Electron-proton scattering

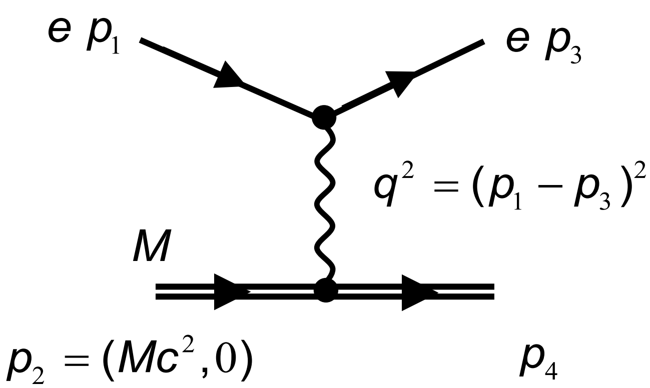
1. Elastic ep-scattering and the proton radius
2. Deep-inelastic (DIS) electron proton scattering

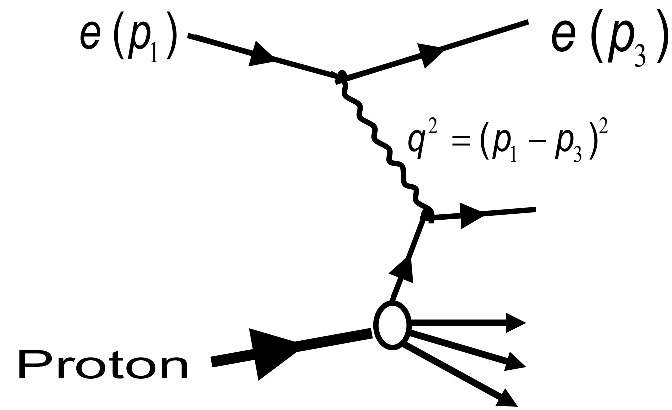
(1) Is largely a recap of PEP4 (we will not discuss all details)

For reference:
This section of the lecture follows closely chapter 7 and 8 of M. Thomson's Modern Particle Physics however provide a few more recent experimental results.

Electron-proton scattering

Relativistic electron-proton scattering probe the structure of the proton:





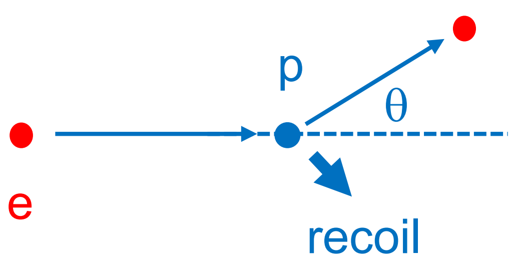
- at "low energy" - elastic scattering is dominant process: "virtual photon" probes the proton as whole and provides proton properties like the charge radius (PEP4)
- At "high energy" – inelastic scattering: proton breaks up. Understood as the elastic scattering of the electron on point-like charged proton constitutes, i.e. quarks.

Relevant quantity to distinguish between the diff. regimes is $Q^2=-q^2$ of virtual photon:

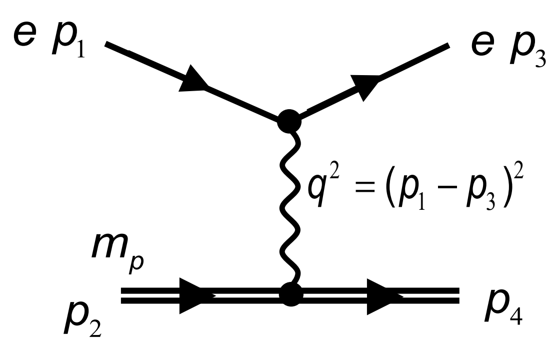
$1/Q^2 >> r_p^2$: proton appears point like (Rutherford)
$1/Q^2 \approx r_p^2$: proton charge distribution resolved (Rosenbluth)
$1/Q^2 << r_p^2$: probe internal proton structure (DIS)

1. Elastic ep-scattering and the proton radius

In the limit that the proton can be treated as a point-like spin ½ particle (Dirac fermion, ignore inner degrees of freedom) one can use Feynman rules to write down the matrix element (assume highly relativistic electrons $E_1 >> m_e$):



In general only electron quantities measured.



$$p_1 = (E_1, 0, 0, E_1)$$
$$p_2 = (m_p, 0, 0, 0)$$
$$p_3 = (E_3, 0, E_3 \sin\theta, E_3 \cos\theta)$$
$$p_4 = (E_4, \vec{p}_4)$$

If one ignores p recoil: $q^2 = (0, \vec{p}_1 - \vec{p}_3)^2 = \vec{q}^2$

$$\mathcal{M}_{fi} = \frac{e^2}{q^2}\left[\bar{u}(p_3)\gamma_\mu u(p_1)\right]\left[\bar{u}(p_4)\gamma^\mu u(p_2)\right]$$

Following the prescription of the QED theory lecture one can determine the average matrix element summed over all final state spin states:

$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{8e^4}{q^4}\left[(p_1 p_2)(p_3 p_4) + (p_1 p_4)(p_2 p_3) - m_p^2 (p_1 p_3)\right]$$

$$= (p_2 p_4)$$

Using
$$Q^2 = -q^2 = -(p_1 - p_3)^2 = 4E_1E_3 \sin^2\left(\frac{\theta}{2}\right)$$
$$p_4 = p_2 + p_1 - p_3$$
$$p_1^2 = p_3^2 = m_e^2 \approx 0 \qquad p_2^2 = p_4^2 = m_p^2$$

One finds:
$$\left\langle |\mathcal{M}|^2 \right\rangle = \frac{m_p^2 e^4}{E_1 E_3 \sin^2\left(\frac{\theta}{2}\right)} \left[\cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{2m_p^2}\sin^2\left(\frac{\theta}{2}\right)\right]$$

Resulting in the differential cross section in the lab:
$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \left(\frac{E_3}{E_1}\right)\left[\cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{2m_p^2}\sin^2\left(\frac{\theta}{2}\right)\right]}$$

Reminder in CMS:
$$\frac{d\sigma}{d\Omega_f} = \frac{1}{64\pi^2 s}\frac{p_f}{p_i}|\mathcal{M}_{fi}|^2$$

In lab for fixed target:
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \approx \frac{1}{64\pi^2}\left(\frac{E_3}{m_\mathrm{p} E_1}\right)^2 \langle |\mathcal{M}_{fi}|^2 \rangle$$

(See e.g. Thomson, Ch 3)

Often called “Dirac cross section“:
e-scattering at a “point-like” proton (academic case!)

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \left(\frac{E_3}{E_1}\right)\left[\cos^2\left(\frac{\theta}{2}\right) + \frac{Q^2}{2m_p^2}\sin^2\left(\frac{\theta}{2}\right)\right]}$$

We can recognize different pieces known to us:

- Rutherford cross section for scattering of a scalar particle on a Coulomb potential:

$$\left.\frac{d\sigma}{d\Omega}\right|_{`Rutherf} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)}$$

- Term E_3/E_1 accounts for the electrons energy loss due to the proton recoil.

- Mott cross section: for relativistic electron scattering w/ spin ½ at a Coulomb potential of a point-like particle in the limit $Q^2 \ll m_p^2$ and $E_1 < m_p$ (see PEP4):

$$\left.\frac{d\sigma}{d\Omega}\right|_{`Mott} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \left(\frac{E_3}{E_1}\right)\cos^2\left(\frac{\theta}{2}\right)$$

Term $\sim \frac{Q^2}{2m_p^2}\sin^2\left(\frac{\theta}{2}\right)$ describes the magnetic interaction between the spin of the electron and the proton spin (relevant only for large Q^2)

In case of electron scattering at an extended charge distribution the Mott cross section needs to be corrected by the form factor of the charge distribution:

$$\frac{d\sigma}{d\Omega} = \left.\frac{d\sigma}{d\Omega}\right|_{Mott} \cdot \left|F(\vec{q})\right|^2$$

(discussed in PEP4)

With the form factor being the Fourier transform of the charge distribution:

$$F(\vec{q}) = \int \rho(\vec{r}) e^{i\vec{q}\vec{r}} d\vec{r}$$

$$= F(\vec{q}^2)$$

for spherical symmetric charge distributions $\rho(r)$ (integration over the polar angle possible) → F is a function of q^2

For $\vec{q}\,\vec{r} \ll \hbar$ one can expand the integrand and obtains:

$$F(\vec{q}^2) = 1 - \frac{1}{6}\vec{q}^2 \left\langle r_p^2 \right\rangle + \ldots$$

where $\left\langle r^2 \right\rangle = \int r^2 \rho(r) d^3 r$ is the mean quadratic charge radius

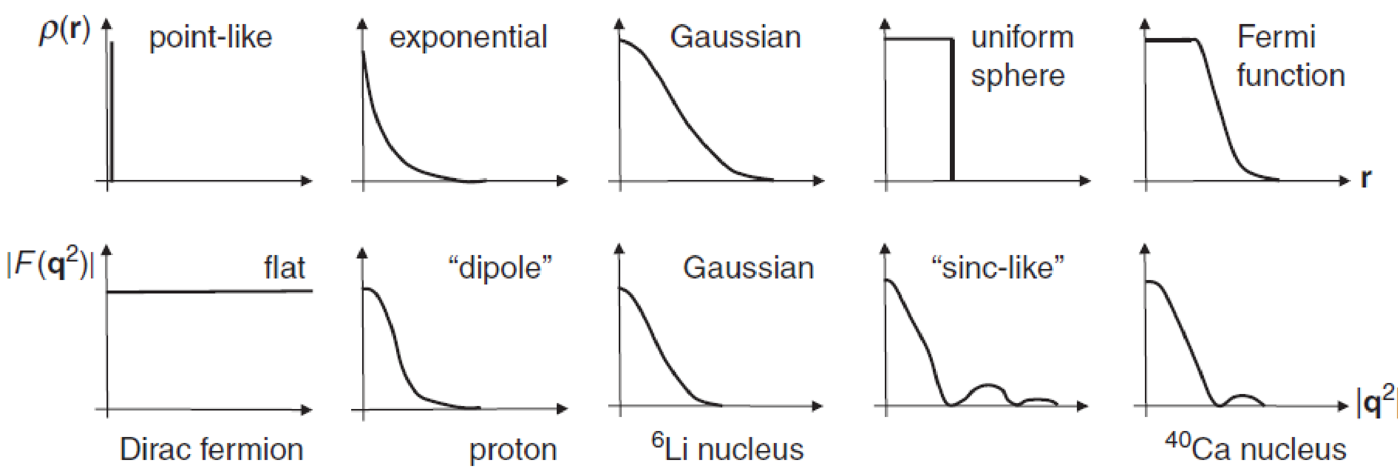
$\left\langle r^2 \right\rangle$ can thus be determined from the gradient of the form factor $F(\vec{q}^2)$ at $\vec{q}^2 \to 0$

$$\left\langle r^2 \right\rangle = -6 \left.\frac{dF(\vec{q}^2)}{d\vec{q}^2}\right|_{\vec{q}^2=0}$$

formula used to extract the proton charge radius (see below)

Form factors for different charge distributions:

Ladungsverteilung	$\rho(r)$	Formfaktor $F(\boldsymbol{q}^2)$	
Punkt	$\delta(r)/4\pi$	1	konstant
exponentiell	$(a^3/8\pi)\cdot\exp(-ar)$	$\left(1+\boldsymbol{q}^2/a^2\hbar^2\right)^{-2}$	Dipol
Gauß	$(a^2/2\pi)^{3/2}\cdot\exp(-a^2r^2/2)$	$\exp\left(-\boldsymbol{q}^2/2a^2\hbar^2\right)$	Gauß
homogene Kugel	$\begin{cases} 3/4\pi R^3 & \text{für } r \le R \\ 0 & \text{für } r > R \end{cases}$	$3\,\alpha^{-3}\,(\sin\alpha-\alpha\cos\alpha)$ mit $\alpha = \lvert\boldsymbol{q}\rvert R/\hbar$	oszillierend



Possible three-dimensional charge distributions and the corresponding form factors plotted as a function of $\mathbf{q}^2$.

