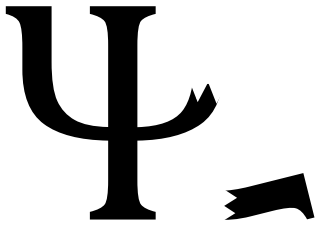
$$i\hbar \frac{\partial}{\partial_t} \Psi(t) = H(t)\Psi(t)$$

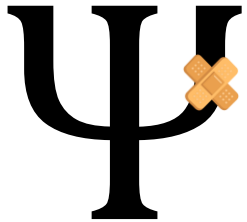
and the physics of measurement

Karen Wadenpfuhl
25.5.2021

Outline

Ontology
The Measurement Problem





Copenhagen Interpretation
Spontaneous Collapse Theories
Bohmian Mechanics

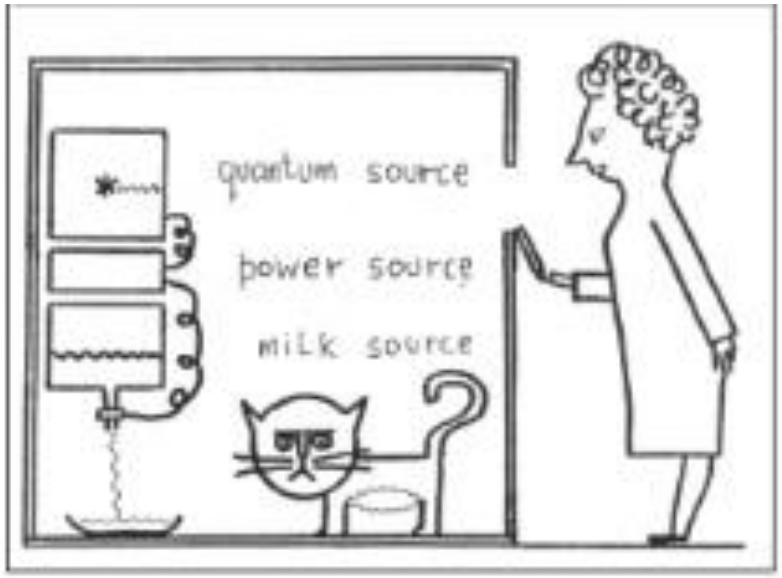
Ontology

ontology (phil.) – the study of the fundamental nature of reality

„[S]ince any physical theory is written in terms of mathematical entities that obey given laws of motion, it should be clear what connection there is between the entities in the mathematics and the entities in the world. [...]
These mathematical entities represent the ontology of the theory: what is real according to the theory."

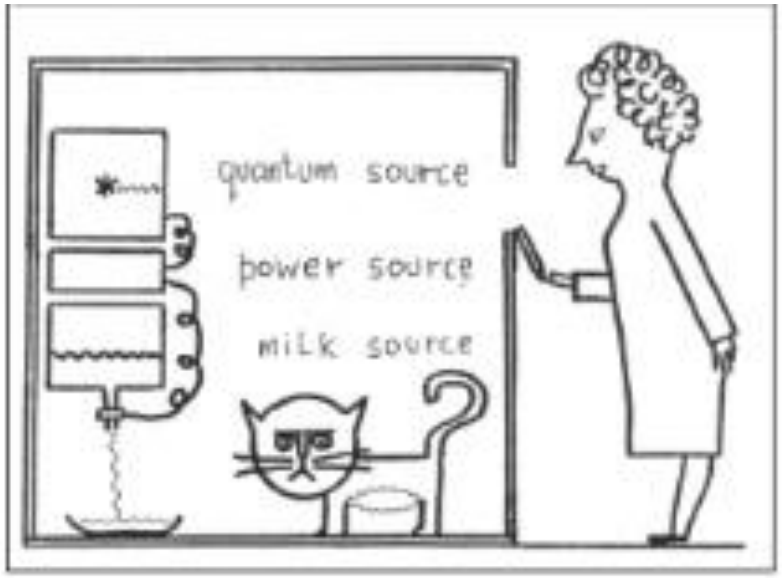
(Valia Allori)

The Measurement Problem



[J. Phys. A.: Math. Theor. 40, 2919 (2007)]

The Measurement Problem



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$$|\Psi\rangle_{t=0} = |e\rangle \;\rightarrow\; |\Psi\rangle_t = \alpha|g\rangle + \beta|e\rangle$$

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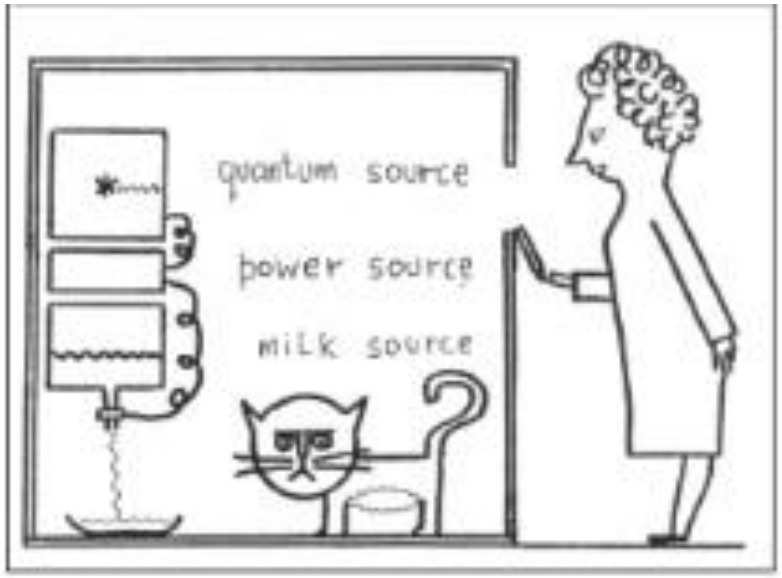

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$$|\Psi\rangle_{t=0} = |e\rangle \;\rightarrow\; |\Psi\rangle_t = \alpha|g\rangle|milk\rangle + \beta|e\rangle|no\ milk\rangle$$

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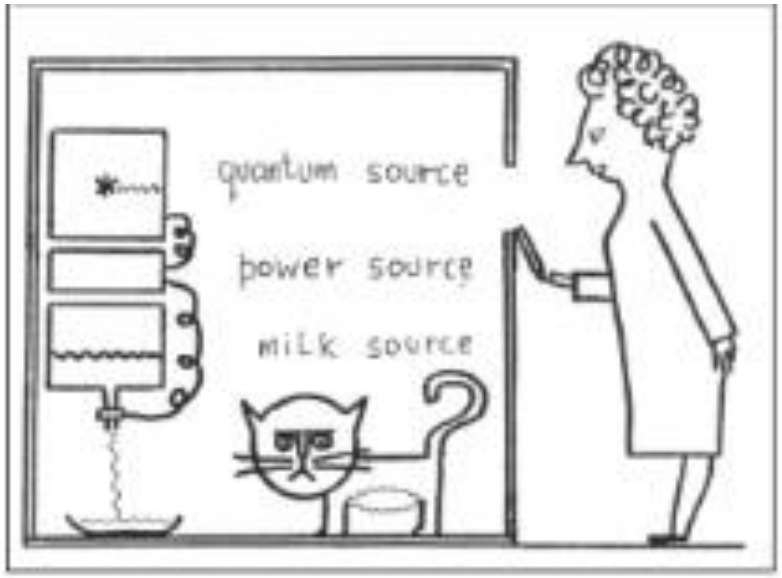

