Fractals in Atomic, Molecular and Optical Physics

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Abstract

In 1982 Benoît B. Mandelbrot came up with the idea of fractals. Nowadays this concept is widely used in physics. In the following its application to Atomic, Molecular and Optical Physics (AMO) through fractional quantum mechanics is shown.

1 Fractals

Mandelbrot described fractals as self-similar objects [8]. But the concept of the fractal dimension (FD) is more widely applicable. The FD is computed by calculating an exponent, which gives the relation between the scaling of the decreasing side length of a measure of an object and the scaling of the then resolved object compared to the original object as a whole. By doing so, one finds the FD of complicated objects, such as the Sierpinski Triangle (Figure 1), to be non-integer numbers. There are several ways for an intuition on the FD. First, the FD is telling the roughness of an object, which helps to describe complicated objects without unnecessary simplifications. Another approach: the FD tells how much more information one can get, by increasing the resolution while looking at an object. As an example one can name clouds: they show a FD between one and two, varying between different types of clouds, which even makes them distinguishable for satellites [3]. [7]

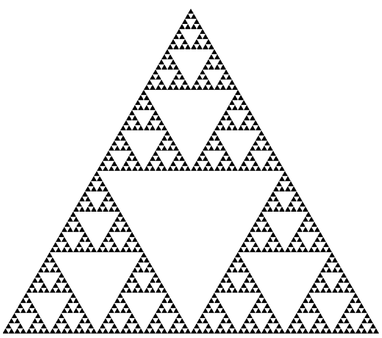

Figure 1: The Sierpinski Triangle [1]

2 Fractional Quantum Mechanics

Quantum Mechanics (QM) can be derived through the Feynman Path Integral, by integrating over all Brownian Paths (BP) from an initial state to a final state. By generalizing the step size from a normal distribution to a Lévi-Distribution (heavy-tailed, as it allows higher values), one arrives at the Lévi Flight (LF) and finds Fractional Quantum Mechanics (FQM). All LFs are self-similar and their FD turns out to be the so-called Stability Parameter α. It defines the appearance of the trajectories and can take any values within $0 < \alpha \leq 2$ (in case of the BP $\alpha = 2$). FQM then is characterized by the FD α. This appears in the most important resulting equations: The Dispersion Relation states, that the energy is proportional to the momentum to the power of α. The Fractional Schrödinger Equation includes the Fractional Hamiltonian, which differs from QM through the derivative with respect to space. It enters as a fractional derivative of the order α. [7]
Potential Realizations for tests of FQM have been developed and one experimental result has been obtained: Heavy nuclides were treated as rigid rotors and FQM with $\alpha \neq 2$ was used to calculate their eigenenergies. Compared with experimental results, the root mean square errors were mostly less than two percent. This is a better result, than what one finds through Taylor expansions up to second order. [4, 7]
In the limit of $\alpha \to 2$ FQM reproduces the well-known QM. Further tests still need to be realized. [7]

3 Outlook on Fractals

As further applications of fractals in AMO: Optical scattering and diffraction was approached by using fractals already in the early 2000s [2, 5]. And in biophysics it was found, that the FD applied to organic molecules, while giving really simple calculations, incorporates all known to be important features for describing molecular complexity [6].

References

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