(from Thomson, Modern Particle Physics)

e - scattering on an extended proton

Following the introduction of the form factors for Mott scattering two form-factors are introduced to account for the finite size of the proton:

$G_E(Q^2)$ related to the charge distribution $\rho(r)$

$G_M(Q^2)$ related to the magnetic moment distribution $\mu(r)$

The elastic electron-proton cross section can be written as;

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4\left(\frac{\theta}{2}\right)} \cdot \left(\frac{E_3}{E_1}\right)\left[\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\left(\frac{\theta}{2}\right) + 2\tau G_M^2 \sin^2\left(\frac{\theta}{2}\right)\right] \quad \text{w/} \quad \tau = \frac{Q^2}{4m_p^2}$$

Remarks:
Form factors depend on the Q^2 = 4-vector of the virtual photon (FF in the Mott cross section were dependent on 3-vector $\vec{q}$) and therefore cannot be simply interpreted as the Fourier transform of the charge / magnetic moment distribution.

However in the limit $Q^2 << m_p^2$ ($Q^2 \approx \vec{q}^{\,2}$) the Fourier transform is recovered:

$$G_E(Q^2) \approx G_E(\vec{q}^{\,2}) \approx \int \rho(r)\, e^{i\vec{q}\vec{r}}\, d\vec{r}$$

$$G_M(Q^2) \approx G_M(\vec{q}^{\,2}) \approx \int \mu(r)\, e^{i\vec{q}\vec{r}}\, d\vec{r}$$

Rsoenbluth / Dirac cross section was obtained for a Dirac fermion with g=2:

$$\vec{\mu} = 2 \cdot \frac{q}{2m} \vec{S}$$

The proton however is a composed object and experimentally the g-factor is

$$g_p = +5.58 \qquad \vec{\mu} = 2.79 \cdot 2 \cdot \frac{q}{2m} \vec{S}$$

To correctly describe the experimental observation the magnetic moment distribution has to be normalized correspondingly:

$$G_E(0) \approx \int \rho(r)\, e^{i\vec{q}\vec{r}}\, d\vec{r} = 1$$

$$G_M(0) \approx \int \mu(r)\, e^{i\vec{q}\vec{r}}\, d\vec{r} = 2.79$$

i.e., if one assumes the same shape for G_E and G_M, one expects G_M to be scaled up by a factor 2.79.

Determination of $G_E(Q^2)$ and $G_M(Q^2)$

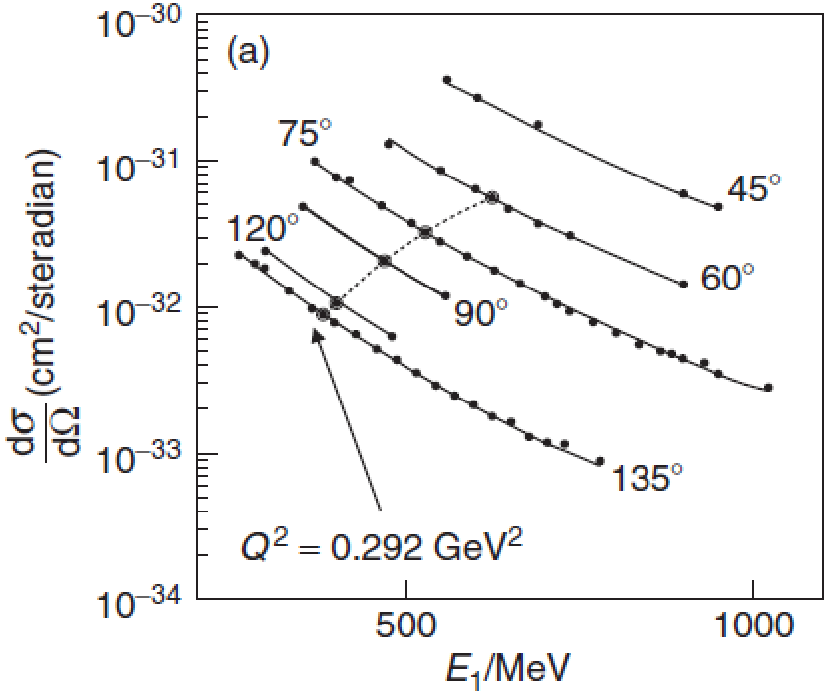
Although one expects similar shape for the two form factors, G_E and G_M should be determined independently. Dividing the Rosnbluth formula by the Mott cross section one obtains:

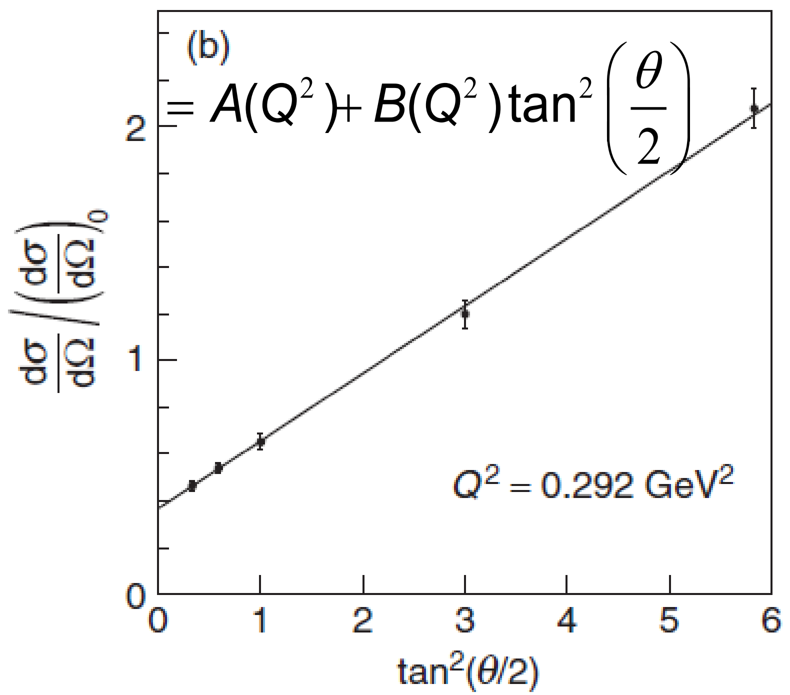
$$\frac{d\sigma}{d\Omega} \Big/ \left.\frac{d\sigma}{d\Omega}\right|_{`Mott} = \left[\frac{G_E^2 + \tau G_M^2}{1+\tau} + 2\tau G_M^2 \tan^2\left(\frac{\theta}{2}\right) \right] = A(Q^2) + B(Q^2)\tan^2\left(\frac{\theta}{2}\right)$$

While low Q^2 data determines $G_E^2(Q^2)$ and high Q^2 data determines $G_M^2(Q^2)$ one can obtain $G_E^2(Q^2)$ and $G_M^2(Q^2)$ for general Q^2 using the **Rosenbluth separation**

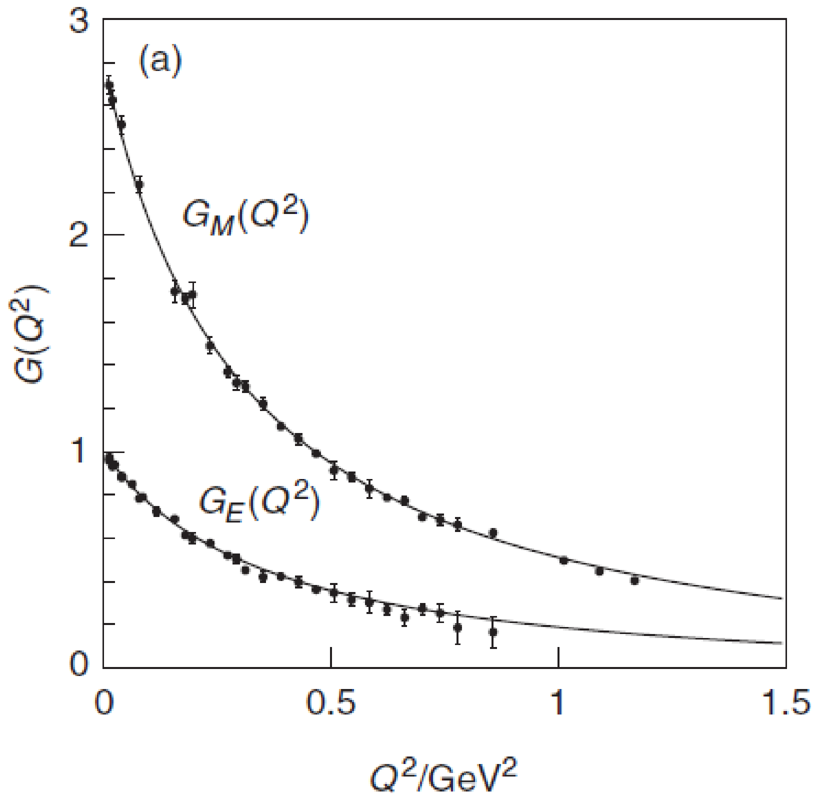
Cross section is measured for different electron energy E_1 and different scattering angle θ → plot the Mott normalized cross section as function of $\tan\theta/2$

Rosenbluth separation





(from Thomson, Modern Particle Physics)



Taken from Thomson,
Data from E. B. Hughes et al. (1965)

The solid line is a dipole form factor model:

$$G(Q^2) = \left(1 + \frac{Q^2}{0.71\,\mathrm{GeV}^2}\right)^{-2}$$

= form factor of exponential charge distr.

$\rho(r) = e^{-ar}$ with a=4.27 fm^{-1}

$$0.71\mathrm{GeV}^2 = a^2\hbar^2 \rightarrow a = 4.27\mathrm{fm}^{-1}$$

One also finds that G_E and G_M follow the same dipole shape (scaled).

From the exponential distribution one can determine the proton charge radius, defined as $\langle r^2 \rangle^{1/2}$ one finds r_p = 0.81 fm.

Instead of fitting the form factor shape one can also extrapolate to $Q^2 = 0$ an determine the slope at $Q^2 = 0$ of the measured behavior (see above)

$$\langle r^2 \rangle = -6\left[\frac{dG(Q^2)}{dQ^2}\right]_{Q^2=0}$$

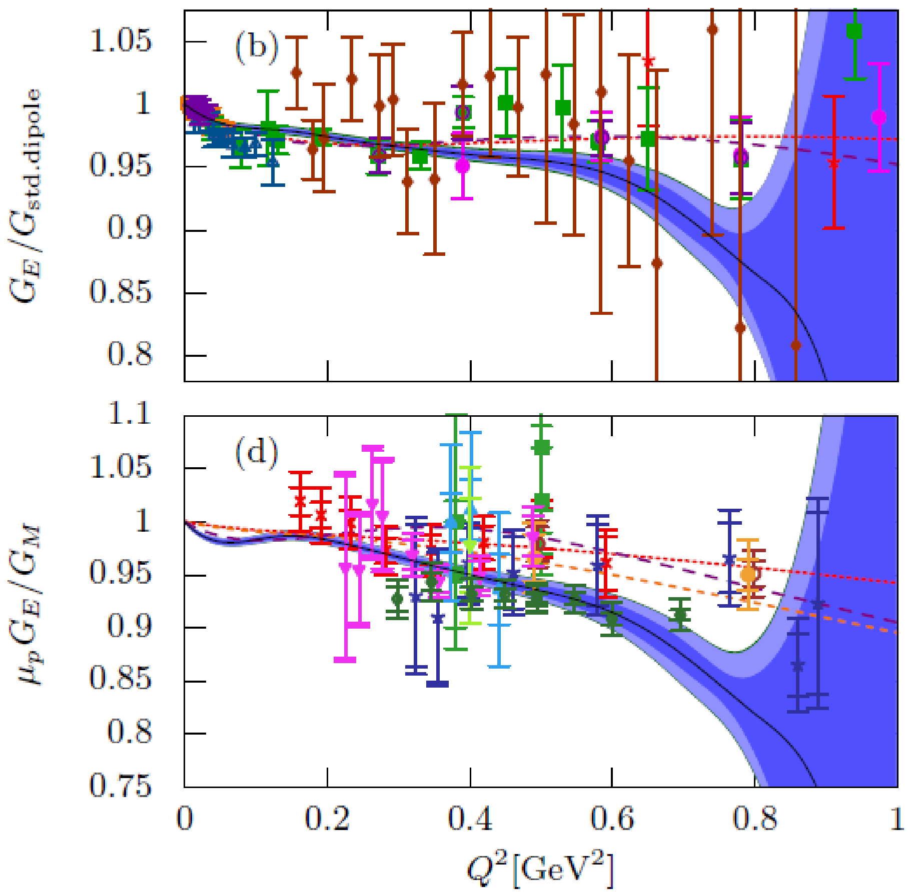
done in many recent measurements.

