3. Electric and magnetic dipole moments

(follows lecture from S. Westhoff)

3.1 Phenomenology

In non-relativistic electrodynamics the interaction of a particle (with Spin $\vec{S}$) with an electric and magnetic field is described by the Hamiltonian

$$\mathcal{H} = -\mu \frac{\vec{S}}{|\vec{S}|} \cdot \vec{B} - d \frac{\vec{S}}{|\vec{S}|} \cdot \vec{E}$$

μ: magnetic dipole moment (MDM)
d: electric dipole moment (EDM)

Transformation properties under parity (P) and time reversal (T):

$$P: \quad \vec{B} \to +\vec{B}, \quad \vec{E} \to -\vec{E}, \quad \vec{S} \to +\vec{S}$$

$$T: \quad \vec{B} \to -\vec{B}, \quad \vec{E} \to +\vec{E}, \quad \vec{S} \to -\vec{S}$$

Contribution of μ is P-even and T-even $\xrightarrow{\text{CPT}}$ CP-conserving
d is P-odd and T-odd $\xrightarrow{\text{CPT}}$ CP-violating

In relativistic quantum electrodynamics MDMs and EDMs are induced by the following effective operators:

$$-\mu_e \frac{\vec{S}}{|\vec{S}|}\cdot\vec{B} \quad \Leftrightarrow \quad e\left(\bar{e}\gamma_\mu e\right)A^\mu + a_e \frac{e}{4m_e}\left(\bar{e}\sigma_{\mu\nu}e\right)F^{\mu\nu}$$

$$-d_e \frac{\vec{S}}{|\vec{S}|}\cdot\vec{E} \quad \Leftrightarrow \quad + d_e \frac{i}{2}\left(\bar{e}\sigma_{\mu\nu}\gamma_5 e\right)F^{\mu\nu}$$

The coupling $e(\bar{e}\gamma_\mu e)A^\mu$ induces the magnetic moment with the gyromagnetic factor $g \;\; = 2$.

The dipole operators induce an anomalous magnetic moment a_e with

$$\mu_e = g_e \frac{e}{2m_e} \quad \text{and} \quad \left(g_e - 2\right) = 2a_e$$

and a CP-violating electric dipole moment d_e.

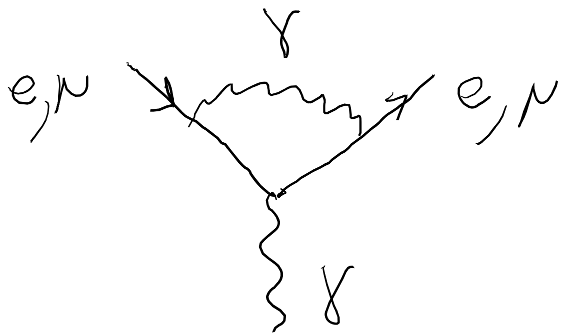
In quantum field theory a_e and d_e are induced by quantum corrections to the interaction of the lepton with a static electromagnetic background field.

These corrections can be calculated from loop-diagrams at zero momentum transfer.

3.2 Anomalous magnetic moment (g-2)

3.2.1 Phenomenology:

a) Electromagnetic contributions (dominant)



$$a_\ell^{QED} = \frac{\alpha}{2\pi}$$ (Schwinger, 1948)

b) Vacuum polarization

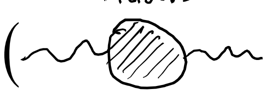
leptonic

hadronic

Hadronic contribution is not calculable in perturbative QCD.

Instead: Optical theorem*):

$$\sigma\left(e^+e^- \to \gamma^* \to \text{hadrons}\right) =$$

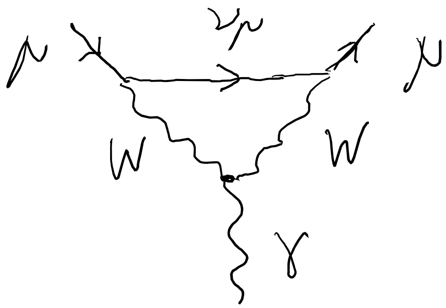
$$\frac{4\pi\alpha}{s}\,\Im\Big($$  $$\Big)(s)$$

→ "measure" hadronic contributions a_μ^{had}

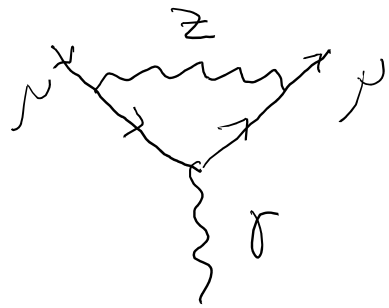
(availability and treatment of data leads to uncertainties)

*) Total cross section ~ imaginary part of fw scattering amplitude

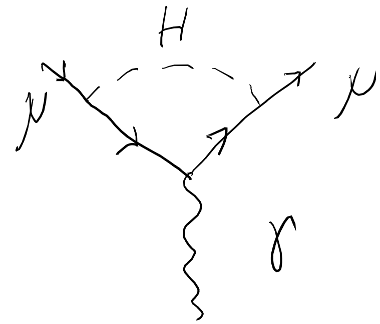
b) Weak contributions



$$\sim \frac{m_e^2}{M_W^2}, \frac{m_\mu^2}{M_W^2}$$



$$\sim \frac{m_e^2}{M_W^2}, \frac{m_\mu^2}{M_W^2}$$



$$\sim \frac{m_e^4}{M_W^4}, \frac{m_\mu^4}{M_W^4}$$

For the electron the weak-contributions are irrelevant!

<u>For the muon:</u>

$$a_\mu^{EW} = \frac{\sqrt{2}\, G_F^2\, m_\mu^2}{48\pi^2}\left(5 + \left(-1 + 4\sin^2\theta_w\right)^2\right) + O\left(m_\mu^4 / M_W^4\right)$$

$$\approx \left(194.82 \pm 0.02\right) \times 10^{-11} \qquad \text{(Jegerlehner)}$$

<u>Summary:</u>

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{pol} + a_\mu^{had} + a_\mu^{EW} = 1.16591786(66) \cdot 10^{-3}$$

$$a_e^{SM} = a_e^{QED} + \underbrace{a_e^{pol} + a_e^{had} + a_e^{EW}}_{\text{small compared to muon}} = 1.15965218073(28) \cdot 10^{-3}$$

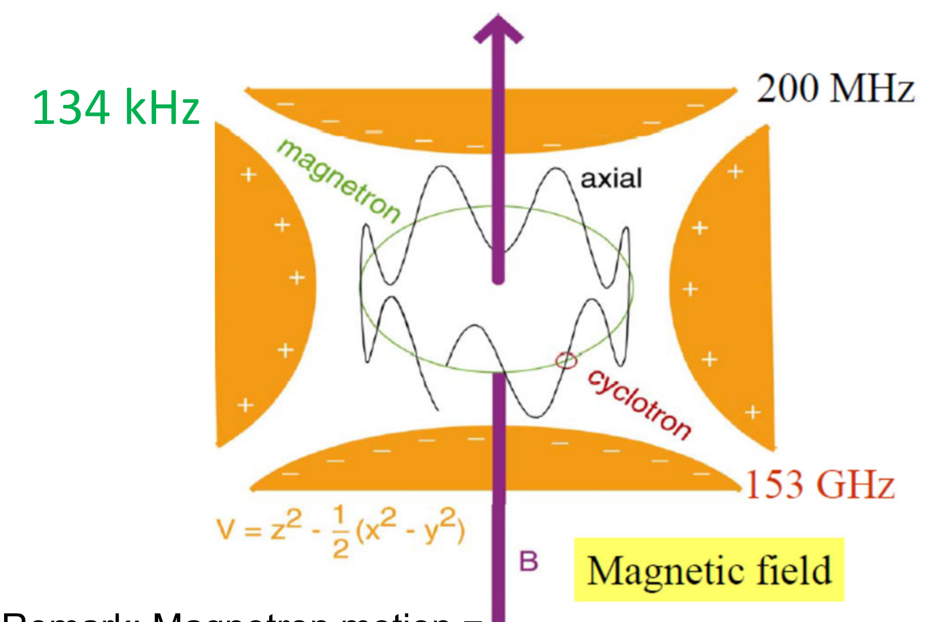
3.3.2. Electron (g-2) measurement: (Gabrielse et al., 2006 + 2008)

Experimental method: Quantum cyclotron

Bind a single electron in a "artificial atom" made of a penning-trap, put into a strong external magnetic field.

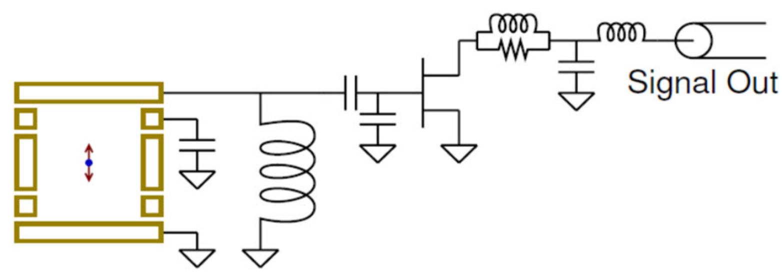
Cool-down of "artificial atom" to ~70 mK: only certain energy levels and circular cyclotron radii are allowed anymore → quantum cyclotron with discrete ladders of energy levels spaced by hf_c (f_c cyclotron frequency).

Energy levels also depend on electron spin – different for spin ⇑ and ⇓.



Remark: Magnetron motion = effect of radial E-field in centre

Single electron stored for several weeks!



A flip of spin/B configuration ↑⇓ to ↑⇑ $\rightarrow$ shift of cyclotron levels by hf_s

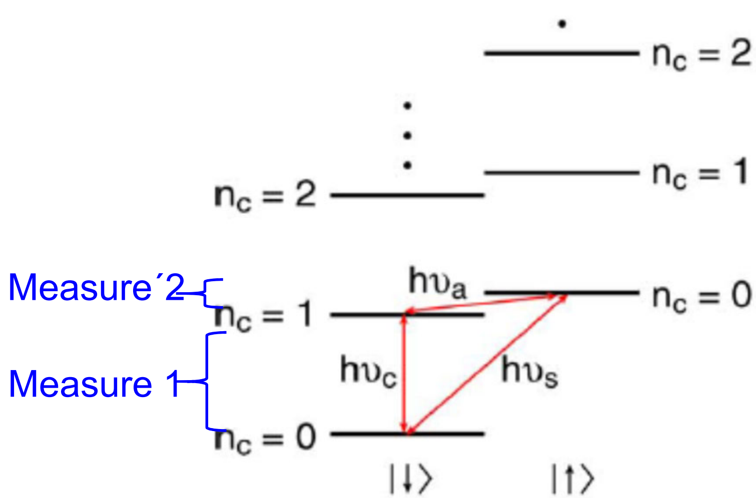
$$f_s = \frac{g}{2} f_c \qquad \text{(spin revolution frequency)}$$

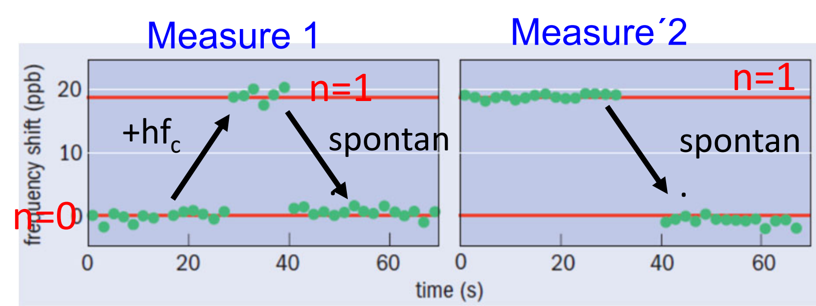
Measurement:

In addition to the cyclotron and the magnetron motion (see Figure) the electron also perform axial oscillations inside the cavity of the trap (f_z).

