Trapped-Ion Quantum Computer

Seminar: Your passion for physics

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Heidelberg University

November 2, 2021

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- 1996 J. A. Jones and M. Mosca: first physical implementation of a quantum algorithm (NMR, 2 qbits)

Quantum computing basics

Qubits and Quantum Circuits [1]

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- q(u)bit: QM 2-level system (2D complex Hilbert space $\mathcal{H}_1$)
  general state:
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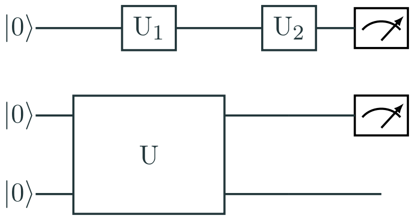
- quantum circuit example



Example: Deutsch-Josza algorithm [2],[3]

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- classically: 2 to $2^{n-1} + 1$ queries
- quantum mechanically: 1 query (exponential speed-up!)

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- Hadamard gate: $\mathrm{H} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

  $\mathrm{H}\left|0\right\rangle = \frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}}$
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- (n+1)-qubit gate $\mathrm{U_f} : \left|\mathrm{x}, \mathrm{y}\right\rangle \mapsto \left|\mathrm{x}, \mathrm{y} \oplus \mathrm{f}(\mathrm{x})\right\rangle$
  $\mathrm{x} \in [0, 2^{\mathrm{n}} - 1]$, $\mathrm{y} \in \{0, 1\}$

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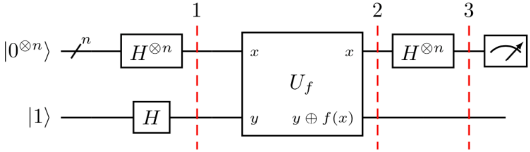

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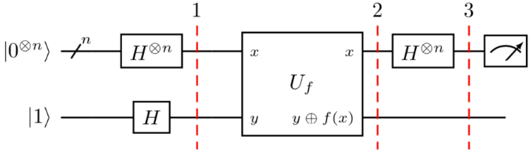

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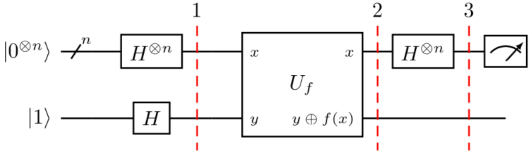

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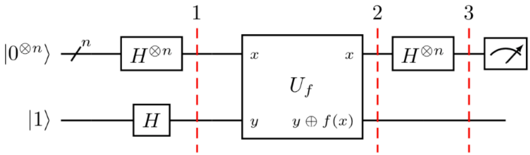

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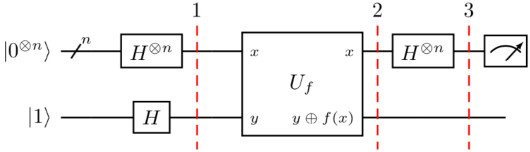

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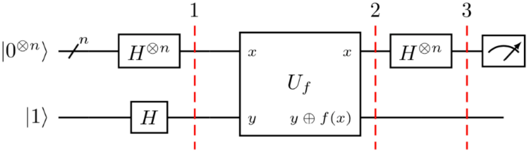

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   f constant $\implies P(|0\rangle^{\otimes n}) = 1$ f balanced $\implies P(|0\rangle^{\otimes n}) = 0$

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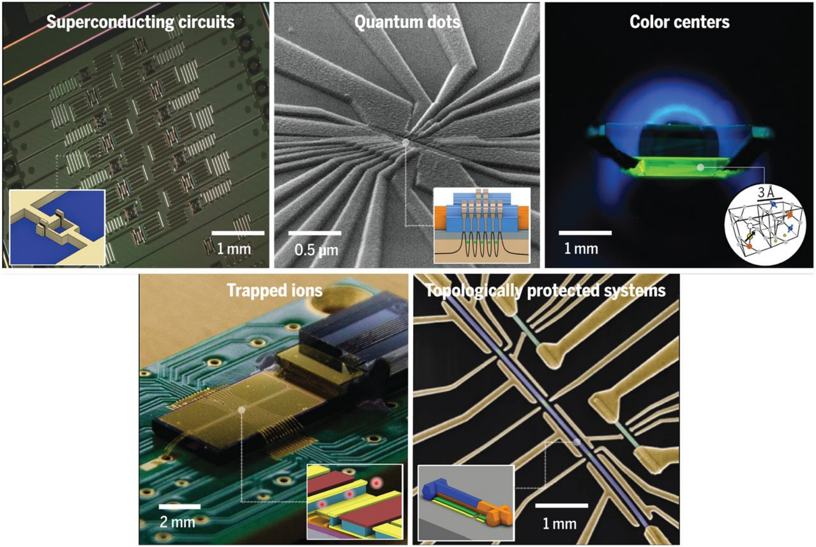
Different existing platforms