Most precise determination of the proton charge radius from ep scattering:

A1 collaboration at Mainz Microtron (MAMI):

electron beams of 150 – 855 MeV, liquid hydrogen target and 3 high resolution spectrometers
Variation of scattering angles and beam energy in more than 500 settings (J.C. Bernauer et al. PRL 105 (2010) 242001 and PRC 90 (2014) 015206)

https://arxiv.org/pdf/1307.6227



Authors determine a proton electric charge radius of

$$r_p = 0.879(7)\,\text{fm}$$

Consistent with earlier ep scattering Results but inconsistent with the most recent results from spectroscopy.

Proton radius from Lamb shift determined in hydrogen spectroscopy

2s-2p transition frequency (Lamb shift) is influenced by the overlap of the 2s orbital with H^+ (proton) charge distribution (p-orbitals have no overlap).

Sensitivity to the proton radius is low (difficult to make precise measurement):

$$\Delta E^{FS}_{ns-np} \sim \frac{(Z\alpha)^4}{3n^3} \cdot m_r^3 \cdot \langle r_p^2 \rangle$$

reduced mass

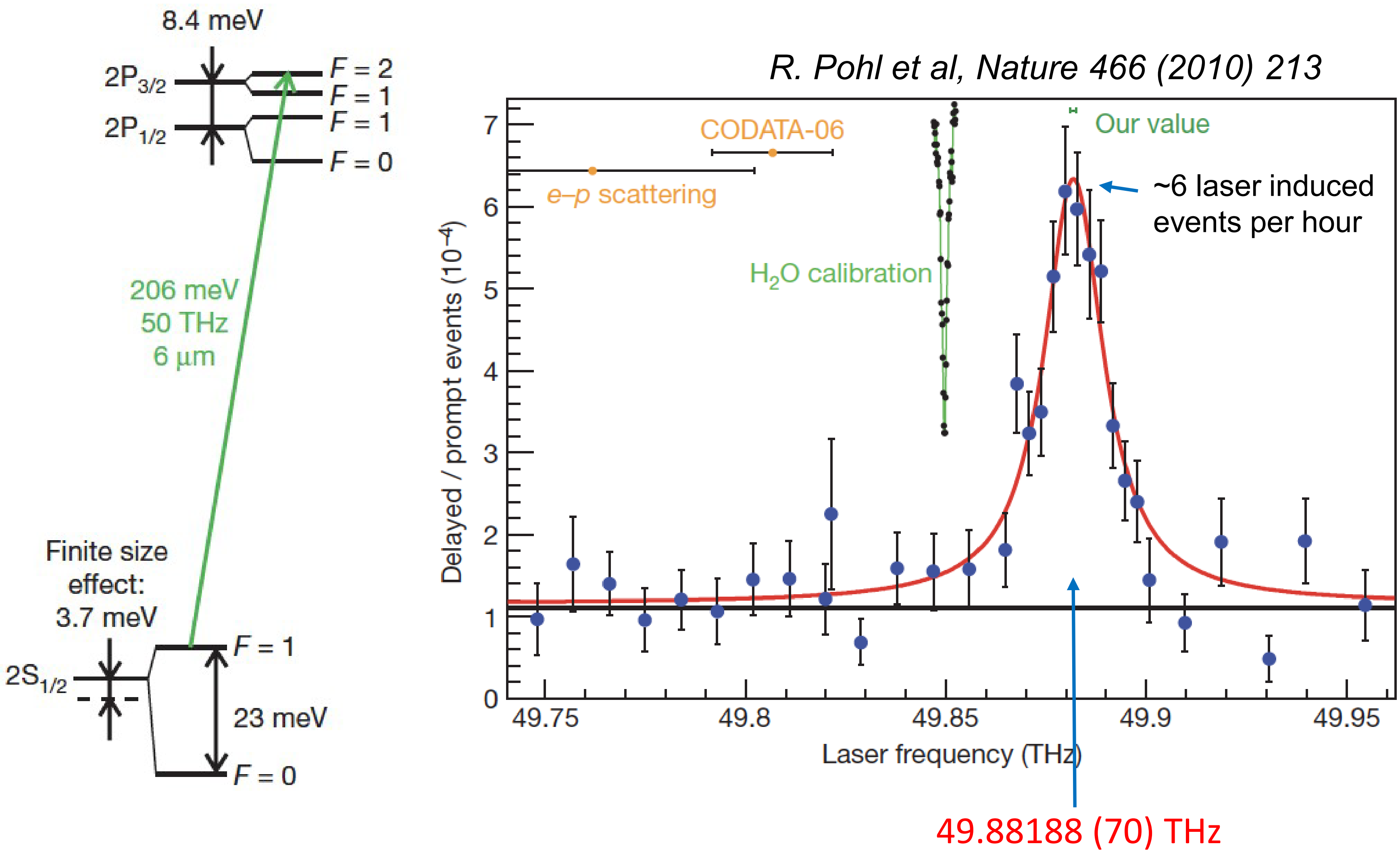
However w/ muonic hydrogen w/ $m_\mu \approx 200\ m_e$ the effect is about 10^7 times larger: Measurement of 2s-2p splitting in muonic hydrogen allows precise determination of the proton charge radius r_p.

Muonic-hydrogen (μp) experiment at PSI:

μ^- stopped in a hydrogen target → highly excited μp atoms (n≈14):
The excited atoms mostly de-excite to the 1s ground state.
About 1% of de-excitation also populate the stable 2s state.

Using laser light (~6μm) to induce the 2s-2p transition
→ de-excitation to 1s ground state → emission of 1.9 keV X-ray

Method: measure the emission of X-rays as a function of the laser tuning.



r_p = 0.84184 (67) fm
much more precise, but 5σ below CODATA value of 0.8768 (69) fm

= "Proton radius puzzle"

Recent summary of the proton radius data:

https://doi.org/10.3390/universe9040182

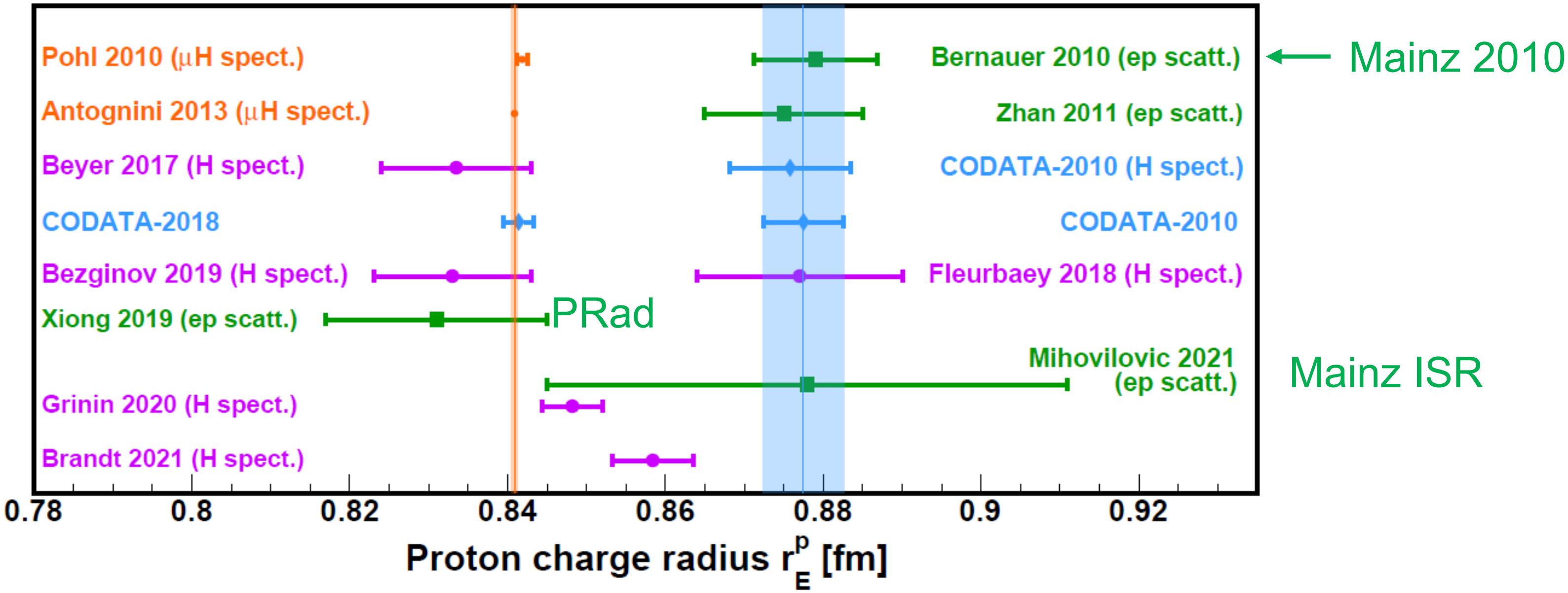


Figure 1. The proton charge radius determined from *ep* elastic scattering, hydrogen spectroscopic experiments, as well as world-data compilation from CODATA since 2010. The muonic spectroscopic measurements [19,20] are shown in orange dots, ordinary hydrogen spectroscopic results [12–16] are shown in purple dots, electron scattering measurements [2–4,6] are shown in green squares, and blue diamonds show the CODATA compilations [18,57].

New hydrogen results since 2010: improvement from new laser techniques and better control of systematic.

New ep scattering since 2010: new Mainz measurement using ISR techniques to access lower Q^2 values; new results from PRad (Jlab, windowless target).

In addition different theoretical revisions (TPE, radiative corrections, dispersion relations to interpret FF).

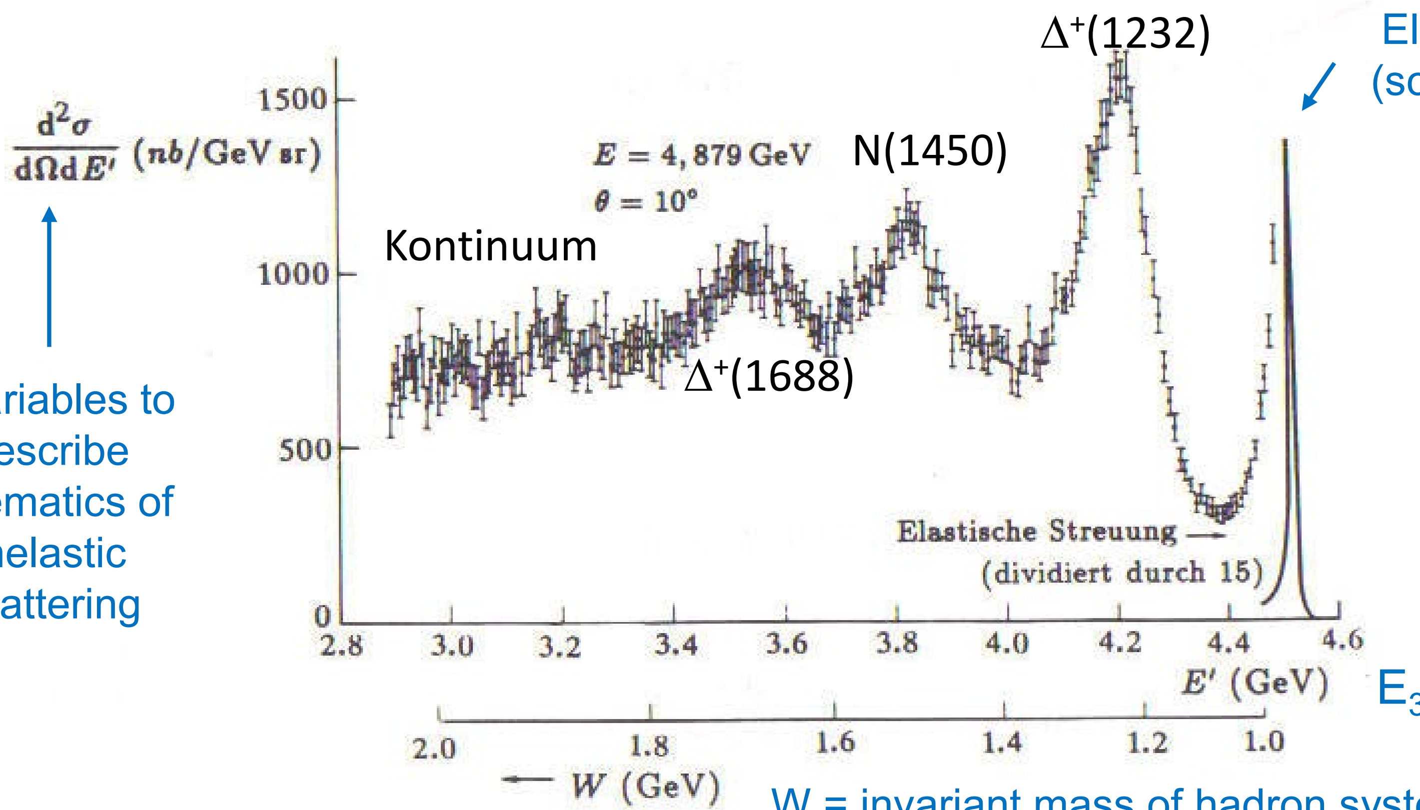
→ very active field!

