2.6 Excitation Spectrum of He-II: Landau Model

Landau's concept of critical velocity

superconductors ⟶ energy gap

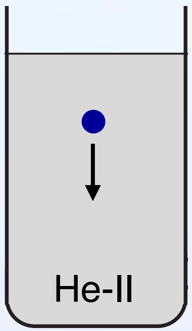
superfluid He-II ⟶ no energy gap, but velocity gap!

comment:

phonons can be excited at arbitrary small energies

Landau's Gedankenexperiment: dropping a massive sphere in He-II at $T = 0$

let's assume that sphere generates one excitation with energy $\mathcal{E}$ and momentum $\boldsymbol{p}$



How fast must this sphere fall in He-II to generate dissipation ?

energy conservation

$$\frac{1}{2}\mathcal{M}v^2 = \frac{1}{2}\mathcal{M}v'^2 + \mathcal{E} \qquad (1)$$

momentum conservation

$$\mathcal{M}\boldsymbol{v} - \boldsymbol{p} = \mathcal{M}\boldsymbol{v}' \qquad (2)$$

not all combinations of $\mathcal{E}$ and $\boldsymbol{p}$ fulfill both conservation law's at the same time, even if the direction of the excitation is not fixed

2.6 Excitation Spectrum of He-II: Landau Model

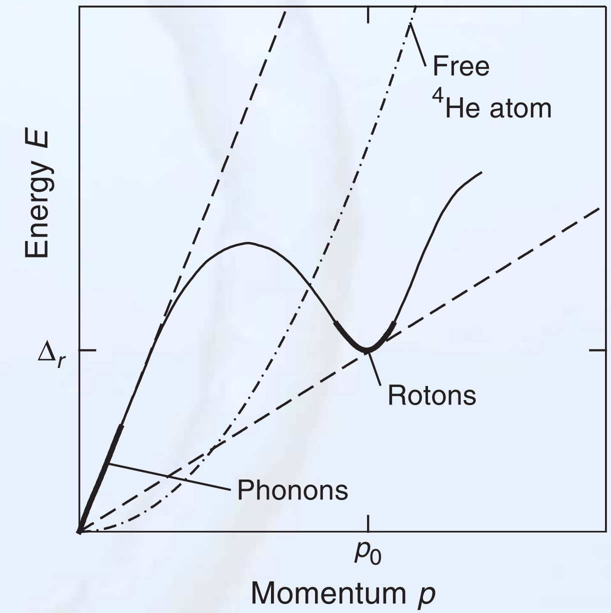
let's test this: square (2) and divide by $2\mathcal{M}$ $\longrightarrow$ $\frac{1}{2}\mathcal{M}v^2 - \boldsymbol{v}\cdot\boldsymbol{p} + \frac{1}{2\mathcal{M}}p^2 = \frac{1}{2}\mathcal{M}v'^2$

comparison with (1) results in: $\boldsymbol{v}\cdot\boldsymbol{p} - \frac{1}{2\mathcal{M}}p^2 = \mathcal{E}$

with $\boldsymbol{v} \parallel \boldsymbol{p}$

mass of sphere is large

$\Rightarrow$ $$v_\mathrm{c} = \frac{\mathcal{E}}{p}$$ independent of nature of excitation



phonons: $v_\mathrm{c} = 238\,\mathrm{m\,s^{-1}}$

rotons: $v_\mathrm{c} \approx 60\,\mathrm{m\,s^{-1}}$

free ^{4}He atom: $v_\mathrm{c} = 0$ $\longleftarrow$ do not exist in He-II

- no excitation for $v < v_\mathrm{c}$
- no dissipation $\longrightarrow$ superfluidity
- for $v \geq v_\mathrm{c}$ sudden onset of dissipation, laminar $\longrightarrow$ turbulent flow

2.7 Motion of Ions in He-II

Testbed for the generation of excitations and the critical velocity

type of ions:

- electrons (–) : zero-point motion ⟶ bubbles $r = 19$ Å
- $^4He^+$, H_2^+ (+) : attract He atoms ⟶ snowballs $r \approx 7$ Å
- other ions (–, +) : properties depend on wave function