Because of the coupling the axial motion depends on n_C and the spin-orientation $\rightarrow$ allows to measure f_c and f_s from the axial motion

- One measures f_c and $f_a = f_s - f_c$ to determine $g/2 = 1 + f_a / f_s$.
- f_c is measured using the so called quantum-jump spectroscopy: electron in lowest energy level + tuned micro-wave photon $\rightarrow$ 1st excited a state.





Coupled system:

n_c=1 detunes the axial frequency w/r to n_c =0.

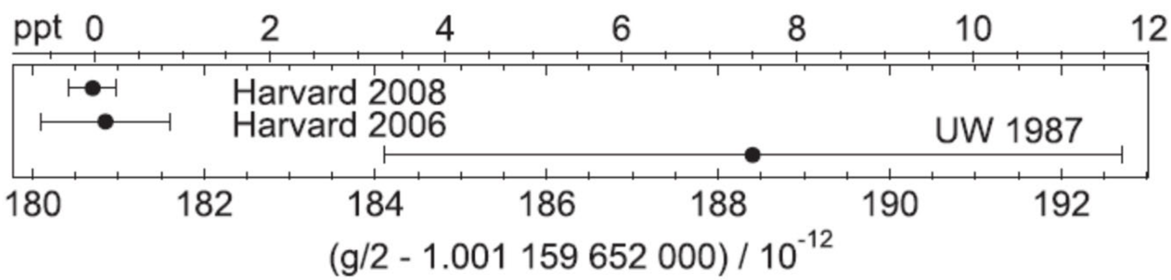
Axial frequency is used to indicate state.

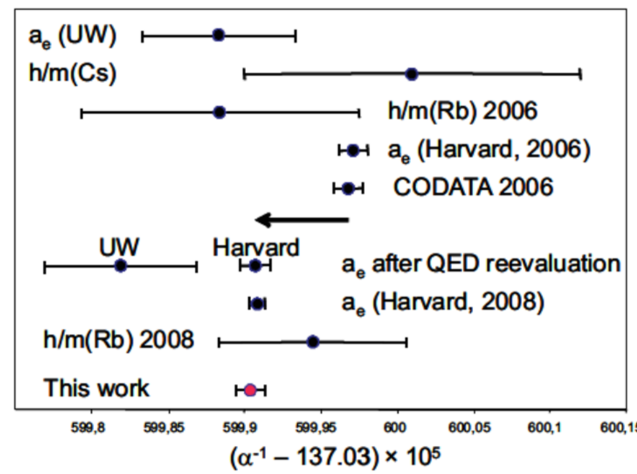
Measurement 1: microwave to excite electron (n_c=0, ⇓) to (n_c=1, ⇓)

Measurement 2: prepare electron in (n_c=0, ⇑) $\rightarrow$ microwave signal to make transition to (n_c=1, ⇓)

PRL **100,** 120801 (2008)

$$g/2 = 1.001\,159\,652\,180\,73(28) \quad [0.28\ \text{ppt}]$$





Using QED calculation to determine α:

$$\frac{g}{2} = 1 + C_2\left(\frac{\alpha}{\pi}\right) + C_4\left(\frac{\alpha}{\pi}\right)^2 + C_6\left(\frac{\alpha}{\pi}\right)^3 + C_8\left(\frac{\alpha}{\pi}\right)^4 + C_{10}\left(\frac{\alpha}{\pi}\right)^5 + \ldots + a_{\mu\tau} + a_{\text{hadronic}} + a_{\text{weak}},$$

$$\begin{aligned}\alpha^{-1} &= 137.035\,999\,084\,(33)\,(39) \quad [0.24\ \text{ppb}][0.28\ \text{ppb}],\\ &= 137.035\,999\,084\,(51) \quad [0.37\ \text{ppb}]. \qquad (5)\end{aligned}$$

Agrees well with the value from spectroscopy and recoil measurement but has a 20 times smaller error

A triumph of QED

3.2.3 Muon (g-2) measurement:

Measurement principle

- store polarized muons in a storage ring with magnetic dipole field B: revolution with cyclotron frequency ω_c

$$\omega_c = \frac{eB}{m_\mu c} = 2\frac{eB}{2m_\mu c}$$

- measure spin precession ω_s around the magnetic dipole field relative to the direction of cyclotron motion:

$$\omega_s = g\frac{eB}{2m_\mu c}$$

$$\omega_a = \omega_s - \omega_c$$

- Taking in addition to the magnetic dipole field also the effect of the electrical focusing fields into account one obtains for ω_a

$$\vec{\omega}_a = -\frac{e}{m_\mu c}\left[a_\mu \vec{B} - \underbrace{(a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta}\times\vec{E}}_{}\right] \quad \text{mit} \quad a_\mu = \frac{g-2}{2}$$

Effect of electrical focusing fields (relativistic effect)

$= 0$ for $\gamma = 29.3$

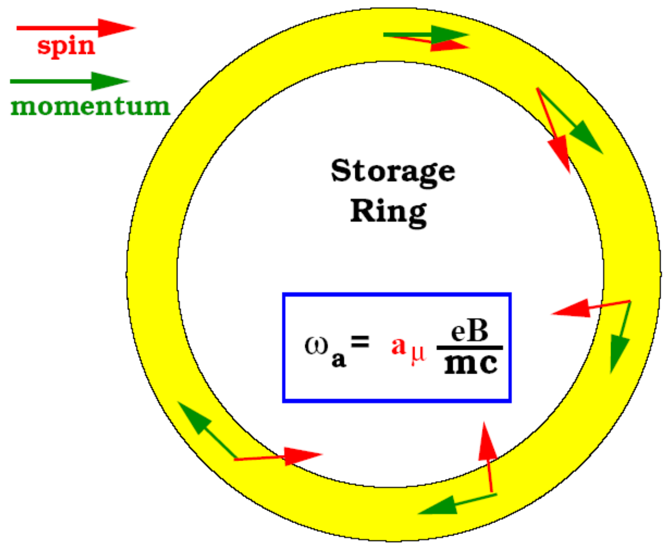
$\Leftrightarrow \; p_\mu = 3.094$ GeV/c

Illustration:

$$\omega_C = 2\frac{eB}{2mc}$$

$$\omega_S = g\frac{eB}{2mc}$$

$$\omega_a = \omega_S - \omega_C$$



(exaggerated ~20x)

First measurements:

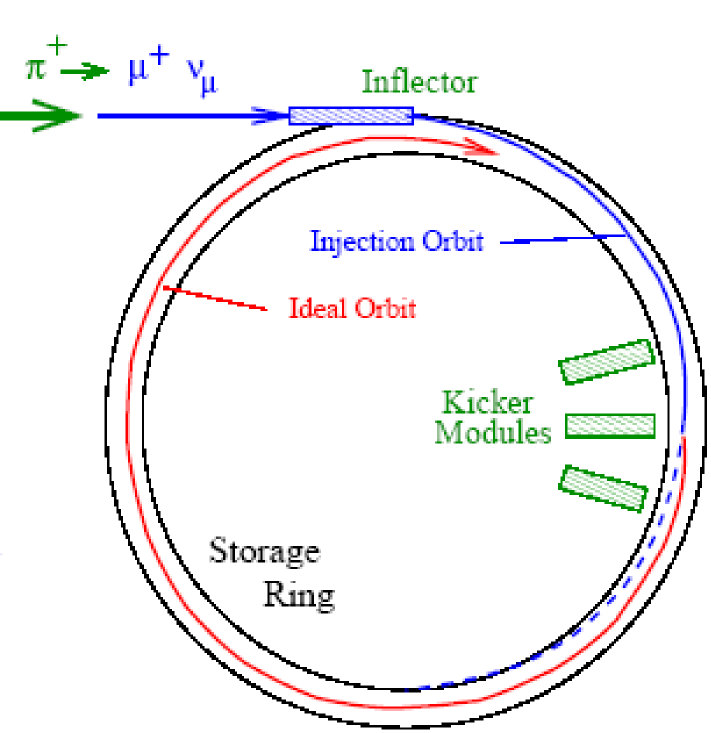
CERN 70s

$$a_{\mu^-} = 0.001165937(12)$$

$$a_{\mu^+} = 0.001165911(11)$$

More recent: BNL g-2.
Currently: Same storage ring at FNAL

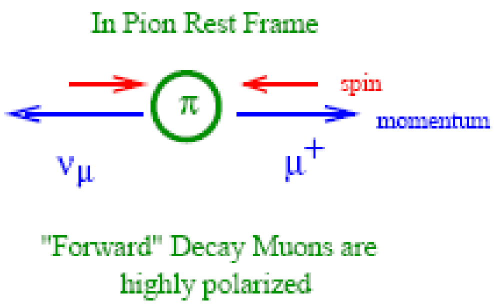
$(g-2)_\mu$ Experiment at BNL

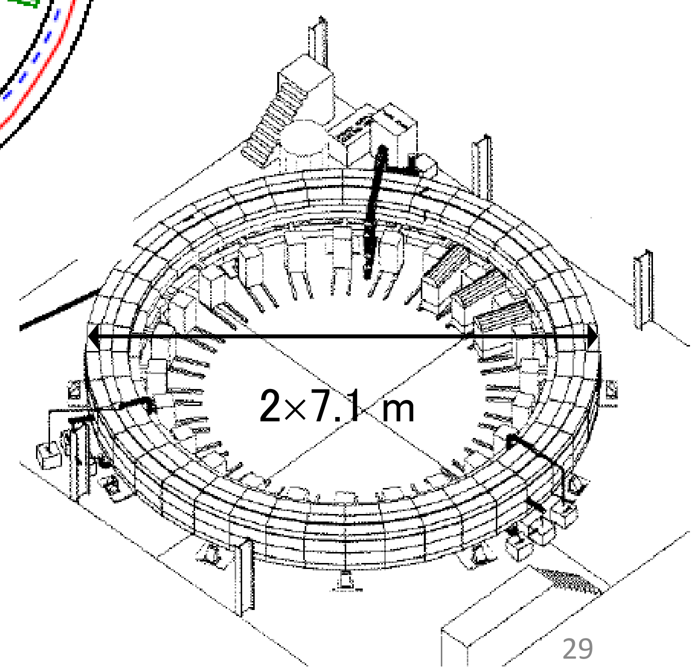


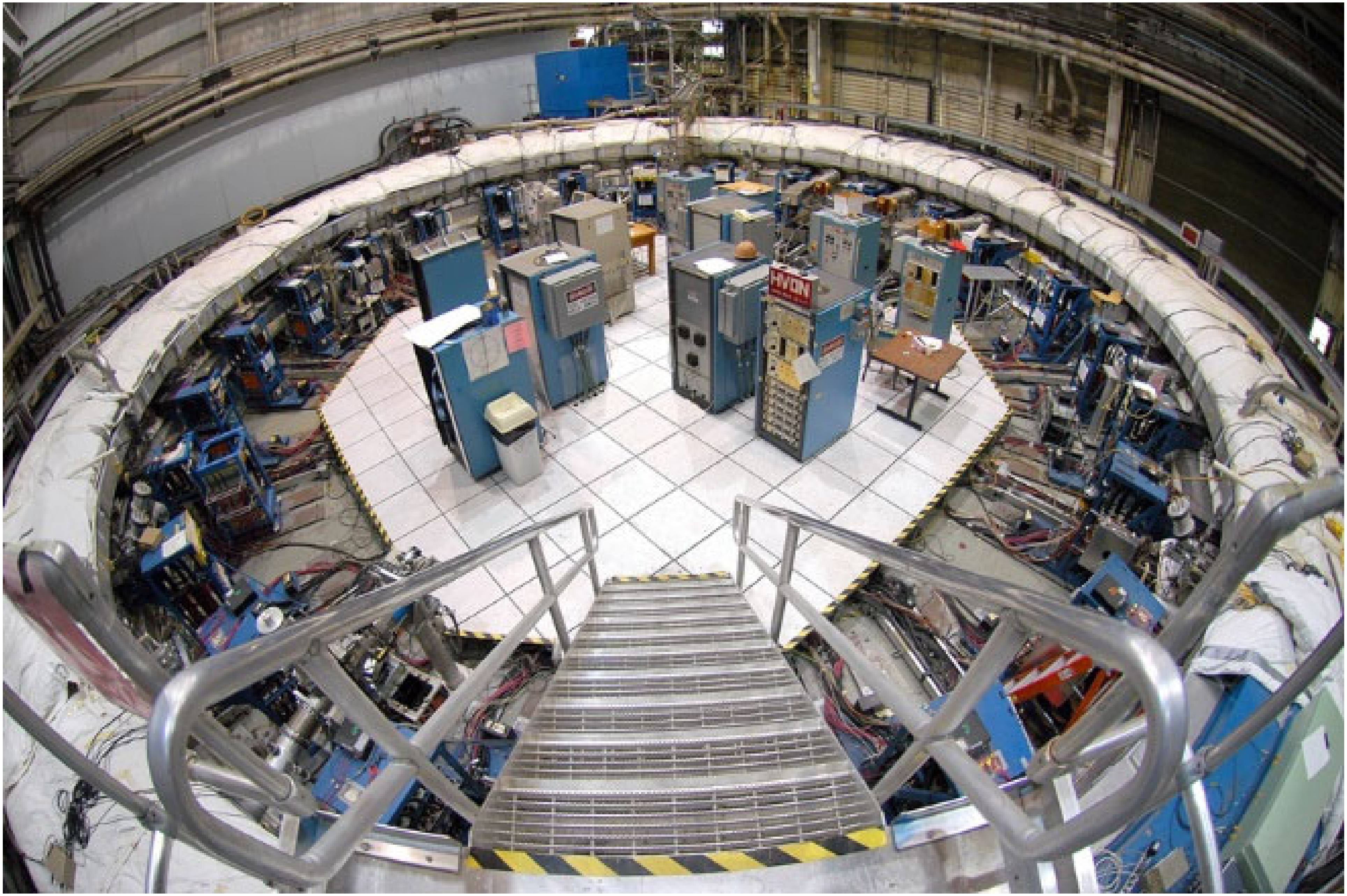
E=24GeV

1 μ / 10^9 protons on target

6x10^{13} protons / 2.5 sec



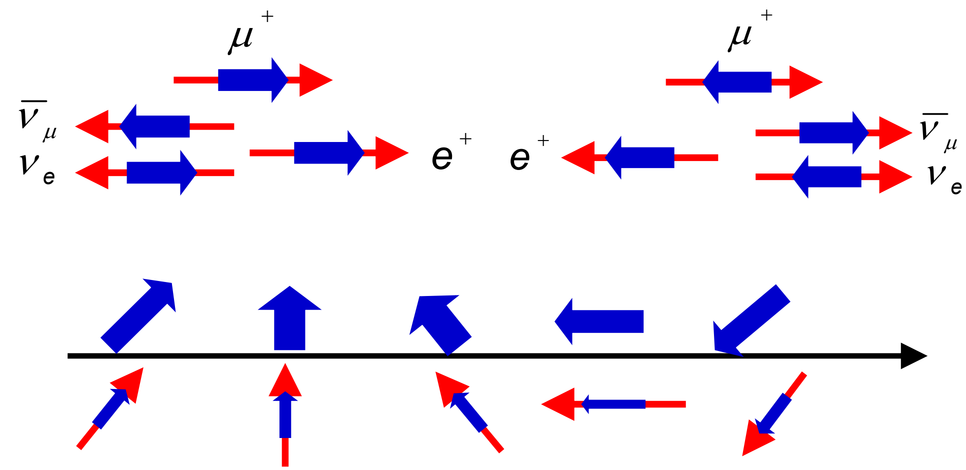


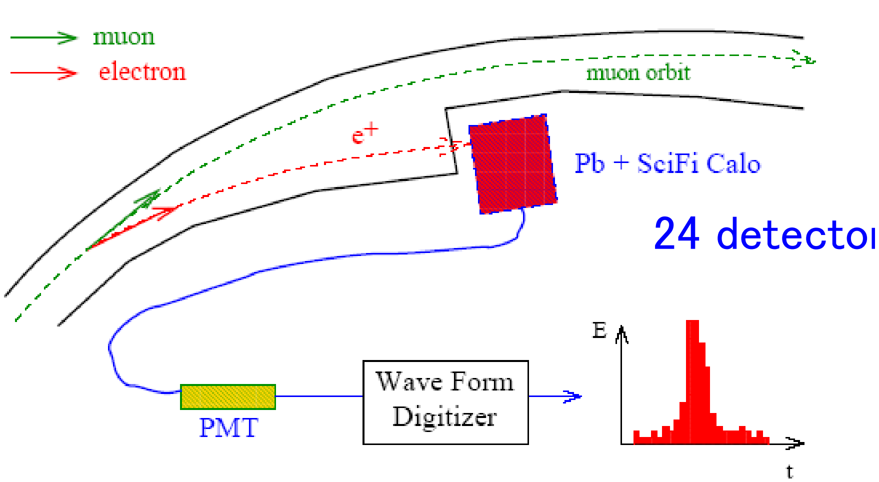

HVON

“V-A” structure of weak decay:

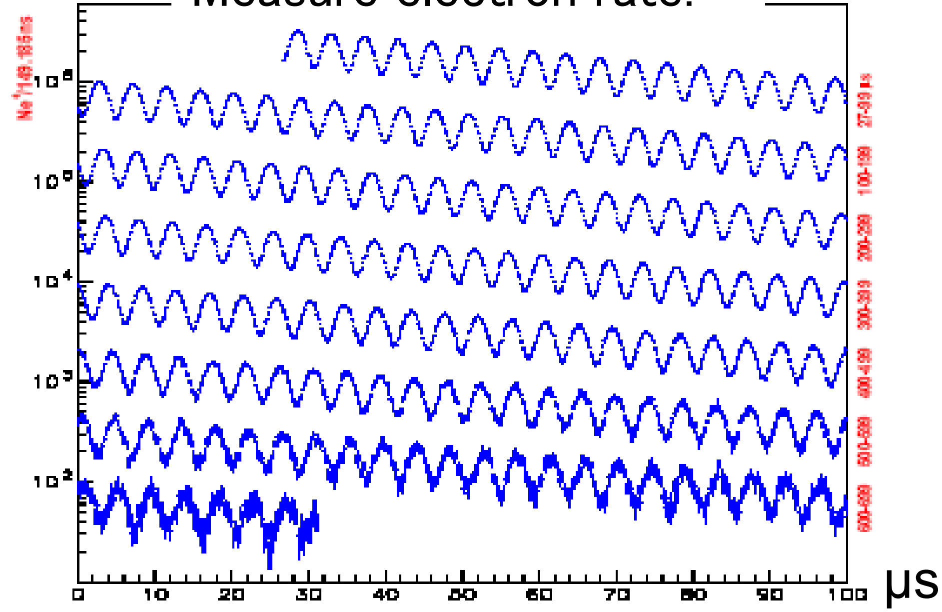
Use high-energy e^+ from muon decay to measure the muon polarization: electron direction align with the muon spin.

Weak charged current couples to LH fermions (RH anti-fermions)





Measure electron rate:



$$N(t) = N_0\, e^{-\lambda t}\left[1 + A\cos(\omega_a t + \varphi)\right]$$

$$\frac{\omega_a}{2\pi} = 229023.59(16)\text{Hz}$$

(0.7ppm)

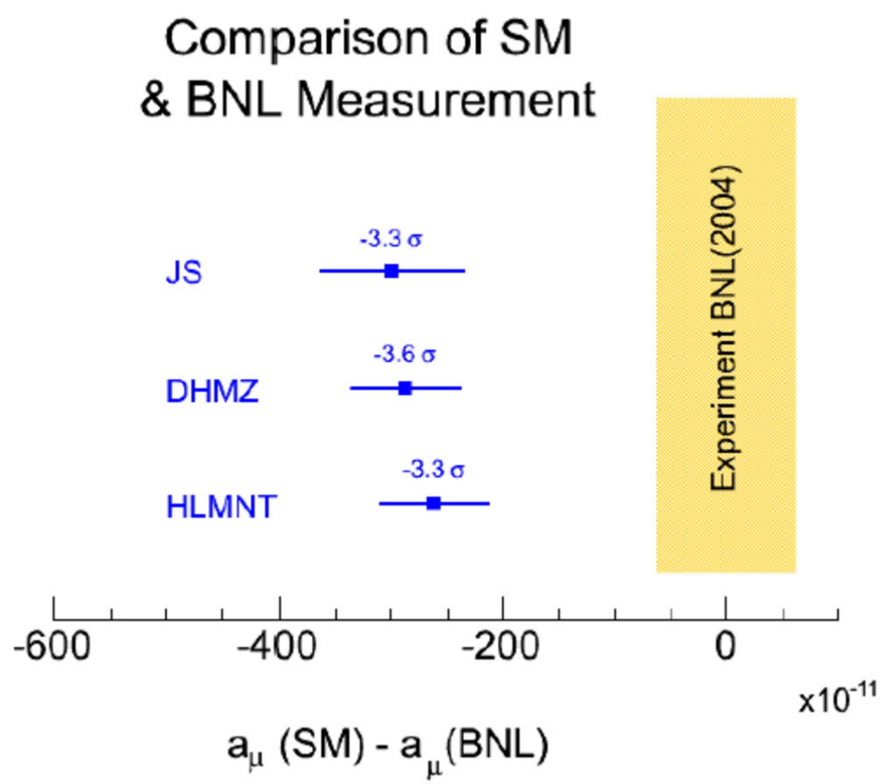
To convert into a_μ need B-field.
B-field is determined with NMR:

$$a_\mu = \frac{\omega_a / \omega_p}{\mu_\mu / \mu_p - \omega_a / \omega_p}$$

$\mu_{\mu+}/\mu_p$=3.183 345 39(10) *)

$$a_{\mu^+} = 11659203(8)\times 10^{-10}(0.7ppm)$$
$$a_{\mu^-} = 11659214(8)\times 10^{-10}(0.7ppm)$$
$$a_\mu = 11659208(6)\times 10^{-10}(0.5ppm)$$

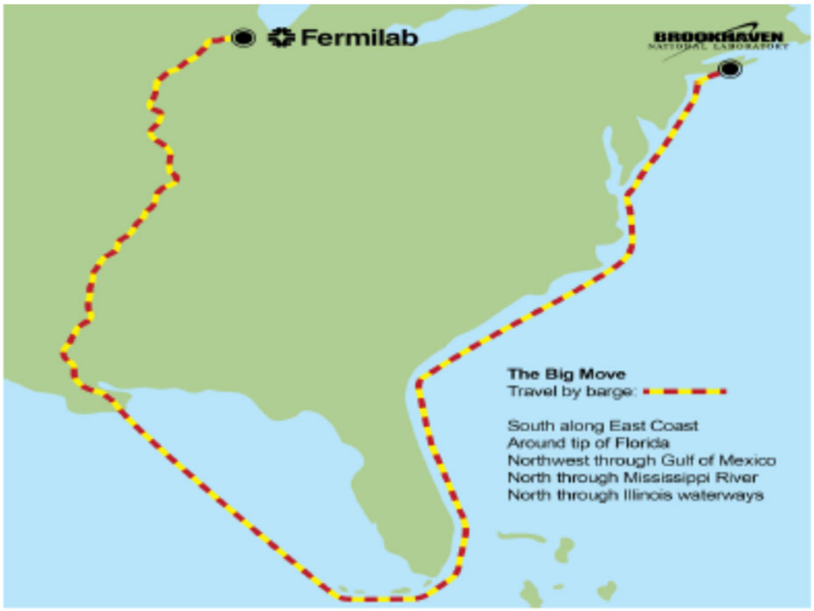
*) Measured via ground state hyperfine structure of muonium: ., PRL **82**, 711 (1999).