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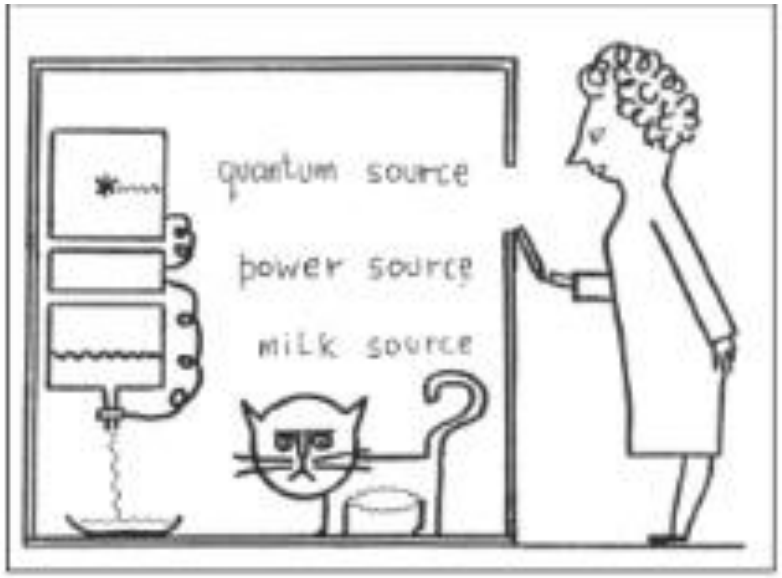

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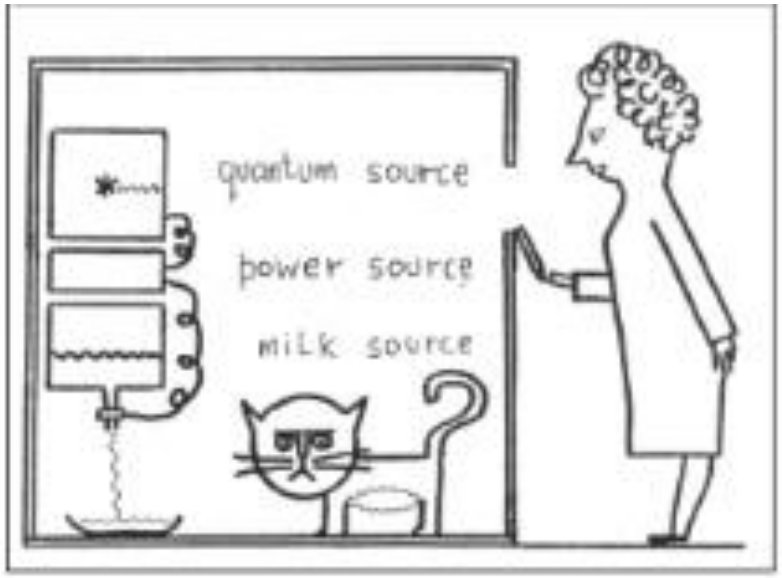


[J. Phys. A.: Math. Theor. 40, 2919 (2007)]

collapse of the wave function

$$|\Psi\rangle_{t=0} = |e\rangle \;\rightarrow\; |\Psi\rangle_t = \alpha|g\rangle|milk\rangle|full\rangle|happy\rangle + \cancel{\beta|e\rangle|no\ milk\rangle|hungry\rangle|sad\rangle}$$

Born's rule: $|\Psi\rangle = \sum_i c_i|i\rangle \Rightarrow P_i = |\langle i|\Psi\rangle|^2 = |c_i|^2$

The Measurement Problem

„Either the wavefunction, as given by the Schrödinger equation, is not everything, or it is not right."

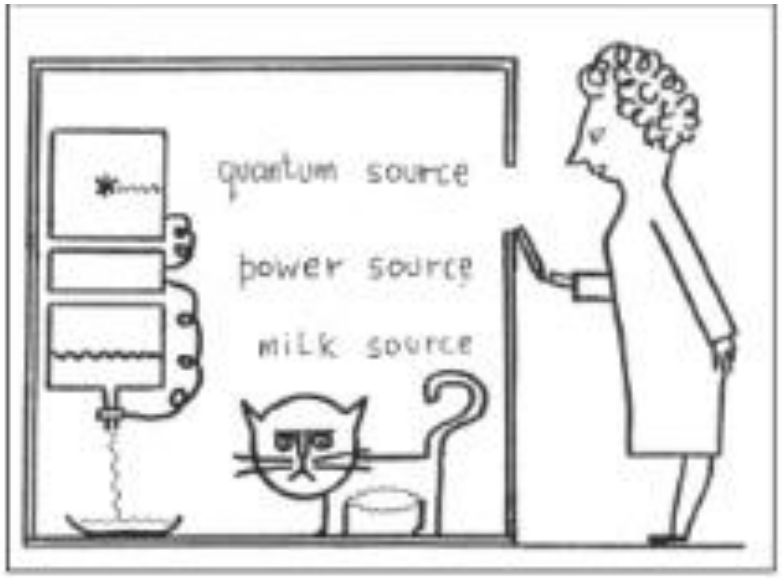
(John S. Bell)

Copenhagen Interpretation

quantum world

e.g. quantum states

$$\Psi = \sum_i c_i \, \psi_i$$



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classical world

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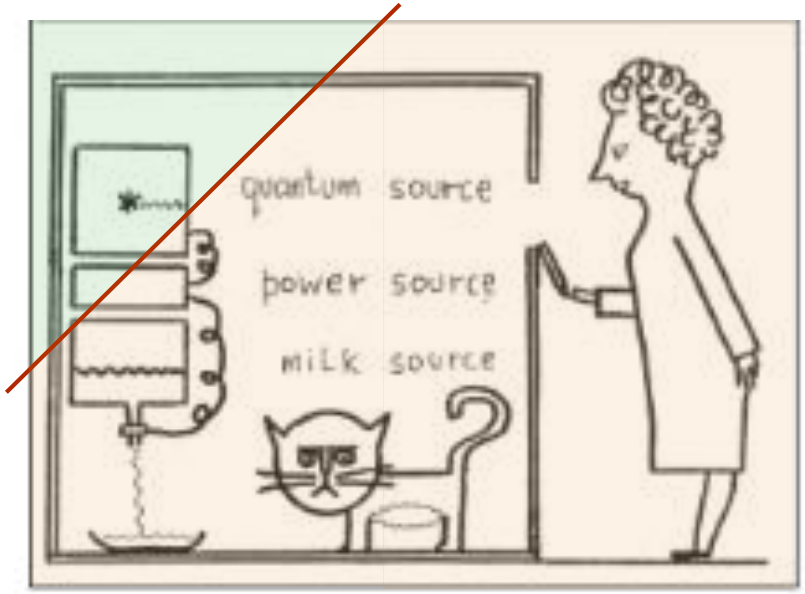