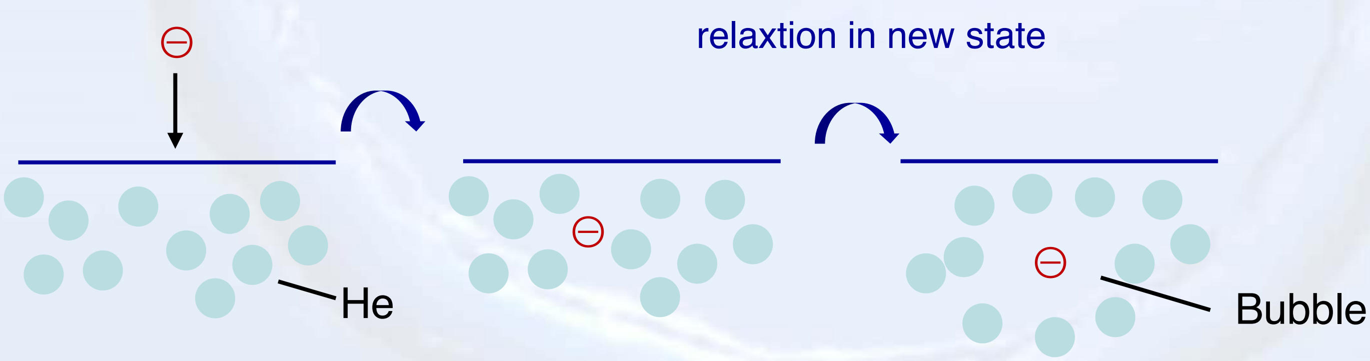
Electrons in liquid He

electrons need energy to be emerged in helium ~ 1 eV, which means they need more that 1 eV of kinetic energy to enter liquid He.

comment:

similar to work function of electrons in metals

bubble formation

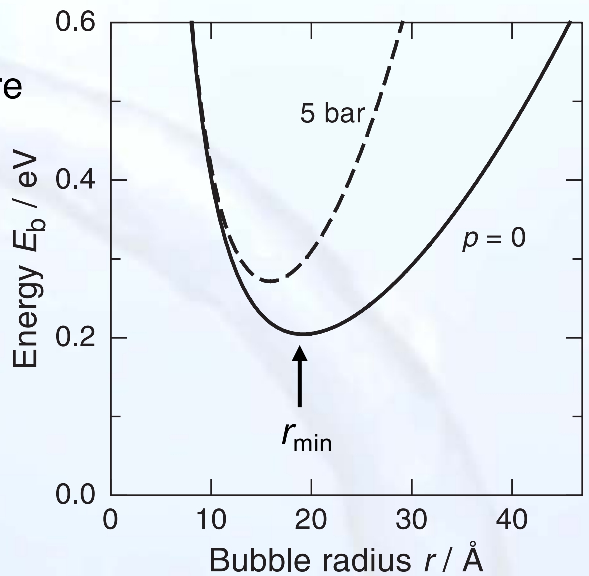


2.7 Motion of Ions in He-II

Energy of bubble

$$E_\mathrm{b} = \frac{h^2}{8mr^2} + 4\pi r^2 \alpha + \frac{4}{3}\pi r^3 p$$

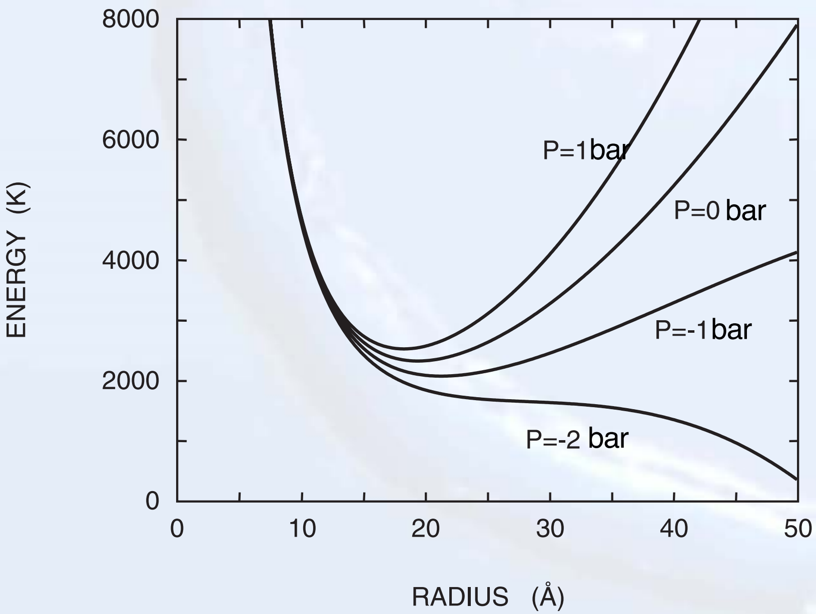
zero-point energy; surface tension $\alpha = 3.41\ \mu\mathrm{J/cm^2}$; volume; applied pressure

