The Measurement Problem and Alternative Formulations of Quantum Mechanics

Karen Wadenpfuhl

Starting from the measurement problem we re-visit the Copenhagen Interpretation of quantum mechanics and recall some criticism raised against it. We then briefly introduce the spontaneous collapse models, where the time evolution of the wave function naturally contains random localisations in configuration space, and the pilot-wave theory, where the wave function guides deterministic system trajectories through configuration space.

It was discovered soon after the first postulation of the Schrödinger equation [1, 2] that a consequent application thereof leads to superpositions of macroscopic objects like cats [3], as shown in Figure 1. Such macroscopic superpositions are in direct conflict with human experience of the everyday world, since we perceive only single, well-defined states of the objects surrounding us. When assuming Born's rule [5] to hold one finds that the wave function immediately after a measurement must consist exclusively of the eigenstate corresponding to the eigenvalue of the observable that was measured, irrespective of whether or not it had been in a superposition before. It then follows that the dynamics of a measurement is not accounted for by Schrödinger's equation but any superposition is *reduced* to a single, definite state - the one corresponding to the measurement outcome. The question as how to get from superpositions to a single state of the wave function during the process of a measurement has come to be known as the *measurement problem*.

Historically, one of the immediate responses to the measurement problem which soon found broad acceptance in the physics community at the time [6] was the so-called Copenhagen Interpretation, championed notably by Bohr [7] and Heisenberg [8] amongst many others. According to the Copenhagen Interpretation our world consists of two distinct realms, a microscopic quantum world with the Schrödinger equation as its law of motion which leads to superposition states of the wave function - and the classical world described through classical concepts where everything possesses single, unambiguous states and the idea of superpositions is ridiculous. It is the process of a measurement, the interaction of a quantum state with a classical object, which irreversibly collapses the wave function to a single, definite outcome. This is known as the *collapse* or *reduction postulate*.

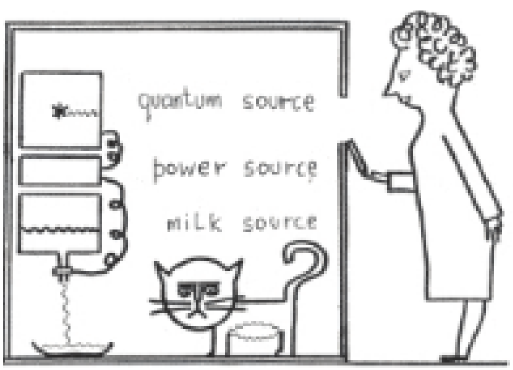

FIG. 1: Schrödinger's cat in the version of John Bell, taken from [4]. A cat is trapped in an box with a radioactive atom that has a certain probability to decay within a finite time interval dt, $0 < P_{decay} < 1$. In case the atom decays it activates a detector which releases some milk that the cat drinks. After an hour or so the cat then is in a superposition of being full and being hungry.

However, it is often criticised that the Copenhagen Interpretation fails to give a precise definition of the boundary between quantum and classical as well as of measurements [9]. Additionally, some regard this interpretation of quantum mechanics to be incomplete [3, 10]. This motivated many physicists to come up with new approaches to quantum mechanics, each resolving the measurement problem in a different fashion.

Well-known among those alternative approaches are the Spontaneous Collapse Theories, the general mechanism of these theories was first proposed by Ghirardi, Rimini and Weber (GRW) in 1986 [11]. Here, the Schrödinger equation is modified such that the time evolution of a quantum state naturally contains random localisations of the wave function in configuration space with localisation width σ and stochastic localisation rate λ. Even though the impact of a collapse on a single particle is assumed to be almost negligible - GRW estimated $\sigma \sim 10^{-7}$m and $\lambda \sim 10^{-16}\text{s}^{-1}$ - this is no longer the case for macroscopic, entangled systems where any superposition ceases on time- and length scales not accessible to the human senses. The probability for a collapse to occur and its consequences for the system are determined by σ and λ which allows to test the parameter space (σ, λ) against experiments [12] (and the predictions of standard quantum mechanics). It should be noted that GRW initially did not propose any mechanism to explain the reason for the occurrence of these spontaneous localisations but researchers have since devised different models to explain their origin. An example is the Diósi-Penrose model which assumes the localisations to be induced by the coupling of matter to a noisy gravitational field [13].

