The measurement problem

Your passion for physics: What are you curious about? | Prof. Dr. Thomas Pfeifer | 2021/22 | Niclas Göring

Abstract: Standard quantum mechanics predicts the existence of macroscopic superpositions. The contradiction to the experimental fact that they are not observed is called the measurement problem (MP). Different solutions to the MP give rise to different interpretations of quantum mechanics (QM).

1. Measurement problem

The MP was first pointed out in the famous gedanken-experiment – Schrödinger's cat [10]. Maudlin [8] was the first to formulate it rigorously as the contradiction of the following three assumptions:

(A) The wave function completely describes the system.

(B) The wave function always evolves according to Schrödinger's equation.

(C) Measurements have unique outcomes described by Born's rule.

The contradiction arises when applying the QM rules to the measurement device. We assume that a radioactive particle can either be in a decayed $|d\rangle$ or not-decayed $|\neg d\rangle$ state. By (A), in principle, we are able to find a wave function describing our whole measurement device. It can be in the states $\{|0\rangle, |\text{yes}\rangle, |\text{no}\rangle\}$ corresponding to no measurement at all, a decay measurement or a no-decay measurement. Assuming the measurement device works, i.e. it points to 'yes', when the particle decays $|0\rangle |d\rangle \to |\text{yes}\rangle |d\rangle$ or to 'no' when the particle does not decay $|0\rangle |\neg d\rangle \to |\text{no}\rangle |\neg d\rangle$, the MP arises when looking at a superposition $c_1 |\neg d\rangle + c_2 |d\rangle$ of the radioactive particle. By (B), we evolve the state $(c_1 |\neg d\rangle + c_2 |d\rangle) |0\rangle$ as indicated above resulting in the final state

$$|\psi_{\text{final}}\rangle = c_1 |\neg d\rangle |\text{no}\rangle + c_2 |d\rangle |\text{yes}\rangle . \tag{1}$$

This state corresponds to the measurement device pointing to 'no' and 'yes' at the same time. But this is in contradiction to (C). The measurement device can only point to 'no' i.e., the final state is $|\neg d\rangle |\text{no}\rangle$ *or* point to 'yes' i.e., the final state is $|d\rangle |\text{yes}\rangle$ (but not to 'no' and 'yes' at the same time).

2. Three Solutions

The MP is solved by negating one of the three assumptions giving rise to a categorization of the different interpretations of QM:

(¬A) Hidden variable theories (e.g. Bohmian mechanics)

(¬B) Collapse theories (e.g. Copenhagen, GRW)

(¬C) Many-worlds theories

(¬B) Copenhagen [9, 2]. The Copenhagen interpretation solves the MP by the collapse postulate: The wave function collapses into an eigenstate of the operator corresponding to the measured observable. Introducing the observer into the axioms of QM is philosophically troublesome [1]. For that, the following two interpretations of QM are interpretations without an observer.

(¬B) GRW theory [9, 6]. The simplest collapse interpretation without an observer is the GRW theory. Here, the collapse is not induced by a measurement but happens randomly with rate λ. The collapse rate scales with the system size, such that macroscopic superpositions, like (1), collapse very quickly.
The theory's details are mainly concerned with formulating a physical law for the collapse. Between two collapses, the wave function evolves according to Schrödinger's equation. When a collapse happens at time T and position X, the wave function is multiplied by the collapse operator L_X

$$\psi \to \frac{L_X \psi}{\|L_X \psi\|} \quad L_X := \left(\frac{1}{\pi a^2}\right)^{\frac{3}{4}} e^{-\frac{(q-X)^2}{2a^2}} \tag{2}$$

which is a Gaussian with width a, the collapse length, a new natural constant, indicating how well the particle is localized after the collapse.

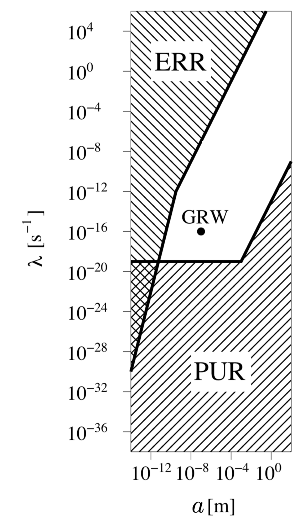

Figure 1: Parameter plot for the collapse- rate λ and length a with an experimental (ERR) and theoretical (PUR) excluded region [7].

Depending on how one chooses the values for the two new natural constants, the theory makes different predictions from standard QM, giving rise to experimental tests (c.f. figure 1).

(¬A) Bohmian mechanics [5, 6]. As a hidden variable theory, Bohmian mechanics introduces particle positions in addition to the wave function to describe the physical system. A macroscopic superposition as (1) cannot exist since the particles the measurement device is made of, have definite positions all times.
The theory's details are mainly concerned with formulating a dynamical law for the particle positions

$$\dot{\mathbf{q}}(t) = \mathbf{v}^{\psi}(t) = \frac{1}{m}\nabla S \quad \leftrightarrow \quad \psi = Re^{\frac{i}{\hbar}S}. \tag{3}$$

The phase of the wave function determines the velocity of the particle while the wave function always evolves according to Schrödinger's equation. The guidance law can be either thought of as the particle 'surfing' on the wave function [3] or as a generalization of de-Broglie's law.
A possible experimental test is the distribution of arrival times. A particle is trapped in a potential and released to fly to a detector which records the time it arrives. To date, there is no way to calculate first arrival times with standard QM while it is possible to calculate them with Bohmian mechanics [4, 11].

Further details and advanced topics are discussed in [6].

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