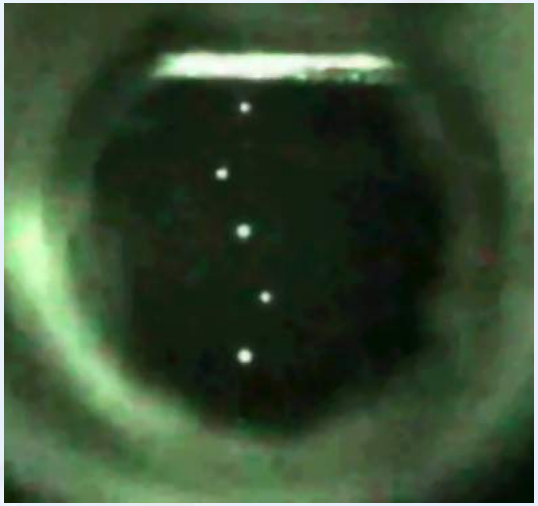


bubble size:

$$\frac{\partial E}{\partial r} = 0 \longrightarrow r_\mathrm{min}(p=0) = 19\ \text{Å}$$

size depends on pressure

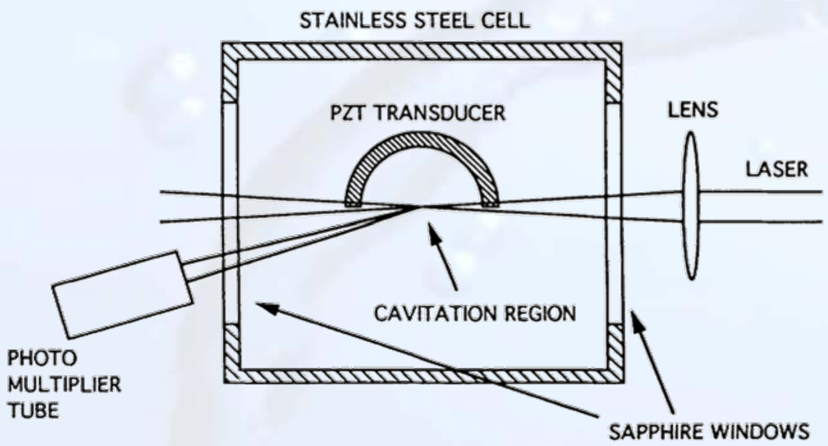