2. Deep-inelastic electron proton scattering

Elastic scattering: no excitation of inner degrees of freedom, no proton break-up

$\Downarrow$ Increase energy transfer $\nu = E_1 - E_3$ from electron to proton beyond the level of the proton recoil ($q^2 \neq \vec{q}^{\,2}$)

<u>Inelastic scattering</u>:



2 variables to describe kinematics of inelastic scattering

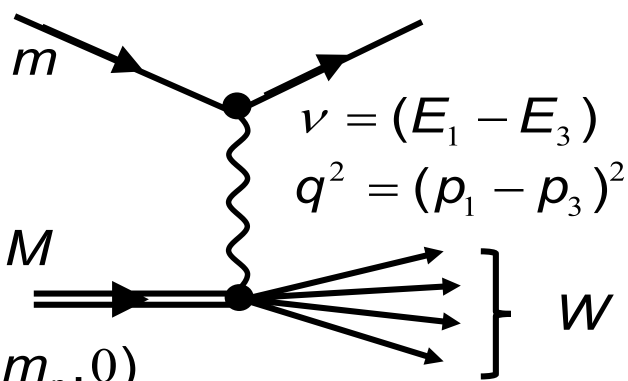
W = invariant mass of hadron system

Observations:

- Excitations ($\Delta^+(1232)$, N(1420), …) of the proton
- At higher energy transfer (smaller E_3) one observes a continuum, cannot be explained by the Q^2 dependence of a compact proton w/ $F(Q^2) \sim 1/Q^4$. This would lead to a strong suppression $\sim 1/Q^8$ $\rightarrow$ here: proton breaks up.

Kinematics of inelastic scattering:

$p_1 = (E_1, \vec{p}_1)$ $\quad$ $p_3 = (E_3, \vec{p}_3)$



$\nu = (E_1 - E_3)$

$q^2 = (p_1 - p_3)^2$

$p_2 = (m_p, 0)$

W = invariant mass of hadronic system

Elastic: W = m_p

with

$q = (p_1 - p_3) = (\nu, \vec{p}_1 - \vec{p}_3)$

$\rightarrow \quad p_2 q = m_p \nu \quad \rightarrow \quad \boxed{\nu = \frac{p_2 q}{m_p}}$

W always $\geq m_p$. Reason: baryon number conservation.

$$W^2 = (p_2 + q)^2 = m_p^2 + 2 p_2 q + q^2$$

Define a new Lorentz invariant dimensionless variable (important to describe parton distributions in the proton): **Bjorken x**

$$\boxed{x = \frac{Q^2}{2p_2 q} = \frac{Q^2}{2m_p \nu}}$$

Using the mass W of hadronic system one can rewrite x:

$$x = \frac{Q^2}{Q^2 + W^2 - m_p^2} \quad \Rightarrow \quad 0 \le x \le 1$$

$x = 1$ for elastic scattering $W = m_p$

Another dimensionless variable is the inelasticity y: $$\boxed{y = \frac{p_2 q}{p_2 p_1}}$$

In the rest frame of the proton $p_2 = (m_p, 0, 0, 0)$

$$y = \frac{m_p(E_1 - E_3)}{m_p E_1} = 1 - \frac{E_3}{E_1}$$

$$0 \le y \le 1$$

One finds with $s = (p_1 + p_2)^2$ the following useful relations:

$$y = \left(\frac{2m_p}{s - m_p^2}\right)\nu \quad \text{and} \quad Q^2 = (s - m_p^2)xy$$

Out of Q^2, x, y, ν
2 variables needed to define kinematics!

Express the Rosenbluth formula of elastic scattering with these variables:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2)\right]$$

with $f_2(Q^2) = \dfrac{G_E^2(Q^2) + 2\tau G_M^2(Q^2)}{1+\tau}$ and $f_1(Q^2) = G_M^2(Q^2)$ $\qquad \tau = \dfrac{Q^2}{4m_p^2}$

Remark: While y appears on the RH side, it is a function of Q^2 only as the scattering is elastic (x=1) !

Modified Rosenbluth formula can be generalized for inelastic scattering by replacing the two form factors f_1 and f_2 by so called structure functions $F_1(x,Q^2)$ and $F_1(x,Q^2)$. (structure functions $F_{1,2}$ should depend on 2 variables to reflect the inelastic case)

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) \frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

For deep-inelastic scattering where $Q^2 \gg m_p^2 y^2$

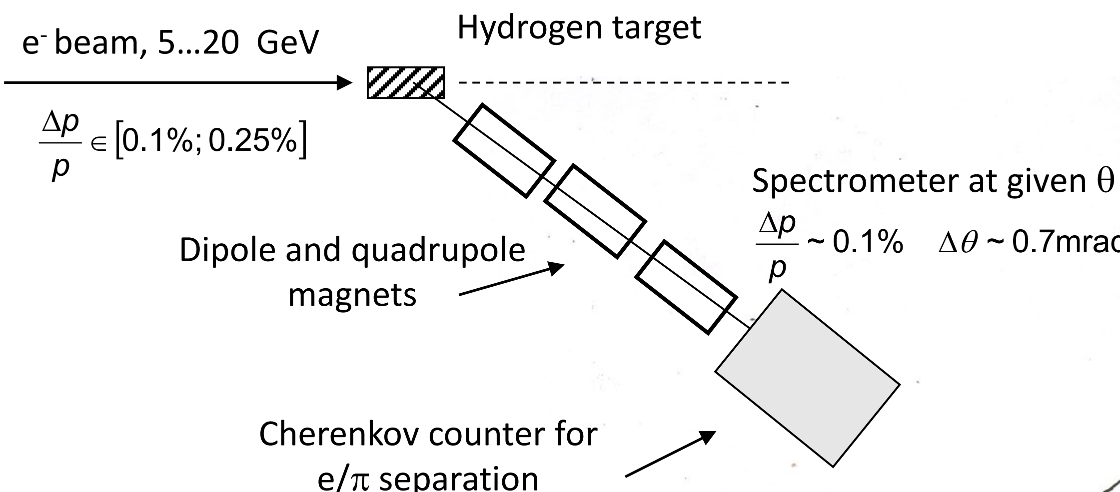
$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[(1 - y) \frac{F_2(x,Q^2)}{x} + y^2 F_1(x,Q^2)\right]$$

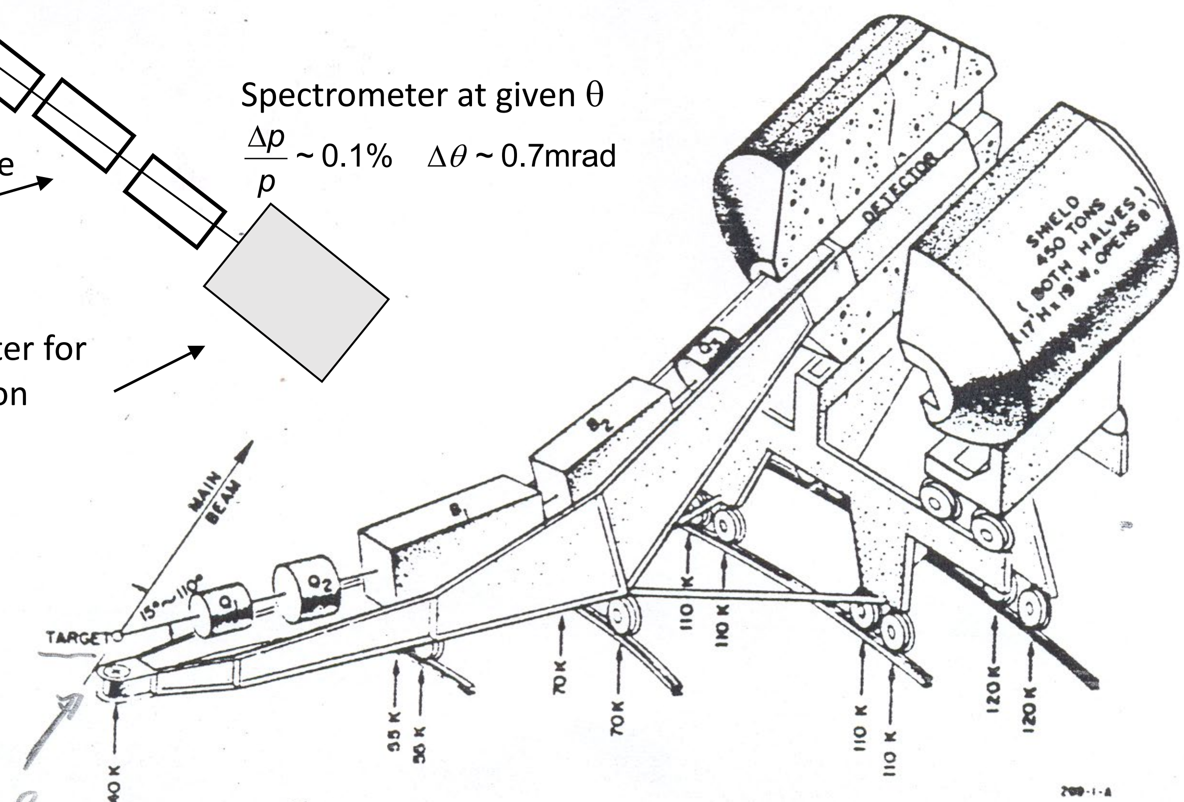
First measurements of the structure functions (SLAC & MIT, 1969):

(J. Friedman, H. Kendall and R. Taylor, Nobel prize 1990)

