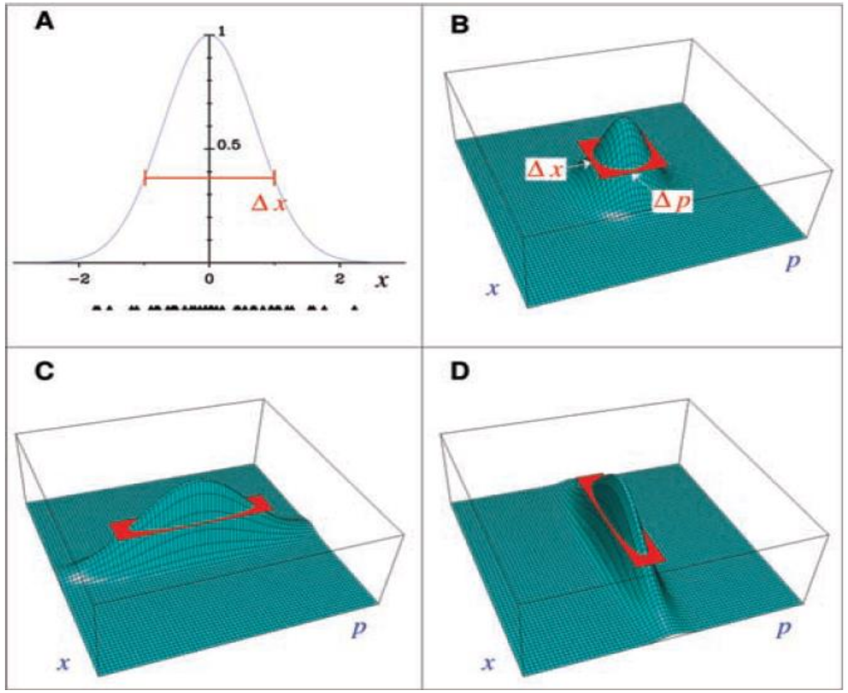


Vittorio Giovannetti. (2004) Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

Quantum Metrology

BEYOND THE STANDARD QUANTUM LIMIT

LEE JUNHEE

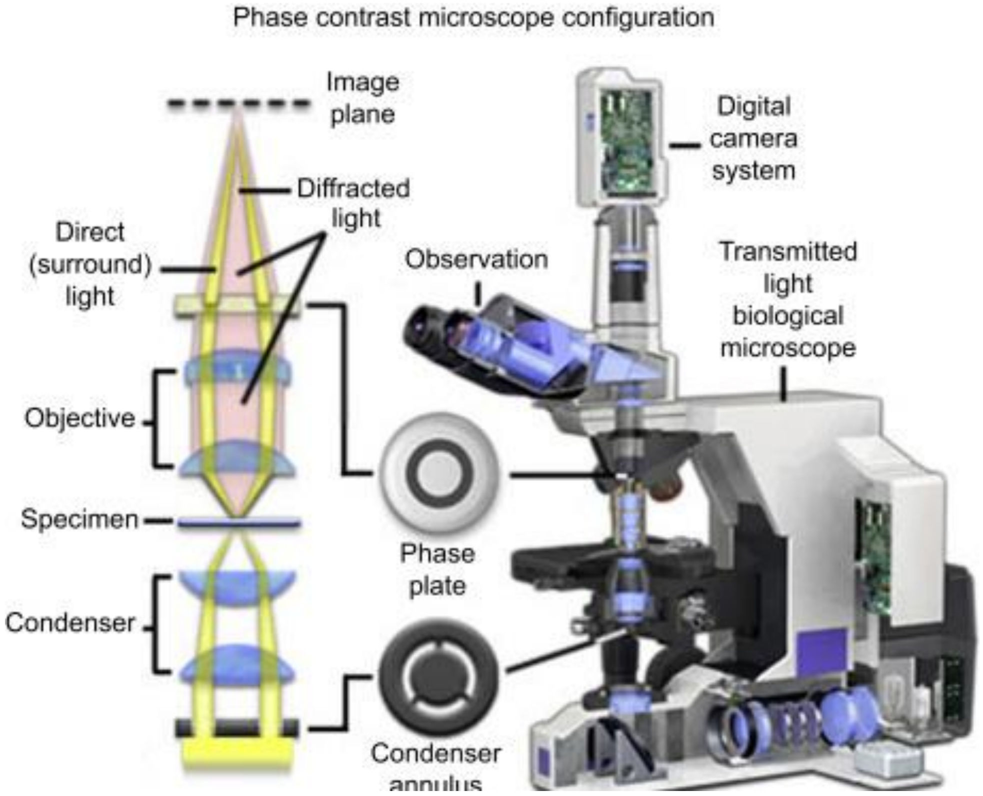
Outline

1. Measurements Limits
   - Familiar limit: Rayleigh’s criteria.
   - Heisenberg uncertainty & Standard Quantum limit
2. Standard Quantum Limit
   - Definition
   - Typical Example in Optical Experiment set up
3. Classical Measurement Scheme
   - Mach-Zehnder Interferometer

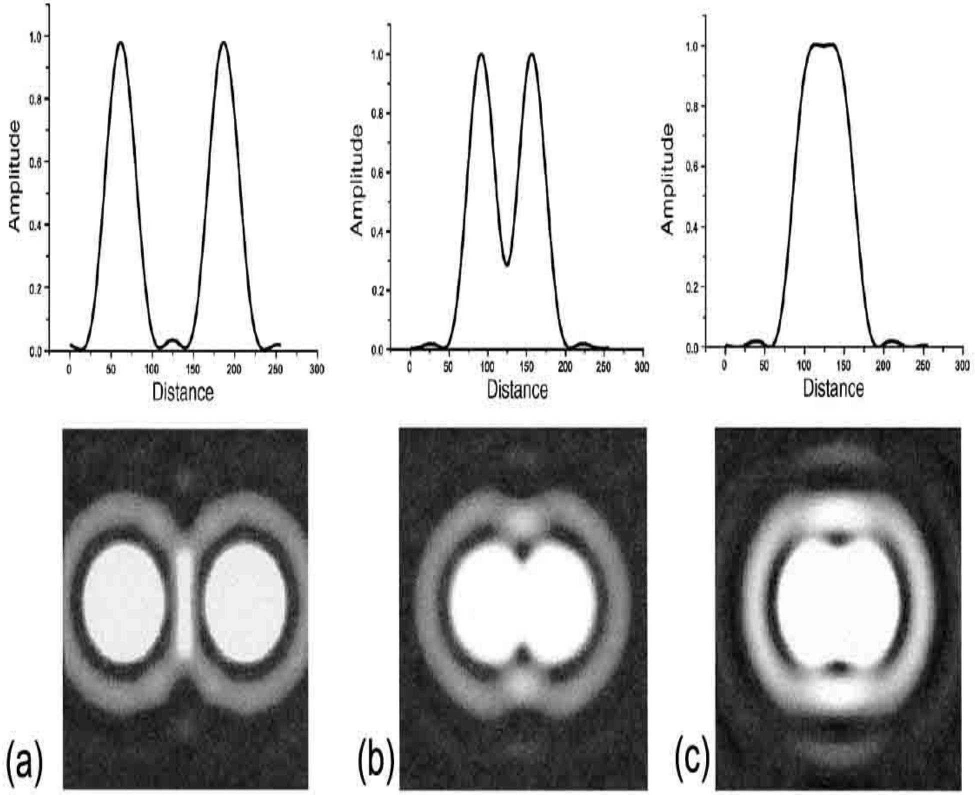
Outline

4. Quantum Measurement Scheme (Squeezed State)
   - Definition
   - Experimental realization
   - Beyond SQL
   - Application Ex: LIGO
5. Quantum Measurement Scheme (N00N State)
   - Definition
   - Experimental Realization
   - Advantages
   - Application Ex: Protein Concentration
6. Summary and Back to Heisenberg uncertainty