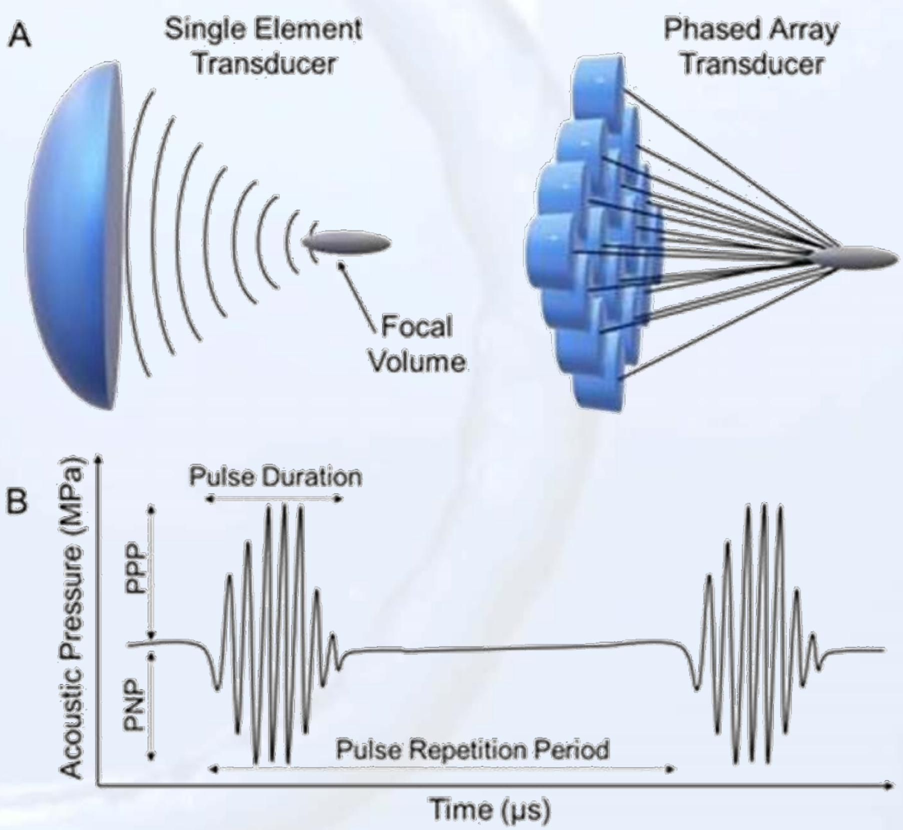


exploding bubbles at negative pressure

2.7 Motion of Ions in He-II

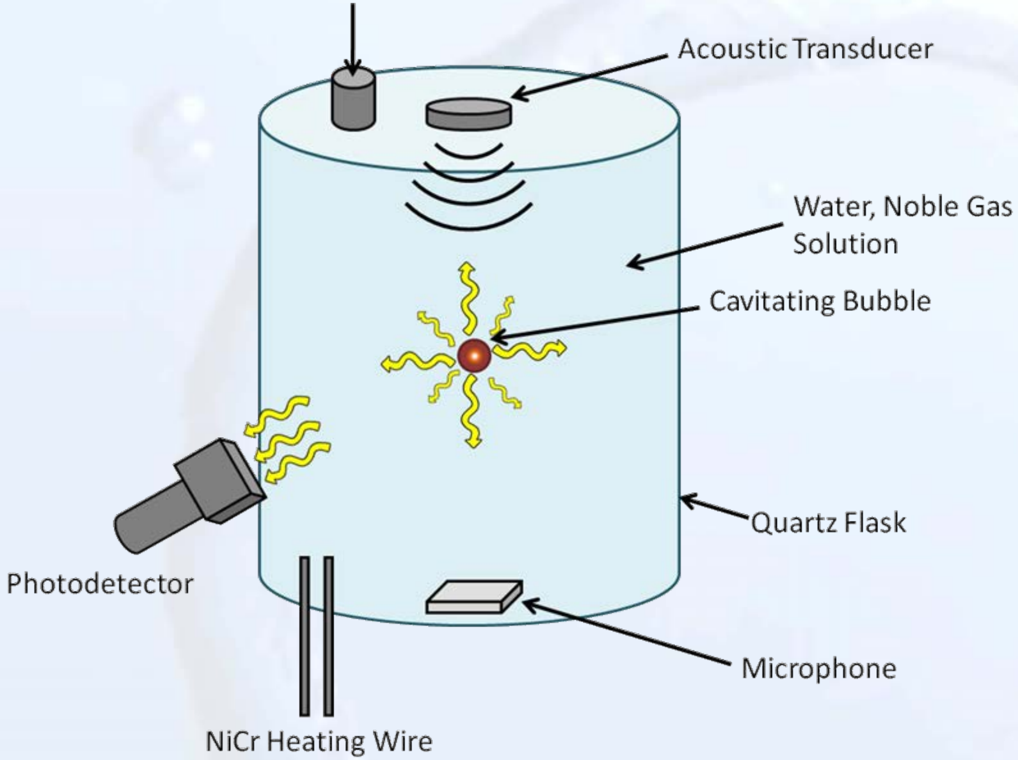
Creation of negative pressure and observation of bubbles