Source: DOI: 10.1126/science.abb2823

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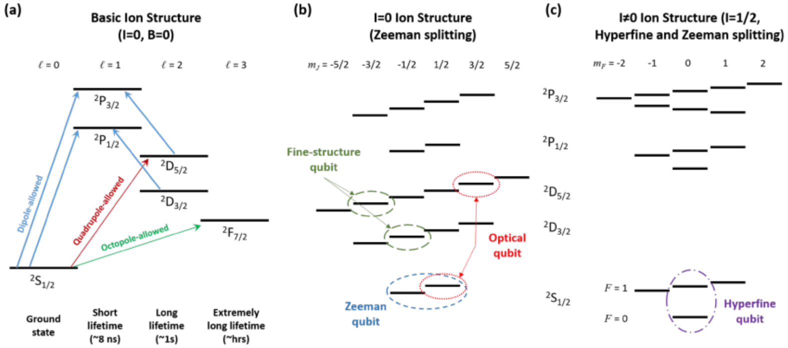
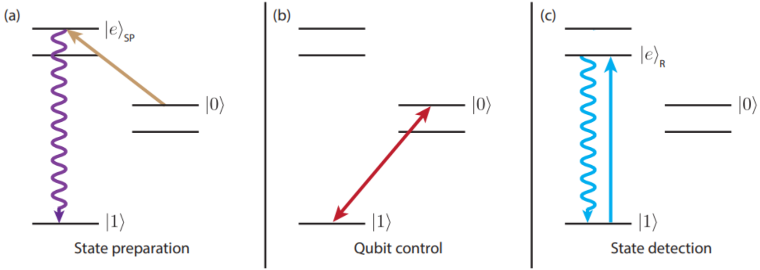
5. A qubit-specific measurement capability

Physical qubits in ions [5]



Level structure in singly-ionized earth-alkali and typical qubit choices. Source: [5]

Preparation, Control and Read-out



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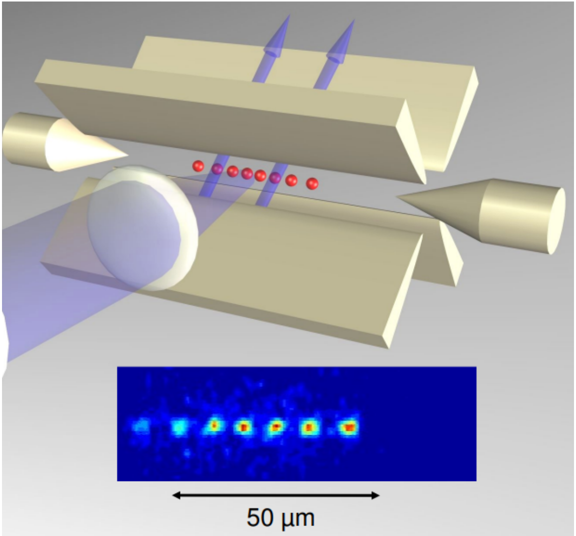
Trapping ions



Linear Paul trap. Source: [6]

Coupling different states: typical realizations [5]

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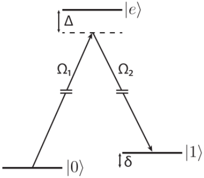
- Raman gates: stimulated Raman coupling using 2 lasers



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Single-qubit gates: Rabi oscillation [1],[6]

$$\mathrm{R}(\theta,\phi) = \exp(\mathrm{i}\tfrac{\theta}{2}(\mathrm{e}^{\mathrm{i}\phi}\sigma_+ + \mathrm{e}^{-\mathrm{i}\phi}\sigma_-)) = \begin{bmatrix} \cos\frac{\theta}{2} & \mathrm{i}\mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2} \\ \mathrm{i}\mathrm{e}^{-\mathrm{i}\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