1.Optical Microscopy

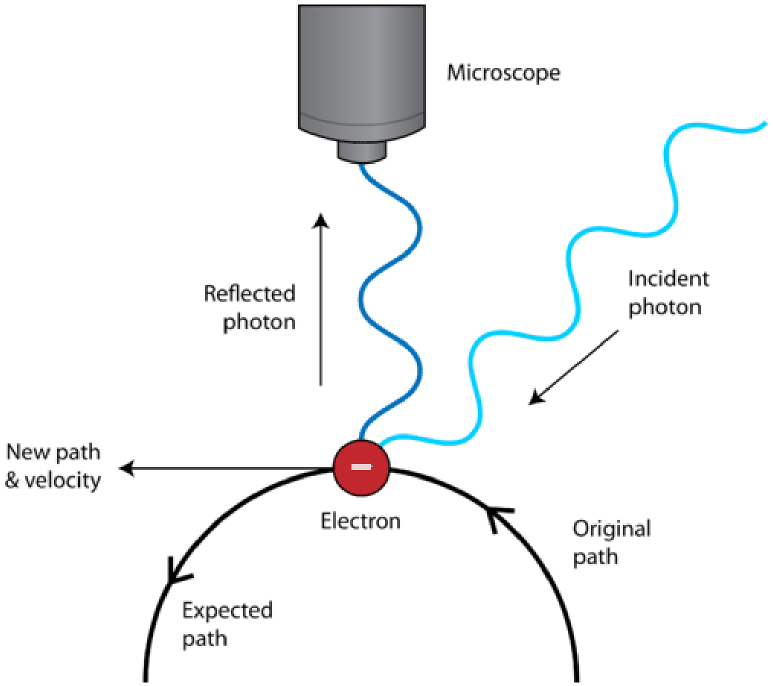


https://www.sciencedirect.com/topics/engineering/optical-microscope



https://www.globalsino.com/EM/page1982.html

1. Heisenberg Uncertainty



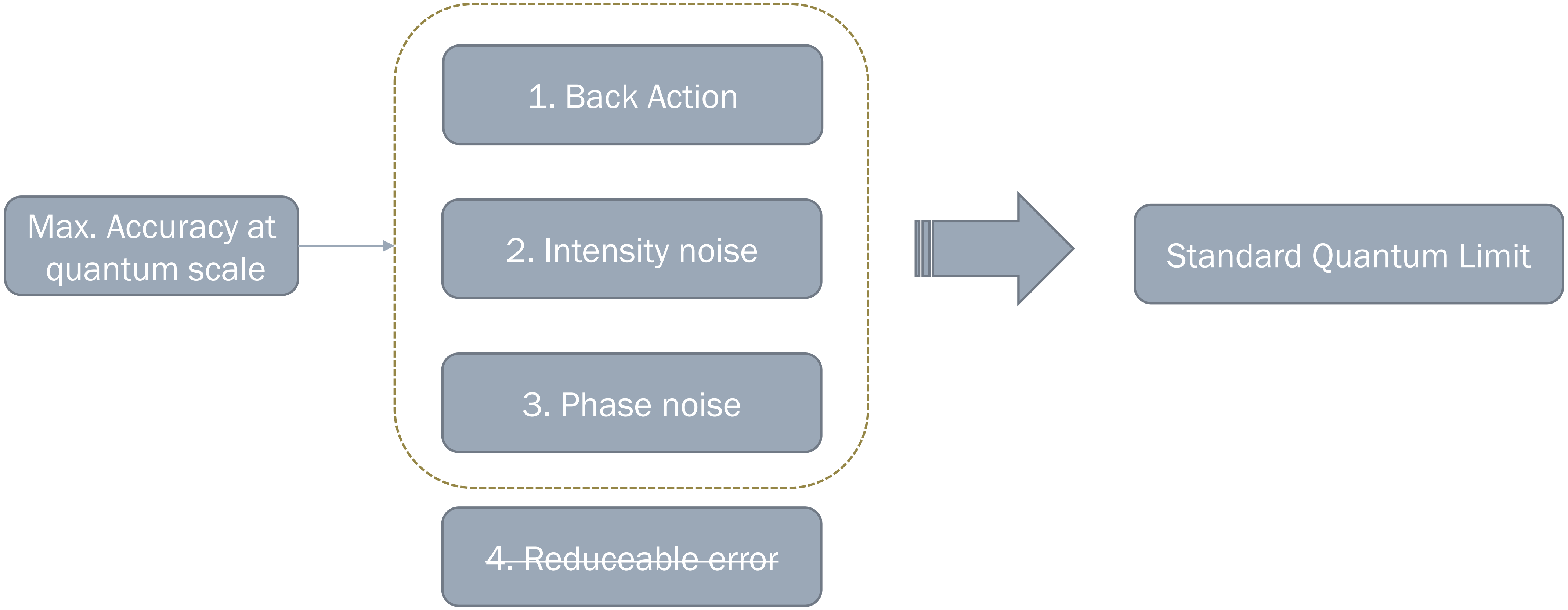
https://www.ck12.org/c/chemistry/heisenberg-uncertainty-principle/lesson/Heisenberg-Uncertainty-Principle-CHEM/

Fundamental limits of Nature

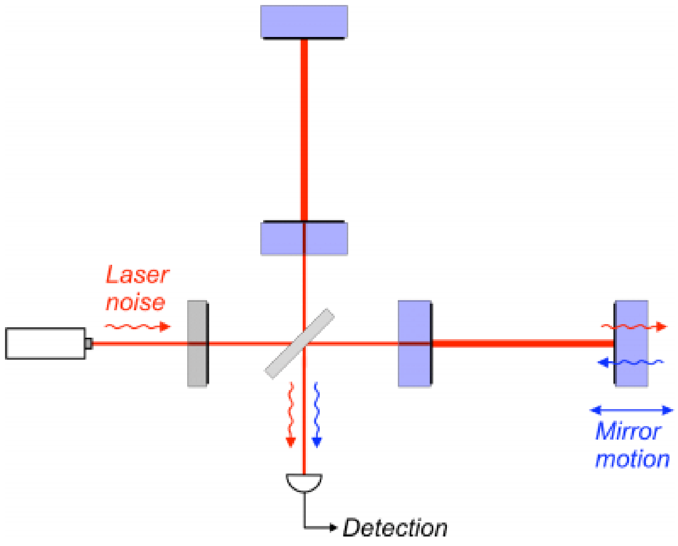
No matter the way you measure the system

Action of Measurement affects to state

2. (Standard) Quantum Limit



2. Ex) SQL in interferometer



1. Particle nature: Shot noise
2. Phase noise: Radiation pressure

http://moriond.in2p3.fr/J07/trans/saturday/heidmann.pdf

Non-fundamental limits for measurement

Relevant to quantum system and difficult to reduce

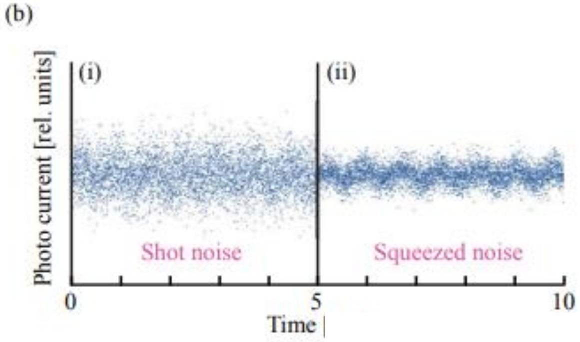
Relevant to classical measurement scheme