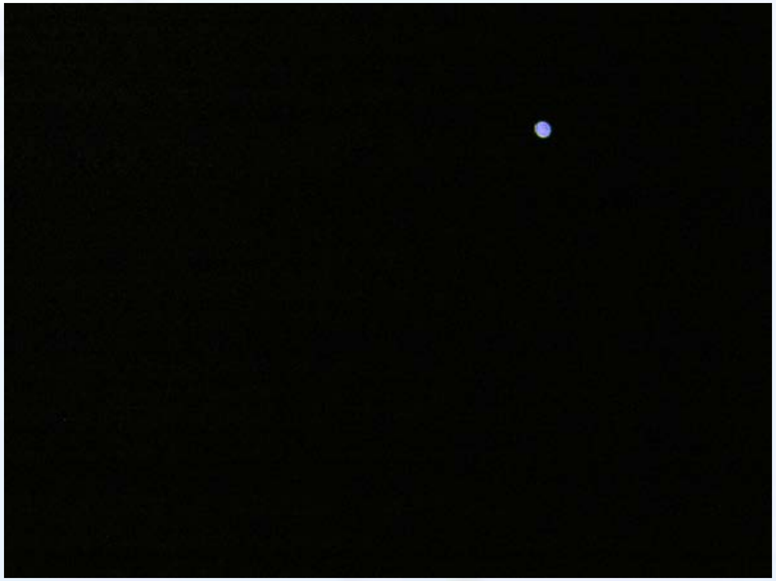


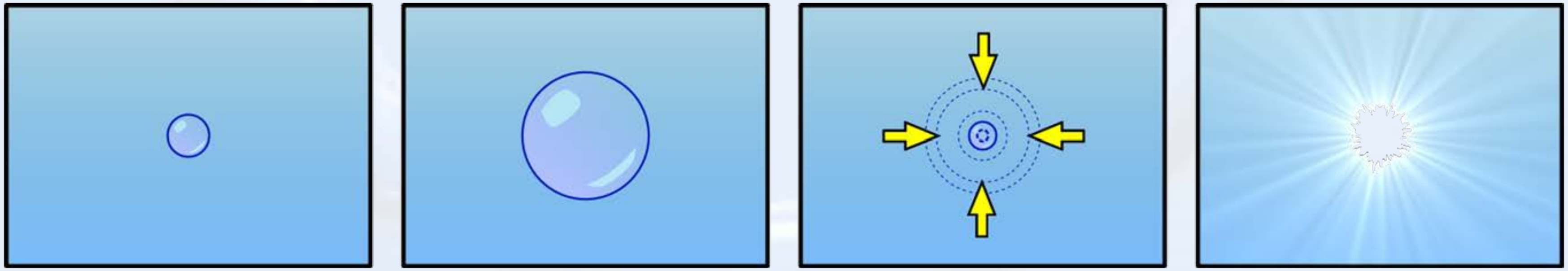
Generation of Bubbles and Cavitation Processes

Sonoluminescence



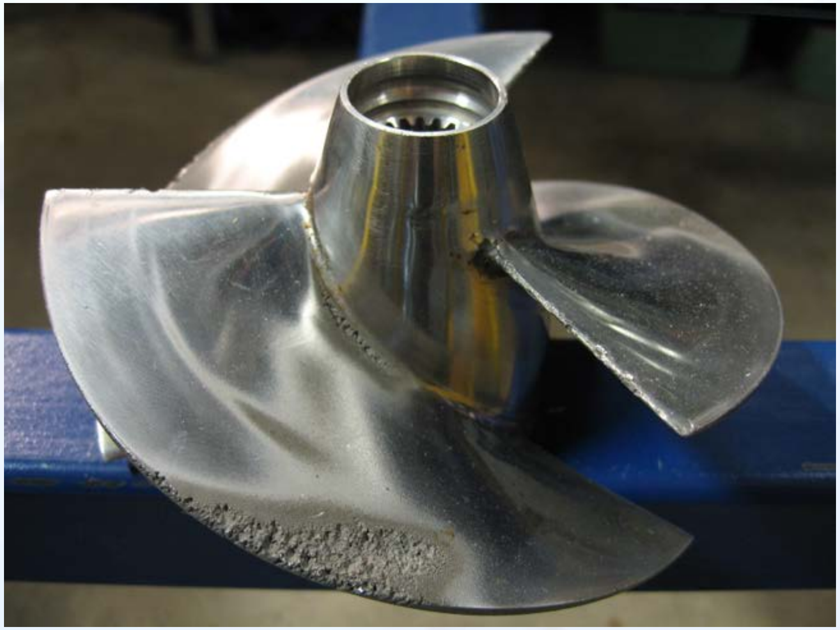
Sonoluminescence in water





Generation of Bubbles and Cavitation Processes

Collapsing bubbles are of great technical importance



Extracorporeal shockwave therapy using cavitation processes

2.7 Motion of Ions in He-II

Acceleration of ions in constant field

→ constant drift velocity is reached $\overline{v}_\mathrm{d} = \dfrac{q\mathcal{E}}{6\pi\eta r}$

Stokes law of viscos friction

constant electrical field $\mathcal{E}$

mobility:

$$\mu = \frac{\overline{v}_\mathrm{d}}{\mathcal{E}} = \frac{q}{6\pi\eta r}$$

snowball (electrons 4π)

collision partners: phonons, rotons, ^{3}He, …

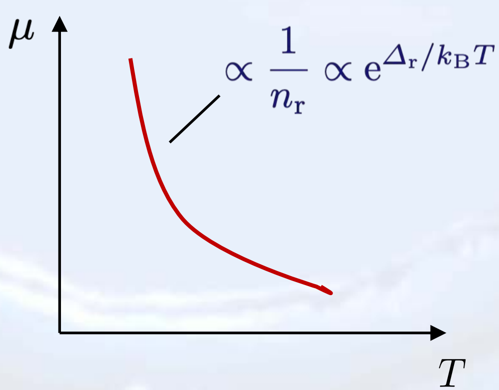
impurities, which at some level are always present