Another approach to the measurement problem is the pilot-wave theory, the fundamental idea of which was first presented by deBroglie [14] in the 1920s and then rediscovered by Bohm [15, 16] in 1952. In the pilot-wave theory one has particles with definite positions at all times and the wave function, which guides these particles along their trajectories through configuration space. The Schrödinger equation, which governs the time evolution of the wave function, is deterministic and as such every system follows its unique, deterministic trajectory. Hence it is only our lack of knowledge of the initial state of the system which prohibits us from computing the outcome of a particular measurement in advance and one can show that we cannot know the initial state better than $|\Psi(t_0)|^2$ [17]. In this theory the probabilistic nature of standard quantum mechanics is not a fundamental property of nature but arises from incomplete knowledge of the initial state of the system including the measurement apparatus - which would require the knowledge of the positions of roughly $N_A \sim 10^{23}$ particles. Hence the proponents of the pilot-wave theory often depict it as taking a role similar to that of Newtonian mechanics in the framework of statistical mechanics.

REFERENCES

[1] Schrödinger, E.: *Quantisierung als Eigenwertproblem*, Ann. der Physik **79**, 361-376 and 489-527; **80**, 437-490; **81**, 109-139 (all 1926)

[2] Schrödinger, E.: *An Undulatory Theory of the Mechanics of Atoms and Molecules*, The Physical Review **28**, 1049-1070 (1926)

[3] Schrödinger, E.: *Die gegenwärtige Situation in der Quantenmechanik*, Die Naturwissenschaften **23**, 807-812, 823-828 and 844-849 (1935)

[4] Bassi, A., and Ghirardi, G.: *The Trieste Lecture of John Stewart Bell*, J. Phys. A.: Math. Theor. **40**, 2919 (2007)

[5] second footnote in Born, M.: *Zur Quantenmechanik der Stoßvorgänge*, Zeitschrift für Physik **37**, 863-867 (1926)

[6] Bacciagaluppi, G., and Valentini, A.: *Quantum Theory at the Crossroads: Reconsidering the 1927 Solvay Conference*, Cambridge University Press, Cambridge (2009)

[7] Bohr, N.: *The Quantum Postulate and the Recent Development of Atomic Theory*, Nature **121**, 580-590 (1928)

[8] Heisenberg, W.: *Physics and Philosophy: The Revolution in Modern Science*, Chapter III; London: George Allen and Unwin, Ltd. (1959)

[9] Bell, J.S.: *Against 'measurement'*, Physics World **3** (8), 33 (1990)

[10] Einstein, A., Podolsky, B., and Rosen, N.: *Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?*, Phys. Rev **47**, 777 (1935)

[11] Ghirardi, G.C., Rimini, A., and Weber, T.: *Unified dynamics for microscopic and macroscopic systems*, Phys. Rev. D **34**, 470 (1986)

[12] Feldmann, W., and Tumulka, R: *Parameter diagrams of the GRW and CSL theories of wavefunction collapse*, J. Phys. A: Math. Theor. **45**, 065304 (2012)

[13] Diósi, L: *Models for universal reduction of macroscopic quantum fluctuations*, Phys. Rev. A **40**, 1165 (1989)

[14] deBroglie, L.: *La mécanique ondulatoire et la structure atomique de la matière et du rayonnement*, Journal de Physique et le Radium **8** (5), 225-241 (1927)

[15] Bohm, D.: *A suggested Interpretation of the Quantum Theory in Terms of Hidden Variables, I*, Phys. Rev. **85** (2), 166-179 (1952)

[16] Bohm, D.: *A suggested Interpretation of the Quantum Theory in Terms of Hidden Variables, II*, Phys. Rev. **85** (2), 180-193 (1952)

[17] Dürr, D., Goldstein, S., and Zanghi, N.: *Quantum equilibrium and the origin of absolute uncertainty*, J. Stat. Phys **67**, 843-907 (1992)

[18] Norsen, T.: *Foundations of Quantum Mechanics*, Springer (2017)

[19] Dürr, D., and Lazarovici, D.: *Understanding Quantum Mechanics*, Springer (2020)

[20] Bricmont, J.: *Making Sense of Quantum Mechanics*, Springer (2016)

[21] Bell, J.S.: *Speakables and Unspeakables in Quantum Mechanics*, Cambridge University Press, Cambridge (1987)