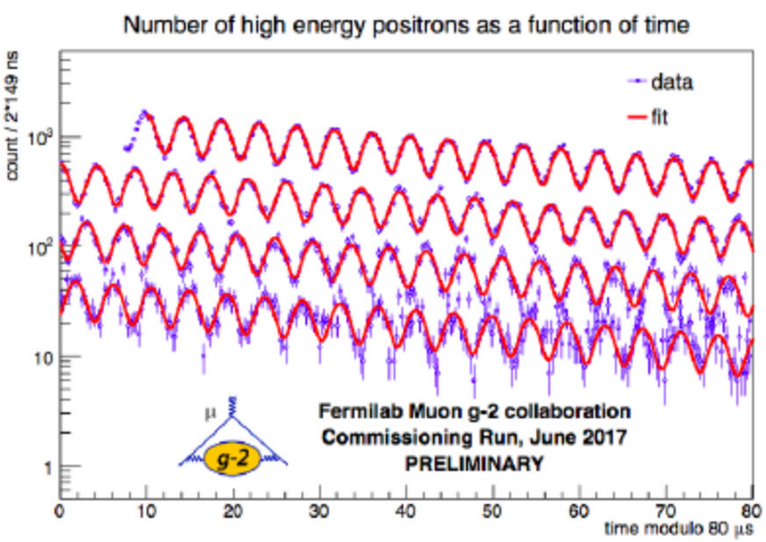


Remark: Standard Model determination differ in the determination of the hadr. corrections.

Theoretician finally converged on one value.

New FermiLab g-2 experiment: https://muon-g-2.fnal.gov/





Improvements:

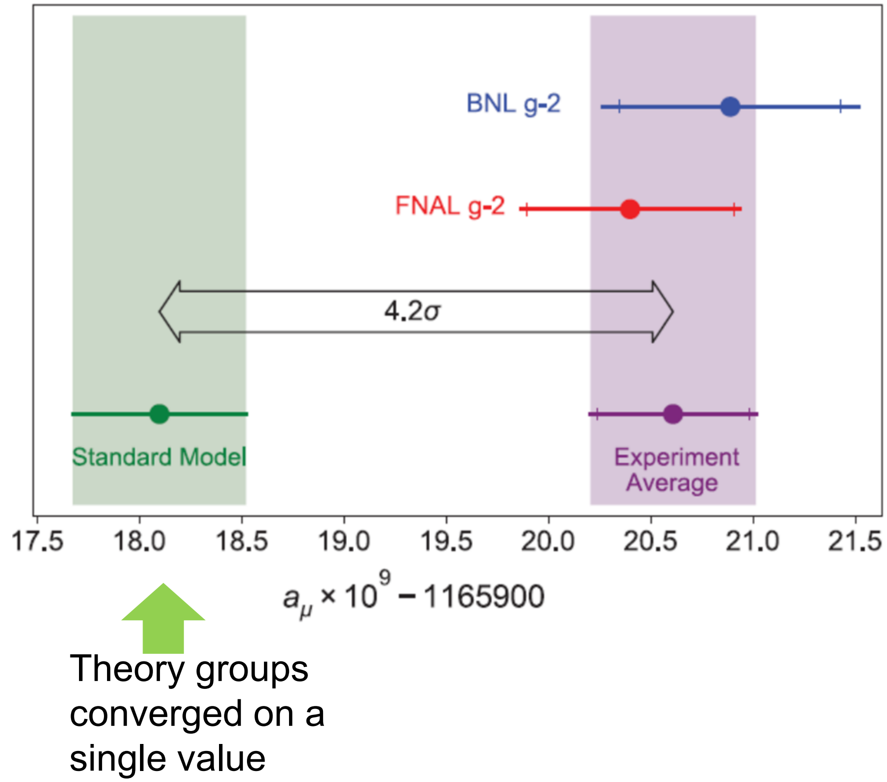
- better B-field homogeneity (x2)
- higher number of muons injected
- beam profile measurem. at injection

$$a_\mu(FNAL) = 11\ 659\ 2040(54)\times 10^{-11}(0.46ppm)$$

$$(0.434_{stat} \pm 0.157_{syst})\ \text{ppm}$$

4.2σ deviation from the SM value.

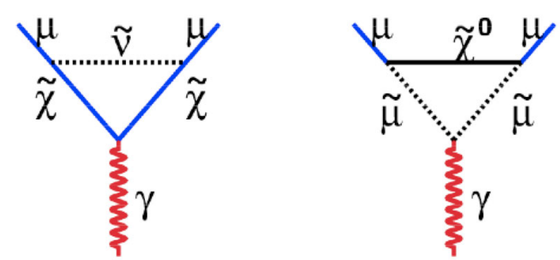
(FNAL g-2 collaboration can reduce the error by another factor of 2)



Currently the most exciting deviation of measurements from the SM prediction (together with the R_K and P_5 measurements)

Potential new physics contributions?

Measurement systematic?

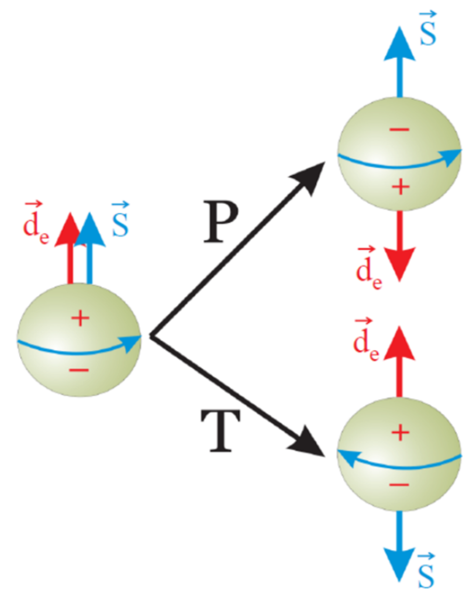


3.3 Electric Dipole Moment

3.3.1 Phenomenology

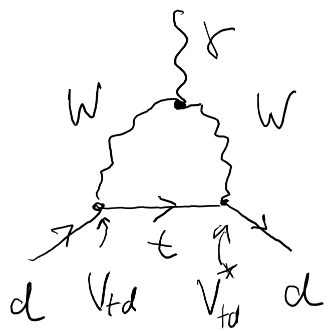
EDMs are CP-violating quantities. In the SM the only source*) of CP violation is the phase δ of the CKM matrix.

(axial vectors don't change sign under parity)



a) EDM in the quark sector (d-quark $\rightarrow$ neutron EDM)

at one loop:



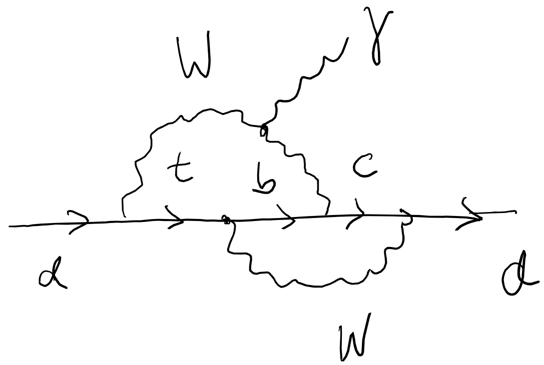
V_{td} complex

$$d_d^{(1)} \sim \frac{e}{16\pi^2} G_F m_d \Im\left(V_{td} V_{td}^*\right) = 0$$

1-loop   helicity suppression   =0, no phase

*) assuming zero neutrino masses and not considering a CPV θ-term in QCD.

at 2-loop:



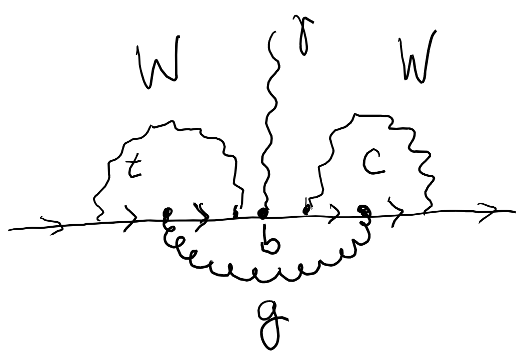
$$d_d^{(2)} \sim \frac{e}{\left(16\pi^2\right)^2} G_F^2 m_d m_c^2 \cdot \Im\left(V_{td}V_{tb}^* V_{cb} V_{cd}^*\right) \neq 0$$

Jarlskog Invariant J ~ 10^{-5}

But: $\sum_{diagrams} d_d^{(2)} = 0$!

(anti-symmetrie of J, E.P. Shabalin 1981)

at 3-loop:



$$d_d^{(3)} \sim \frac{e}{\left(16\pi^2\right)^2} \frac{g_s^2}{16\pi^2} G_F^2 m_d m_c^2 \cdot J \neq 0$$

$$\Longrightarrow \quad d_d \approx 10^{-34} e \cdot \text{cm}$$

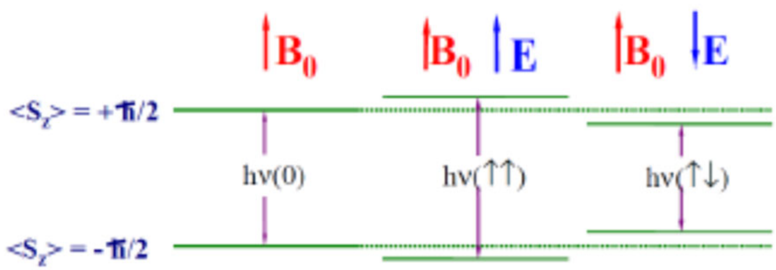
For the down quark an EDM is only induced at 3-loop level.

Most recent masurement of the neutron EDM

Phys. Rev. Lett. 124, 081803 (2020)

Neutron EDM:
Measure spin precession in B and E field of ultra-cold trapped neutrons.