0.7 K < *T* < 1.8 K: rotons should dominate however, difficult to observe because of other excitations / impurities

mobility for roton scattering

$$\mu \propto \frac{1}{\eta} \propto \frac{1}{\tau} \propto \frac{1}{n_\mathrm{r}}$$

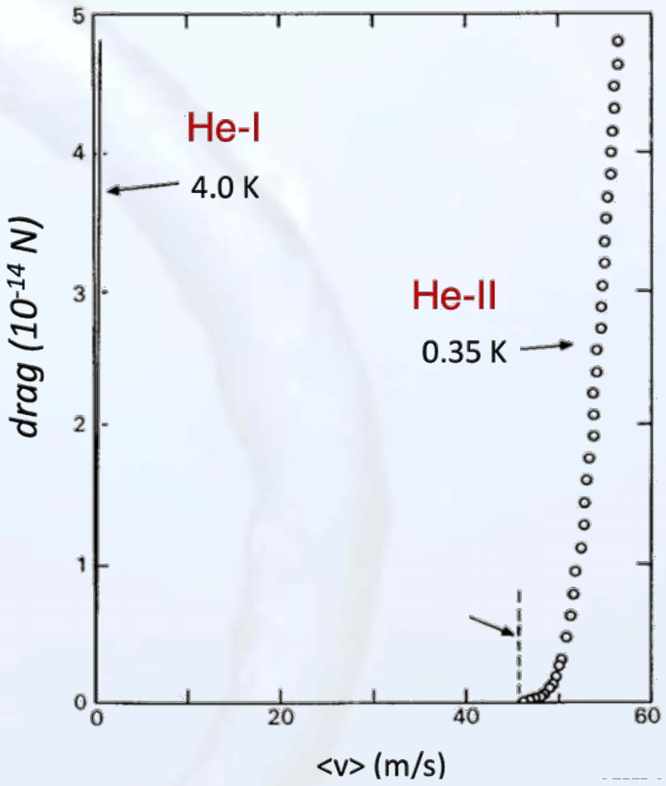
$$\eta = \frac{1}{3}\varrho v^2 \tau = \frac{1}{3}\varrho v \ell$$



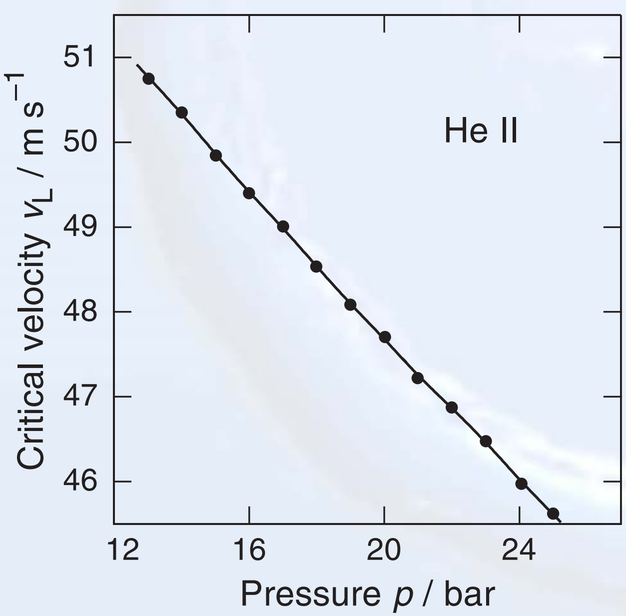
2.7 Motion of Ions in He-II

in ultra-pure He-II under pressure ions can be accelerated up to Landau velocity

- negative ions accelerated in electric field under high pressure
- drag is measured by time-of-flight method
- in He-I: drag proportional to velocity
- in He-II: drag is not observable until critical velocity is reached



pressure dependence of v_c



- $v_L \mathrel{\widehat{=}} v_c$ Landau velocity
- roton pair production
- $p \uparrow \longrightarrow v_L \downarrow$ since $\Delta_r(p)$ decreases with pressure