2.5 Questions

1. Rayleigh's Criteria
2. Standard Quantum Limit
3. SQL in Interferometer

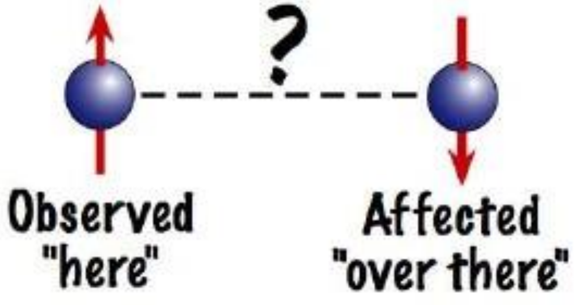
3.Beyond the SQL

Quantum Enhanced Measurement



a. Squeezed state

b. NOON State



Quantum Non-Demolition Detection

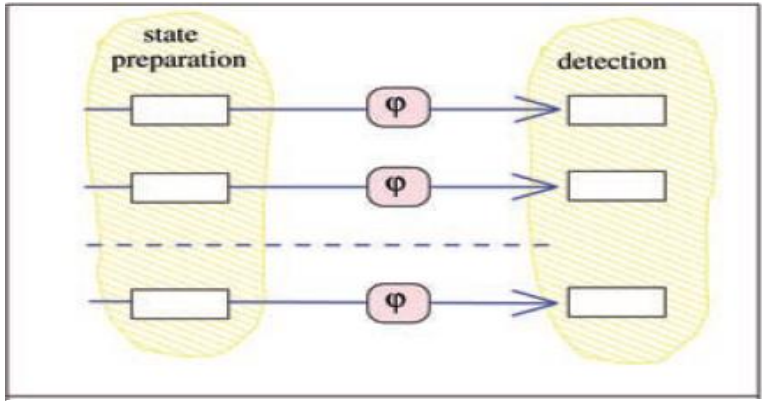
$$\sigma_{A_1}\sigma_{A_2} \geq | \frac{1}{2}[\hat{A}(t_1),\hat{A}(t_2)]| = 0$$

→ No backaction

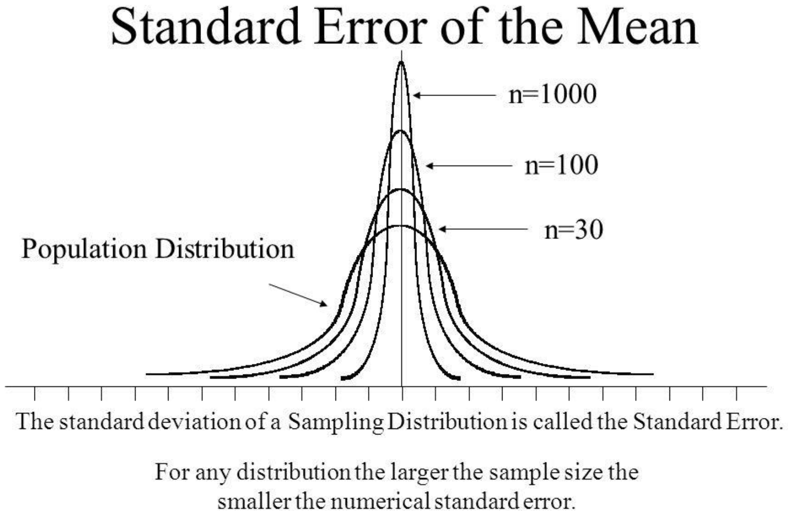
→ Enable consequent measurement

3. Classical vs Quantum Measurement

Classical Measurement



Vittorio Giovannetti. (2004) Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

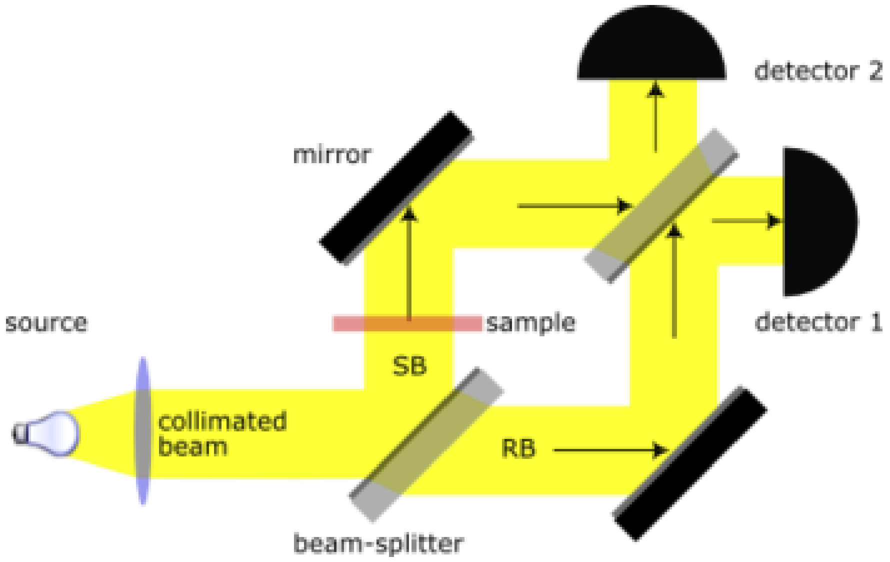


- N Independent physical system
- N separate measurement → Stat.

$$\sigma \propto \frac{1}{\sqrt{N}}$$

3. Classical measurement

Classical MZI



https://en.wikipedia.org/wiki/Mach%E2%80%93Zehnder_interferometer

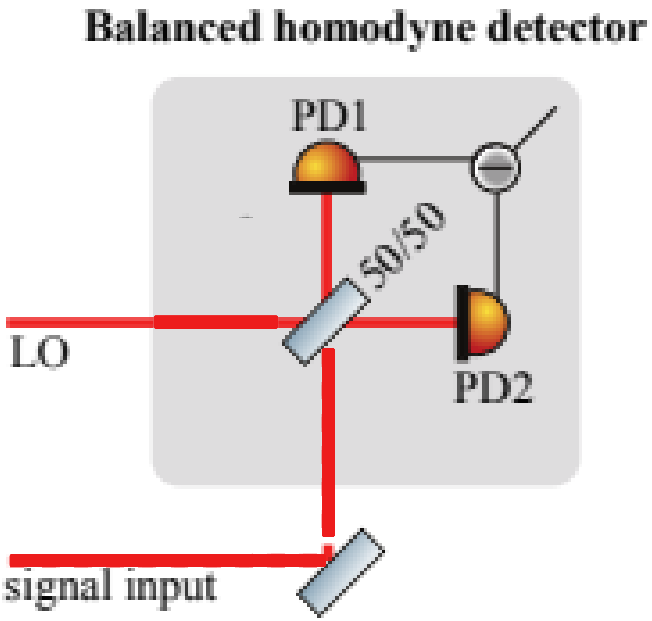
$$\varphi_1 = \frac{1}{\sqrt{2}}(\hat{R}\hat{T} + \hat{T}\hat{S}\hat{R})\ |\varphi_0 >= \frac{i}{\sqrt{2}}(1 + e^{i\theta})\ |\varphi_0 >$$

$$\varphi_2 = \frac{1}{\sqrt{2}}(\hat{R}\hat{S}\hat{R} + \hat{T}\hat{T})|\varphi_0 >= \frac{1}{\sqrt{2}}(1 - e^{i\theta})\ |\varphi_0 >$$

$$p_1(\mathrm{t}) = |< \varphi_1|\varphi_0 > |^2 = cos^2\left(\frac{\theta}{2}\right)$$

$$p_2(\mathrm{t}) = |< \varphi_2|\varphi_0 > |^2 = sin^2\left(\frac{\theta}{2}\right)$$

3. Classical measurement(Conti.)



R.Schnabel (2017) Squeezed states of light
and their applications in laser interferometers