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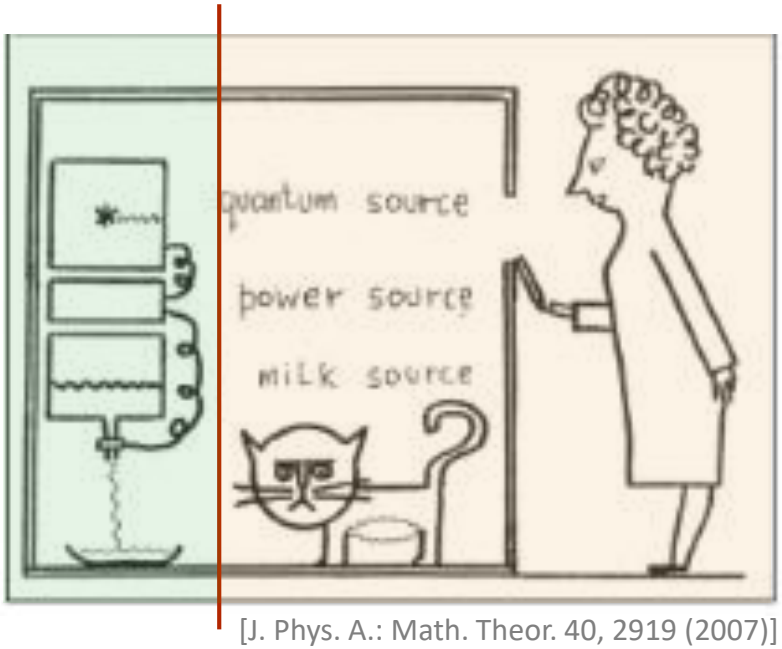

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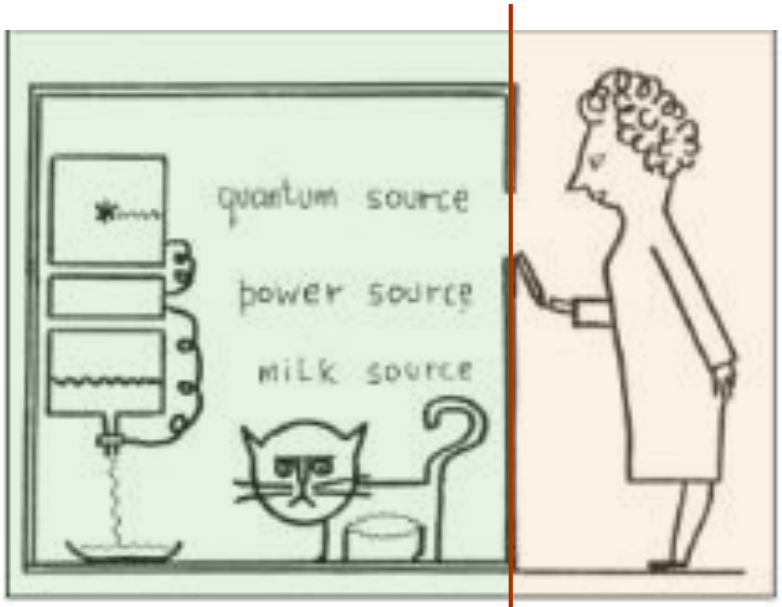

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classical world

e.g. measurement apparati

$$\Psi = \psi_j$$

Copenhagen Interpretation

„... we ourselves produce the results of measurement."

(Ernst Pascual Jordan)

Copenhagen Interpretation

shifty split

quantum world

e.g. quantum states

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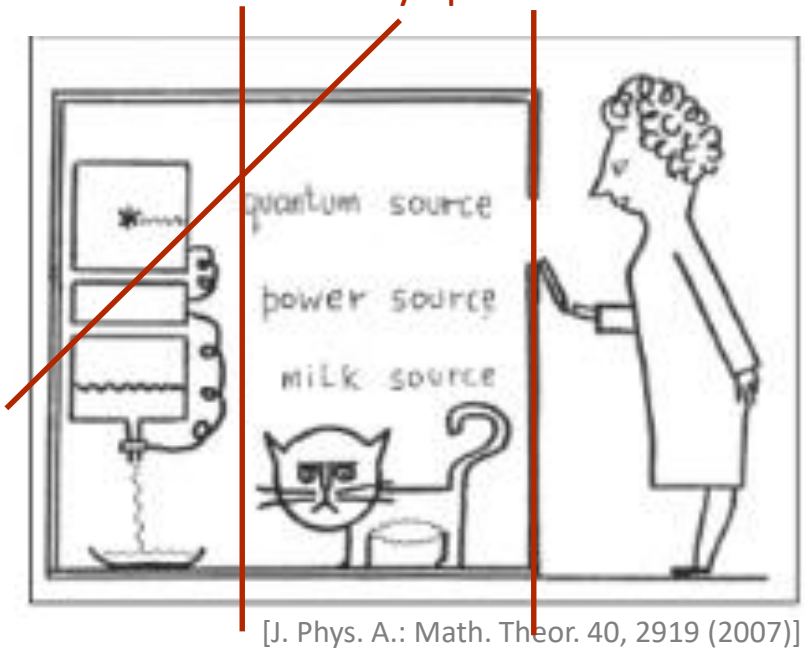

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&\rightarrow \quad |\Psi\rangle_t = \alpha|g\rangle|milk\rangle && + \beta|e\rangle|no\ milk\rangle \\
&\rightarrow \quad |\Psi\rangle_t = \alpha|g\rangle|milk\rangle|full\rangle && + \beta|e\rangle|no\ milk\rangle|hungry\rangle \\
&\rightarrow \quad |\Psi\rangle_t = \alpha|g\rangle|milk\rangle|full\rangle|happy\rangle && + \beta|e\rangle|no\ milk\rangle|hungry\rangle|sad\rangle
\end{aligned}
$$

?

Copenhagen Interpretation

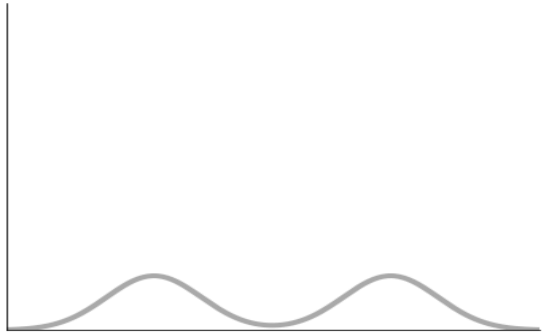
„What exactly qualifies some physical system to play the role of ‚measurer'? Was the wave function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system … with a PhD?
…are we not obliged to admit that more or less ‚measurement-like' processes are going on more or less all the time, more or less everywhere?"

(John S. Bell)

Spontaneous Collapse Theories

$$\Psi(x,t)$$

$$i\hbar\partial_t\Psi(x,t) = H(t)\Psi(x,t)$$



[Phys. Rev. D, **34** (2), pp. 470 (1986)]

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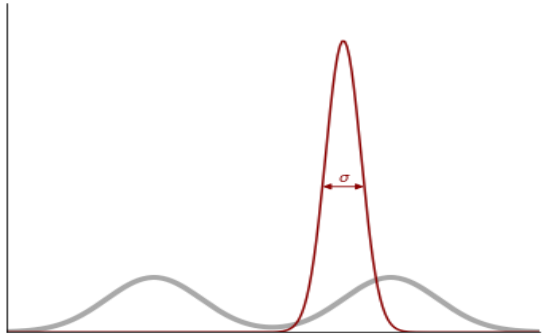

additionally:

random localisations with **localisation width $\boldsymbol{\sigma}$**

$$g_r(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{(x-r)^2}{2\sigma^2}\right)$$

[Phys. Rev. D, **34** (2), pp. 470 (1986)]