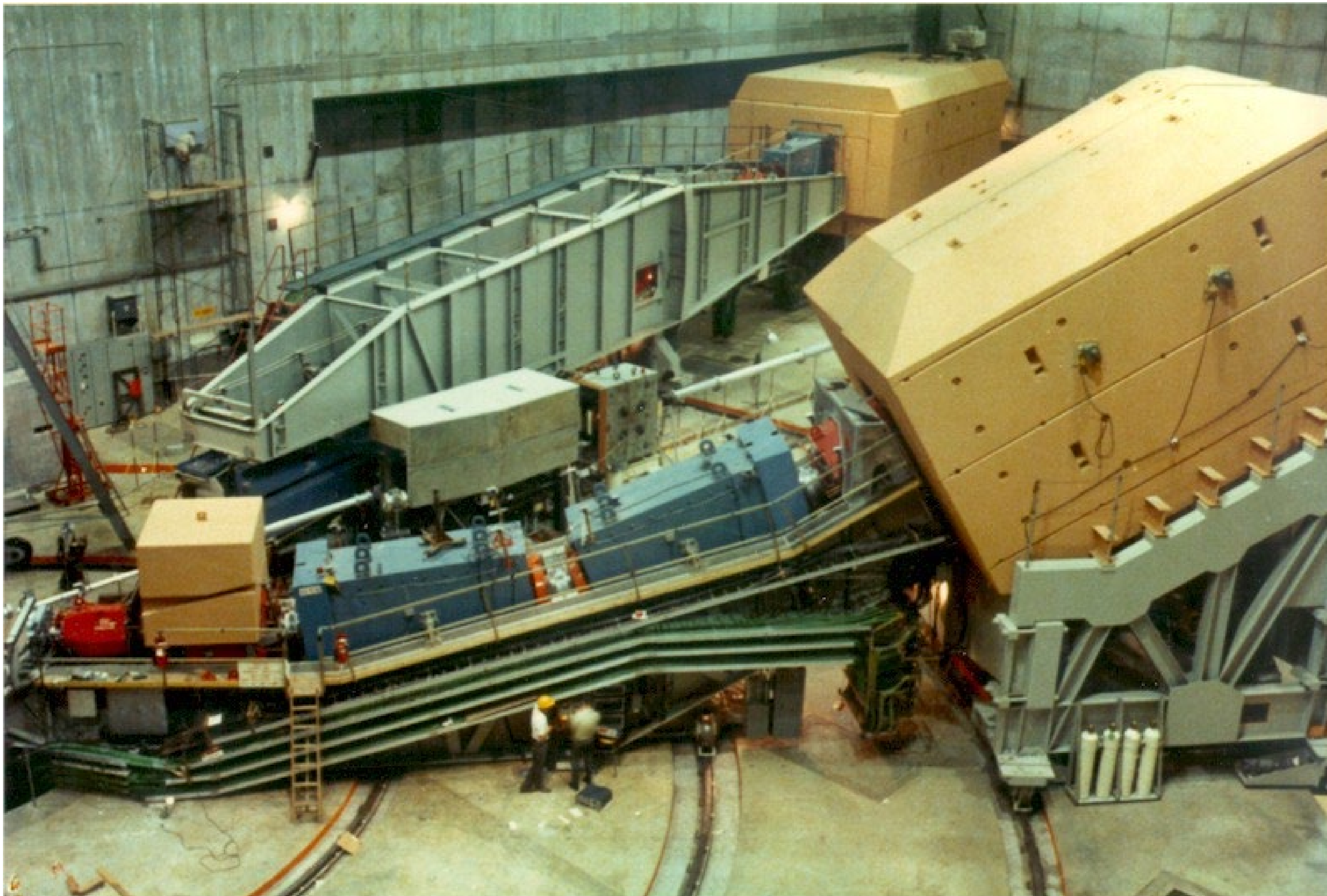
For fixed target electron-proton scattering the necessary kinematic variables x, Q^2, y can all be determined form the electron system: E_1, E_3, e scattering angle θ

Electron beam from 2 miles LINAC



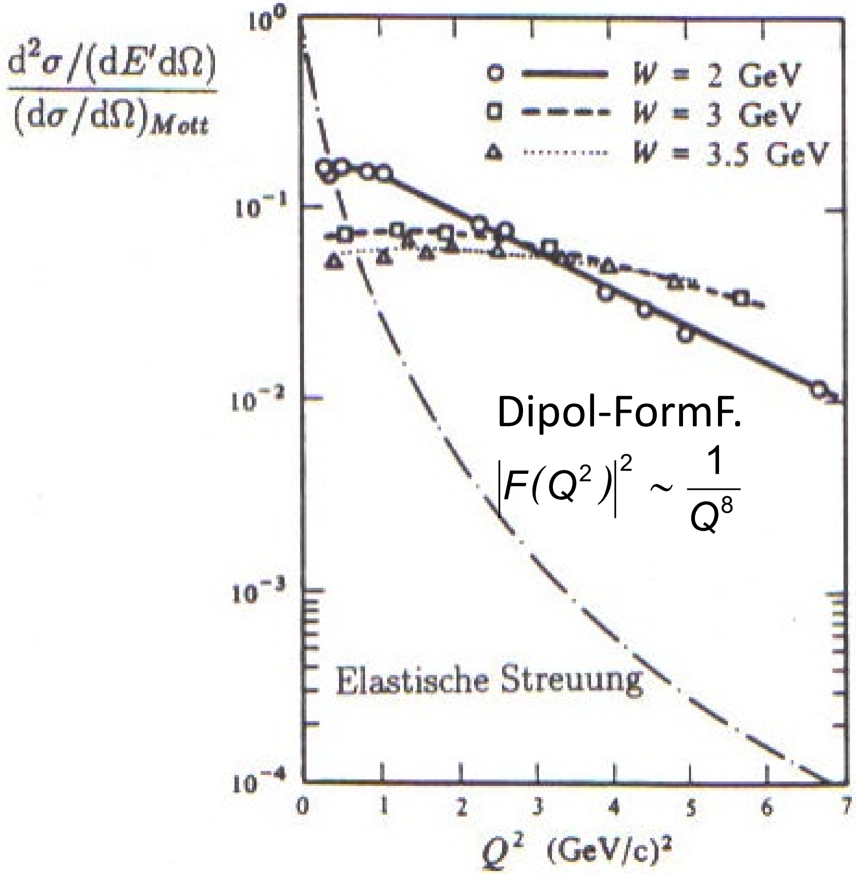


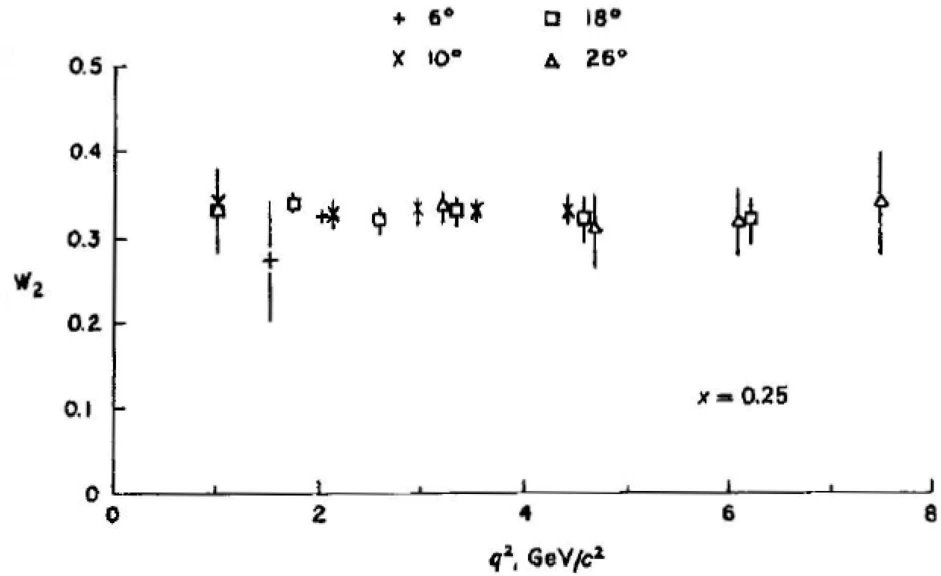
"8 GeV spectrometer"



in front: 8 GeV spectrometer, in back: 20 GeV spectrometer

Cross section and structure function F_2





Two observations/findings:

1) Bjorken scaling:
$F_{1,2}$ are functions of x and not (x,Q^2)

$$F_1 = F_1(x) \quad F_2 = F_2(x)$$

(no explicit Q^2 dependence)

The lack of Q^2 dependence suggests the scattering on point-like constituents. The Callan-Gross relation: Spin ½ constituents

2) F_1 and F_2 are not independent:

$$F_2(x) = 2x F_1(x)$$

= Callan Gross relation

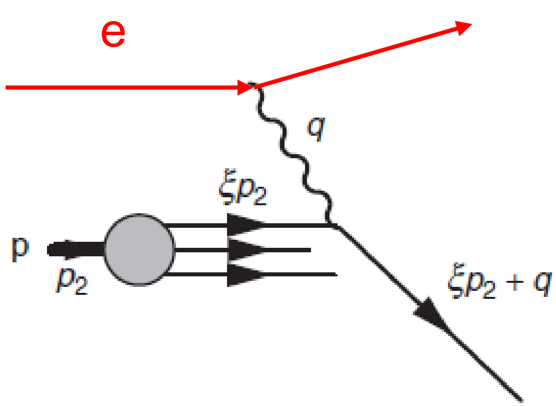
Confirms quarks as point-like constituents of the proton.

DIS ep-scattering in the parton model

Feynman, 1969

Both observations become clear if the scattering is discussed in the parton model.

Parton model is formulated in the infinite momentum frame: the proton has a very large (infinite) energy $E_P >> m_p$ and its mass can be neglected: $p_2 = (E_2, 0, 0, E_2)$



In this model the proton is a "stream of partons" (constituents). The transverse momentum of the partons can be neglected.

4-momentum of struck quark

$$p_q = \xi p_2 = (\xi E_2, 0, 0, \xi E_2)$$

ξ = proton 4-momentum fraction carried by quark

Invariant mass of the quark after interaction:

$$(\xi p_2 + q)^2 = \xi^2 p_2^2 + 2\xi p_2 q + q^2 = m_q^2$$

Quark mass before interaction $\xi^2 p_2^2$

Possible only if $2\xi p_2 q + q^2 = 0$

$$\xi = -\frac{q^2}{2p_2 q} = \frac{Q^2}{2p_2 q} = x$$

$$\xi = x$$

Process possible only if momentum fraction carried by quark equals the Bjorken variable x!

Interesting finding:

defined by electron kinematics

Inelastic cross sections measured as function of the Bjorken variable x and the structure functions $F_{1,2}$ are related to the momentum distribution of the quarks.

The kinematic variables of the underlying e-quark scattering process are related to the kinematic variables of the electron-proton scattering process.

e-proton kinematics:

$$s = (p_1 + p_2)^2 \approx 2p_1p_2$$

$$y = \frac{p_2q}{p_2p_1} \qquad x = \frac{Q^2}{2p_2q}$$

e-quark kinematics:

$$s_q = (p_1 + \xi p_2)^2 \approx 2xp_1p_2 = xs$$

$$\xi = x$$

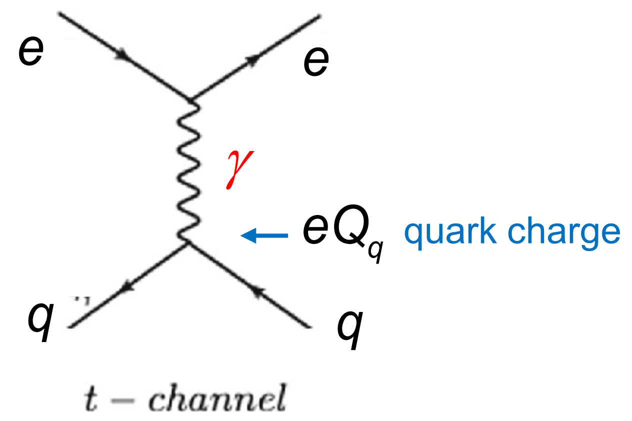
$$y_q = \frac{p_qq}{p_qp_1} = \frac{xp_2q}{xp_2p_1} = y \qquad x_q = 1 \quad \text{(elastic)}$$

To calculate the total electron-proton cross section in the parton model the fundamental e-quark cross section eq→eq is needed. However a similar cross section has already been calculated as t-channel contribution of $e^+e^- \rightarrow e^+e^-$ (Bhabha scattering) ☺ .

Cross section of the fundamental eq→eq process:

$$\langle |M_{fi}|^2 \rangle = 2Q_q^2 e^4 \left(\frac{s_q^2 + u_q^2}{t_q^2} \right)$$

(see lecture on ee annihilation)



Diff. cross section in CMS frame (θ* is scattering angle in CMS frame) :

$$\frac{d\sigma_{eq}}{d\Omega^*} = \frac{Q_q^2 e^4}{8\pi^2 s_q} \frac{1 + \frac{1}{4}\left(1 + \cos\theta^*\right)^2}{\left(1 - \cos\theta^*\right)^2}$$

$$\frac{d\sigma_{eq}}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4} \left[1 + \left(1 + \frac{q^2}{s_q} \right)^2 \right]$$

$$\boxed{\frac{d\sigma_{eq}}{dQ^2} = \frac{4\pi\alpha^2 Q_q^2}{Q^4} \left[(1-y) + \frac{y^2}{2} \right]}$$

Lorentz invariant form – use:

$$\frac{d\sigma_{eq}}{dq^2} = \frac{d\sigma_{eq}}{d\Omega^*} \left| \frac{d\Omega^*}{dq^2} \right|$$

with $q^2 / s_q = -x_q y_q = -y$

and

$$\left[1 + (1-y)^2 \right] = 2\left[(1-y) + \frac{y^2}{2} \right]$$

To calculate the deep-inelastic electron-proton cross section from the fundamental (elastic) electron-quark cross section one needs to sum over all possible quark flavor and weight the contribution with the probability to find a corresponding quark with the correct parton momentum fraction x.