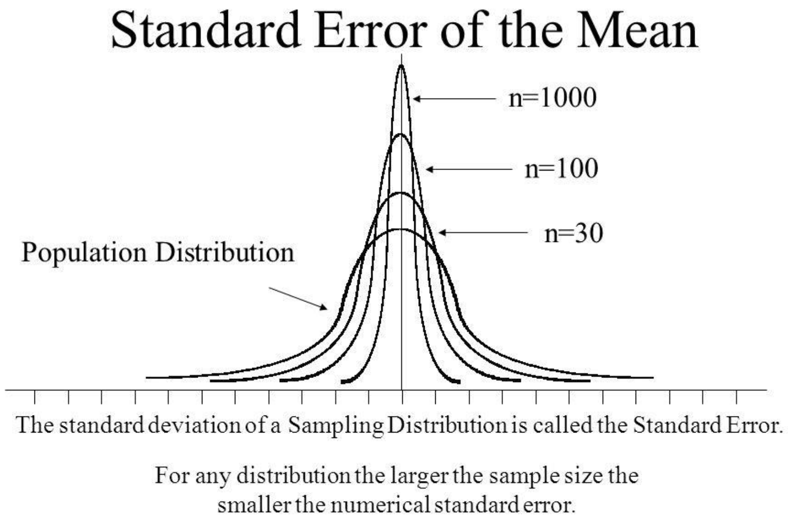
$I = I_1 - I_2$ *is used to decode* information.

$N_1 = Np_1(\mathrm{t}) = N\cos^2\left(\frac{\theta}{2}\right) \rightarrow I_1$ *is intensity at det.* 1

$N_2 = Np_2(\mathrm{t}) = N\sin^2\left(\frac{\theta}{2}\right) \rightarrow I_2$ *is intensity at det.* 2

→ $I = I_1 - I_2 \propto \mathrm{N}\cos(\theta)$

3. Classical measurement(Conti.)



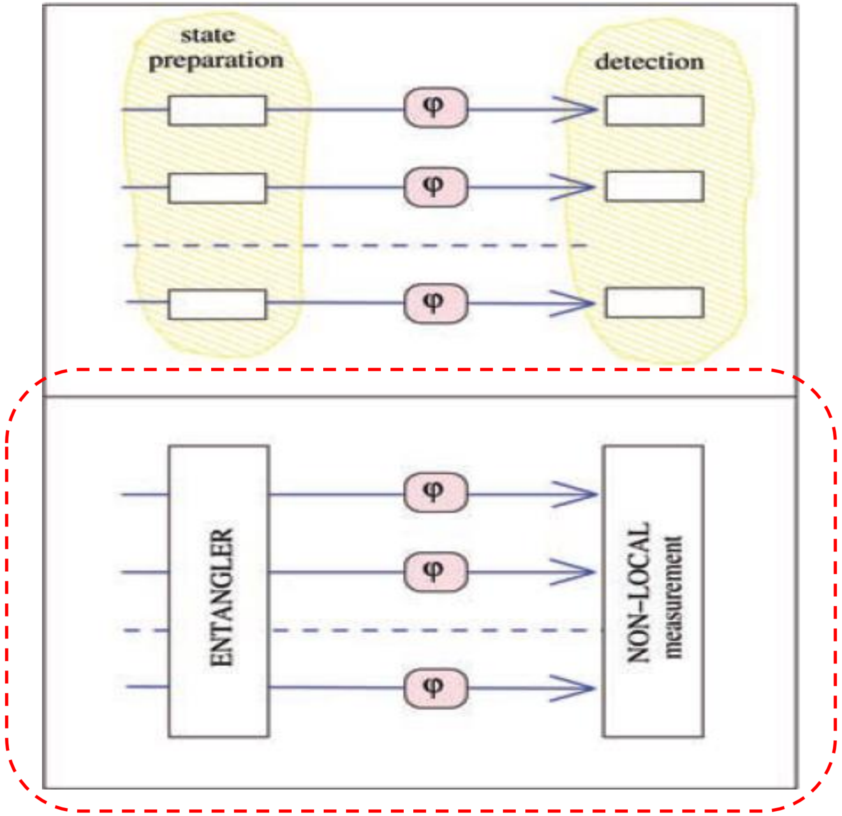
$$\sigma \propto \frac{1}{\sqrt{N}}$$

Information is encoded in $\theta, not\ intensity$

■ Error propagation

→ $\Delta\theta = \frac{\Delta I}{\frac{\partial I}{\partial \theta}} = \frac{1}{\sqrt{N}}$ $\therefore \Delta I = \sqrt{< I^2 > - < I >^2}$

4. Classical vs Quantum Measurement



Vittorio Giovannetti. (2004) Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

Highly correlated input

- Entangled or Squeezed state

Collective measurement

- Encompasses all the systems

4. Coherent state

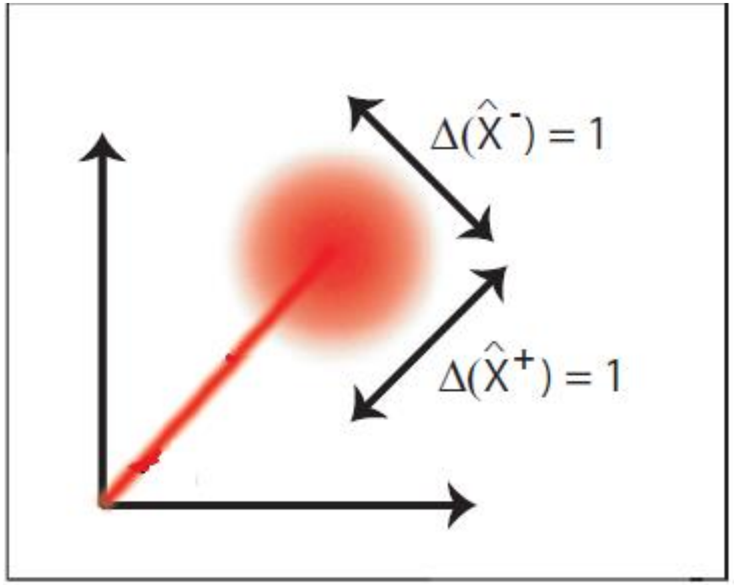
Definition

- The coherent state refers to a super -position of states of the quantized EM field.

- Often, coherent light is thought of as light emitted by sources in-phase.

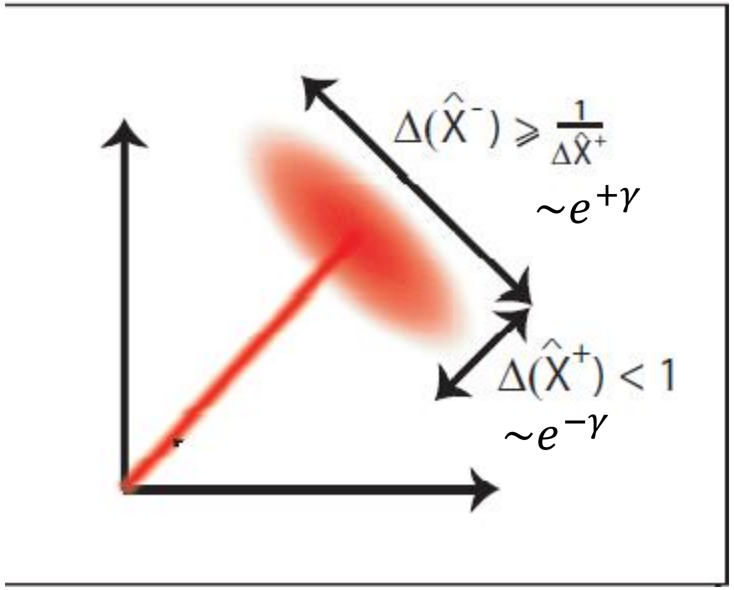
$$|\alpha> = e^{-\frac{|\alpha|^2}{2}} \sum \frac{\alpha^n}{\sqrt{n!}} |n>$$