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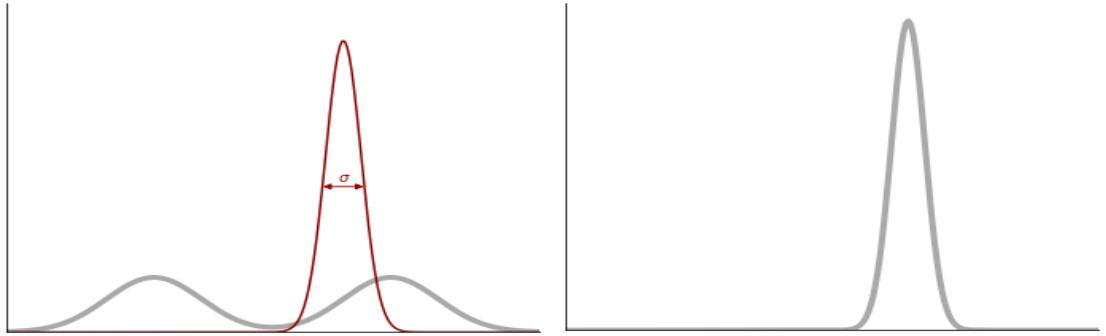

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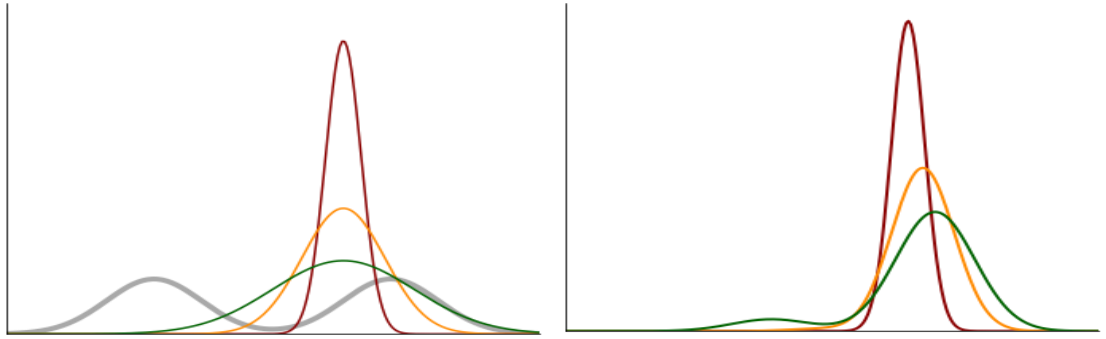

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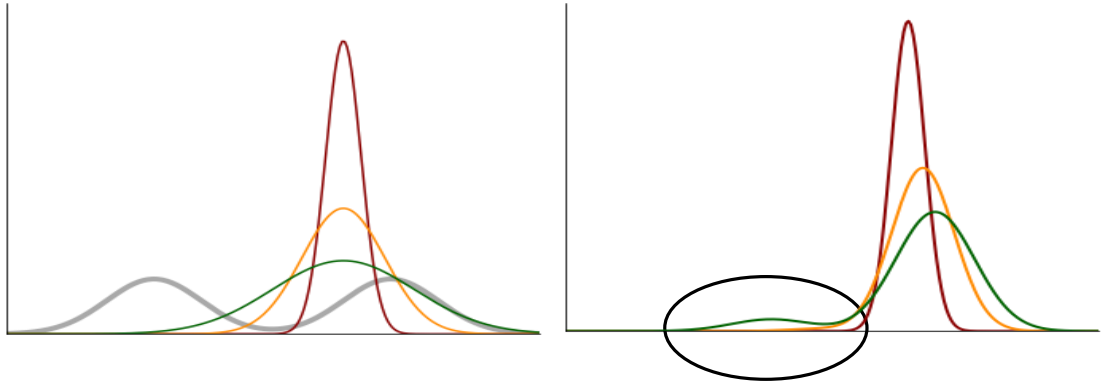

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FAPP collapse

For
All
Practical
Purposes

[Phys. Rev. D, **34** (2), pp. 470 (1986)]

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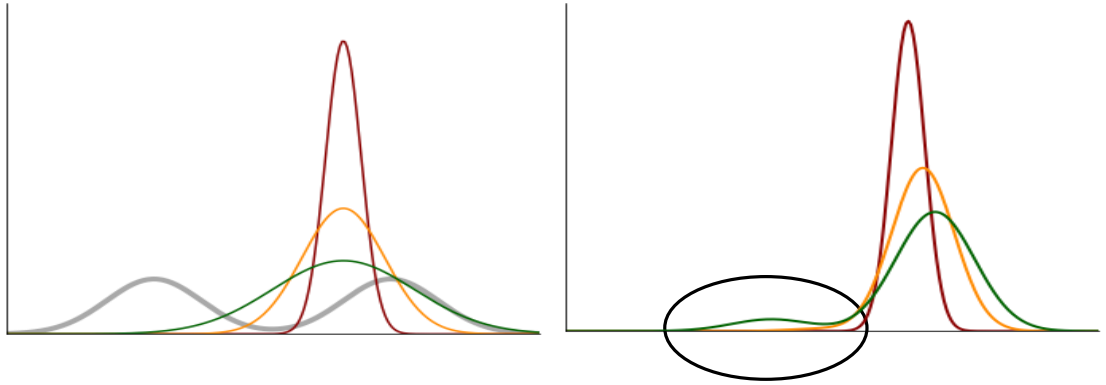

FAPP collapse

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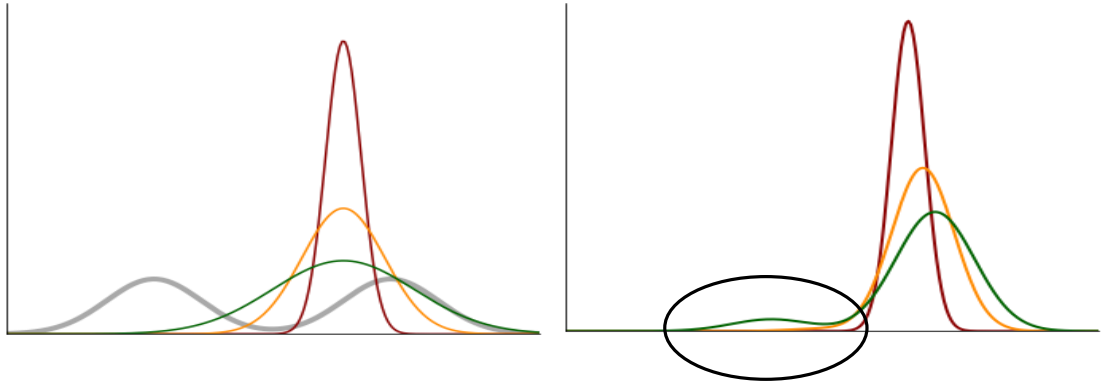

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FAPP collapse

For
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$\sigma_{GRW} \sim 10^{-7}\, m$