2.7 Motion of Ions in He-II

T < 0.3 K

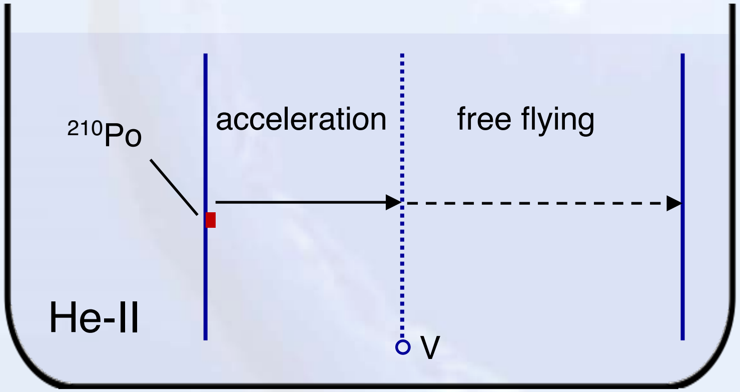
no thermal rotons are excited

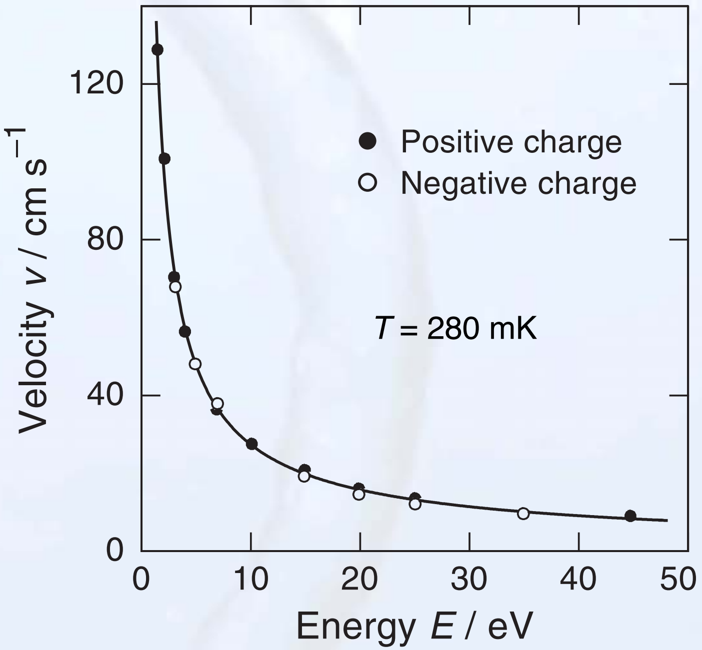
phonons mean free path becomes very large → several cm! $v_c \to 238\,\frac{\mathrm{m}}{\mathrm{s}}$?

experimental answer: no! $\overline{v}_\mathrm{d}$ = 10 … 100 cm/s

in addition: $\overline{v}_\mathrm{d}$ decreases with energy of ions, which means it decreases with accelerating field

Experiment by Rayfield and Reif 1964





→ measurement of ion velocity by time of flight

explanation:

- ▶ creation of vortex rings and trapping of ions
- ▶ experiment observes motion of vortex rings

2.7 Motion of Ions in He-II

vortex rings

kinetic energy of vortex ring: He-II $\varrho \to \varrho_s$

$$E_{vr} = \int \frac{1}{2}\varrho_s v_s^2 \mathrm{d}V = \frac{1}{2}\varrho_s \kappa^2 r \left[\ln\left(\frac{8r}{a_0}\right) - \frac{7}{4}\right] \quad \propto r$$

momentum of vortex ring $p_{vr} = \pi \varrho_s \kappa r^2$

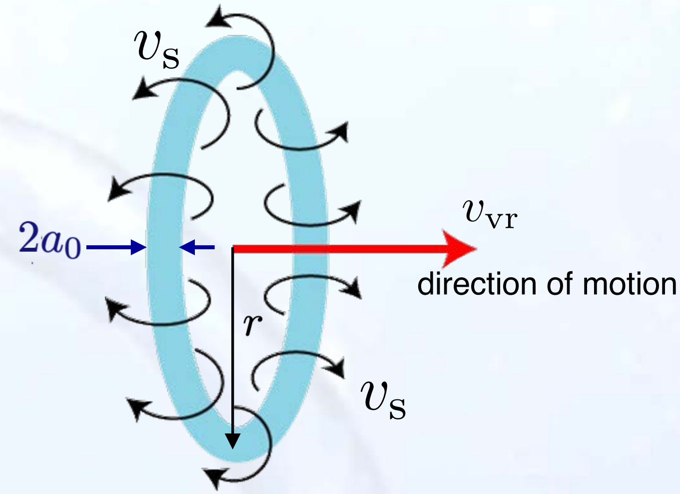
$$v_{vr} = \frac{\partial E}{\partial p_{vr}} = \frac{\kappa}{4\pi r}\left[\ln\left(\frac{8r}{a_0}\right) - \frac{1}{4}\right]$$