Phase space

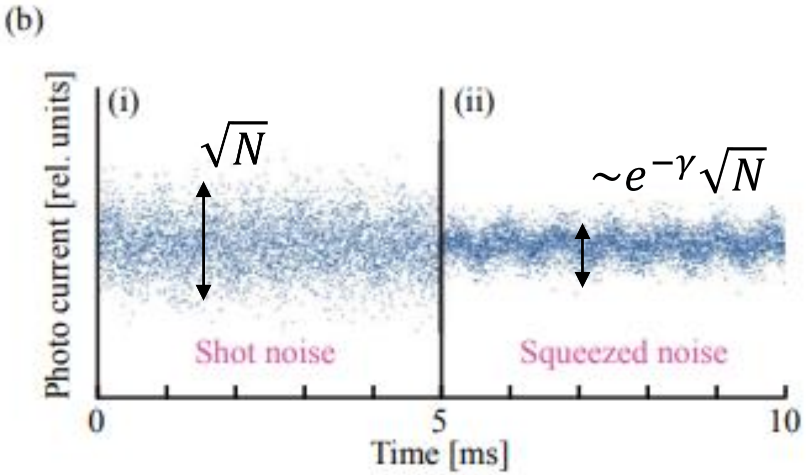


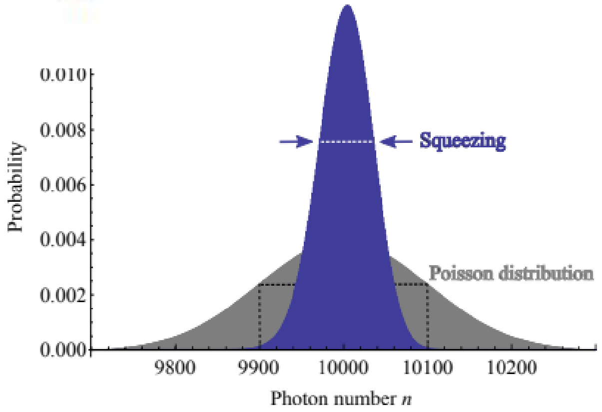
(Left) https://physik.uni-paderborn.de/en/silberhorn/forschung/quantum-networking/continuous-variables

4. Squeezed state



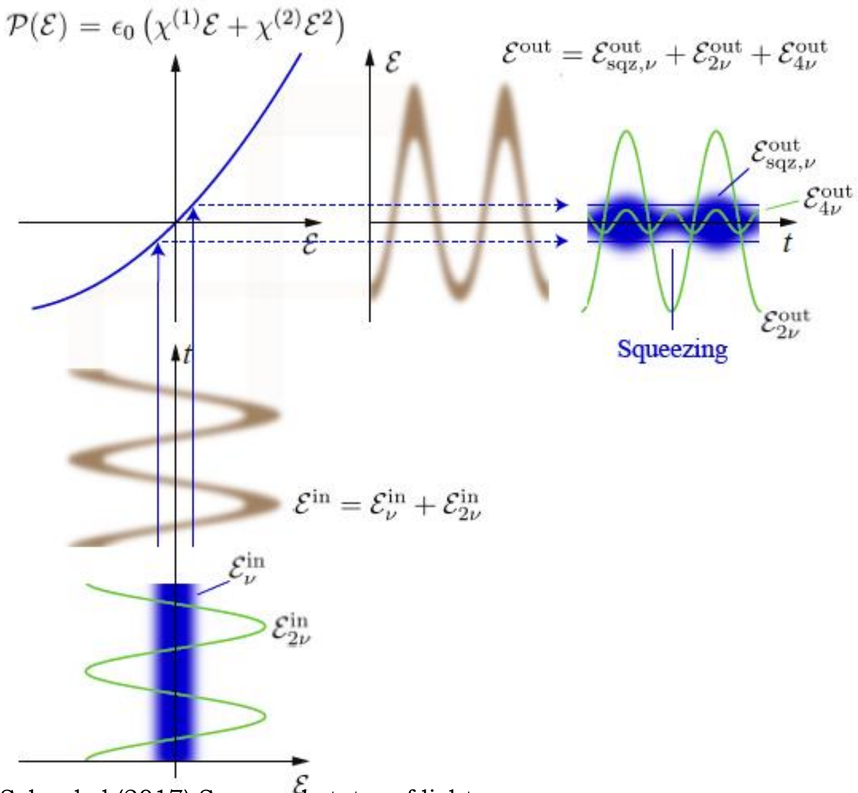
(Left) https://physik.uni-paderborn.de/en/silberhorn/forschung/quantum-networking/continuous-variables





(For middle & right) R.Schnabel (2017) Squeezed states of light and their applications in laser interferometers

4.Generation of Squeezed States



R.Schnabel (2017) Squeezed states of light
and their applications in laser interferometers

1. $\varepsilon^{input} = \varepsilon^{in}_{\nu} + \varepsilon^{in}_{2\nu}$

2. $P(\varepsilon) = \epsilon_0(x^{(1)}\varepsilon + x^{(2)}\varepsilon^2)$

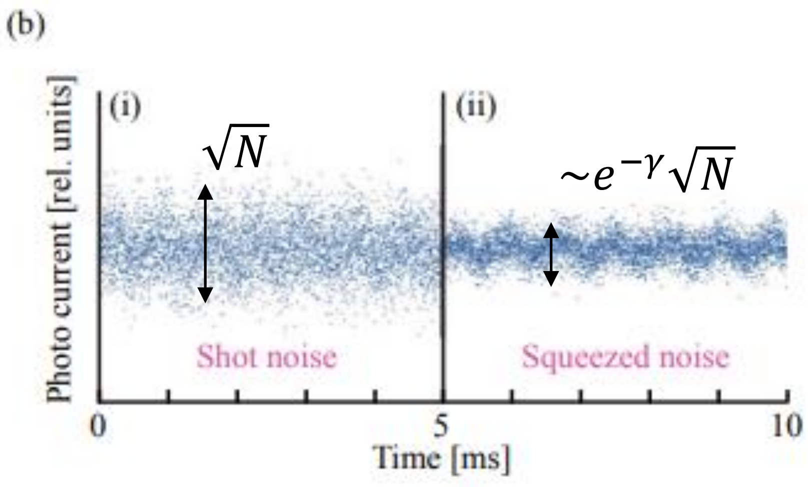
↓

(dielectric polarization)

↓

3. $\varepsilon^{out} = \varepsilon^{out}_{sqz,\nu} + \varepsilon^{out}_{2\nu} + \varepsilon^{out}_{4\nu}$

4. Squeezed state

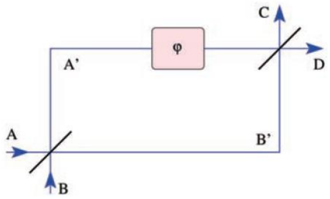


R.Schnabel (2017) Squeezed states of light and their applications in laser interferometers

$$\rightarrow (Coherent)\ \Delta\theta = \frac{\Delta I}{\frac{\partial I}{\partial \theta}} \propto \frac{\Delta N}{\frac{\partial N}{\partial \theta}} = \frac{1}{\sqrt{N}}$$