$\lambda_{GRW} \sim 10^{-16}\, s^{-1}$

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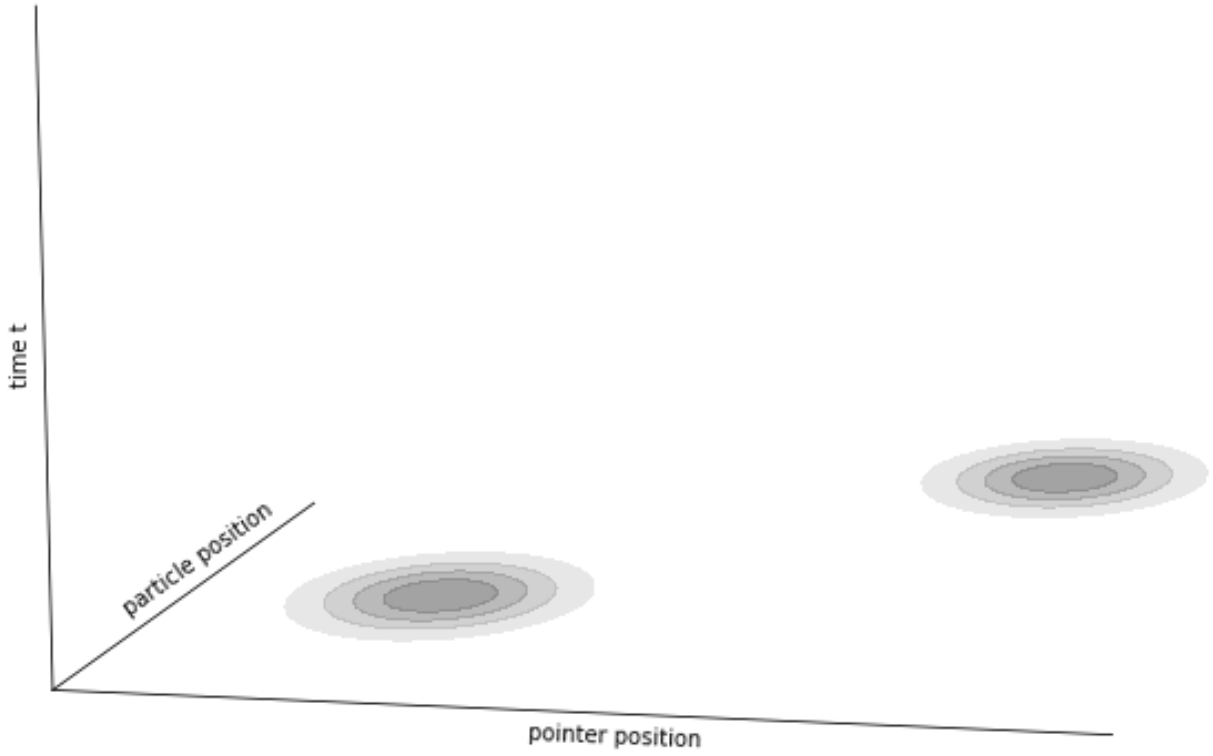
Spontaneous Collapse Theories

…and the measurement problem?



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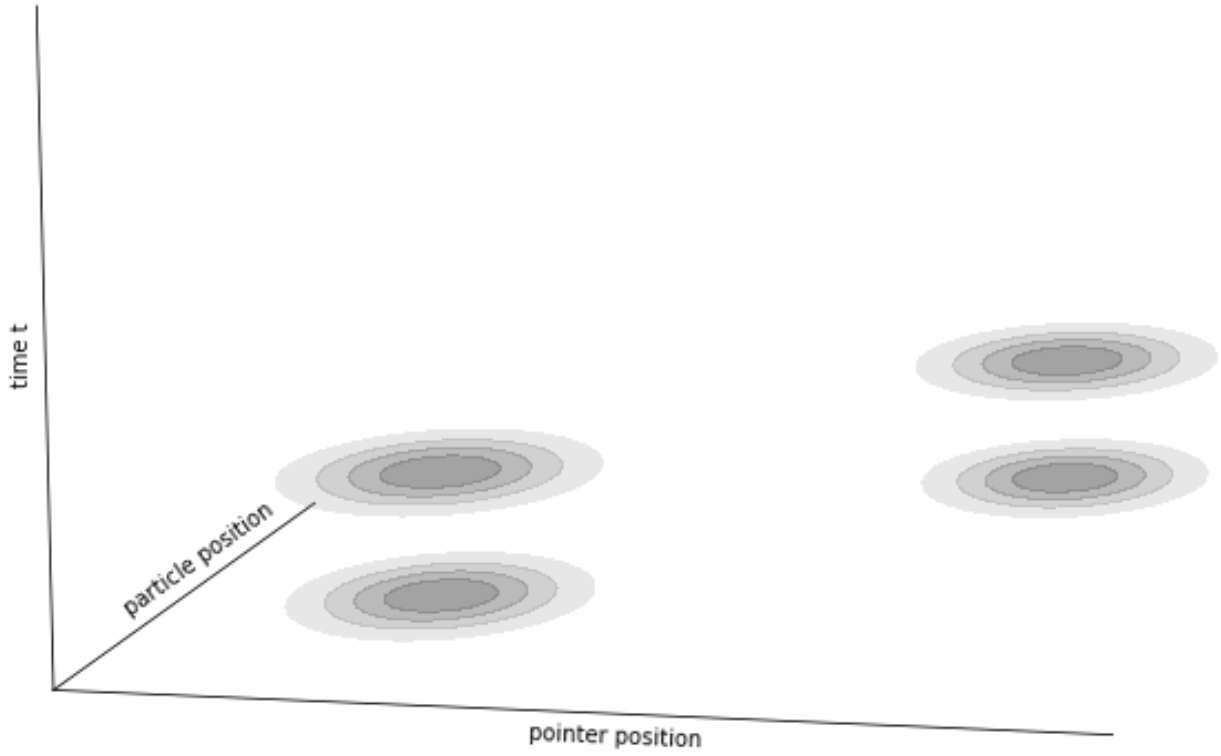

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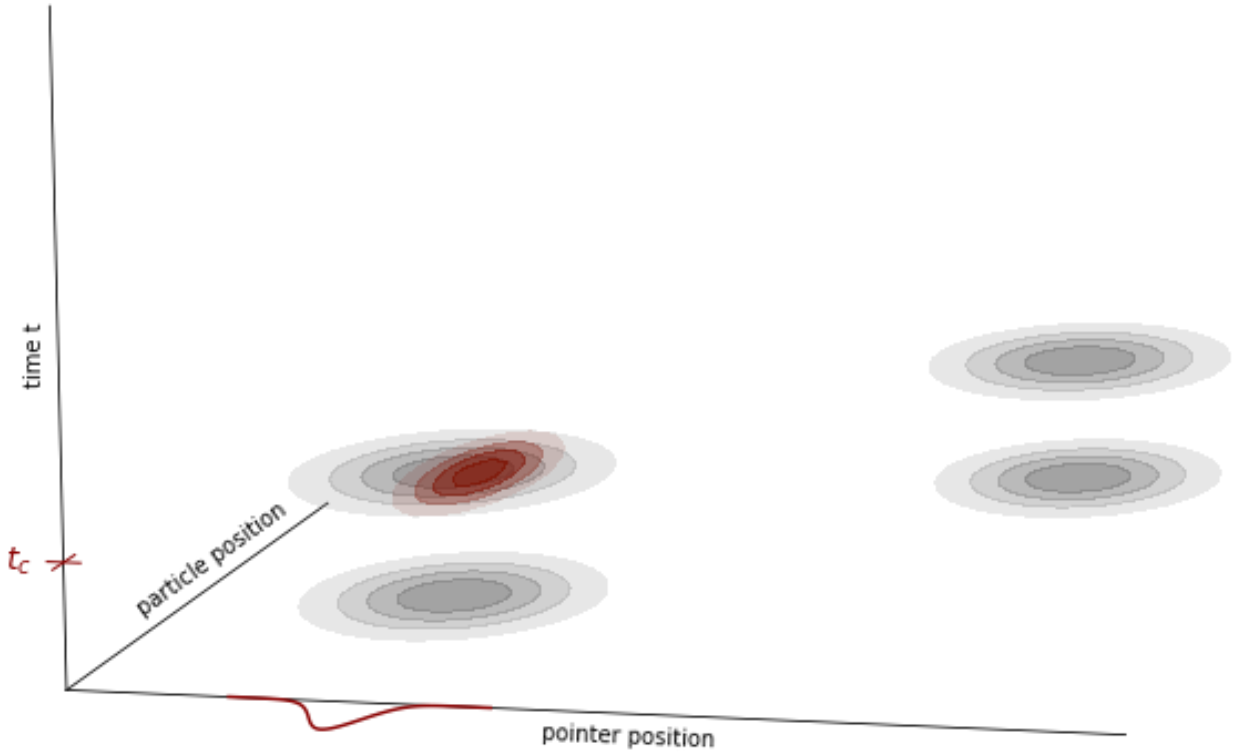