The probability density $q_i(x)$ for a quark of flavor i is defined such that $q_i(x)dx$ Gives the probability to find a quark of flavor I carrying a proton momentum fraction $\in [x, x+dx]$.

The DIS electron-proton cross section in the parton model s then given by:

$$\frac{d\sigma_{ep}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\cdot\sum_i Q_i^2 q_i(x)$$

Comparison w/ the phenomenological result defines the structure functions:

$$\frac{d\sigma_{ep}}{dxdQ^2} = \frac{4\pi\alpha^2}{Q^2}\left[(1-y)\frac{F_2(x)}{x}+y^2F_1(x)\right]$$

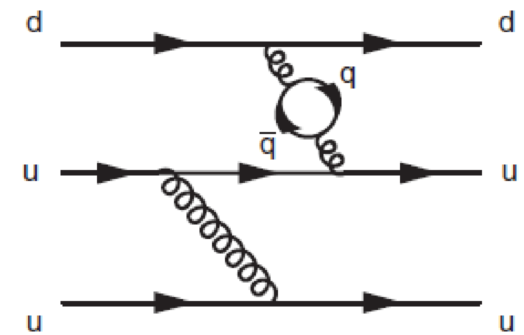
$$\boxed{F_2(x) = 2xF_1(x) = x\sum_i Q_i^2 q_i(x)}$$

Parton model predicts Bjorken scaling (elastic scattering on point-like constituents (no explicit Q^2 dependence) and Callan-Gross relation (spin ½ partons).

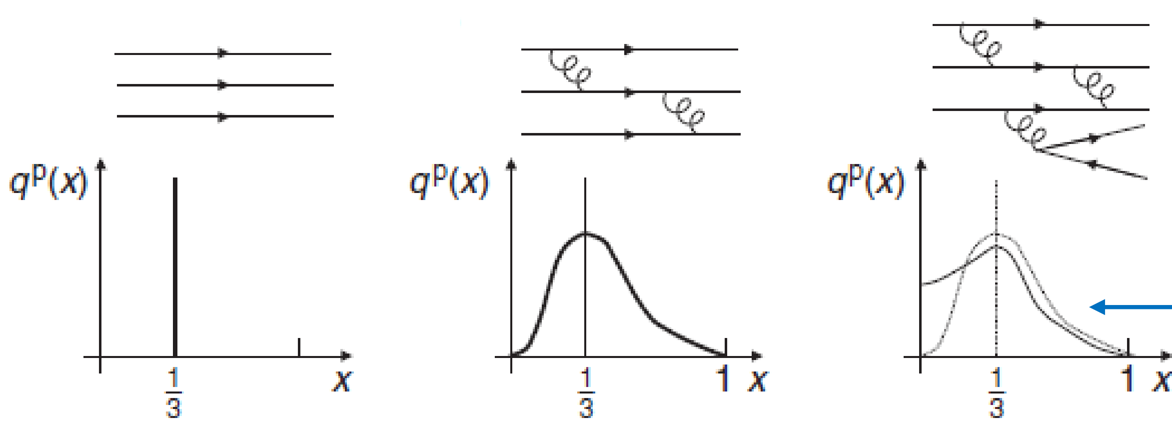
Parton distributions / parton densities

In static quark model, proton is made-up from 2 u-quarks and 1 d-quark (=**valence quarks**). If there was no interaction between the quarks one simply would assume that each quark carries 1/3 of the proton momentum.

In reality the proton is a dynamic system: quarks are bound strongly by exchanging gluons. Gluons could also – shortly – convert into additional $q\bar{q}$ pairs.



This leads to the presence of additional **$q\bar{q}$** pairs (in addition to the 3 valence quarks): **sea quarks** – most frequently **$u\bar{u}$** and **$d\bar{d}$**, but also **$s\bar{s}$** and even $c\bar{c}$ and $b\bar{b}$ pairs (strongly suppressed).



Dynamic effects lead to modified quark momentum distributions q(x).

Please note that the peak at 1/3 ignores that the gluons also carry momentum (see below).

Structure functions for e-nucleon scattering:

For the e-proton scattering the structure function $F_2(x)$ is thus given by:

$$F_2^{ep}(x) = x\sum_i Q_i^2 q_i(x) \approx x\left[\frac{4}{9}u(x)+\frac{1}{9}d(x)+\frac{4}{9}\bar{u}(x)+\frac{1}{9}\bar{d}(x)\right]$$

neglect s-quarks

where $u, \bar{u}, d, \bar{d}$ are the parton density distributions of the u, d quark and anti-quarks of the proton (sum of valence and sea quarks).

A similar expression could also be written down for DIS electron-neutron scattering (measurement done usings deuterons and correcting for proton)

$$F_2^{en}(x) = x\sum_i Q_i^2 q_i(x) \approx x\left[\frac{4}{9}u^n(x)+\frac{1}{9}d^n(x)+\frac{4}{9}\bar{u}^n(x)+\frac{1}{9}\bar{d}^n(x)\right]$$

Isospin symmetry relates the parton densities of proton and neutron:

$$u^n = d^p = d, \quad \bar{u}^n = \bar{d}^p = \bar{d},$$
$$d^n = u^p = u, \quad \bar{d}^n = \bar{u}^p = \bar{u}$$

To calculate the proton / neutron momentum carried by the quarks one should integrate the structure functions over x:

$$\int F_2^{ep}(x)dx = \frac{4}{9}f_u + \frac{1}{9}f_d \quad \text{with} \quad f_u = \int\left[u(x)+\bar{u}(x)\right]dx$$

$$f_d = \int\left[d(x)+\bar{d}(x)\right]dx$$

and for the neutron

$$\int F_2^{en}(x)dx = \frac{4}{9}f_d + \frac{1}{9}f_u$$

Experimentally one finds for the two integrals:

$$\int F_2^{ep}(x)dx \approx 0.18 \qquad \int F_2^{en}(x)dx \approx 0.12$$

Solving for the integrals of the u and d quarks: one gets:

$$f_u \approx 0.36 \quad \text{and} \quad f_d \approx 0.18 \quad \Longrightarrow \quad f_u + f_d \approx 0.54$$

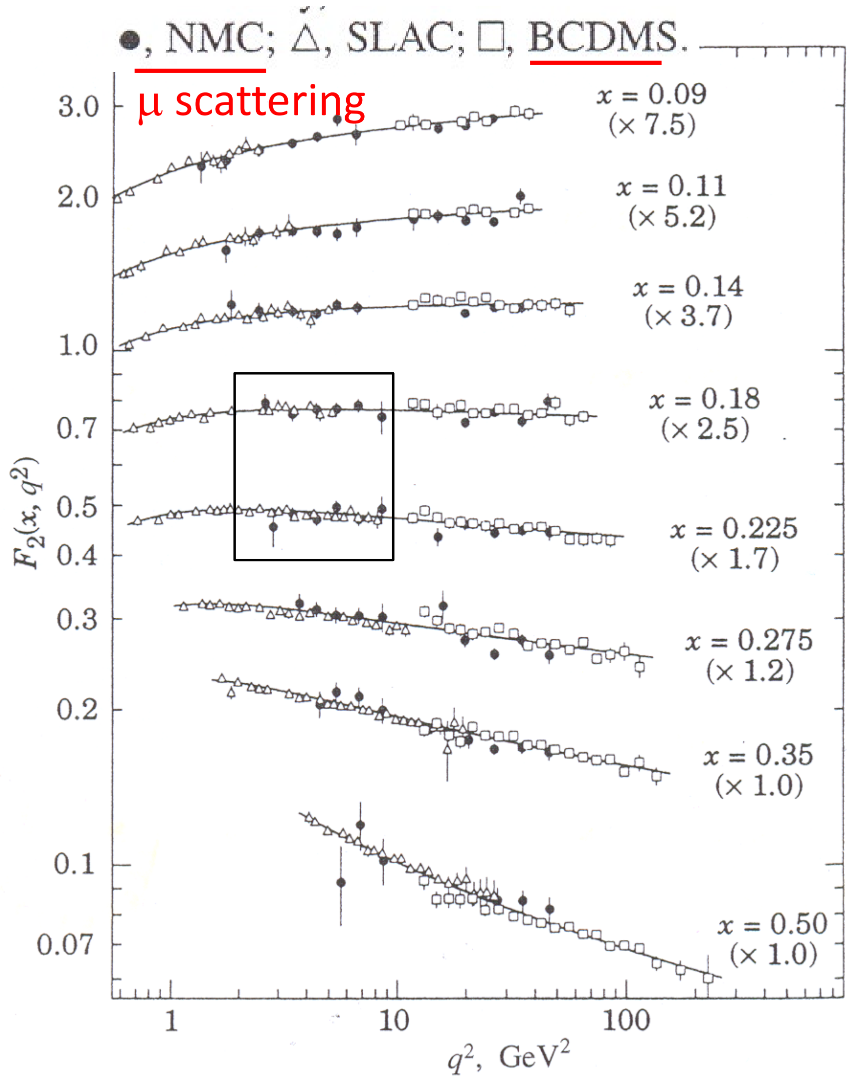
This means that the sum of the quarks (u,d) carry only ~50% of the proton momentum fraction: rest is carried by …???? The gluons!

Precision determination of F_2 and of the parton distributions

After the first SLAC measurements many different DIS experiments have been conducted to determine F_2 and of the parton distribution of the proton: Instead of electron also muons and neutrinos (CC interactions) have been used

A summary of early F_2 measurements is shown in the plot: it covers a much extended Q^2 range and much different x-values than the early SLAC measurements (range given in the box)

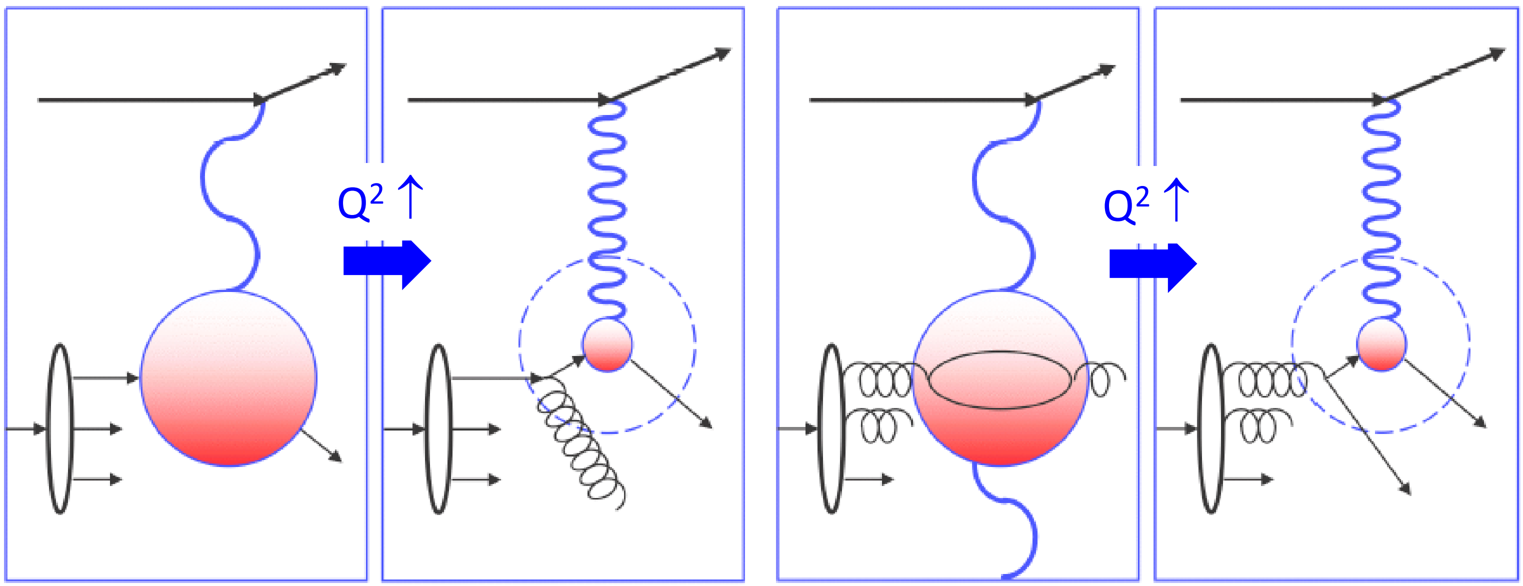
Scaling violation:
What is clearly noticeable is that F_2 (scaled in the plot to avoid overlap) is has indeed very little Q^2 dependence for the early SLAC measurements (box). However at different Q^2 values and for different x-values the predicted "scaling behavior" is violated and F_2 is a clear function of both (x, Q^2).
Reason: large dynamic effects between quarks ignored by simple parton model.



