrt{6}}(|r\overline{r}\rangle + |g\overline{g}\rangle - 2|b\overline{b}\rangle)$$

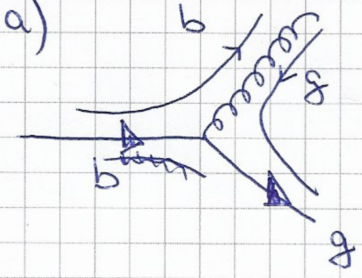
Write down all tree-level Feynman diagrams for a) and indicate the gluon wave function. Each diagram results in a factor c_i that is proportional to the overlap of the gluon wave function and the quark color. For a quark-gluon vertex, the color factor is then

$$C_F = \frac{1}{2}\sum c_i^2$$

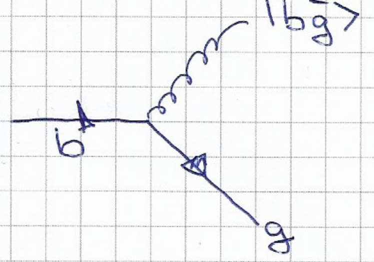
c) In a similar way, chose your favorite gluon and calculate T_F.

Solution:

2.1 a) $|b\bar{g}\rangle = 4_6$



b)



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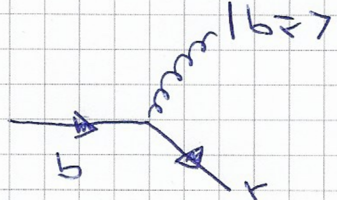

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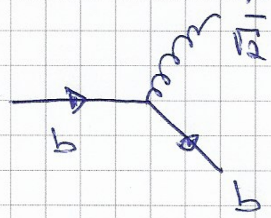
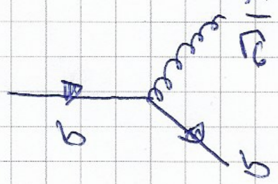
$\frac{1}{\sqrt{2}}\left(|r\bar{r}\rangle - |g\bar{g}\rangle\right) \sim 0$



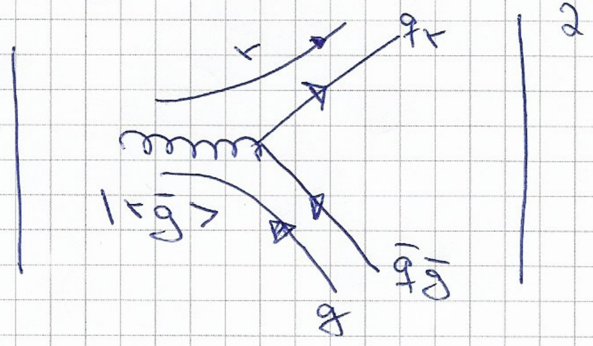
$\frac{1}{\sqrt{6}}\left(|r\bar{r}\rangle + |g\bar{g}\rangle - 2|b\bar{b}\rangle\right) \sim \frac{2}{\sqrt{6}}$



$C_F \sim \frac{1}{2}\left| \quad \right|^2 = \frac{1}{2}\left(1 + 1 + \frac{4}{6}\right) = \frac{1}{2}\frac{16}{6} = \frac{4}{3}$

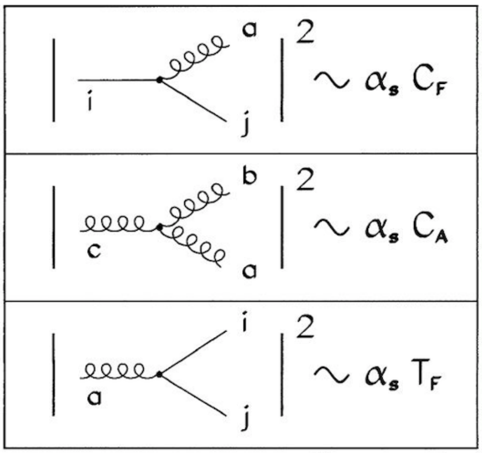
red quark in initial state: $C_F = \frac{1}{2}\left(1 + 1 + \frac{1}{2} + \frac{1}{6}\right) = \frac{1}{2}\frac{16}{6} = \frac{4}{3}$
[gluons $4_1, 4_2, 4_7, 4_8$ involved]

2.1.b) T_F : gluon splitting into a $q, \bar{q}$ pair:



$\sim 1 \Rightarrow T_F = 1/2$

N.B.: The 3-gluon vertex has a more complicated structure.



2.3 Froissart bound (1 Point)

The Froissart bound is given by $\sigma \leq \frac{\pi}{m_\pi^2} \ln^2 \frac{s}{s_0}$, with m_π the mass of the lightest exchanged meson, e.g. the pion, $\sqrt{s}$ the center-of-mass energy, and s_0 an unspecified constant usually taken at the hadronic mass scale, i.e. 1 GeV.

a) Calculate the black disk limit for the proton-proton cross section.

b) Calculate the Froissart bound for proton-proton collisions at a center-of-mass energy of $\sqrt{s} = 13$ TeV.

c) Look up the total cross section for proton-proton scattering at the LHC center-of-mass energy of $\sqrt{s} = 13$ TeV, e.g. on page 10 of the cross section review of the particle data book.

d) Assuming that the number N of gluons increases with decreasing Bjorken-x as $N \sim (1/x)^\lambda$, the total cross section increases as $\sigma_{\text{tot}} \sim s^\lambda$, with $\lambda \approx 1/3$.
At which center-of-mass energy would the Froissart bound be exceeded?

Solution:

a) $\sigma_{\text{geo}} = 2\pi R^2$, with $R = 0.8\text{fm}$
$\sigma_{\text{geo}} = 2\pi \cdot 0.8^2 \text{fm}^2 = 4\text{fm}^2 = 40\text{mb}$.

b) $\sigma \leq \frac{\pi}{m_\pi^2} \ln^2 \frac{s}{s_0} = \frac{\pi}{140^2 (\text{MeV})^2} \ln^2 \left(s/(1\text{GeV}^2)\right)$
This is the cross section bound in natural units. In order to convert it into units of an area, one has to multiply by $(\hbar c)^2 = (200 \text{ MeV} \cdot \text{fm})^2$
$\sigma \leq \frac{\pi}{140^2 (\text{MeV})^2} \cdot (200 \text{ MeV} \cdot \text{fm})^2 \ln^2 \left(s/(1\text{GeV}^2)\right)$
$\sigma \leq \frac{200^2 \pi}{140^2} \text{fm}^2 \ln^2 \left(s/(1\text{GeV}^2)\right)$
$1 \text{ b} = 10^{-28} \text{m}^2$
$1 \text{ mb} = 10^{-31} \text{m}^2$;
$\sigma \leq 64\text{mb} \ln^2 \frac{s}{1\text{GeV}}$
At LHC, $\sqrt{s} = 13\text{TeV} = 13000$ GeV.
$\sigma \leq 64\text{mb} \ln^2 (13000^2) = 22971$ mb.

c) $\sigma_{pp}(13\text{TeV}) \approx 100\text{mb}$, and thus far from the Froissart bound

d) $\sigma_{\text{tot}} = C \cdot s^{1/3}$, at LHC $\sigma_{\text{tot}} = 100\text{mb}$.

$C = \frac{100\text{mb}}{(13000\text{GeV})^{1/3}}$

$s = 1.75 \cdot 10^{12}\text{GeV}^2$

$\sqrt{s} = 1.3 \cdot 10^{6}\text{GeV} = 1300\ \text{TeV}$.