(2 n / cm^3; 180 s)

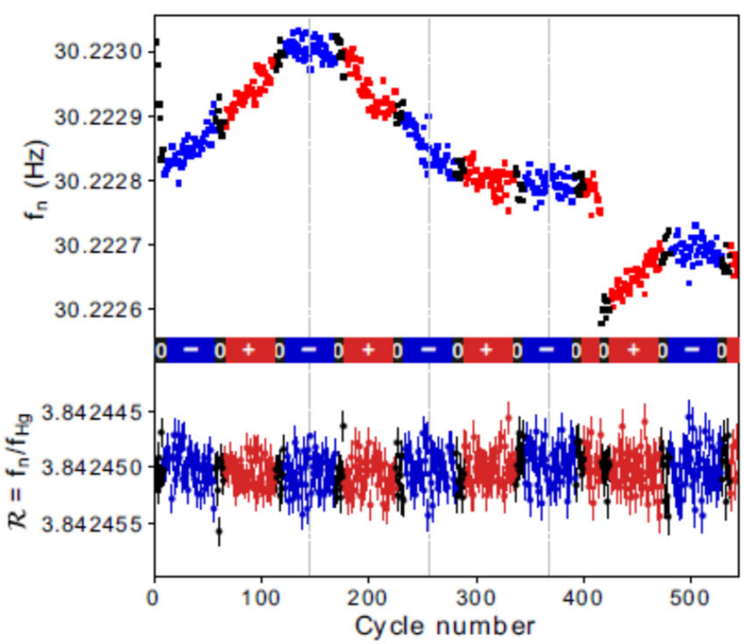


$$h\nu_{\uparrow\uparrow} = 2\,(\mu B + d_n E)$$
$$h\nu_{\uparrow\downarrow} = 2\,(\mu B - d_n E)$$

$$h\Delta\nu = 4\, d_n E$$

Sensitivity:

$$\sigma(d_n) = \frac{\hbar}{2\alpha E T \sqrt{N}}$$

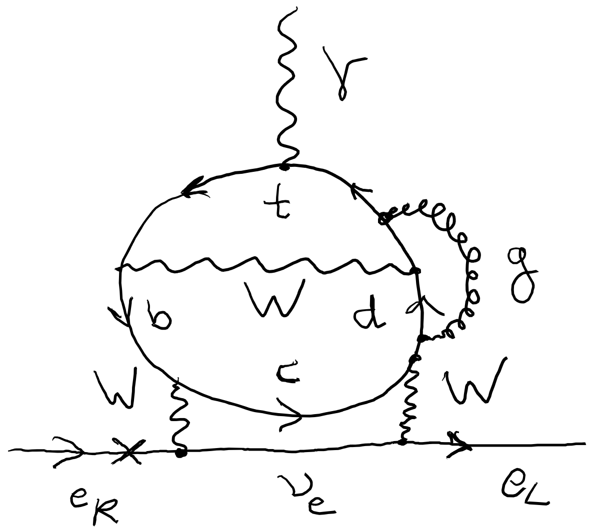


← Using Hg co-magnetometry as a reference frequency measurement

Using the E-field strength (11KV) the measured frequency shifts for the 2 polarities are converted into a measurement for the electrical dipole moment of the free neutrons:

$$d_n = (0.0 \pm 1.1_{\rm stat} \pm 0.2_{\rm sys}) \times 10^{-26}\ e{\cdot}{\rm cm}.$$
$$|d_n| < 1.8 \times 10^{-26}\ e{\cdot}{\rm cm}\ \ (90\%\ {\rm C.L.}).$$

b) Lepton EDM only induced at 4-loop level:



$$d_e \sim \frac{e}{\left(16\pi^2\right)^3}\frac{g_s^2}{16\pi^2} G_F^3 m_e m_c^2 m_s^2 \cdot J \neq 0$$

$$d_e \approx 10^{-44}\, e \cdot \mathrm{cm} \ll d_d$$

(Pospelov & Ritz, 2013)

New physics in electron EDM d_e ?

Current experimental upper bound on electron EDM:

$$\left|d_e\right| < 8.7 \times 10^{-29}\, e \cdot \mathrm{cm} \qquad \text{(ACME 2013 using ThO - see below)}$$

Consider new physics at a scale Λ with CP-violating (complex) couplings:

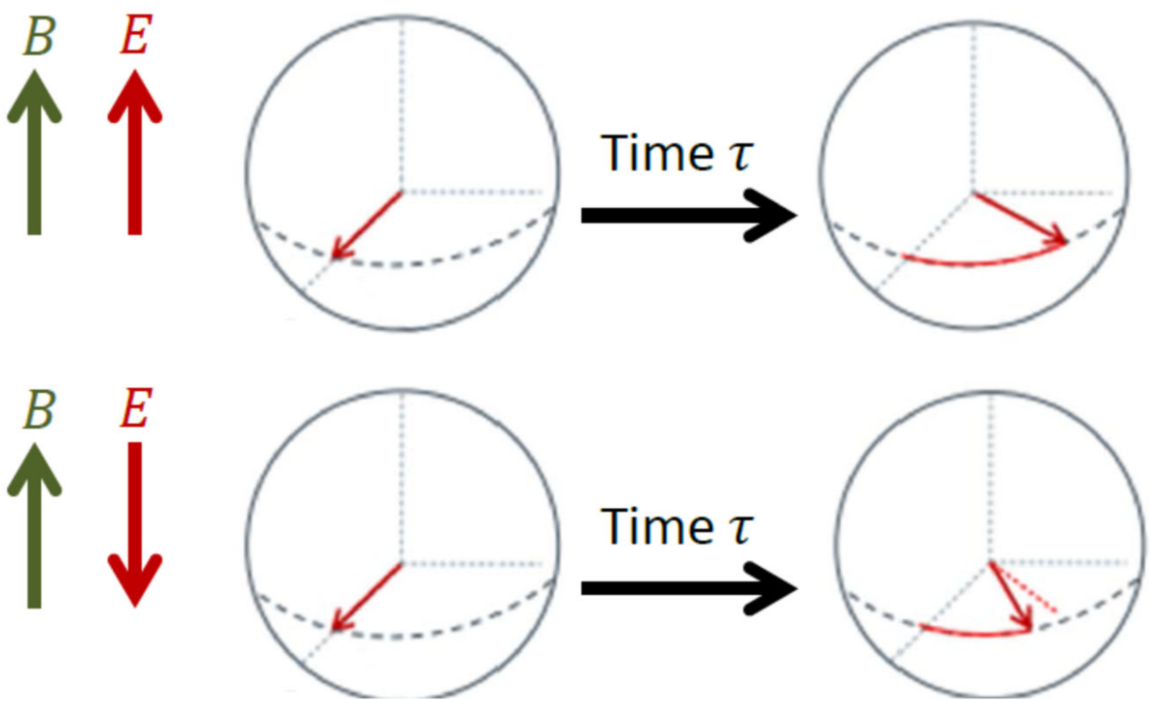
$$d_e^{NP} \sim \frac{C_{NP}}{\Lambda^2} m_e \left(\bar{e}\sigma_{\mu\nu}\gamma_5 e\right) F^{\mu\nu}$$

Assuming $C_{NP} \sim 1$ (NP at tree level) or $C_{NP} \sim 1/(16\pi^2)$ (NP at loop level) with a CP violating phase of O(1), ACME probes New Physics at scales $\Lambda \sim 300$ (30) TeV

c) <u>Principle of electron EDM measurement:</u>

$$H = -\mu \cdot B - d \cdot E$$

additional phase due to energy difference



$$\phi_{+} = \mu B\tau + dE\tau$$

$$\phi_{-} = \mu B\tau - dE\tau$$

A bound electron with magnetic and electrical dipole moment inside a magnetic and electrical field experiences an energy shift. If it evolves in time, it aquires an additional phase ϕ.

ACME Collaboration
Science 343 (2014)
EPJconf/20135702004

<u>Problems:</u>

- A bound electron in an atom / molecule is not experiencing a net electrical field - as there is no net acceleration the net electrical field experienced by the electron should be zero!

- For heavy nuclei, electrons move at relativistic speed near the nucleus. Lorentz contraction causes d_e to spatially vary over the orbit. While the mean <E> is zero the mean <d.E> is not.

- Not only that the effective E-field defined as d.E_{eff} = <d.E> is non zero in atoms/molecules, it is also much larger than achievable in laboratory.
For ThO: E_{eff}=84 GV/cm (scales with Z^3).

- Only unpaired electrons can create an EDM. And, since the relativistic contraction occurs only near the nucleus, the atoms/molecules must have unpaired electrons penetrating the core.

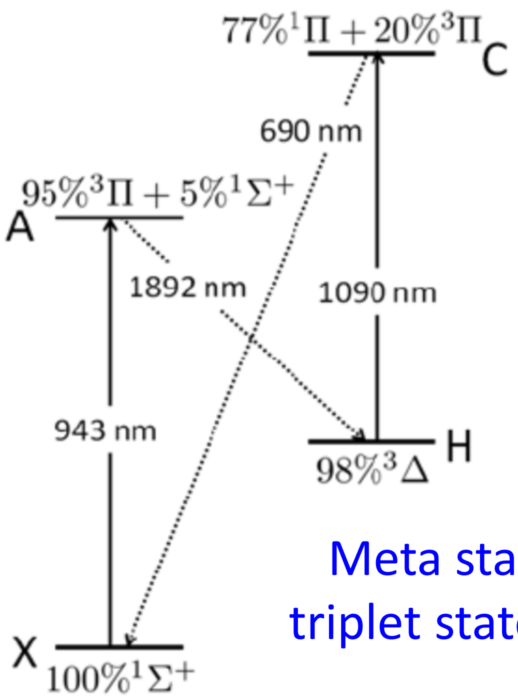
- Di-atomic polarizable molecules are advantageous. The polarization induced by an moderate outside field (<100 V/cm) leads to very strong effective E-field.

Measurement is difficult! Due to time reasons I will not discuss is but I will keep the slides (p 41 – 50) in the notes.

Thorium Oxide

ACME Collaboration
Science 343 (2014)
EPJconf/20135702004

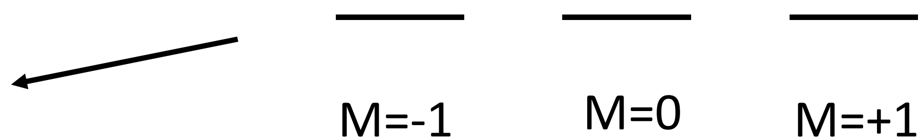
ThO effective E-filed = 84 GV/cm



Meta stable
triplet state J=1

Ground state

In absence of an applied E-field the so-called Ω-doubled states are parity states with P=±1 (spitting of a few hundred kHz)

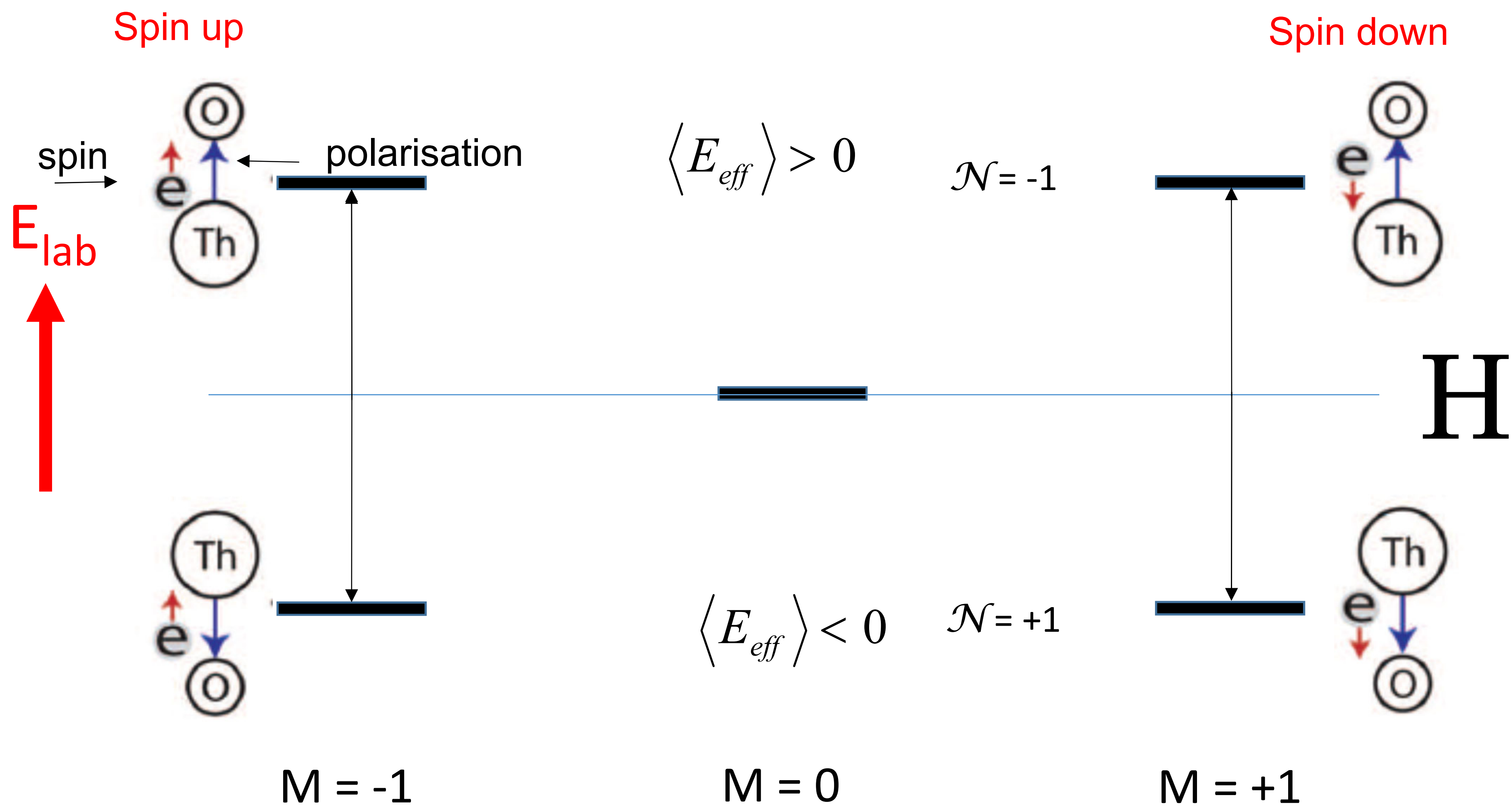


Degenerated states with
P±1 (parity) sublevels.