“Qualitative explanation” of observed scaling violation

Exact quantitative description (DGLAP) is the topic of next semester!

Scattering at large x $\rightarrow$ mostly valence quark, at small x $\rightarrow$ mostly sea quark
Changing Q^2 one can change “the resolution” of the virtual photon (λ):



$\Rightarrow$ F_2 or fixed (large) x $\downarrow$

$\Rightarrow$ F_2 for fixed (small) x $\uparrow$

Scaling violation is a clear manifestation of radiative effects predicted by QCD. PDFs (and structure functions) depend on Q^2 and x.

PDF = Parton Distribution/density Functions

Precise measurement of PDFs at HERA

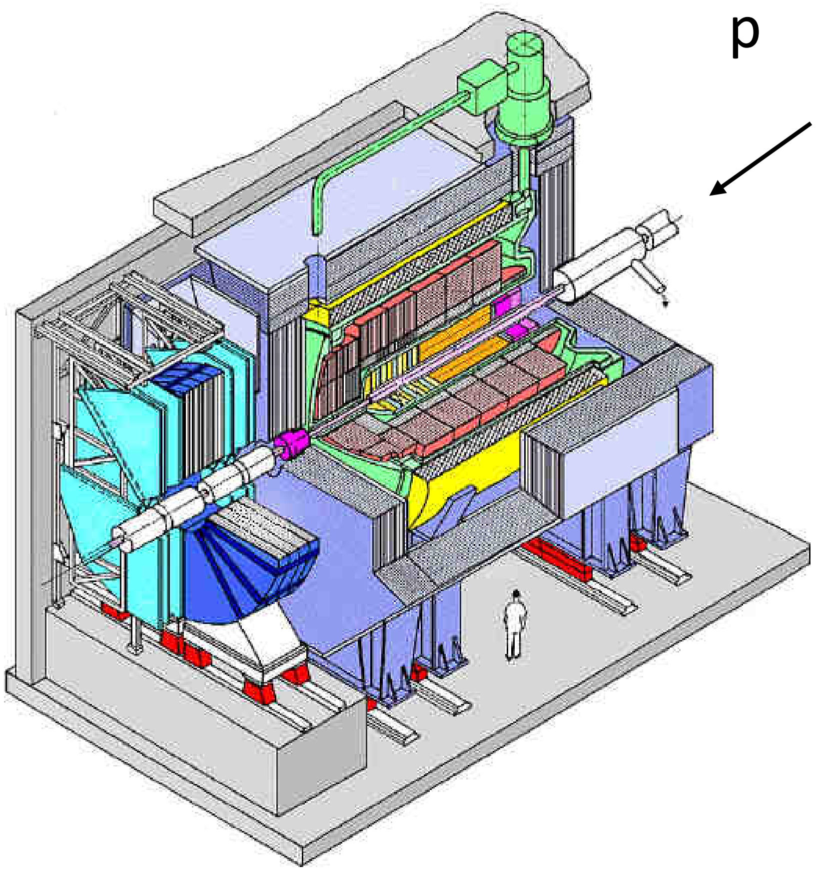
HERA

e → 30 GeV

p ← 900 GeV

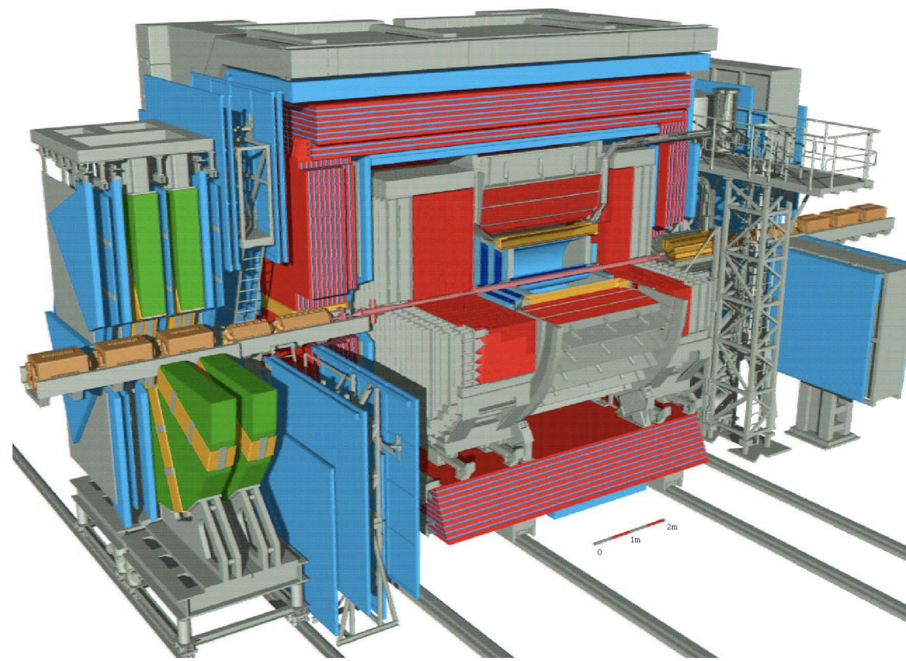
$$s = 4E_eE_p \approx 10^5\,\mathrm{GeV}^2$$