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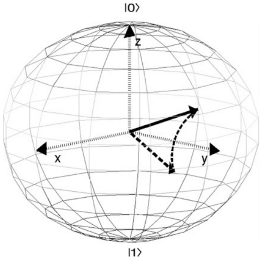
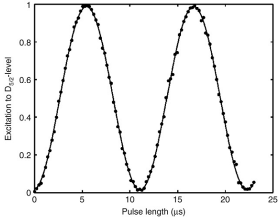


(c) Rotation around y-axis on Bloch sphere



(d) Rabi oscillation in Ca^+ ion (1000 measurements per point)

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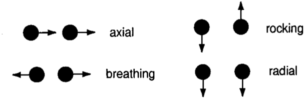

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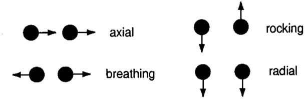

- Preparation: cool the CM mode to the ground state

Entangling gate: Cirac-Zoller gate [5],[7]

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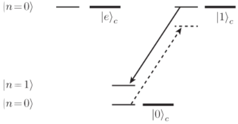

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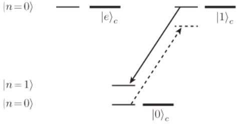

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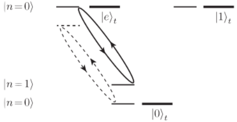

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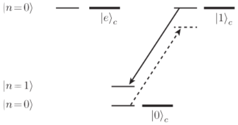

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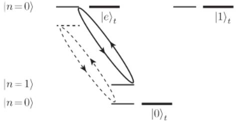

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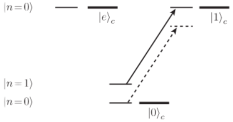

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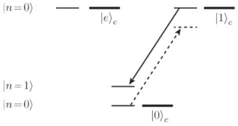

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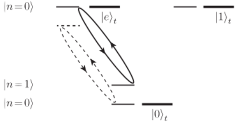

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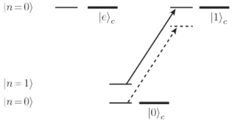

Overall: $|11\rangle \mapsto -|11\rangle$; $\quad |00\rangle, |01\rangle, |10\rangle$ unaltered

Example: IonQ 11 qubit QC [8]

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- Raman (2-photon) coupling

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- Qubits: $|0\rangle = |\mathrm{F} = 0\rangle$ and $|1\rangle = |\mathrm{F} = 1\rangle$ of $^2\mathrm{S}_{1/2}$ in $^{171}\mathrm{Yb}^+$
- Linear surface RF-trap
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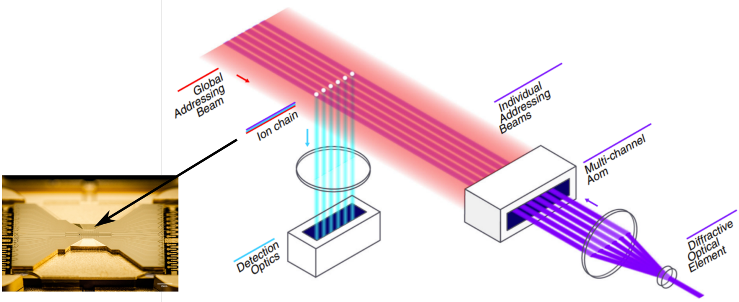

Source: [8], IonQ website

Technical progress [5]

- Fully connected register size up to 20 qubits (2018)
- Coherence time: up to several minutes
- Single-qubit gate time: $\sim 1\ \mu$s
- Single-qubit gate fidelity: $\sim 0.999 \ldots 0.999999$
- Two-qubit gate time: $\sim 10\ \mu$s
- Two-qubit gate fidelity: $\sim 0.98 \ldots 0.998$

Recap: Recipe for quantum supremacy

1. Find a hard problem for a classical computer and
   a quantum algorithm giving you a relevant speed-up
2. Construct a quantum system with well-characterized qubit states
   (trap and cool ions, choose qubit states)
3. Implement control: initialization, 1-qubit gates, 2-qubit entangling gate
   (laser or microwave pulses)
4. Implement a quantum mechanical observer/measurement
   (state-dependent fluorescence)
5. Optimize and hope for the best

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