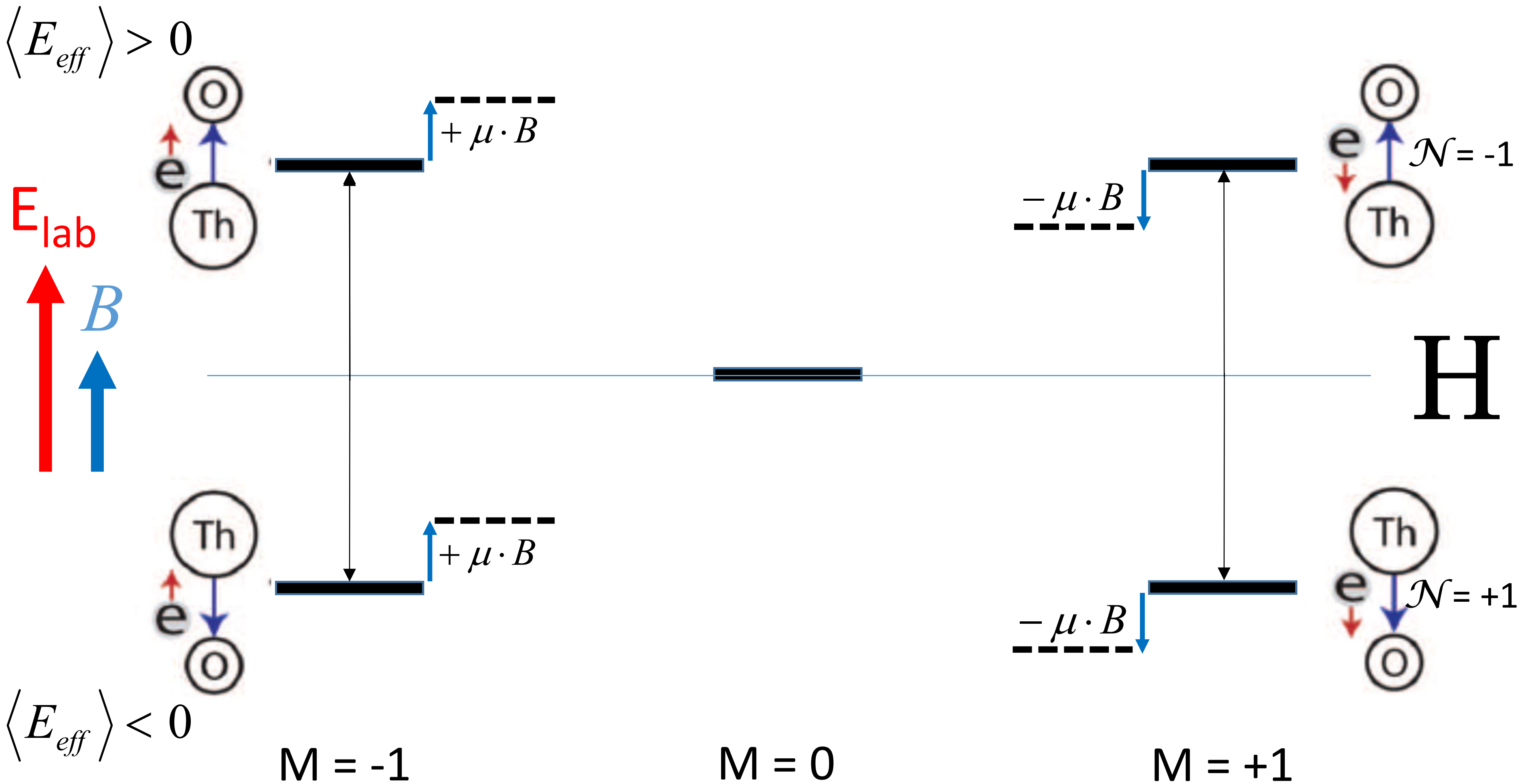
Complicated molecular spectrum.

With small external E-field E_{lab} (>10V/cm)

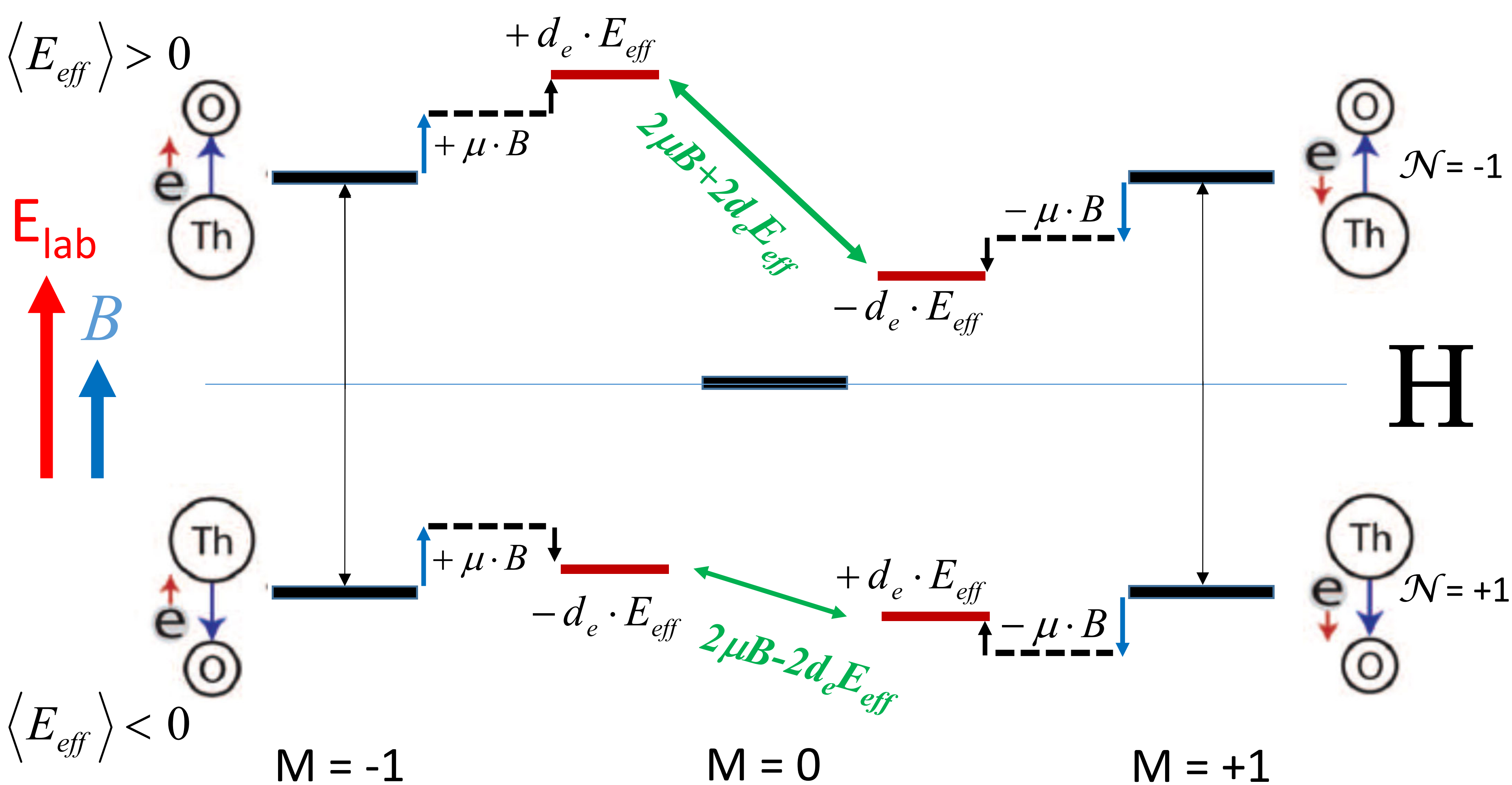
$\rightarrow$ The P=± sublevels with the same M_J mix completely: the resulting eigenstates have complete electrical polarization (N=±1) - The M_J =0 don't mix.