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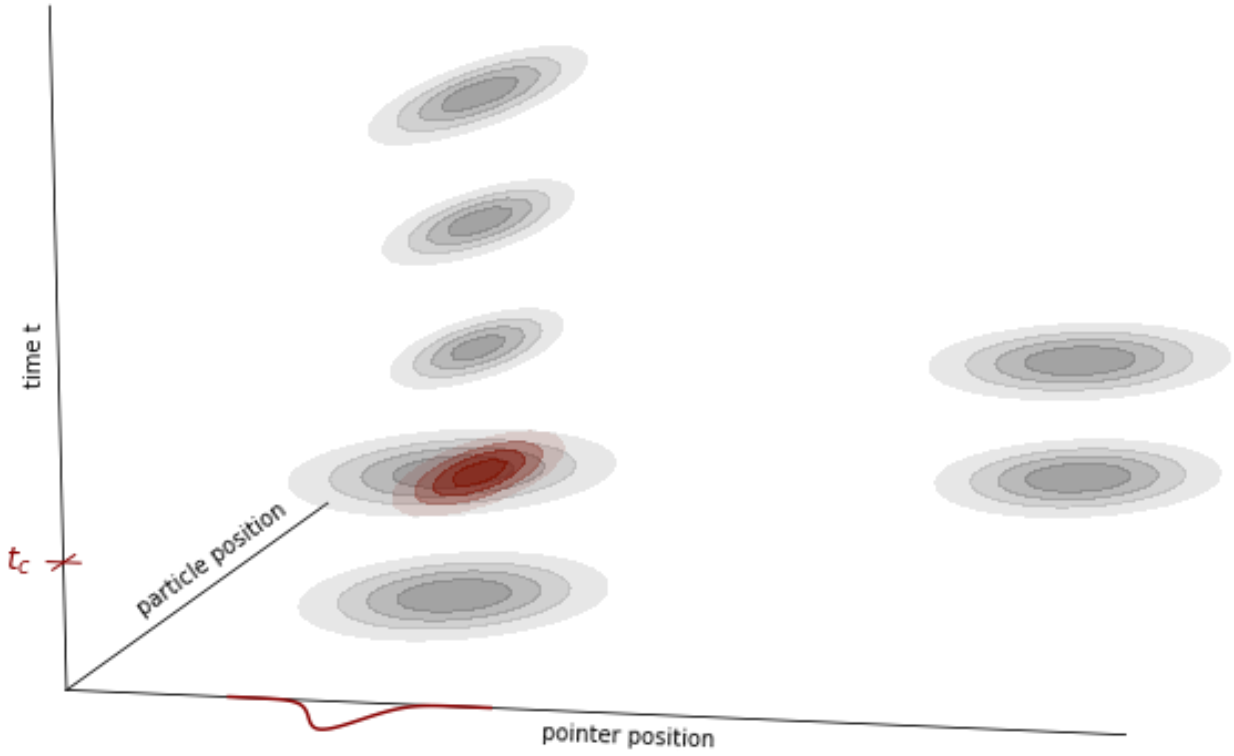

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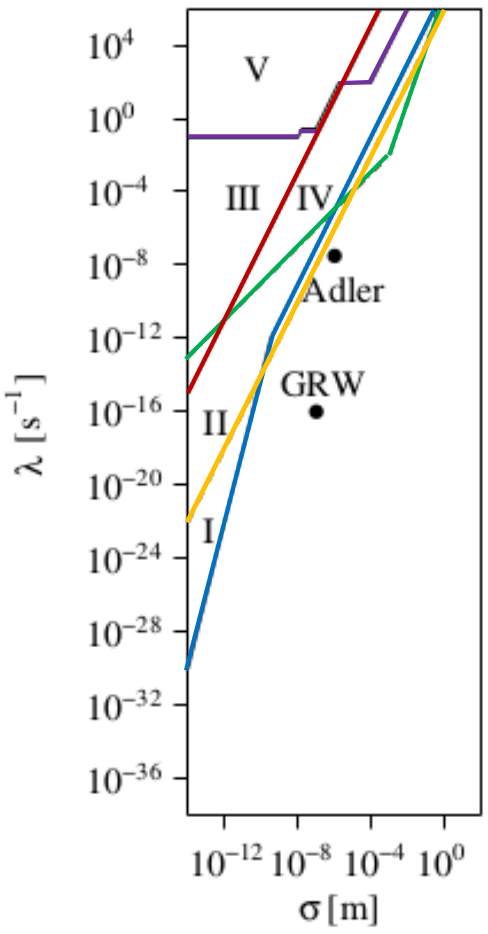

[J. Phys. A: Math. Theor. 45, 065304 (2012)]

I. spontaneous X-ray emission
II. spontaneous warming of the intergalactic medium
III. spontaneous warming of air
IV. decay of supercurrents
V. diffraction experiments

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Spontaneous Collapse Theories

„A piece of matter then is a galaxy of such events."

(John S. Bell)

Bohmian Mechanics (Pilot-wave Theory)

„While the founding fathers agonized over the question

'particle' *or* 'wave'

de Broglie in 1925 proposed the obvious answer

'particle' *and* 'wave'."

(John S. Bell)

Bohmian Mechanics (Pilot-wave Theory)

ontology

wave function: $\Psi(X(t), t)$

postulates

Schrödinger equation: $i\hbar \frac{\partial}{\partial_t}\Psi(t) = H(t)\Psi(t)$

[J. Phys. Radium, **8** (5), pp. 225 (1927)]

[Phys. Rev. **85**, pp.166 and pp.180 (1952)]

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velocity field: $v(t) = \frac{i}{\hbar}\Im\left\{\frac{\left(\frac{\partial\Psi(t)}{\partial x}\right)}{\Psi(t)}\bigg|_{x=X(t)}\right\}$