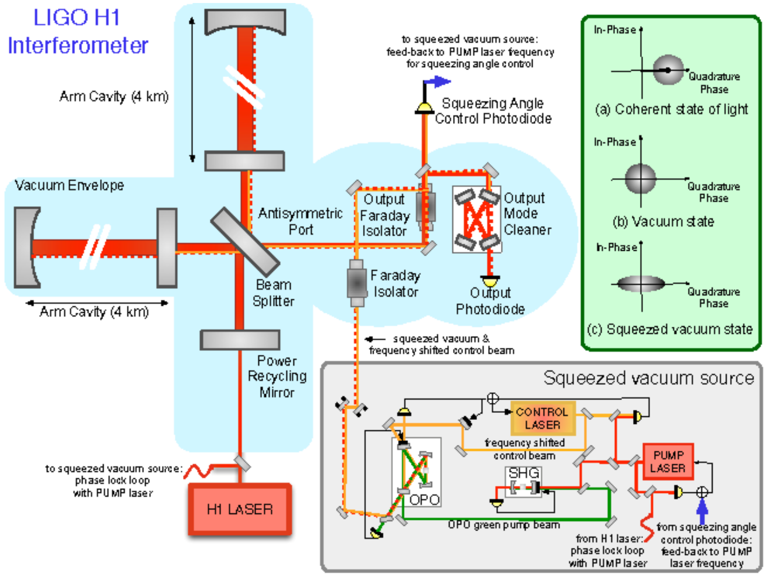
$$\rightarrow (Squeezed)\ \Delta\theta \propto \frac{\Delta N_{squeezed}}{\frac{\partial N}{\partial \theta}} = \frac{e^{-\gamma}\Delta N}{\frac{\partial N}{\partial \theta}} = \frac{e^{-\gamma}}{\sqrt{N}}$$

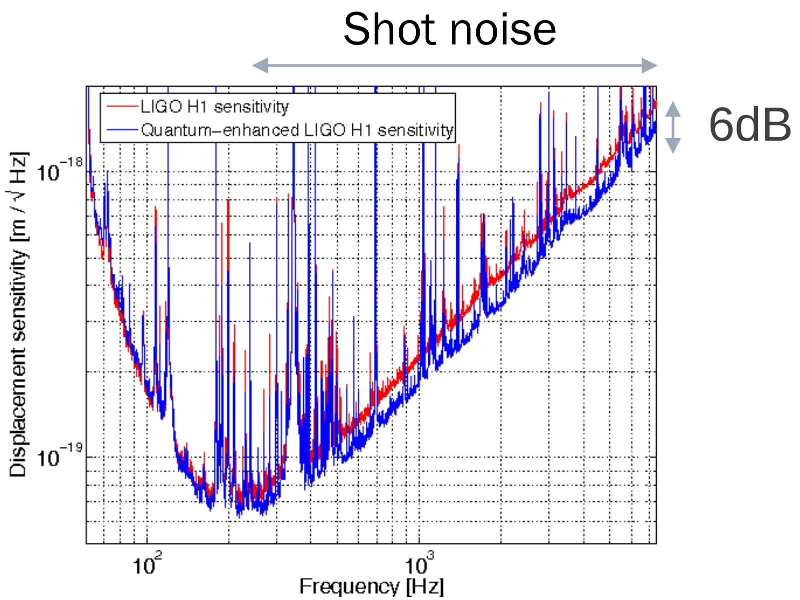
$$c.f)\ (Squeezed)\ \Delta(\theta) \geq \frac{1}{2\sqrt{2}}\left[\frac{1}{<n>^2 + <n>}\right]^{1/2}$$



Vittorio Giovannetti. (2004) Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

4. Application Ex: LIGO





L.Barsotti (2017) Quantum noise reduction in the LIGO gravitational wave interferometer with squeezed states of light

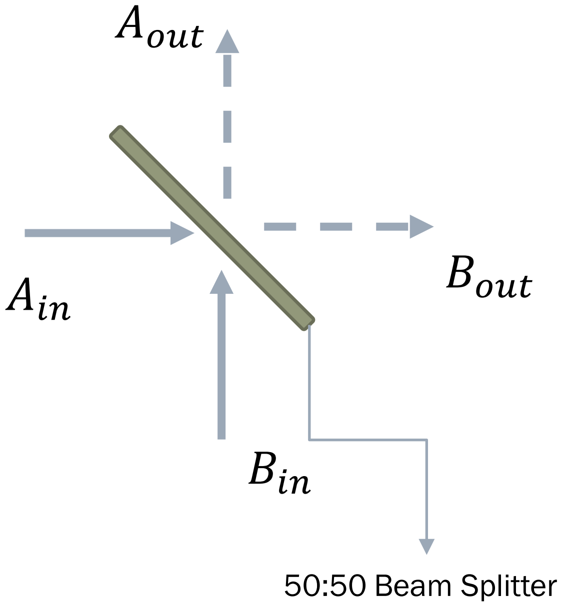
5. N00N State

Definition

$|N00N >= |N >_A|0 >_B + |0 >_A|N >_B$

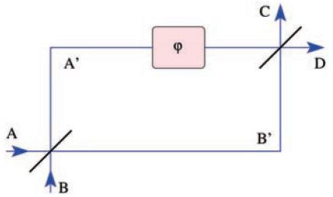
- A superposition of N particles in one mode A with zero particles in another mode *B*
- *Usually, particles are photons*

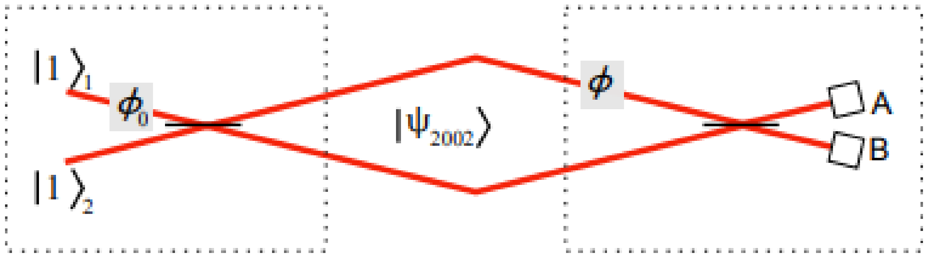
Realization (low N)



- $|1 >_A|0 >_B =$
  $|1 >_{A'}|0 >_{B'} +|0 >_{A'}|1 >_{B'}$
- $|1 >_A|1 >_B =$
  $|2 >_{A'}|0 >_{B'} +|0 >_{A'}|2 >_{B'}$

5. N00N state (Entangled states)





(Above) Vittorio Giovannetti. (2004) Quantum-Enhanced Measurements: Beating the Standard Quantum Limit
(Below) Michael A. Taylor (2016) Quantum metrology and its application in biology

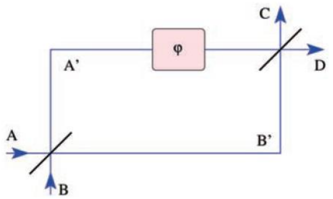
$$|N00N>=|N>_A|0>_B+|0>_A|N>_B$$

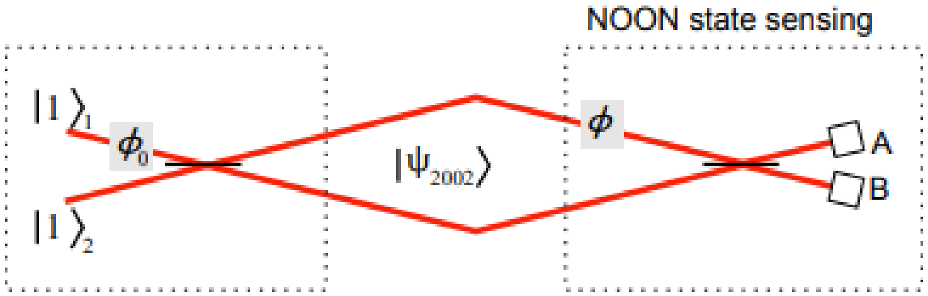
$$|N00N>_{sig}=e^{iN\varphi}|N>_A|0>_B+|0>_A|N>_B$$

$$|\varphi>_{A\ or\ B}=\widehat{HOM}\ |N00N>_{sig}$$

- $|\varphi>_A=e^{iN\varphi}|N>_A|0>_B+|0>_A|N>_B$
- $|\varphi>_B=e^{iN\varphi}|N>_A|0>_B-|0>_A|N>_B$