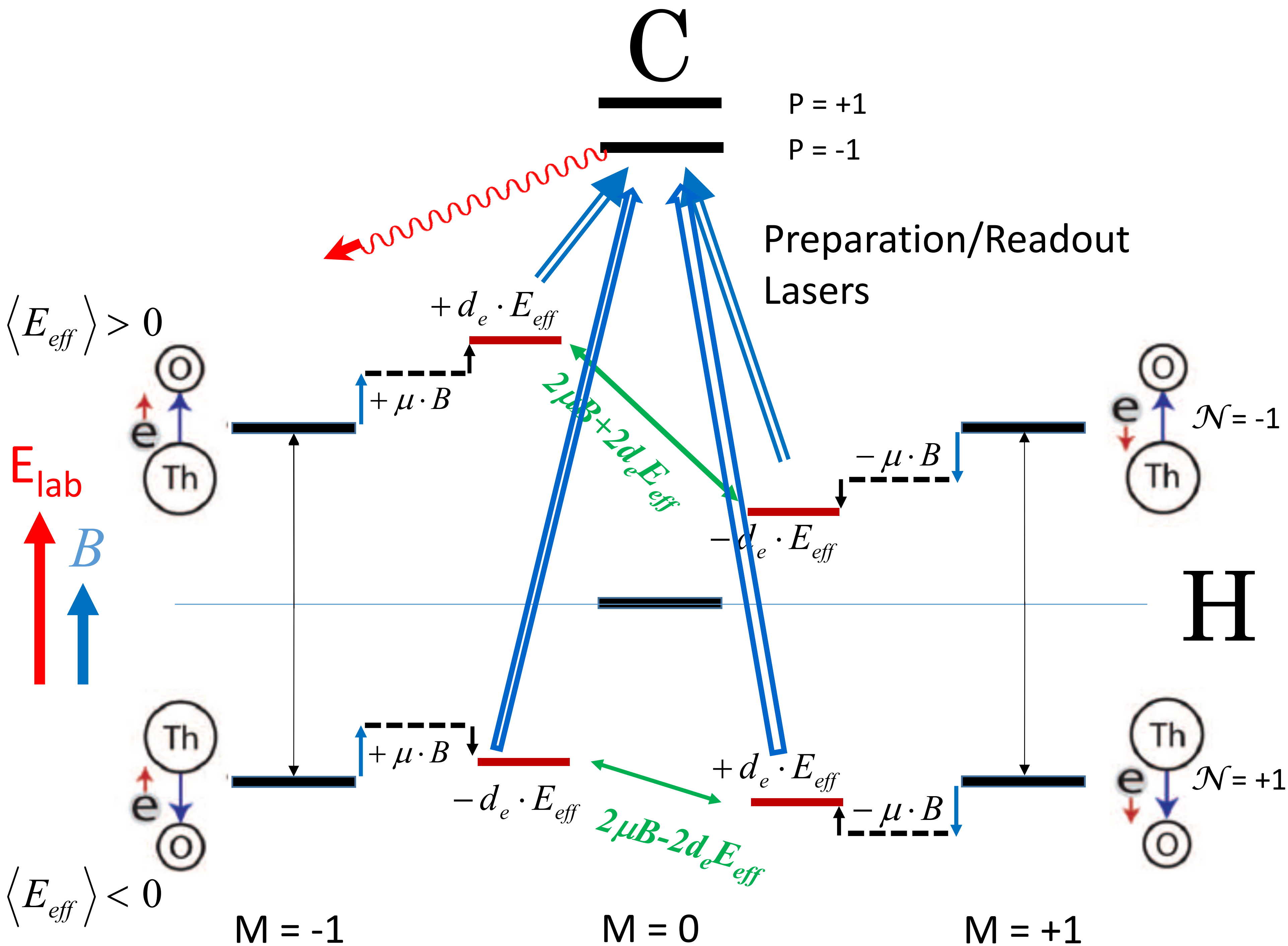


Additional B-field shift the levels



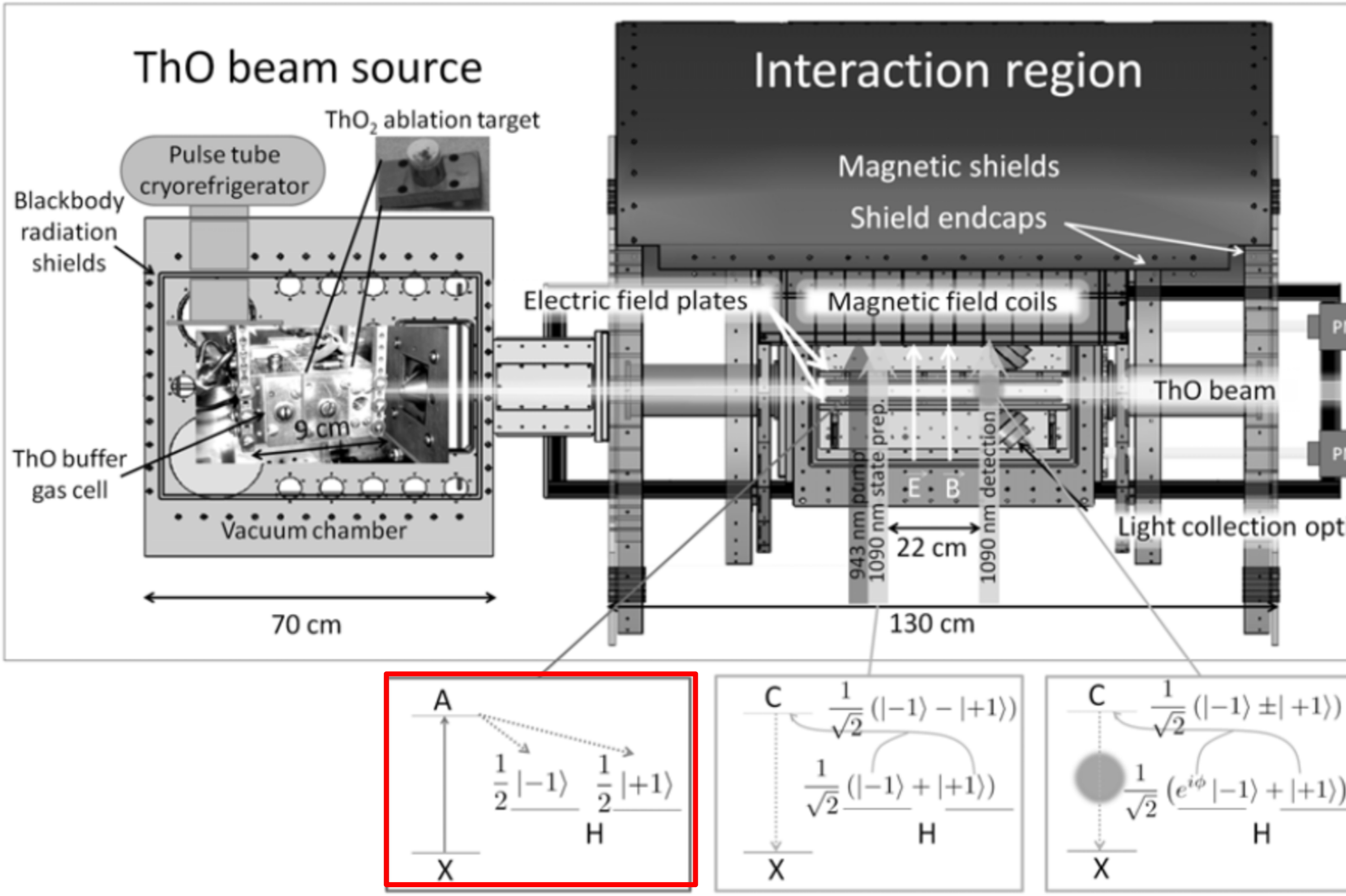
Further level shifting in case of electron EDM: $\sim d_e \frac{\vec{s}\vec{E}_{eff}}{|\vec{s}|}$





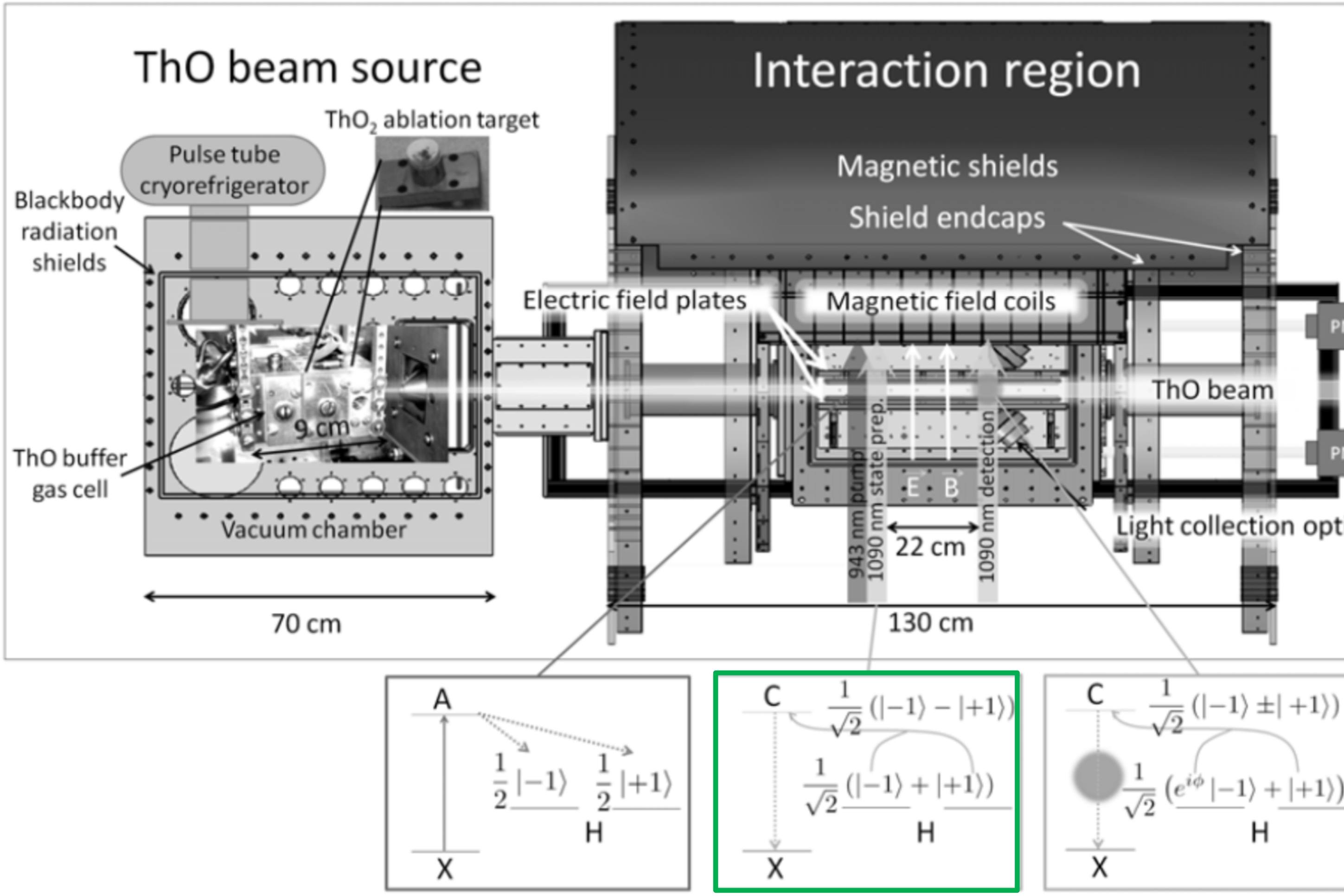
C
P = +1
P = -1
Preparation/Readout Lasers
$\langle E_{eff} \rangle > 0$
$+d_e \cdot E_{eff}$
$+\mu \cdot B$
$2\mu B+2d_eE_{eff}$
$-\mu \cdot B$
$-d_e \cdot E_{eff}$
$\mathcal{N}$= -1
E_lab
B
H
Th
O
e
$+\mu \cdot B$
$-d_e \cdot E_{eff}$
$+d_e \cdot E_{eff}$
$-\mu \cdot B$
$2\mu B-2d_eE_{eff}$
$\mathcal{N}$= +1
$\langle E_{eff} \rangle < 0$
M = -1
M = 0
M = +1

Measurement



At the entrance of the field region, the molecules are pumped from the |X> states to the |A> state, where they spontaneously decay to the |H>, equally populating the |J = 1,M = ±1, N=±1 > sublevels