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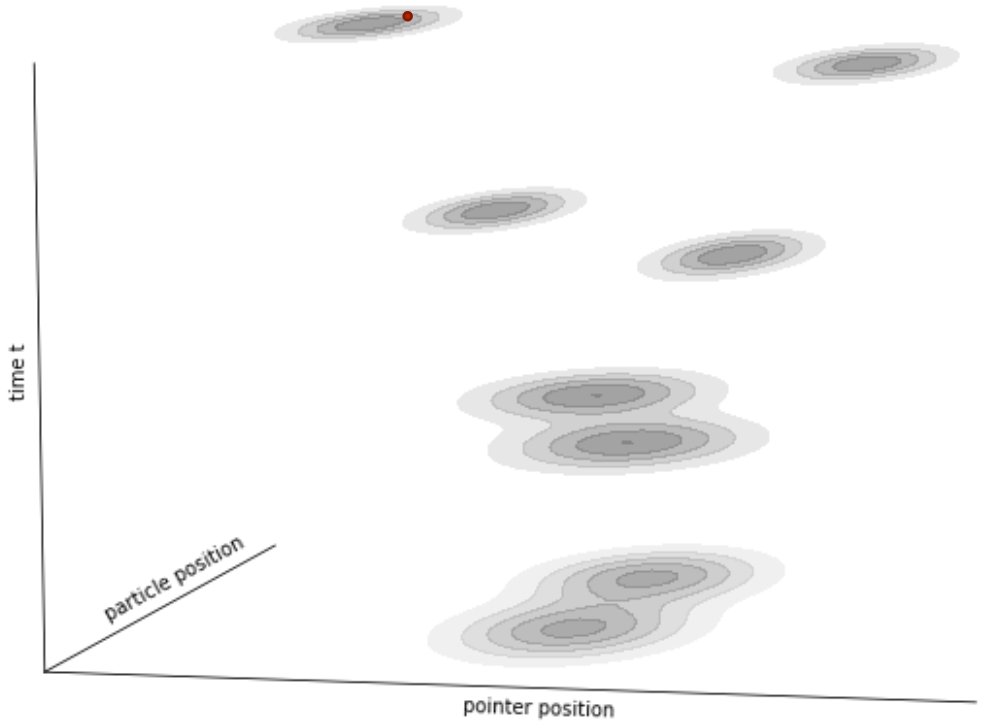
Bohmian Mechanics (Pilot-wave Theory)

…and the measurement problem?



ontology

$\Psi(X(t), t)$ and $X(t)$

postulates

$$i\hbar \frac{\partial}{\partial_t}\Psi(t) = H(t)\Psi(t)$$

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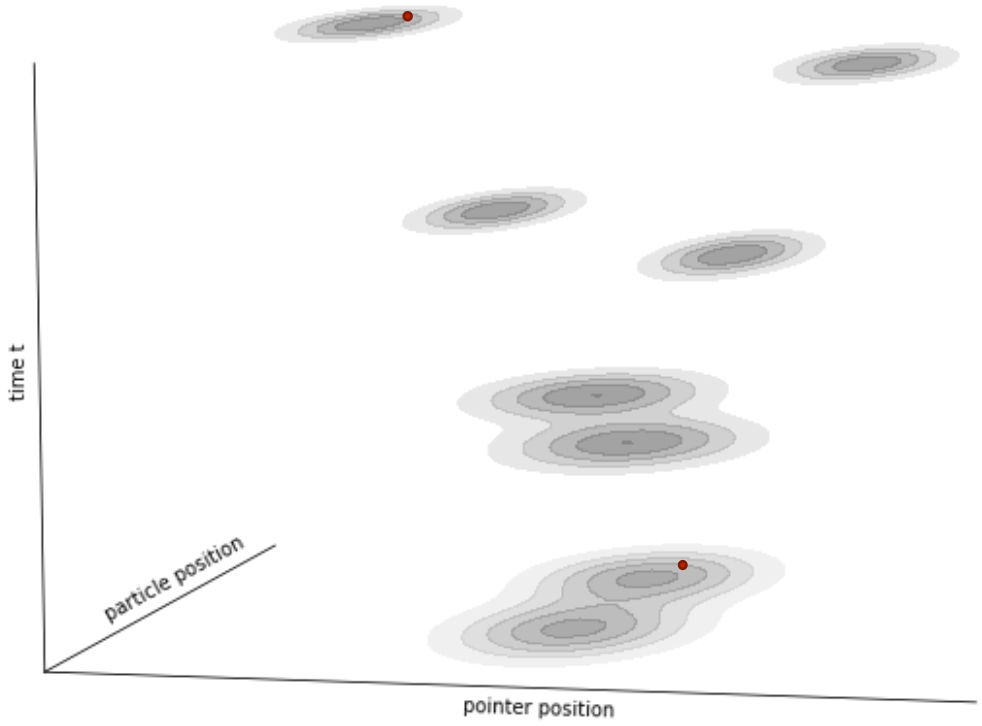

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Bohmian Mechanics (Pilot-wave Theory)

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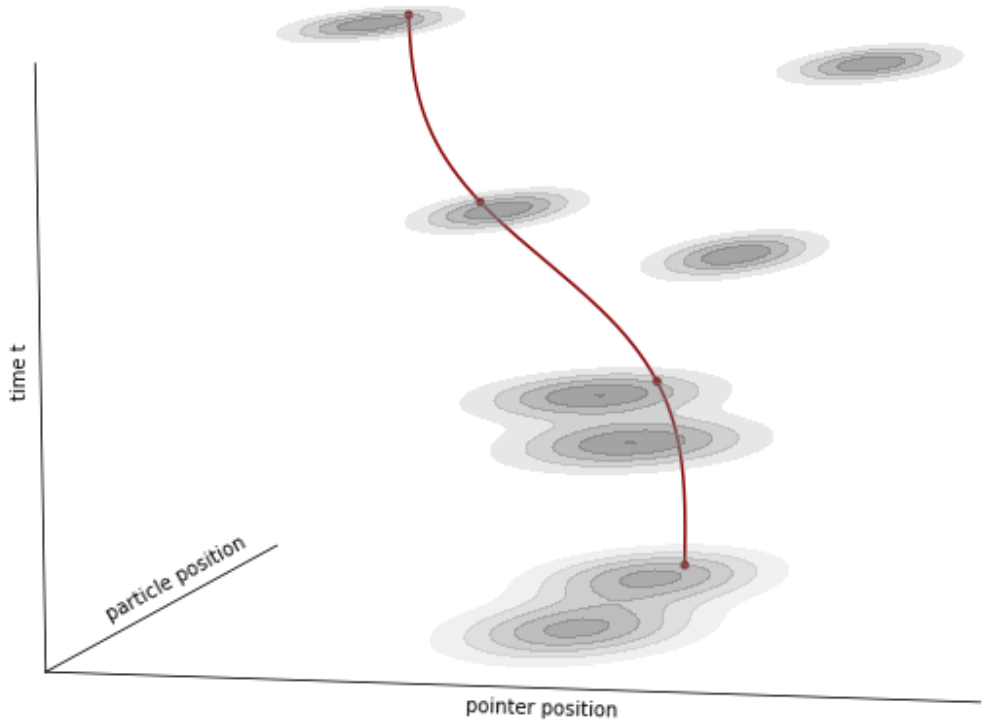

ontology

$\Psi(X(t), t)$ and $X(t)$

postulates

$$i\hbar \frac{\partial}{\partial_t}\Psi(t) = H(t)\Psi(t)$$

$$v(t) = \frac{i}{\hbar}\Im\left\{\left.\frac{\left(\frac{\partial\Psi(\mathrm{t})}{\partial x}\right)}{\Psi(\mathrm{t})}\right|_{x=X(t)}\right\}$$

[J. Phys. Radium, **8** (5), pp. 225 (1927)]

[Phys. Rev. **85**, pp.166 and pp.180 (1952)]