5. N00N state (Entangled states)

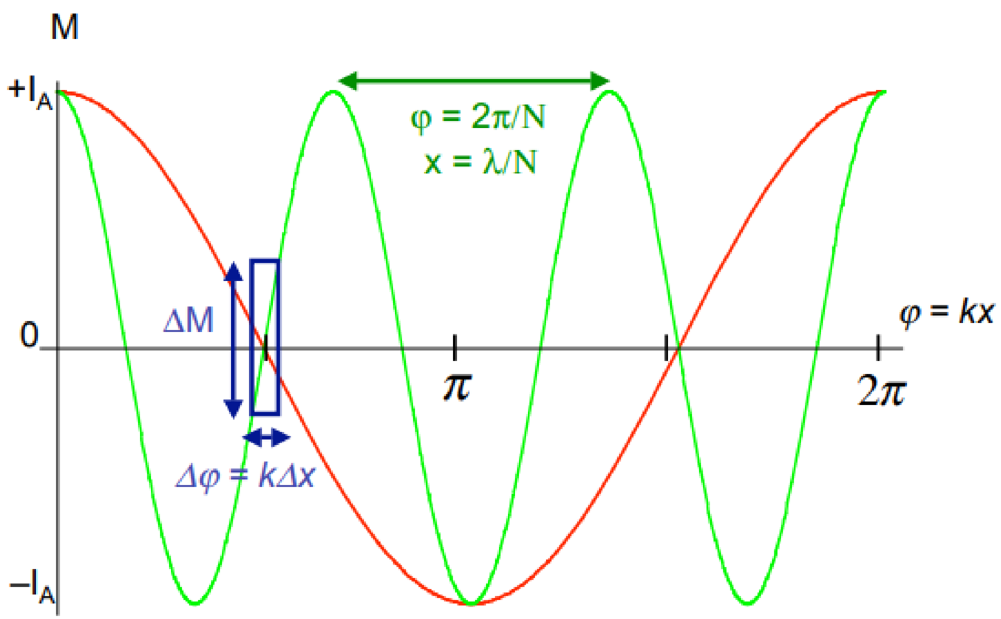




(Above) Vittorio Giovannetti. (2004) Quantum-Enhanced Measurements: Beating the Standard Quantum Limit
(Below) Michael A. Taylor (2016) Quantum metrology and its application in biology

- $p_A = |< N00N|\varphi >_A|^2 = cos^2\left(\frac{N\varphi}{2}\right)$
- $p_B = |< N00N|\varphi >_B|^2 = sin^2\left(\frac{N\varphi}{2}\right)$
- $I = I_a - I_B = I_0 cos(N\varphi)$
- $\Delta\varphi = \frac{\Delta I}{\frac{\partial I}{\partial \varphi}} = \frac{1}{N}$ $\quad \therefore \Delta I = \sqrt{< I^2 > - < I >^2}$

5. Advantages



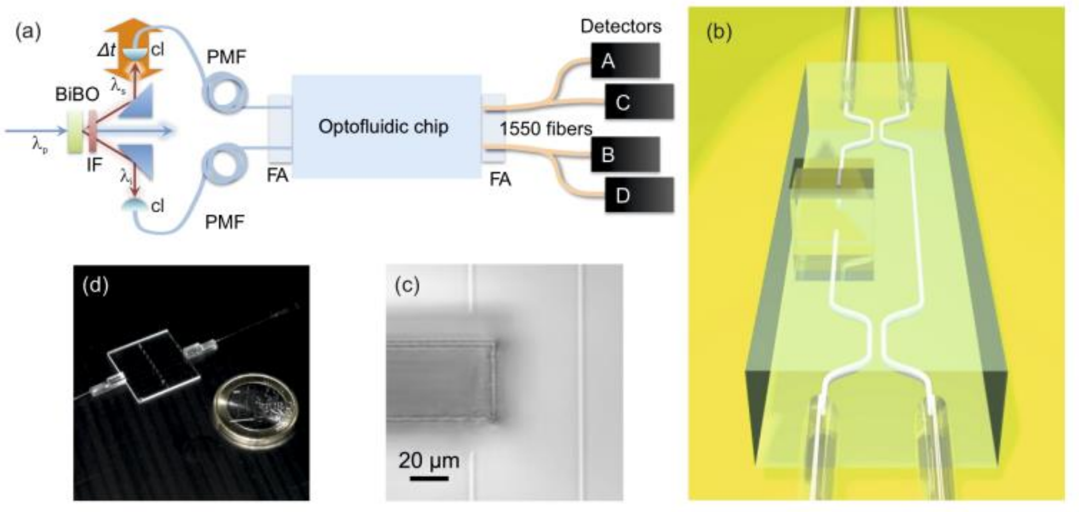
Jonathan P. Dowling. (2008) Quantum Optical Metrology — The Lowdown on High-N00N States,

$$\Delta\varphi = \frac{1}{N}$$

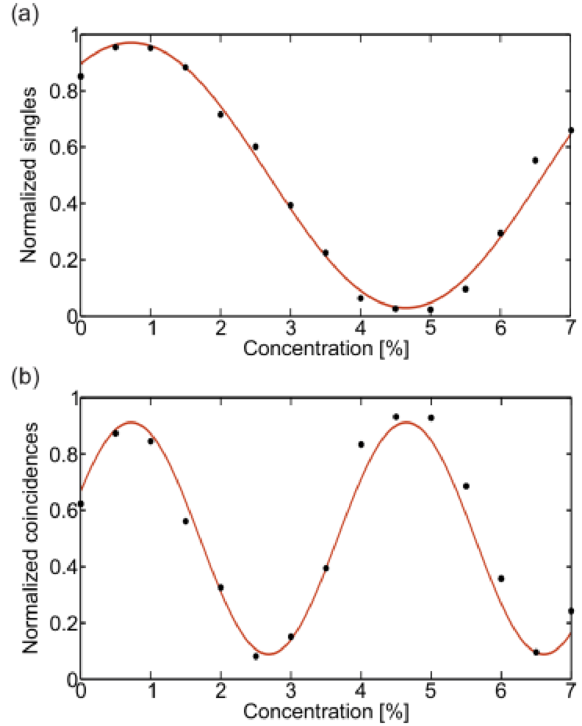
1. Super-sensitivity (minimum detectable signal $\equiv SNR = 1$)

2. Super-resolution (below Rayleigh Criteria)

5. Application Ex: Protein Concentration



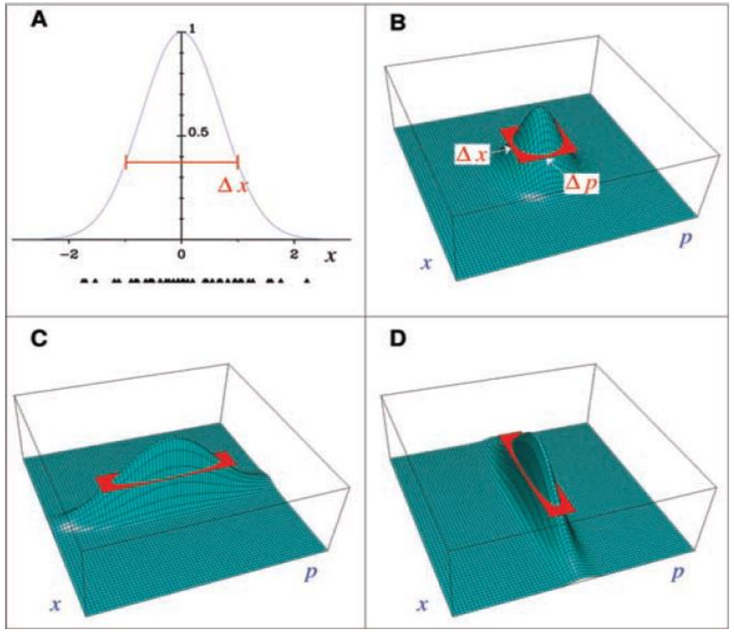
(Both) A.Crespi (2012) Measuring protein concentration with entangled photons