Measurement

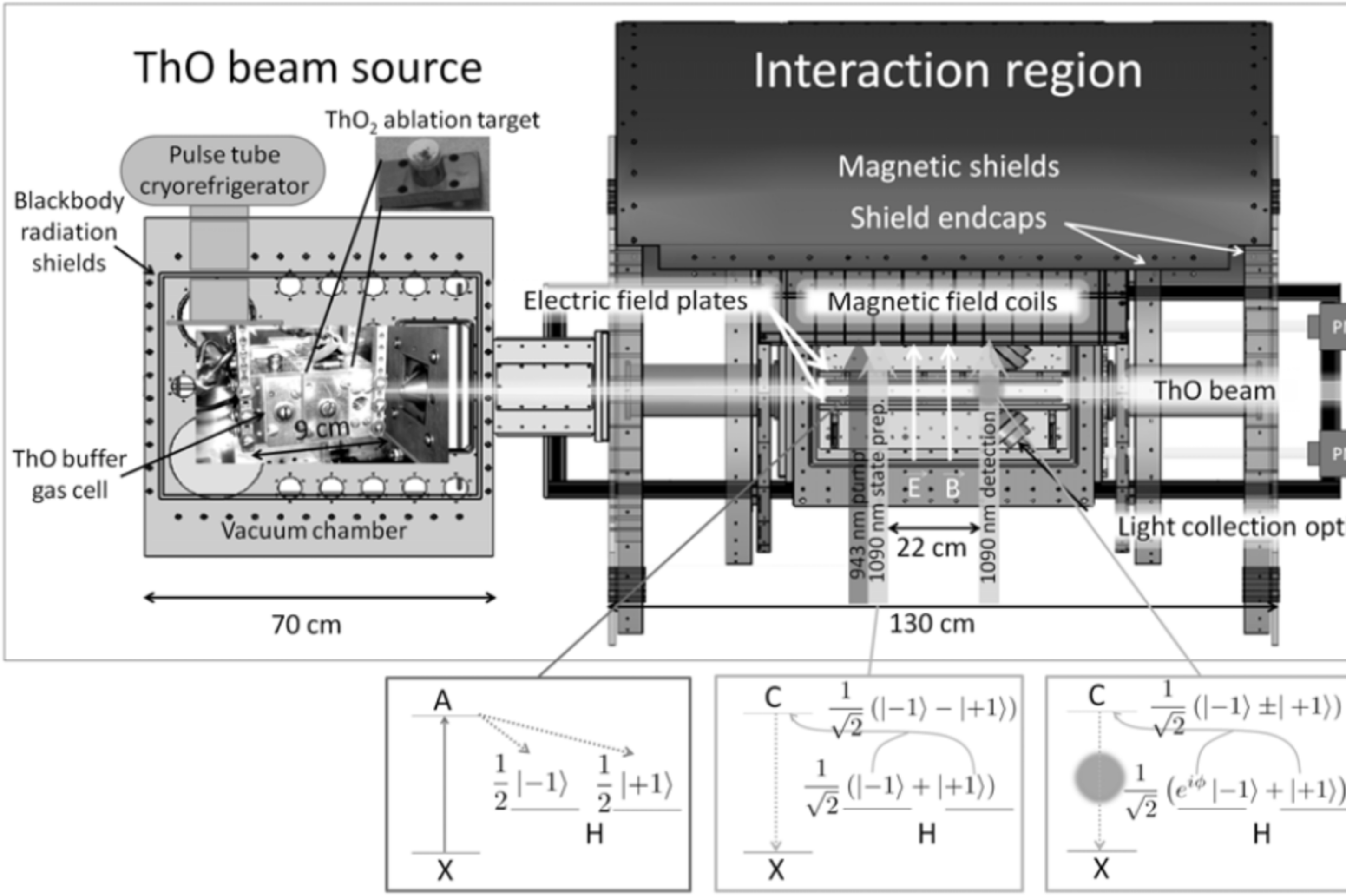


Next, a pure superposition of Zeeman sublevels $|X_N>$ is prepared by pumping out the orthogonal superposition $|Y_N>$ using linearly polarized light resonant with the transition frequency.

$$|X_{\mathcal{N}}\rangle \equiv \frac{1}{\sqrt{2}}(|M_J = +1; \mathcal{N}\rangle + |M_J = -1; \mathcal{N}\rangle)$$

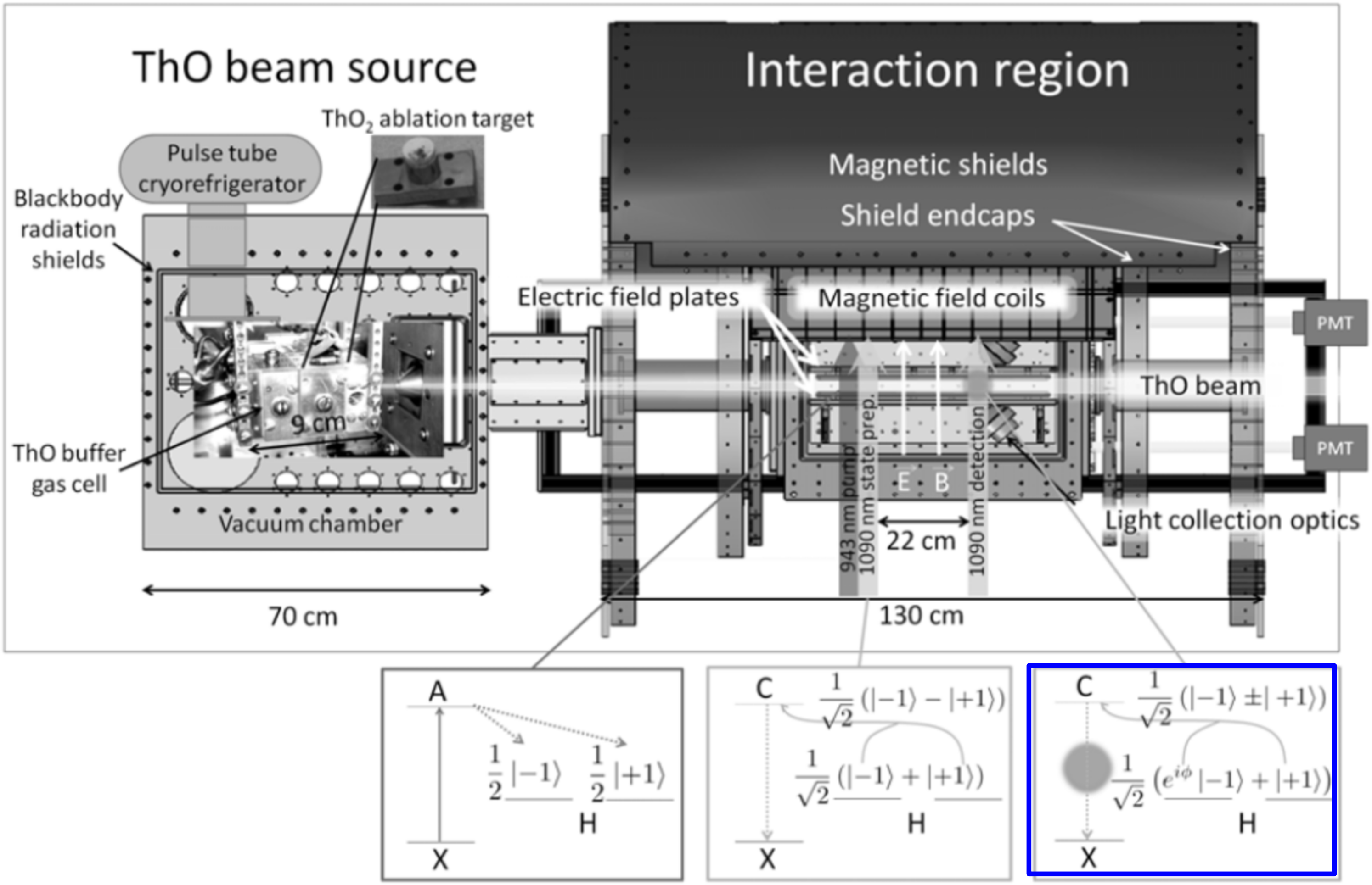
$$|Y_{\mathcal{N}}\rangle \equiv \frac{1}{\sqrt{2}}(|M_J = +1; \mathcal{N}\rangle - |M_J = -1; \mathcal{N}\rangle)$$

Measurement



Next, the molecule state precedes in the applied **E** and **B** fields for approximately 1.1 ms as the beam traverses the 22-cm-long interaction region. The relative phase accumulated between the two Zeeman sublevels depends on EDM d_e.

Measurement



Near the exit of the field region, we read out the final state of the molecules: By exciting the |H, J = 1> → |C, J = 1, M_J = 0> transition with rapidly switched orthogonal (ˆx and ˆy) linear polarizations and detecting the C → X fluorescence from each polarization, the population is projected onto the |X_N> and |Y_N> states.

Determination of the accumulated phase ϕ

$$|\psi_f^{\mathcal{N}}\rangle = \frac{1}{\sqrt{2}}\left(e^{i\phi}\,|M_J = +1;\mathcal{N}\rangle + e^{-i\phi}\,|M_J = -1;\mathcal{N}\rangle\right)$$

$$|X_{\mathcal{N}}\rangle \equiv \frac{1}{\sqrt{2}}\left(|M_J = +1;\mathcal{N}\rangle + |M_J = -1;\mathcal{N}\rangle\right)$$

$$|Y_{\mathcal{N}}\rangle \equiv \frac{1}{\sqrt{2}}\left(|M_J = +1;\mathcal{N}\rangle - |M_J = -1;\ \mathcal{N}\rangle\right)$$

The probability of detecting the molecule in the state $|X_N\rangle$ or $|Y_N\rangle$ is:

$$P_X = \left|\left\langle X_N \middle| \psi_f^N \right\rangle\right|^2 = \cos^2\phi \qquad P_y = \left|\left\langle Y_N \middle| \psi_f^N \right\rangle\right|^2 = \sin^2\phi$$

with

$$\phi = \int_{x=0}^{x=L} \left(d_e \mathcal{E}_{\text{eff}} \mathcal{N} \hat{E} + g_{H,J=1}\mu_B B \hat{B}\right) \frac{dx}{\hbar v} \equiv \phi_{\mathcal{E}} + \phi_B.$$

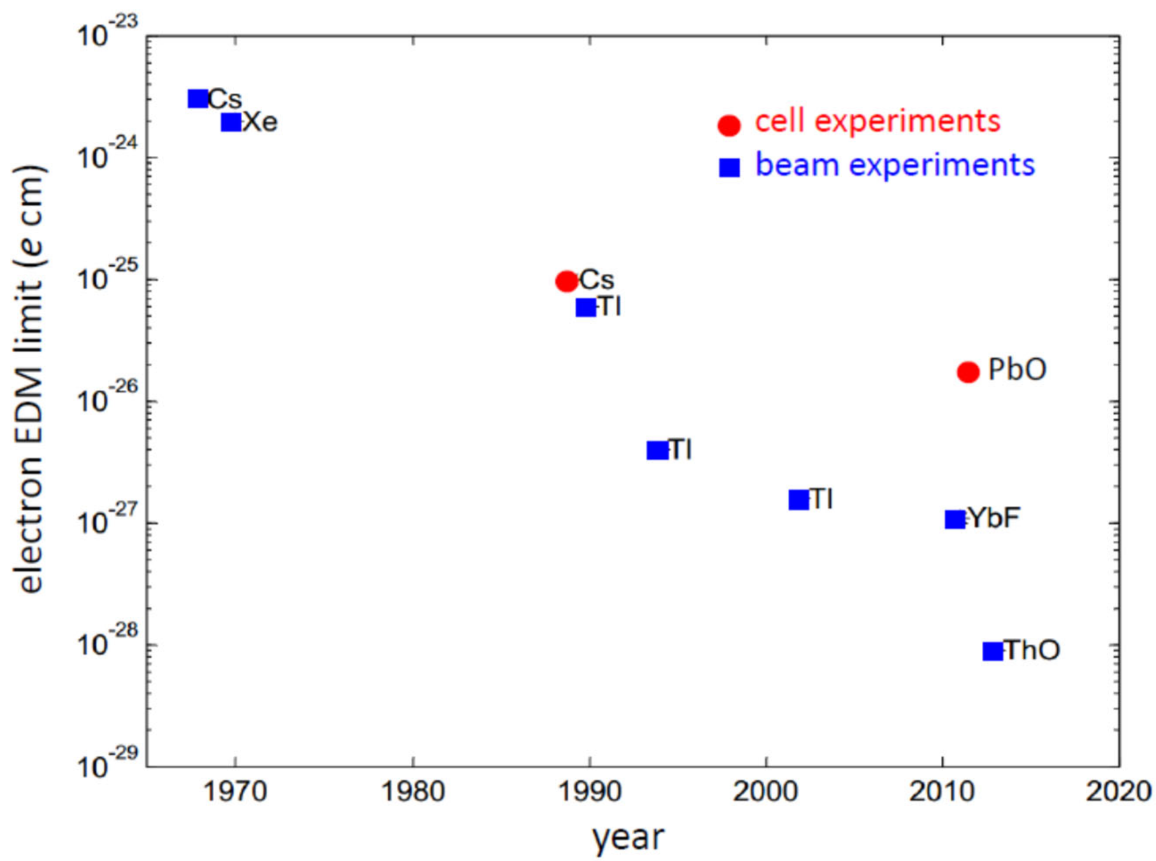
Electron EDM

ACME collaboration, 2014

$$d_e = (-2.1 \pm 3.7_{\text{stat}} \pm 2.5_{\text{syst}}) \times 10^{-29} e\cdot\text{cm}$$

$$|d_e| < 8.7 \times 10^{-29} e\cdot\text{cm}$$

Assuming
E_{eff} =84 GV/cm



Electron EDM and New (BSM) Physics