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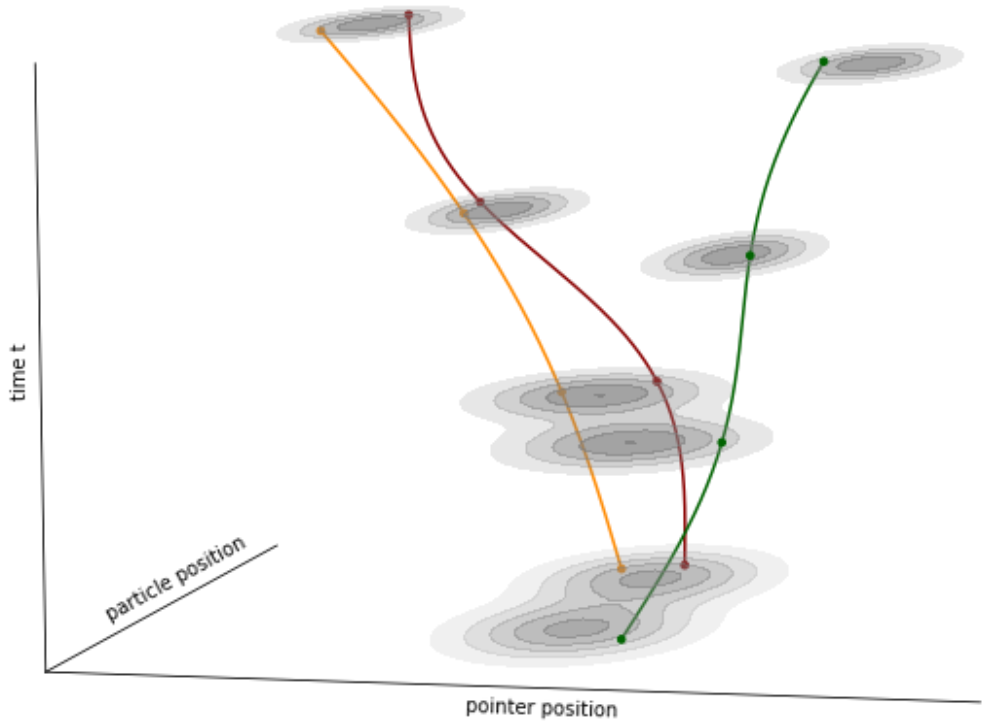

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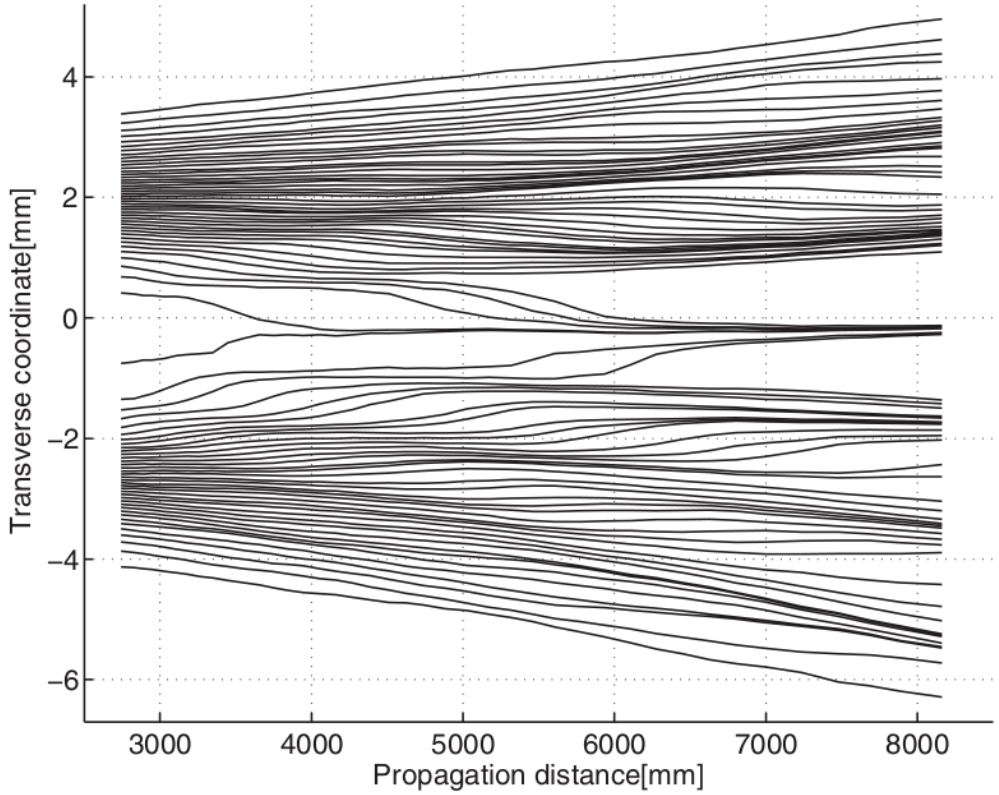
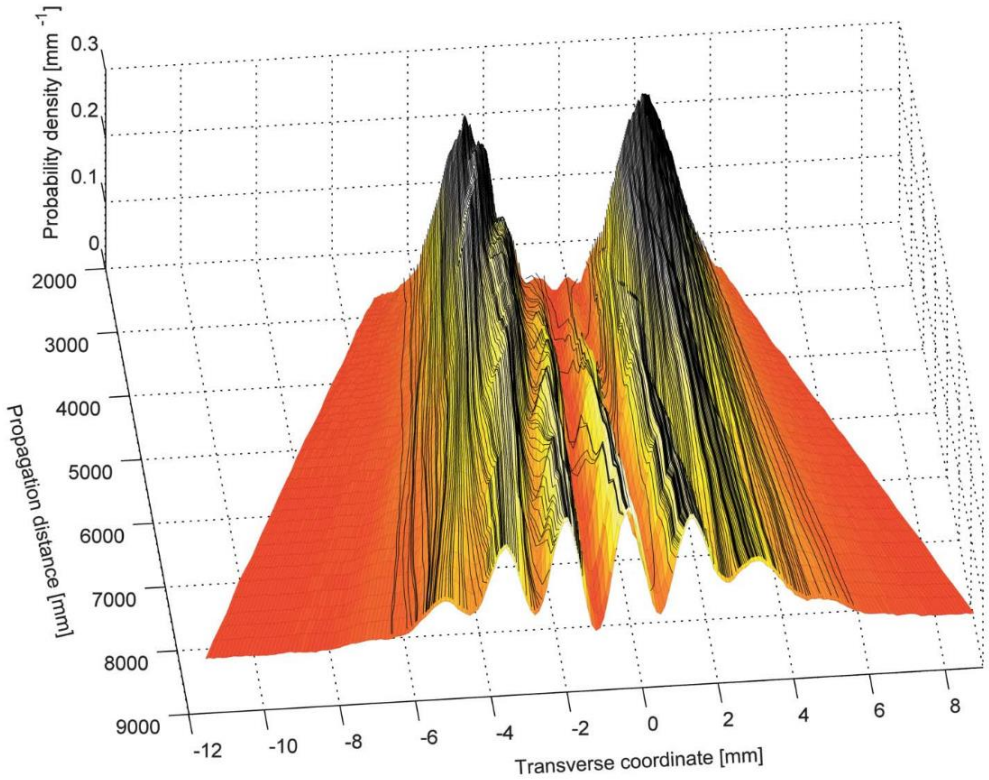
Bohmian Mechanics (Pilot-wave Theory)

„Observing the Average Trajectories of Single Photons in a Two-Slit Interferometer“





[Science **332** (6034), 1170-1173 (2011)]

$i\hbar \frac{\partial}{\partial_t}\Psi(t) = H(t)\Psi(t)$ and the physics of measurement

The Measurement Problem

„Either the wavefunction, as given by the Schrödinger equation, is not everything, or it is not right.“

Copenhagen Interpretation

„… we ourselves produce the results of measurement.“

Spontaneous Collapse Theories

„A piece of matter then is a galaxy of such events.“

Bohmian Mechanics (Pilot-wave Theory)

„While the founding fathers agonized over the question 'particle' *or* 'wave' de Broglie in 1925 proposed the obvious answer 'particle' *and* 'wave'.“