H1 detector



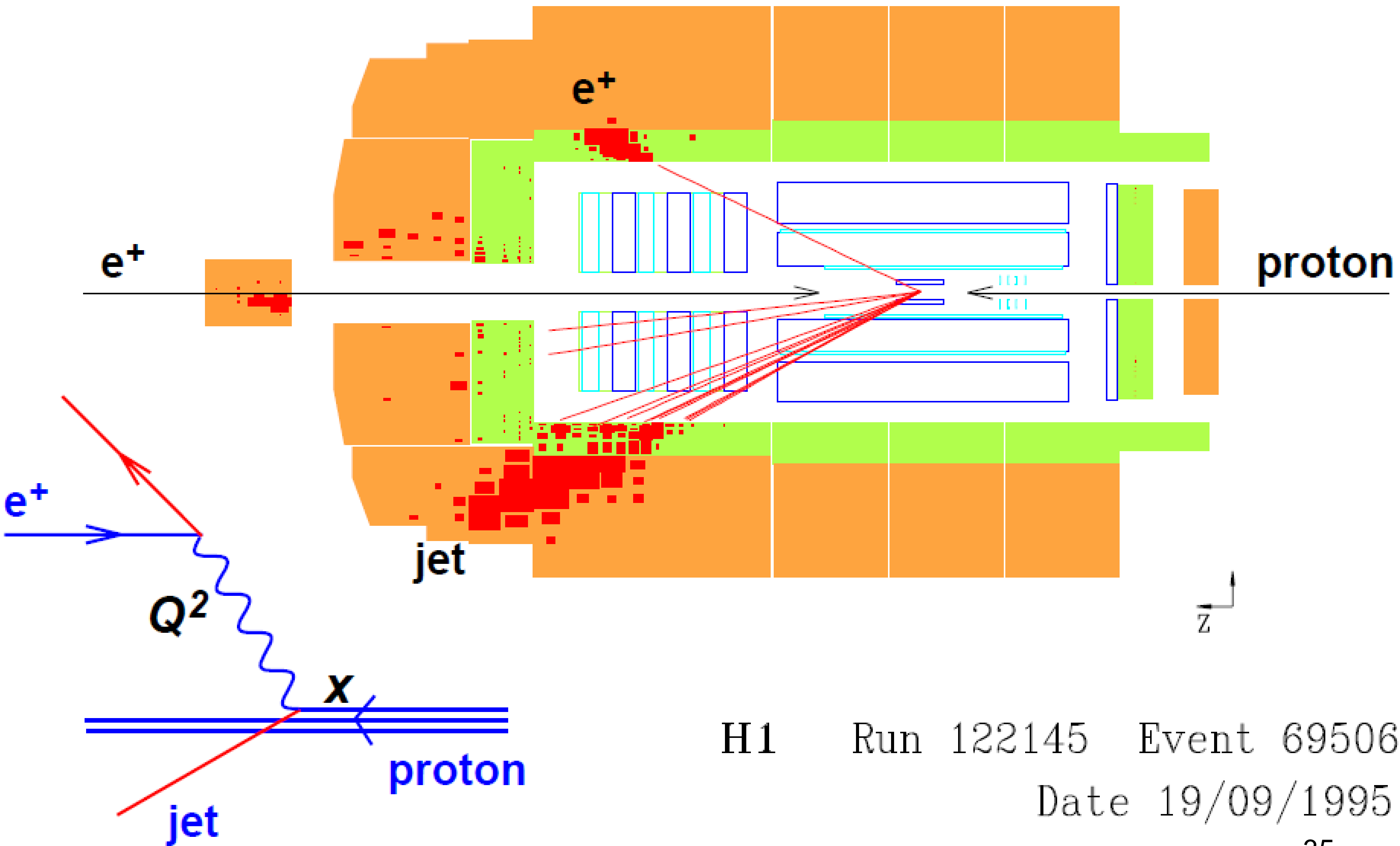
p

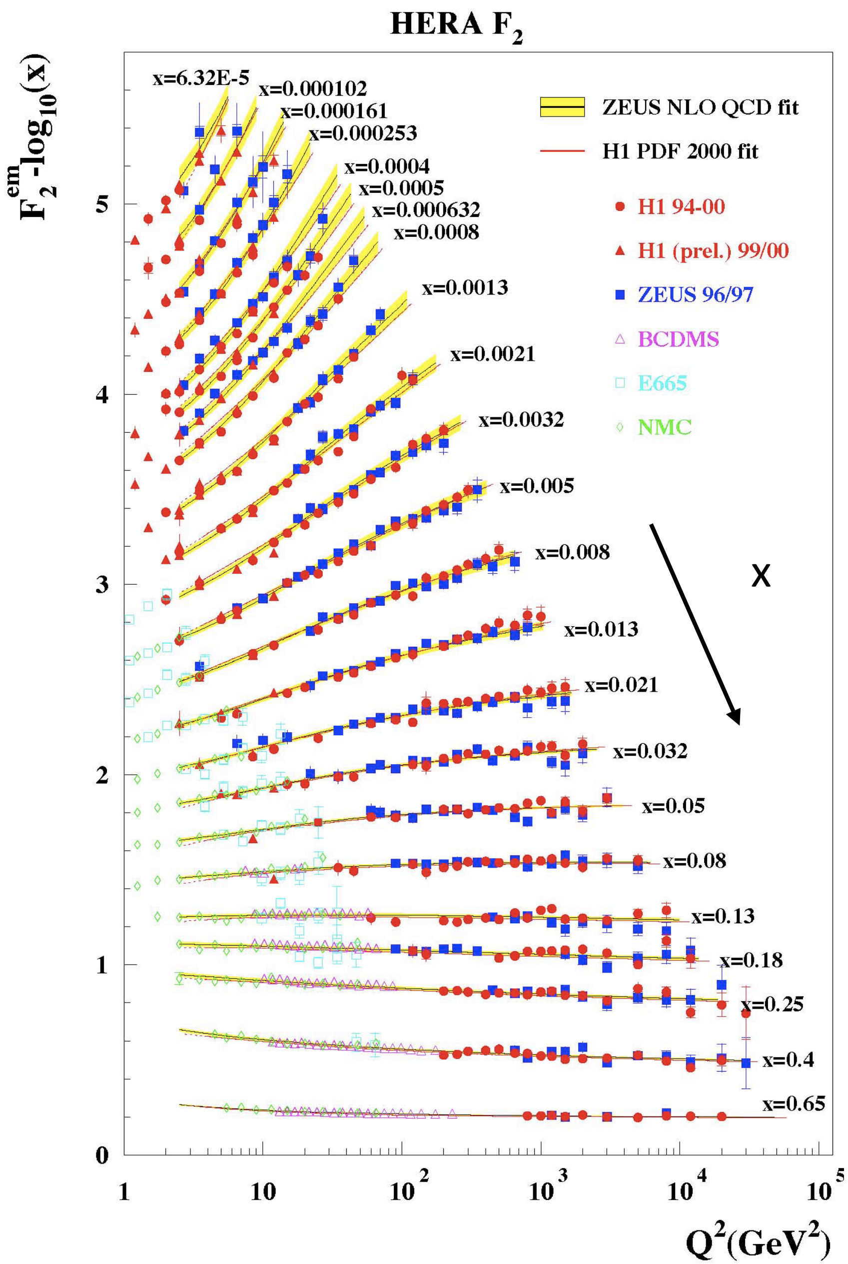
ZEUS detector

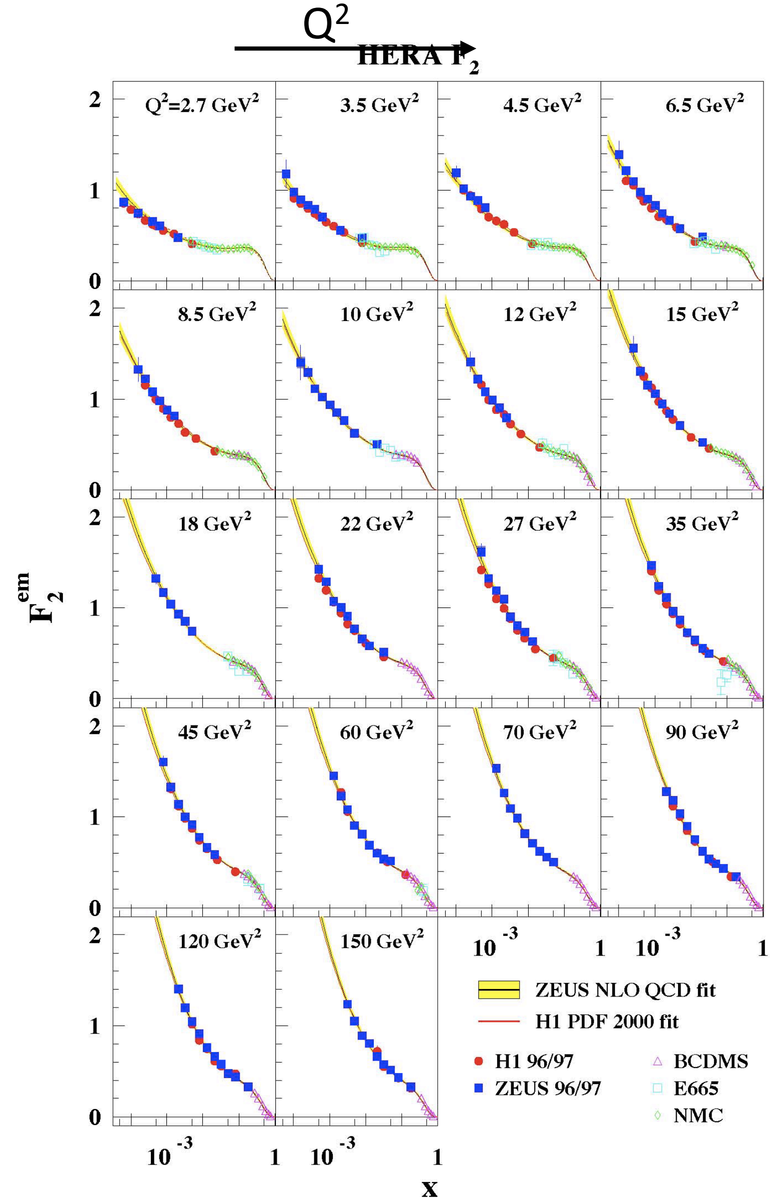


p

Q^2 = 25030 GeV^2, y = 0.56, x=0.50



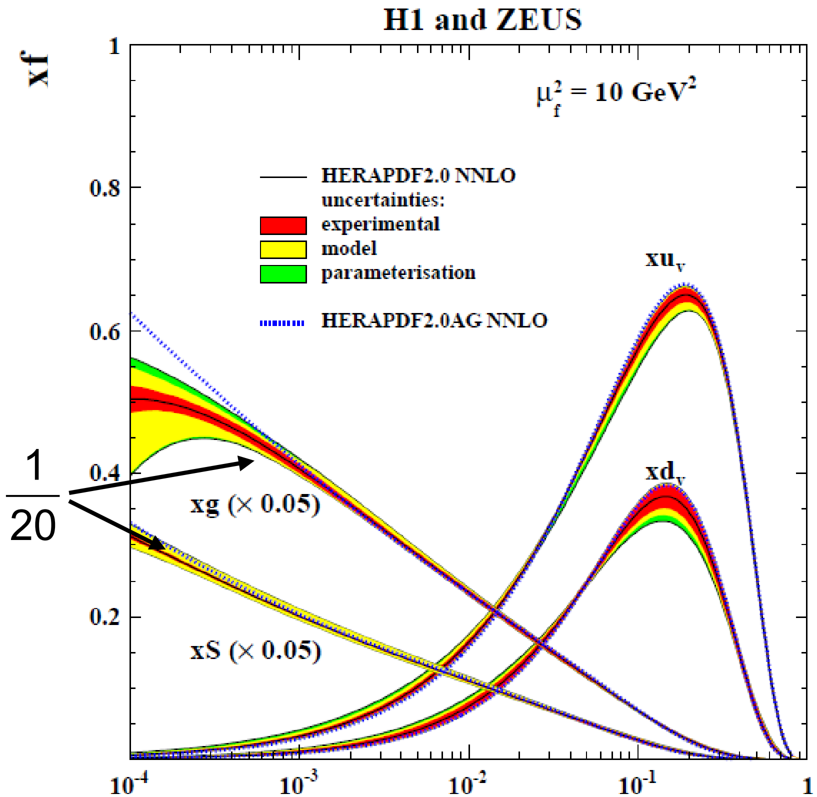




Determination of PDFs relies on factorization

$$d\sigma \sim d\sigma_{eq} \times F_2$$

QCD fit to the data – proton PDFs for a given Q^2 scale



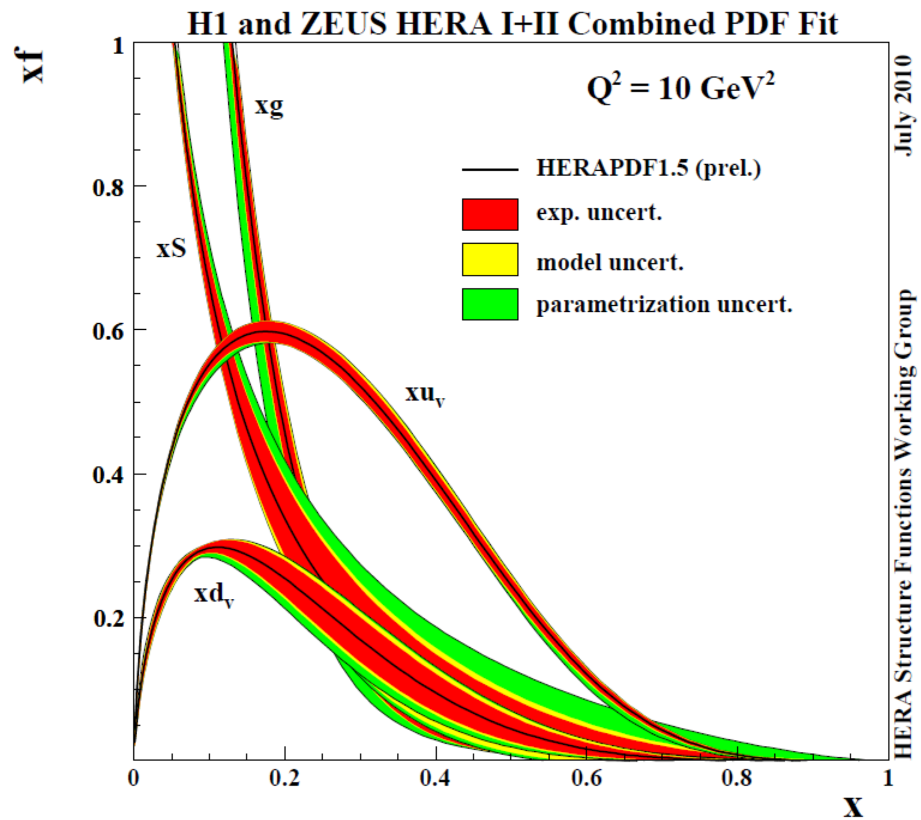


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Figure 23: The parton distribution functions xu_v, xd_v, $xS = 2x(\bar{U}+\bar{D})$ and xg of HERAPDF2.0 NNLO at $\mu_f^2 = 10\,\mathrm{GeV}^2$. The gluon and sea distributions are scaled down by a factor 20. The experimental, model and parameterisation uncertainties are shown. The dotted lines represent HERAPDF2.0AG NNLO with the alternative gluon parameterisation, see Section 6.8.

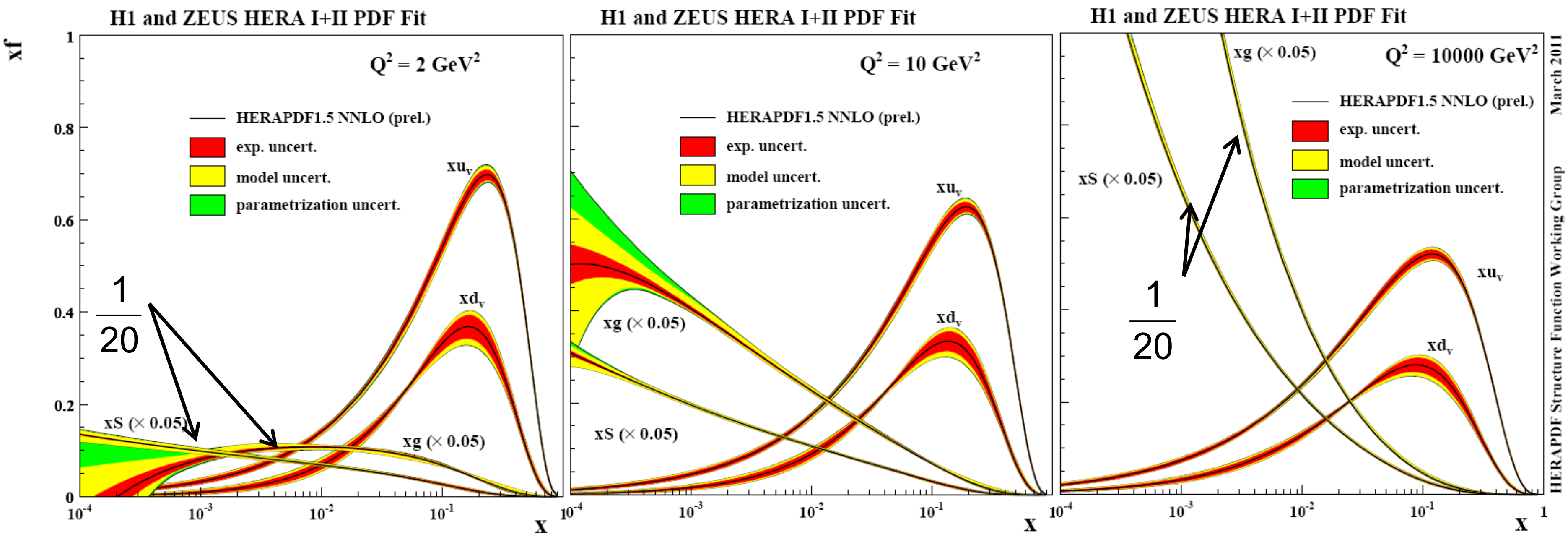
Linear scale for illustration (it is not exactly the same pdf set, but nearly)

Remarks:

- At low x, sea quarks dominate (xS in the plot) the scattering → huge gluon content
- While the proton C

Q2 evolution (predicted by QCD – DGLAP)

DGLAP = Dokshitzer, Gribov, Lipatoiv Altarelli, Parisi



https://www.desy.de/h1zeus/combined_results/

The most dramatic of these [experimental consequences] is that the protons viewed at ever higher resolution would appear more and more as field energy (soft glue), was only clearly verified at HERA ...   F. Wilczek [Nobel Prize 2004]