- 2002 states increases sensitivity by 2

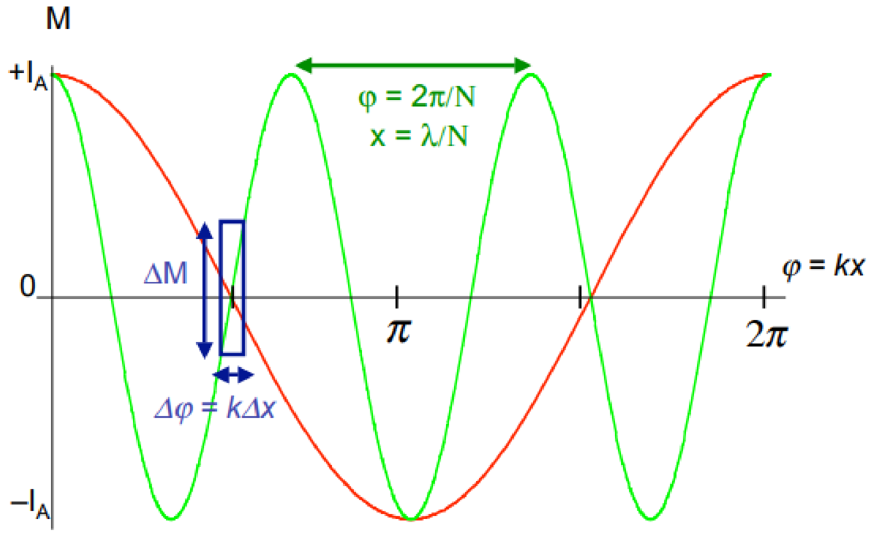
6. Summary

Squeezed state



(Left) Vittorio Giovannetti. (2004) Quantum-Enhanced Measurements: Beating the Standard Quantum Limit

NOON state



(Right) Jonathan P. Dowling. (2008) Quantum Optical Metrology — The Lowdown on High-N00N States,

7. Questions

1. Classical Measurement Scheme
   - Standard Quantum Limit
2. Squeezed Light for Meas.
3. N00N States for Meas.

References *(in year)*

1. Vittorio Giovannetti. (2004) Quantum-Enhanced Measurements:
Beating the Standard Quantum Limit

2. Jonathan P. Dowling. (2008) Quantum Optical Metrology — The Lowdown on High-N00N States

3. A.Crespi (2012) Measuring protein concentration with entangled photons

4. Michael A. Taylor (2016) Quantum metrology and its application in biology

5. L.Barsotti (2017) Quantum noise reduction in the LIGO gravitational wave interferometer with squeezed states of light

6. Roman Schnabel. (2017) Squeezed states of light and their applications in laser interferometers

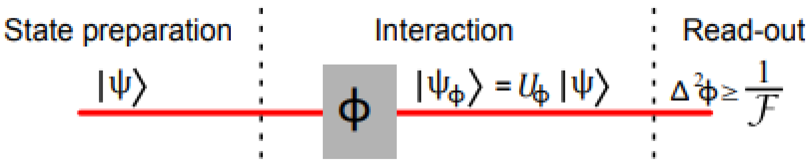
A.1 Error propagation

Measured data $\neq$(directly) Desired Parameter(φ)

$\forall\, function\ s.t$
$f(data) = \varphi$

- Data = Signal + Noise
- Function f does honestly transform (S+N) to $\varphi - space$
- Relation of noise in data and noise in φ
- $\varphi\ = \varphi_0 + \frac{\partial\varphi}{\partial a} da\ \rightarrow\ \sigma_\varphi^2 \approx |\frac{\partial\varphi}{\partial a}|^2 \sigma_a^2$

A.2 N00N-State Measurement



Michael A. Taylor (2016) Quantum metrology and its application in biology

$|\varphi_f> = \ U_\theta|\varphi_i>$

On the signal arm, sample transforms incident light $|\varphi_i>$ to $U_\theta|\varphi_i>$

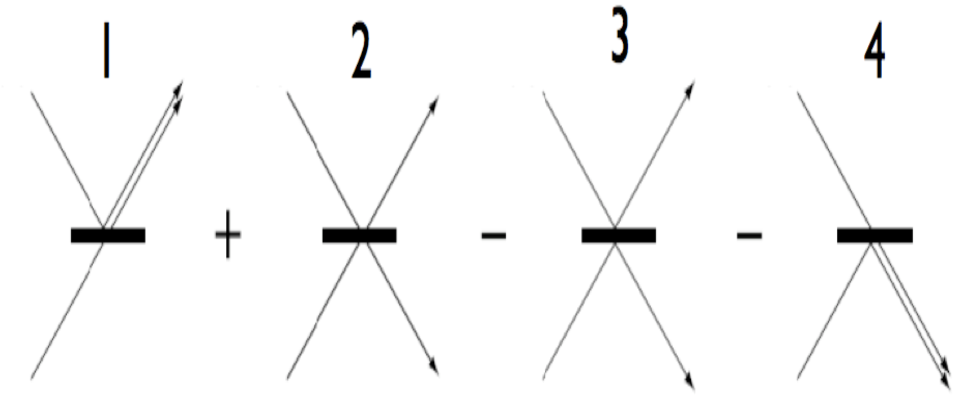
1. Coherent state (usual laser)

$|\alpha_f> = U_\theta|\alpha_i> = e^{i\theta\hat{n}}\,(e^{-\frac{|\alpha|^2}{2}}\sum\frac{\alpha^n}{\sqrt{n!}}|n>) = \ |e^{i\theta}\alpha_i>$

2. N00N state

$|N_f> = U_\theta|N_i> = \ e^{i\theta\hat{n}}\ |N_i> = e^{i\theta N}\ |N_i>$

A.3 Hong-Ou-Mandel effect



1. The photon(up) is reflected and the photon(down) is transmitted.

2. Both are transmitted.

3. Both are reflected.

4. The photon(up) is transmitted and the photon(down) is reflected

https://en.wikipedia.org/wiki/Hong%E2%80%93Ou%E2%80%93Mandel_effect

- Two optical modes are mixed at the B.S and turn to new modes

- $\binom{a'}{b'} = \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\binom{a}{b}$

  →(N00N state) $\binom{a'}{b'} = \frac{1}{\sqrt{2}}\binom{e^{iN\varphi}+1}{e^{iN\varphi}-1}$

A.4 Fisher-Info. & C.R bound

- Unknown Parameter θ is estimated by observations of x → $\text{prob. density}\ f(x;\theta)$
- Var(θ) $\geq \frac{1}{I(\theta)}$ $,where\ I(\theta)$= −n $<\partial_{\theta\theta}\log(f(x;\theta))>$

In Quantum measurement,

- $I = 4\left(<\varphi_f'|\varphi_f'> - |<\varphi_f'|\varphi_f>|^2\right)$

A.4 Fisher-Info. & C.R bound (cont)

- I=4($< n^2 > -| < n > |^2$) (for observation of N)

- For N00N state

  $< n >=< a^+a > = \frac{N}{2}, \quad < n^2 > = < a^+aa^+a > = \frac{N^2}{2},$

- I=4($< n >^2 -|< n > |^2) = N^2$

  → $\Delta(\theta) \geq \frac{1}{N}$