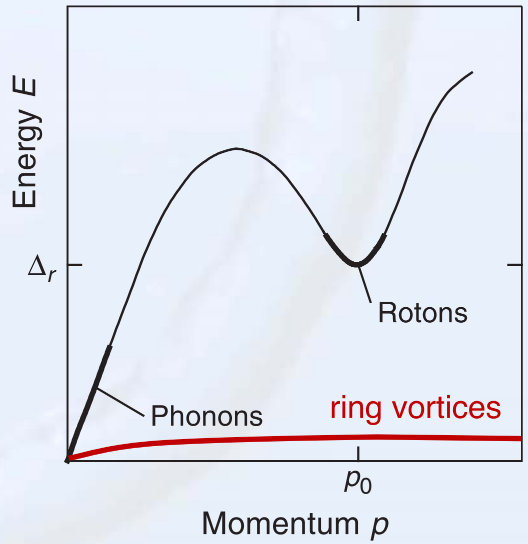
$p_{vr} \propto r^2 \propto E_{vr}^2$ and $v_{vr} \propto 1/E$

as observed

$$\boxed{E_{vr} \propto \sqrt{p_{vr}}}$$

dispersion of vortex ring





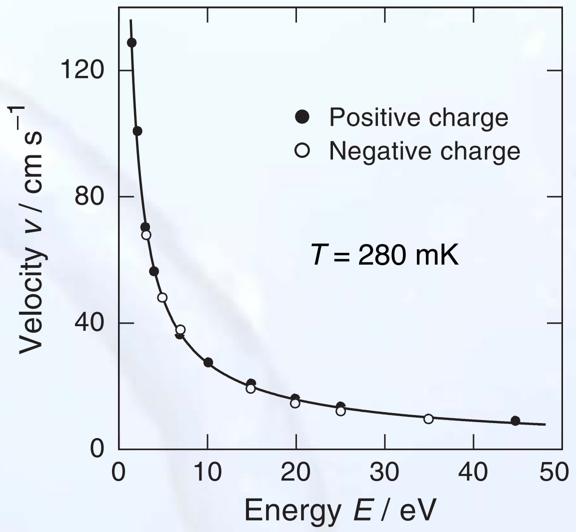
2.7 Motion of Ions in He-II

Explanation of the experiment by Rayfield and Reif

- generation of vortex rings
- ions are captured by vortex ring
- field increases kinetic energy of vortex ring

$$v_{\mathrm{vr}} \propto \frac{1}{r} \propto \frac{1}{E_{\mathrm{vr}}}$$

- theory line with $a_0 = 1.2\ \text{Å}$

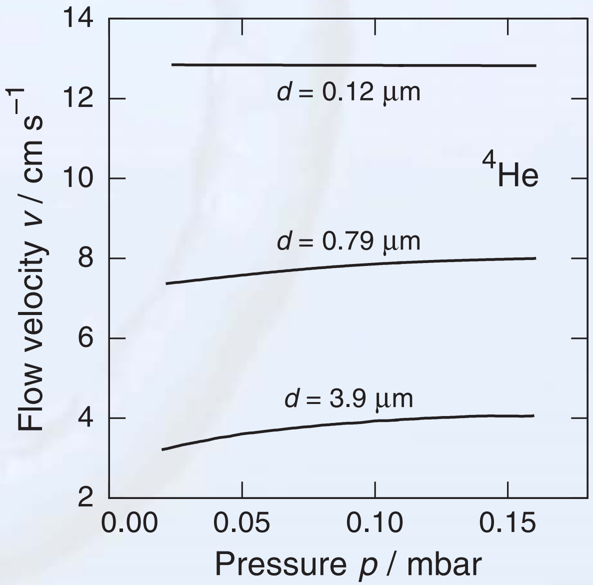


let's revisit the flow experiments through capillaries

because of $E_{\mathrm{vr}} \propto \sqrt{p_{\mathrm{vr}}}$, largest possible vortex is has minimal critical velocity

for capillary with diameter *d*

$$v_{\mathrm{c,vr}} = \frac{\hbar}{m_4 d}\left[\ln\left(\frac{4d}{a_0}\right) - \frac{1}{4}\right] \propto \frac{1}{d}$$



➡ qualitative agreement with flow experiments in capillaries